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TO THE THEORY OF A FREE ELECTRON LASER

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A b s t r a c t

The interaction of an electron beam moving inside the magnetic lattice with the electromagnetic wave is considered. It is shown that under certain conditions the generation (amplification) of the coherent electromagnetic radiation appears. The gain is calculated.

The relativistic electron beams passing through a static, transverse, periodic magnetic field (magnetic lattice) are one of the possible sources of coherent radiation. In recent years free electron lasers of such a kind have been discussed quite widely (see /1,2/ and the cited references). Specific features of such lasers are: 1) a broad frequency range due to the fact that the radiation wavelength λ is connected with the period of the magnetic lattice λ_0 by a relation $\lambda \approx \lambda_0/\gamma^2$ where $\gamma = \frac{\mathcal{E}}{m}$ ($\mathcal{E}(m)$ is theelectron energy (mass)); 2) a possibility to pass into rather short wavelengths (at large values γ).

Early in 1976 the gain in radiation of a CO_2 laser /3/ was achieved in the electron beam with energy $\mathcal{E} = 24$ MeV and very recently the same group has also obtained the generation of infrared radiation ($\lambda = 3.417\,\mu$) /4/.

The presented by authors model of a free electron laser /1-4/(see /5/ as well) is based on the quantum induced magnetic bremsstrahlung although in a realistic situation Plank's constant is not included in the gain expressions. In our opinion, this model is unedequate and does not contain a number of important matters. Below it will be shown that the action mechanism of a free electron laser may be described within the framework of the classical theory, the phase regime playing a dominant role. Both the generation and amplification of electromagnetic waves are possible at certain parameters.

Experiment /3,4/ was made in the magnetic lattice with a helical magnetic field. In such a lattice the electron moves along a helix with a constant speed which is parallel to the axis of the lattice, what is equivalent to the motion in a longitudinal magnetic field. Since the transverse momenta are small ($p_{\parallel} \gg p_{\perp} \sim m$), the incoherent radiation spectrum represents a few separated harmonics which are Doppler-shifted with respect to λ_0 . Broadening of these harmonics is due to the finite number of the lattice wavelengths. If side by side with the magnetic lattice there is a wave propagating along the lattice axis, then an additional coherent (induced) radiation appears that provides a more efficient transfer of the particle energy into the radiation within a broad parameter range. This effect occurs in any types of magnetic lattices, in particular, in a plane lattice. Symmetry of the lattice determines the wave polarization.

We now consider the motion of a relativistic particle in the superposition of the transverse magnetic field and the field of the plane electromagnetic wave propagating along the direction of a particle motion. The axis z is chosen along the lattice axis and the magnetic lattice is taken to be helical

$$F^{31} = H \sin\left(\frac{\nu_0 z}{c}\right), \quad F^{23} = H \cos\left(\frac{\nu_0 z}{c}\right) \qquad (1)$$

And the wave is assumed to be circularly polarized. The wave field strength is represented as follows

$$f^{\mu\nu} = H_w\left(\varkappa^{\mu} \dot{a}^{\nu}(\varphi) - \varkappa^{\lambda} \dot{a}^{\mu}(\varphi)\right) \qquad (2)$$

where $\varkappa^{\mu} = (1,0,0,1)$, $\varphi = \nu\left(t - \frac{z}{c}\right) + \psi$;
$\dot{a}^{\lambda} \equiv da^{\lambda}/d\varphi$, $a^1 = \cos\varphi$, $a^2 = \sin\varphi$

The equations of motion in this field have the following form:

$$\frac{du^0}{d\tau} = \Omega\left(u^1 \sin\varphi - u^2 \cos\varphi\right)$$

$$\frac{du^1}{d\tau} = \Omega \sin\varphi \left(u^0 - u^3\right) + \Omega_0 u^3 \sin\left(\frac{\nu_0 z}{c}\right)$$

$$\frac{du^2}{d\tau} = -\Omega \cos\varphi \left(u^0 - u^3\right) - \Omega_0 u^3 \cos\left(\frac{\nu_0 z}{c}\right)$$

$$\frac{du^3}{d\tau} = \frac{du^0}{d\tau} - \Omega_0\left(u^1 \sin\left(\frac{\nu_0 z}{c}\right) - u^2 \cos\left(\frac{\nu_0 z}{c}\right)\right)$$

$$\frac{d\Omega}{d\tau} = -\frac{\omega_p^2}{2}\,\overline{\left(u^1 \sin\varphi - u^2 \cos\varphi\right)} \qquad (3)$$

where

$$\Omega = \frac{eH_w}{mc}, \quad \Omega_0 = \frac{eH}{mc}, \quad \omega_p^2 = \frac{4\pi e^2 N_e}{mV}$$

τ is the proper time, V is the volume of the electromagnetic radiation bunch, N_e is the number of electrons in this volume. The line in the last equation (3) denotes the averaging over all the electrons in the interaction region. This equation describes the change of the wave field because of the interaction with all the electrons and follows from the energy conservation law.

$$\frac{d}{d\tau}\, mc^2 \sum u^0 = -\frac{d}{d\tau}\,\frac{H_w^2}{4\pi} V \qquad (4)$$

Assuming that Ω is the smooth function of time, we integrate the second and third equations of the set (3)

$$u^1 = -\left[\frac{\Omega}{\nu}\cos\varphi + \frac{\Omega_0}{\nu_0}\cos\left(\frac{\nu_0 z}{c}\right)\right] + a^1$$

$$u^2 = -\left[\frac{\Omega}{\nu}\sin\varphi + \frac{\Omega_0}{\nu_0}\sin\left(\frac{\nu_0 z}{c}\right)\right] + a^2 \qquad (5)$$

As will be seen below, near the resonance the wave frequency $\nu \sim 2\gamma^2 \nu_0$ and of interest is the situation when $\frac{\Omega}{\nu} \ll \frac{\Omega_0}{\nu_0}$.
In order to a transverse displacement of the particle in the lat-

tice does not exceed the oscillation amplitude it is necessary that $a^{1,2} \ll \frac{\Omega_0}{v_0}\frac{1}{2\pi N}$ (here N is the number of the lattice wavelengths), then $u^1(0) = -\frac{\Omega_0}{v_0}$, $u^2(0) = 0$.

Substituting the solutions (5) into equations (3) we get the set of equations

$$\ddot{\phi} = 2\Omega\Omega_0 \sin\phi$$
$$\dot{\Omega} = \frac{\Omega_0\omega_p^2}{2v_0}\overline{\sin\phi} \qquad (6)$$

where

$$\phi = \varphi - \frac{v_0 z}{c} \qquad (7)$$

The terms $\sim 1/\gamma^2$ are discarded in (6). We carry out the substitution in (6)

$$\Omega = \Omega_w y \;, \quad \tau\sqrt{\Omega_0\Omega_w} = s$$

Note that $y(0) = 1$. Then these equations take the form

$$\phi'' = 2y\sin\phi$$
$$y' = \beta\,\overline{\sin\phi} \qquad (6')$$

where $\beta = \frac{\omega_p^2}{2v_0\Omega_w}\sqrt{\frac{\Omega_0}{\Omega_w}}$

The initial condition for $\phi' \equiv \frac{d\phi}{ds}$ (see (7)) is

$$\varkappa \equiv \phi'(0) = \frac{v_0\gamma}{\sqrt{\Omega_0\Omega_w}}(\delta - 1) \qquad (8)$$

here

$$\delta = \frac{v}{2v_0\gamma^2}\left(1 + \frac{\Omega_0^2}{v_0^2}\right)$$

In the centre of the resonance line $\delta = 1$, i.e.

$$\lambda = \frac{\lambda_0}{2\gamma^2}\left(1 + \frac{\Omega_0^2}{v_0^2}\right) \qquad (9)$$

When passing through the lattice the "time" increment is equal to

$$s_N = \frac{2\pi N}{\gamma}\,\frac{\sqrt{\Omega_0\Omega_w}}{v_0} \qquad (10)$$

If $\varkappa \gg 1$, $\frac{s_N}{\varkappa} \ll 1$, what is valid when the wave field is weak, the solution of the set (6') may be found in the analytical form by using the perturbation series expansion in inverse powers of $\varkappa$. In this case

$$y(s) = 1 + \beta s^3 R\left(-\frac{\varkappa s}{2}\right) \qquad (11)$$

where

$$R(z) = \frac{1}{2}\,\frac{\sin z}{z^2}\left(\frac{\sin z}{z} - \cos z\right) \qquad (12)$$

In deriving Eq.(11) the electrons are assumed to be uniformly distributed over the initial phases at $s = 0$. The function (see Fig.1) describes the dependence of a gain on a distance of the resonance (cf. Fig.1b in /3/), the maximum value R being 0.135 at z = 1.3. Taking account of Eqs. (8) and (10) we have that in the maximum R the function $1-\delta$ is $1.3/\pi N$. At a single passing (amplification mode) the resonance line width is determined by the function R (Fig.1). In this case, at I/e half-width $\frac{\Delta v}{v} = \frac{2.2}{\pi N}$, that gives $\frac{\Delta v}{v}$ = 0.4% in agreement with experiment /3/. In the generation mode an additional narrowing of the line takes place due to multiple passings. Estimates show that for experiment /4/ the line width should be twice as narrow, what is in agreement with the observation as well.

The wave is naturally amplified with the same polarization as the initial one. Moreover, this polarization should correspond to the lattice symmetry. From (11) we have for the gain

$$G \equiv y^2 - 1 = 2\eta\left(1 + \frac{\eta}{2}\right) \qquad (13)$$

where

$$2\eta = \frac{R(z)}{2\pi}\,\frac{\omega_p^2\Omega_0^2N^3\lambda_0^4}{\gamma^3c^4}\;, \quad z = \pi N(1-\delta)$$

Formulae presented above may be applied directly to the description of the wave amplification /3/. In the case of a small amplification when $y-1 \ll 1$, in (13) it is possible to confine ourselves to the term linear over η. In this particular case, the dependence G on $\omega_p^2, N, \lambda_0, \gamma$ is in agreement with that found in /3,5/. However, even in this event there are essential differences, first of all, in the fact that Eqs. (11) and (13) determine a form of the resonance function.

In the case of generation a bunch of the electromagnetic radiation moving in optical cavity successively interact with the electron bunches. In the beginning of the generation the wave field is quite small, i.e. $\varkappa \gg 1$ (see (8)), then Eqs.(11)-(13) are applicable. The generation is possible if the gain $G > \varkappa_{ef}$ where $\varkappa_{ef}$ is the total loss coefficient (transmission of mirrors, diffractional losses, etc.). For a confocal cavity when a transverse cross-section of the radiation bunch is $\frac{L\lambda}{4}$ (L is the diatance between mirrors) this inequality may be rewritten as follows

$$\frac{\Omega_0^2}{V_0^2}\left(1+\frac{\Omega_0^2}{V_0^2}\right)^{-1} 16\, R(z)\left(\frac{J}{e}\right)\frac{r_0}{L}\,\frac{(N\lambda_0)^3}{\gamma c^3}\,V_0^2 > \varkappa_{ef} \qquad (14)$$

where $r_0 = \frac{e^2}{mc^2}$ is the classical electron radius, J is the electron current. This expression determines, particularly, a threshold magnitude of the current.

As the wave field strength is increasing, inequalities determining the applicability of Eqs.(11) cease to be fulfilled. In this case the set (6') has been solved numerically for the parameter region close to those cited in /4/. This solution

shows that the gain G which for a weak field (13) was independent on H_ω, begins to fall down with increase H_ω. In Fig.2 the dependence of the gain on H_ω/H is shown. For strong fields the spectrum function is also distorted (see Fig.1). The decrease of the gain gives rise finally to ceasing the wave field increase. After that the generation becomes stationary.

All the presented results concern the electron bunches with the uniform initial distribution over phase. In the course of the electron interaction with the electromagnetic wave and magnetic lattice the phase distribution transforms so that at the electron energy higher than the resonance one (see (9)) the radiation of an electron bunch exceeds the absorption, and at the electron energy lower than the resonance one the absorption dominates. The most efficient transfer of the electron energy in the wave field would take place in the case when at the beginning all the electrons were phased in an appropriate manner, and the increment of the phase when passing the electron through the lattice were $\delta\varphi \ll 1$. In such a situation the gain is

$$G = \frac{\omega_p^2}{V_0^2}\left(\frac{\Omega_0}{\Omega}\right)\frac{2\pi N}{\gamma}\sin\psi \qquad (15)$$

so that at $0 < \psi < \pi$ the resonant loss of the energy by electrons occurs, and at $-\pi < \psi < 0$ we have the opposite situation. This result may be considered as an upper limit for the resonant energy transfer from the particles in the wave and inversely.

After the present paper was completed we were aware of the works /6/ (we are indebted to V.N.Korchuganov who payed our attention to these works). In these papers the classical theory of a free electron laser is discussed. The field of the lattice is replaced by the field of the incident electromagnetic wave, what does not provide a correct description of the phase since a compensation occurs in it. This leads in turn to a significant deformation of all the results, particularly, the resonance condition (9) in /6/ does not involve a factor $\left(1+\frac{\Omega_0^2}{V_0^2}\right)$ which under the experiment condition /3,4/ is 1.5.

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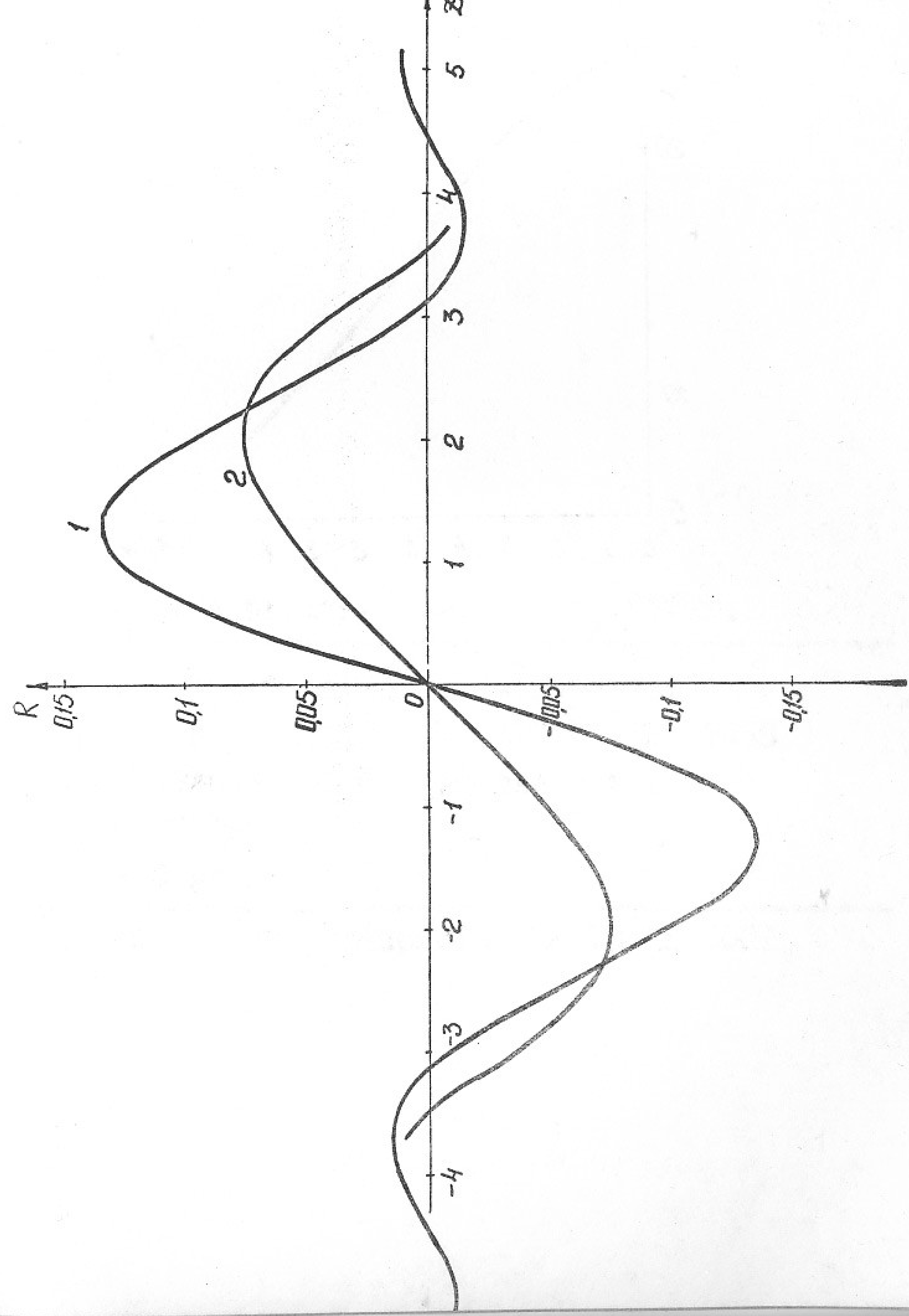
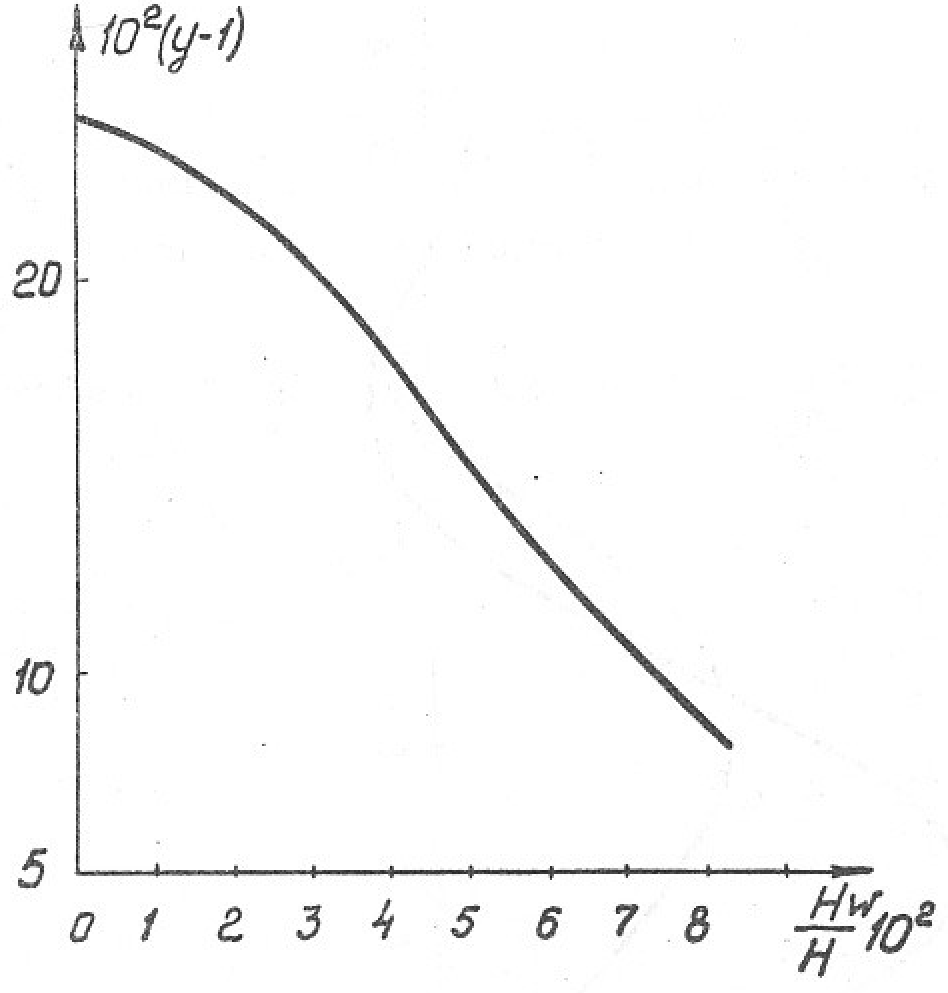
FIGURE CAPTIONS

<u>Fig.1.</u> Function $R(z)$, $z = -\frac{\varkappa s}{2}$; (1)-theoretical curve at weak fields of the wave (formula (12)), (2)-function for a strong field ($\beta = 0.11$, $H\omega/H = 7\cdot10^{-2}$, $s = 2.25$)

<u>Fig.2.</u> Dependence of the gain on the wave field strength ($\beta s^3 = 1.78$)





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