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JACOBY IDENTITY IN THE THEORY OF DIRAC MONOPOLE

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1. It is known that the vector - potential $\bar{A}$ singular along a string is introduced in the theory of Dirac monopole/1/. We shall choose this potential in the Schwinger form/2-4/

$$\bar{A} = \frac{g}{4\pi}\frac{(\bar{n}\bar{\nu})}{r}\frac{[\bar{n}\bar{\nu}]}{[\bar{n}\bar{\nu}]^2} \tag{1}$$

($\bar{n} = \frac{\bar{r}}{r}$, $\bar{\nu}$ - a unit string vector). The magnetic field $\bar{\mathcal{H}} = \operatorname{rot}\bar{A} = \bar{\mathcal{H}}^{(g)} + \bar{\mathcal{H}}^{(f)}$ differs from the purely Coulomb monopole field $\bar{\mathcal{H}}^{(g)} = \frac{g}{4\pi}\frac{\bar{n}}{r^2}$ on the string where the additional term $\bar{\mathcal{H}}^{(f)} = \frac{1}{2}g\bar{\nu}\,\operatorname{sign} z\,\delta(\bar{r}_\perp) \neq 0$ (z -axis has been chosen along the vector $\bar{\nu}$; $\bar{r}_\perp\bar{\nu} = 0$). The complete flow of magnetic field $\bar{\mathcal{H}}$ through closed surface which surround monopole is equal to zero: the flow g of the Coulomb part $\bar{\mathcal{H}}^{(g)}$ of field $\bar{\mathcal{H}}$ is being compensated by equal but opposite flow of $\bar{\mathcal{H}}^{(f)}$ along the string.

The interaction of electric and magnetic charges are described in Dirac theory by Lagrangian

$$L_{int} = e\bar{v}\bar{A} \tag{2}$$

so that the velocity and canonical impulse of electric charge are connected by usual relation

$$\bar{v} = \frac{1}{\mu}(\bar{p} - e\bar{A}) \tag{3}$$

The Poisson brackets of Cartesian coordinates and components of canonical impuls have a standard form

$$\{x_i, x_j\} = \{p_i, p_j\} = 0\ , \ \{p_i, x_j\} = \delta_{ij} \tag{4}$$

too.

Due to (3) and (4) the Poisson brackets of velocity components v_i are equal

$$\{v_i, v_j\} = -\frac{e}{\mu^2}\varepsilon_{ijk}\mathcal{H}_k \tag{5}$$

The right side of this equality is contains the unphysical singular part $\bar{\mathcal{H}}^{(f)}$ of magnetic field $\bar{\mathcal{H}}$ that seems unsatisfactory in so far as the components of velocity have the direct physical sense. But on the other hand due to presence only in (5) of field $\overline{\mathcal{H}}^{(f)}$ the calculation of the double Poisson brakets of v_i is gives

$$\varepsilon_{ijk}\{v_i\{v_j,v_k\}\} = -\frac{2e}{\mu^3}\left(\operatorname{div}\overline{\mathcal{H}}^{(g)} + \operatorname{div}\overline{\mathcal{H}}^{(f)}\right) \equiv 0 \qquad (6)$$

in agreement with known Jacoby identity[5].

It was noted repeatedly[6] that if we assumed instead of (5) the relation

$$\{v_i, v_j\} = -\frac{e}{\mu^2}\varepsilon_{ijk}\mathcal{H}_k^{(g)} \qquad (7)$$

without the introduction of singular potential (1), then we obtained with help (4)

$$\varepsilon_{ijk}\{v_i\{v_j,v_k\}\} = -\frac{2e}{\mu^3}\operatorname{div}\overline{\mathcal{H}}^{(g)} = -\frac{2eg}{\mu^4}\delta(\bar{r}) \neq 0 \qquad (8)$$

in contradiction to Jacoby identity. In virtue of such calculation the conclusion was made sometimes[7] that the removal of string from the monopole theory must led to the inconsistency.

2. In the author's paper[3] the formalism has been proposed in which the singular magnetic field $\overline{\mathcal{H}}$ is not introduced. The interaction of nonrelativistic electric and magnetic charges is being interpreted as the particular kind of remote action. The interaction Lagrangian

$$\mathcal{L}_{int} = -\frac{eg}{4\pi}\bar{n}\,\bar{\Omega} \qquad (9)$$

is expressed in mechanical quantity only but not in term of the singular vector potential $\bar{A}$. The such approach is analogous to description of interaction of two nonrelativistic electric charges with help the interaction energy without introduction of electrostatic field conception.

Let us show that both the relation (7) not containing the singular field $\overline{\mathcal{H}}^{(f)}$ and the Jacoby identity are fulfiled in formalism proposed in[3]. In work[4] has been considered the transition to Cartesian coordinates $x_i = (x,y,z)$ from curvilinear ones $\xi_i = (r,\beta,\alpha)$ used in[3]. As it has been made clear the electric charge's velocity can be represented in the form (3) where $\bar{A} = -\frac{g}{4\pi}(\bar{n}\bar{\nu})\,\bar{\nabla}\alpha$ that coincides as it can be seen with (1). However the relations

$$\{x_i,x_j\} = 0,\ \{p_i,x_j\} = \delta_{ij}$$
$$\{p_i,p_j\} = \varepsilon_{ijk}\operatorname{rot}_k(\bar{\nabla}\alpha)\,\pi_\alpha \qquad (10)$$

are fulfiled instead of (4). It can be easily verified[4] now that due to (10) the singular term $\overline{\mathcal{H}}^{(f)}$ drops out from Poisson brackets of v_i and the equality (7) is obtained. It is necessary to make into account in calculation that

$$\pi_\alpha = \frac{\partial\mathcal{L}}{\partial\dot{\alpha}} = \mu\left[\bar{v}\cdot\bar{\nu}\right]\bar{r}_\perp - \frac{eg}{4\pi}(\bar{n}\bar{\nu})$$
$$\operatorname{rot}(\bar{\nabla}\alpha) = 2\pi\,\bar{\nu}\,\delta(\bar{r}_\perp) \qquad (11)$$

so that

$$\{p_i,p_j\} = -\frac{1}{2}eg\,\varepsilon_{ijk}\nu_k \operatorname{sign} z\,\delta(\bar{r}_\perp) \qquad (12)$$

Now let us consider the double Poisson brackets

$$\varepsilon_{ijk}\{v_i,\{v_j,v_k\}\} = \frac{1}{\mu^3}\{p_i,\{p_j,p_k\}\}\varepsilon_{ijk} -$$
$$-\frac{2e}{\mu^3}\{A_i,\operatorname{rot}_i(\bar{\nabla}\alpha)\} - \frac{2e}{\mu^3}\{p_i,\operatorname{rot}_i\bar{A}\} = \frac{1}{\mu^3}\varepsilon_{ijk}\{p_i,\{p_j,p_k\}\} \qquad (13)$$

It may be seemed that substituting (12) into (13) we obtain (8) again. This conclusion is wrong however because the all quantities it is necessary to expresse in the form of function of canonical coordinates and impulses (but not velocities) when their Poisson brackets are calculated. Mear .hile the canonical impulse π_α has been expressed in term of velocity $\bar{v}$ in the right side of (12). This circumstance has led to error in calculation of double Poisson brackets. On the other hand using (10) we obtain

$$\varepsilon_{ijk}\{p_i,\{p_j,p_k\}\} =$$
$$= 2\{p_i,\operatorname{rot}_i(\bar{\nabla}\alpha)\,\pi_\alpha\} = 2\operatorname{rot}_i(\bar{\nabla}\alpha)\{p_i,\pi_\alpha\} \equiv 0 \qquad (14)$$

as $\{P_2, \pi_\lambda\} = -\frac{\partial P_2}{\partial \lambda} \equiv 0$. This relation coincides with Jacoby identity.

In quantum theory we have

$$[\hat{x}_i, \hat{x}_j] = 0, \; [\hat{P}_i, \hat{x}_j] = -i\delta_{ij}$$

$$[\hat{P}_i, \hat{P}_j] = -i\varepsilon_{ijk}\,\mathrm{rot}_k(\bar{\nabla}\lambda)\,\hat{\pi}_\lambda \tag{15}$$

in analogous to (10).

Due to properties of wave functions of system we can replace[4] the right side of latter commutator in (15) by $i\frac{eg}{2}\varepsilon_{ijk}\nu_k\,\mathrm{sign}\,z\,\delta(\bar{r}_\perp)$ when we act on those functions directly. But it is necessary to act by commutator $[\hat{P}_i, \hat{P}_j]$ on derivative of wave functions with respect to Cartesian coordinates when calculating the double commutators of $\hat{P}_i$. In this case we must use the formulaes (15) and calculation of double commutators under consideration gives

$$\varepsilon_{ijk}[\hat{P}_i, [\hat{P}_j, \hat{P}_k]] = 2\,\mathrm{rot}_i(\bar{\nabla}\lambda)\frac{\partial \hat{P}_i}{\partial \lambda} \equiv 0 \tag{16}$$

in agreement with Jacoby identity.

Consequently there is no inconsistency in proposed in[3] formalism.

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