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ON AN INFLUENCE OF POLARIZATION ON INTERNAL SCATTERING EFFECTS OF ELECTRONS IN STORAGE RINGS

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A b s t r a c t

In order to measure the electron (positron) polarization at the facilities with colliding beams, the most extensive use finds the method based on the dependence of internal scattering effects (ISE) of the particles moving in a storage ring on their polarization. For the bunches of an arbitrary configuration (the vertical sizes of a beam have been previously neglected) the rate of particle loss of the beam, due to ISE, is determined. Some shortcomings of the previous papers are eliminated. An explicit expression for the rate of particle loss in the case of small relative energy losses, where the difference between the cross-section of electron-electron scattering and the Born one should be taken into account, is also derived.

The radiative polarization has begun used in research on a variety of storage rings (the study of the reaction e^+e^- - hadrons /1/, comparison between the anomalous magnetic moments of the electron and the positron /2/ etc.). Due to the use of polarized particles in storage rings a problem of measuring of the polarization of particles in storage rings becomes urgent. Several different methods of such measurements are known. Some of them are based on measuring the azimuthal asymmetry in distribution of the final particles, for example, in the two-particle annihilation of the electron-positron pair /3/ (such an experiment is described in /4/), or in scattering the polarized electrons on the circularly polarized laser photons, or on the polarized electron target /5/ (the first of these methods is used now at Stanford).

However, the method proposed in /6/ which based on the dependences of the internal scattering effects (ISE) in storage rings on the polarization of particles has the most extensive use. This has been applied at Novosibirsk /7/, Orsay /8/ and Stanford /9/. If the scattering in an electron beam occurs so that the particles possessing a large transverse momentum and small longitudinal one in the beam-rest system (RS) are scattered at a large angle and gain a large longitudinal momentum, then due to relativistic transformation, in the laboratory system (LS) the longitudinal momentum can be larger than the permissible deviation of the momentum in storage ring. This scattering - sometimes called the Touschek effect - can be important element of beam loss in a storage ring. Since ISE depend on the polarization, the number of the particles outgoing from the beam depends also on this polarization, that can be registered by the appropriate method.

In /6/ the expression for the number of outgoing particles has been obtained in the approximation when the amplitude of vertical oscillations is quite small (a plane beam) with the polarization effects taken into account. For this case, the assumption has been also made that in LS the ratio of the maximally permissible deviation of the momentum from the equilibrium one to the energy $\eta = \Delta P/\mathcal{E}$ is small and the expansion is carried out in powers of η. But actually these assumptions do not always fulfilled, and therefore the solution of the problem without approximations mentioned above is of interest. Moreover, as this will be seen below, for very small values of η it is necessary to take into account that the cross-section of electron-electron scattering differs from Born one. This paper is devoted to these subjects. An attempt to consider the first one of the mentioned problems was made in /7/, but that result contains an error.

Let two particles in the beam have the momenta $\vec{P}_1$ and $\vec{P}_2$ in LS, respectively. We define the vectors

$$\vec{P} = \frac{1}{2}(\vec{P}_1 + \vec{P}_2) \quad , \quad \vec{q} = \frac{1}{2}(\vec{P}_1 - \vec{P}_2) \qquad (1)$$

In a real situation, the spread of the longitudinal (along a beam motion) momentum can be neglected. The spreads of the transversal momentum are small, compared to the energy of particles in the beam $\mathcal{E}$. In the following, we shall regularly discard terms $\sim |\vec{q}|/\mathcal{E}$, with this accuracy: $\vec{n}\vec{q} = 0$ ($\vec{n} = \vec{P}/|\vec{P}|$).

The Lorentz transformation with the velocity*) $\vec{V} = \frac{2\vec{P}}{\mathcal{E}_1 + \mathcal{E}_2}$, where $\mathcal{E}_1, \mathcal{E}_2$ are the energies of the particles under consideration, performs the transition to their centre-mass-system (CMS). If in LS the polarization of a beam is described by the vector $\vec{\zeta}$,

*) Here and below we use the system $\hbar = m = c = 1$.

then in CMS we have with the accuracy indicated above

$$\tilde{\vec{P}}_{1,2} = \pm\vec{q}, \quad \tilde{\mathcal{E}} = \sqrt{q^2+1}, \quad \tilde{\vec{P}}'_{1,2} = \pm\left(\vec{q}'_\perp + \vec{n}(\vec{q}'\vec{n})\frac{\tilde{\mathcal{E}}}{\mathcal{E}}\right),$$

$$\tilde{\vec{\zeta}}_{1,2} = \vec{\zeta}_{1,2} - \left(1 - \frac{1}{\mathcal{E}}\right)\left[\vec{n}(\vec{\zeta}_{1,2}\vec{n}) + \frac{\vec{q}(\vec{\zeta}_{1,2}\vec{n})}{q^2}\right] \pm \frac{[\vec{\zeta}_{1,2}[\vec{q}\vec{n}]]}{\tilde{\mathcal{E}}} \qquad (2)$$

The values $\tilde{P}_{1,2}, \tilde{\mathcal{E}}, \tilde{\zeta}_{1,2}$ are defined in CMS, $'$ denotes the momenta after scattering. From Eqs.(2) it is seen that the vectors $\vec{\zeta}_{1,2}$ undergo the rotation during transformation into CMS. If in LS the vectors $\vec{\zeta}_1$ and $\vec{\zeta}_2$ coincide, then in CMS the angle between them appears:

$$\sin^2\frac{\varphi}{2} = \frac{q^2}{q^2+1}\left[(\vec{\zeta}\vec{n})^2 + \frac{(\vec{\zeta}\vec{q})^2}{q^2}\right]\frac{1}{\vec{\zeta}^2}$$

In /7/ it is assumed that $\tilde{\vec{\zeta}} = \vec{\zeta}$, as a result, the formulae (5) of this paper are incorrect.

Generally speaking, the vectors $\vec{\zeta}_1, \vec{\zeta}_2$ are different, for example, due to spread in directions of the equilibrium polarization of the electrons in a storage ring. However, since all of the values in the right-hand side of the expression for $\tilde{\vec{\zeta}}$ in (2) are taken in LS, one can see that the small differences of $\vec{\zeta}_1$ from $\vec{\zeta}_2$ do not affect the expression $\tilde{\vec{\zeta}}_{1,2}$.

Taking into account (2), one can find an explicit form of the cross-section for polarized electrons using the invariant cross-sections /10/. Integration over angles of the vector $\vec{q}'$ can be conveniently performed if one takes $\vec{n}$ as an axis of the spherical coordinate system. Since we are interested only in the particles whose longitudinal momentum is changed by a value larger than η, the integration over $q'_{\parallel}$ is performed under condition that $|q'_{\parallel}| \geq \rho\eta$. As a result, we get for the cross-section:

$$\sigma_\eta = \frac{1}{2}\frac{\pi\alpha^2}{q^4(q^2+1)}\left\{A_0 + A_1(\vec{\zeta}\vec{n})^2 + A_2\frac{(\vec{\zeta}\vec{q})^2}{q^2} + A_3\frac{(\vec{\zeta}\vec{n}\vec{q})^2}{q^2}\right\}\vartheta(1-x_0) \quad (3)$$

where
$$x_0 = \frac{\eta}{q}\sqrt{q^2+1}\,;\quad \alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}\,;\quad (\vec{\zeta}\vec{n}\vec{q}) \equiv (\vec{\zeta}[\vec{n}\vec{q}])\,;$$

$$A_0 = (4q^2+1)\ln x_0 + (2q^2+1)^2\left(\frac{1}{x_0^2}-1\right) + q^4(1-x_0),$$

$$A_1 = \left[(4q^2+1)(1-2q^2) - \frac{q^2}{q^2+1}\right]\ln x_0 - q^4(1-x_0) - q^2(2q^2+1)\frac{(1-x_0^2)}{2(q^2+1)},$$

$$A_2 = \left(4q^2+1+\frac{q^4}{q^2+1}\right)\ln x_0 + q^2(2-q^2)(1-x_0) + q^4(2q^2+1)\frac{(1-x_0^2)}{2(q^2+1)}, \quad (4)$$

$$A_3 = (3q^2+1)\ln x_0 - q^2(2+q^2)(1-x_0) + \frac{q^2}{2}(1+2q^2)(1-x_0^2).$$

The number of events of interest per unit time $\nu(\eta)$ for which a relative variation of the longitudinal momentum is larger than η is expressed through the cross-section (3) as follows:

$$\nu(\eta) = 2q\sqrt{q^2+1}\,\frac{\sigma_\eta}{V\varepsilon^2}$$

where V is the beam volume in LS.

It is necessary to average the value ν over the distribution of momenta in a beam. In the case of Gaussian distribution, we have for $\nu(\eta)$

$$\nu(\eta) = \frac{2\pi r_0^2 N^2}{\gamma^2 V\delta_x\delta_z}\int_{q_{min}}^{\infty}\frac{dq}{q^2\sqrt{q^2+1}}\exp\left\{-\frac{q^2}{2\delta_+^2}\right\}(a_+A_+ + a_-A_-) \quad (5)$$

Here
$$q^2_{min} = \frac{\eta^2}{1-\eta^2}\,;\quad \delta_\pm^2 = \left(\frac{1}{\delta_x^2} \pm \frac{1}{\delta_z^2}\right)^{-1};\quad \gamma = \frac{\varepsilon}{m}\,;$$

$$a_\pm = \frac{1}{2}\left[I_0\left(\frac{q^2}{2\delta_-^2}\right) \pm I_1\left(\frac{q^2}{2\delta_-^2}\right)\right];$$

$$A_+ = A_0 + \zeta_\parallel^2 A_1 + \zeta_z^2 A_2 + \zeta_x^2 A_3, \quad (6)$$

$$A_- = A_0 + \zeta_\parallel^2 A_1 + \zeta_x^2 A_2 + \zeta_z^2 A_3,$$

where I_0, I_1 are the modified Bessel functions of the first kind; δ_x, δ_z are the widths of distributions of the radial and vertical momenta, respectively; N is the number of particles in the beam, r_0 is the classical radius of the electron.

For applicability of the formula (5) or analogous formulae for other distributions of the electron in a beam, it is necessary that the value η should be not too small. This is connected with the fact that the cross-section (3) is taken in Born approximation. The applicability of the latter will be guaranteed within the entire interval of variation of q in (5) if $\eta \gg \alpha$. Let us now consider the situation when $\eta \lesssim \alpha$. Then in the case $\delta_x \sim \delta_z$ a relative contribution of the terms depending upon the spin is small. This is the case if $\delta_x \gg \delta_z \gg \eta$, $\delta_z \gg \delta_x \gg \eta$. Only at $\delta_x \gg \eta \gg \delta_z$ or $\delta_z \gg \eta \gg \delta_x$ the dependence ν upon the spin terms becomes essential. Let $\delta = max(\delta_x, \delta_z)$, then ν is given by:

$$\nu = \frac{2\sqrt{\pi}\, r_0^2 N^2}{\gamma^2 V\delta\eta^2}\left\{\ln\frac{2}{\eta} - \frac{3}{2} - \frac{(1+\vec{\zeta}_\perp^2)}{4}a(\xi) + B\left(\frac{1}{\xi}\right)\right\} \quad (7)$$

where
$$B(z) = \sqrt{\pi}\left[\frac{2}{z}e^{z^2}\left(1+\frac{z^2}{2}\right)(1-\Phi(z)) - \int_0^z e^{x^2}(1-\Phi(x))dx\right],\quad \xi = \frac{\alpha}{\eta}.$$

The function $a(\xi)$ takes into account the difference of the cross-section from the Born one (see Fig.3):

$$a(\xi) = \frac{2}{\xi}\int_0^1 J_1\left(2\xi z\, \mathrm{sh}\, z\, \mathrm{ch}\,\frac{1}{z}\right)dz \tag{8}$$

J_1 is Bessel function, $\phi(x)$ is error function, $\phi(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$, at $\xi\to 0$, $a(\xi)\to 1$, at $\xi\gg 1$, $a(\xi)\simeq 1/\xi^2\ln 4\xi$.

In modern storage rings the vector of polarization is directed along the axis z. Assuming $\zeta_{\parallel}=\zeta_x=0$ in (5), we have $\tilde{\Delta}(\eta)=\frac{\nu_0(\eta)-\nu(\eta)}{\nu_0(\eta)}=\zeta_z^2\Delta(\eta)$, where $\nu_0(\eta)$ is the corresponding number of events in the unpolarized beam. In Fig.1 the dependence of the value $\Delta(\eta)$ on η in the limiting case $\delta_z\gg\delta_x$, $\eta\gg\delta_x$ is presented. This plot characterizes a relative contribution of the term $\sim A_2$ to (5). In Fig.(2) the dependence of the function $\Delta(\eta)$ on a parameter δ_z at the fixed δ_x = 0.56 for different values of η is given. If the particles are detected within a given interval $\eta_1 \div \eta_2$, then the measured jump in the counting rate is $\sim\zeta^2\Delta_{eff}(\eta_1,\eta_2)$ $\left[\Delta_{eff}(\eta_1,\eta_2)=\frac{\Delta(\eta_1)\nu_0(\eta_1)-\Delta(\eta_2)\nu_0(\eta_2)}{(\nu_0(\eta_1)-\nu_0(\eta_2))}\right]$. Note that $\Delta_{eff}(\eta_1,\eta_2)\leq\Delta(\eta_1)$. The curve 4 in Fig.2 presents the dependence of the value $\Delta_{eff}(\eta_1,\eta_2)$ for η_1 = 0.18, η_2 = 0.30 on δ_z at the same δ_x = 0.56. The values of the parameters taken here correspond to those which have been used in the experiment on VEPP-2M /7/. Estimates show that for this experiment the use of formula obtained above results in increasing the measured degree of polarization by several percent.

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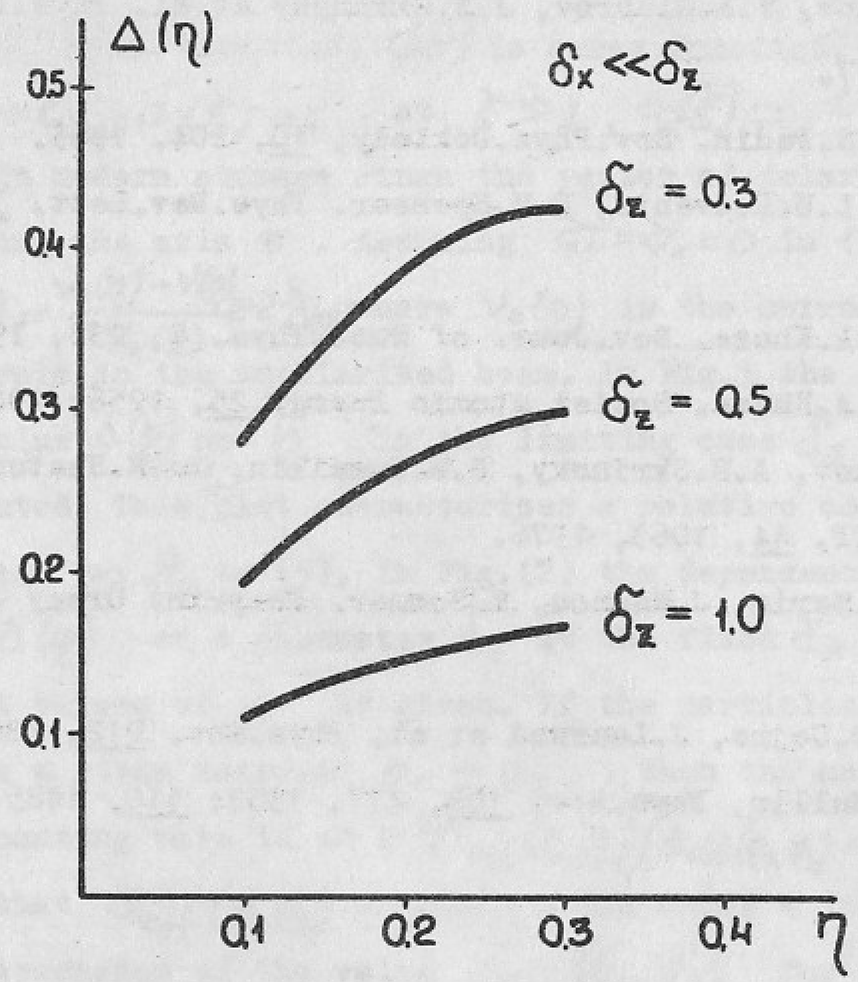

Fig.1 The function $\Delta(\eta)$ at $\delta_z \gg \delta_x, \eta \gg \delta_x$ for different values of δ_z.

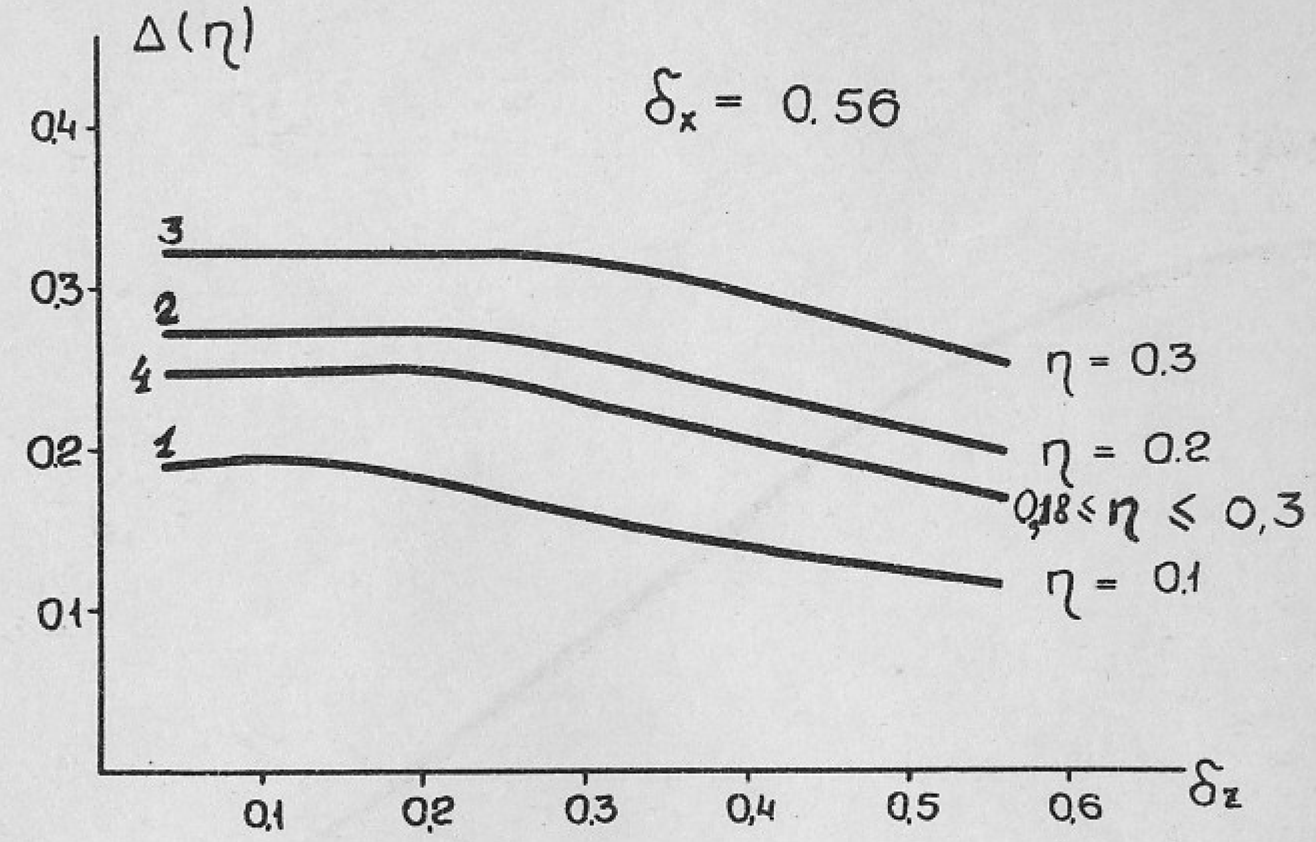

Fig.2 The dependence $\Delta(\eta)$ on the parameter δ_z at $\delta_x = 0.56$ for different values of η.

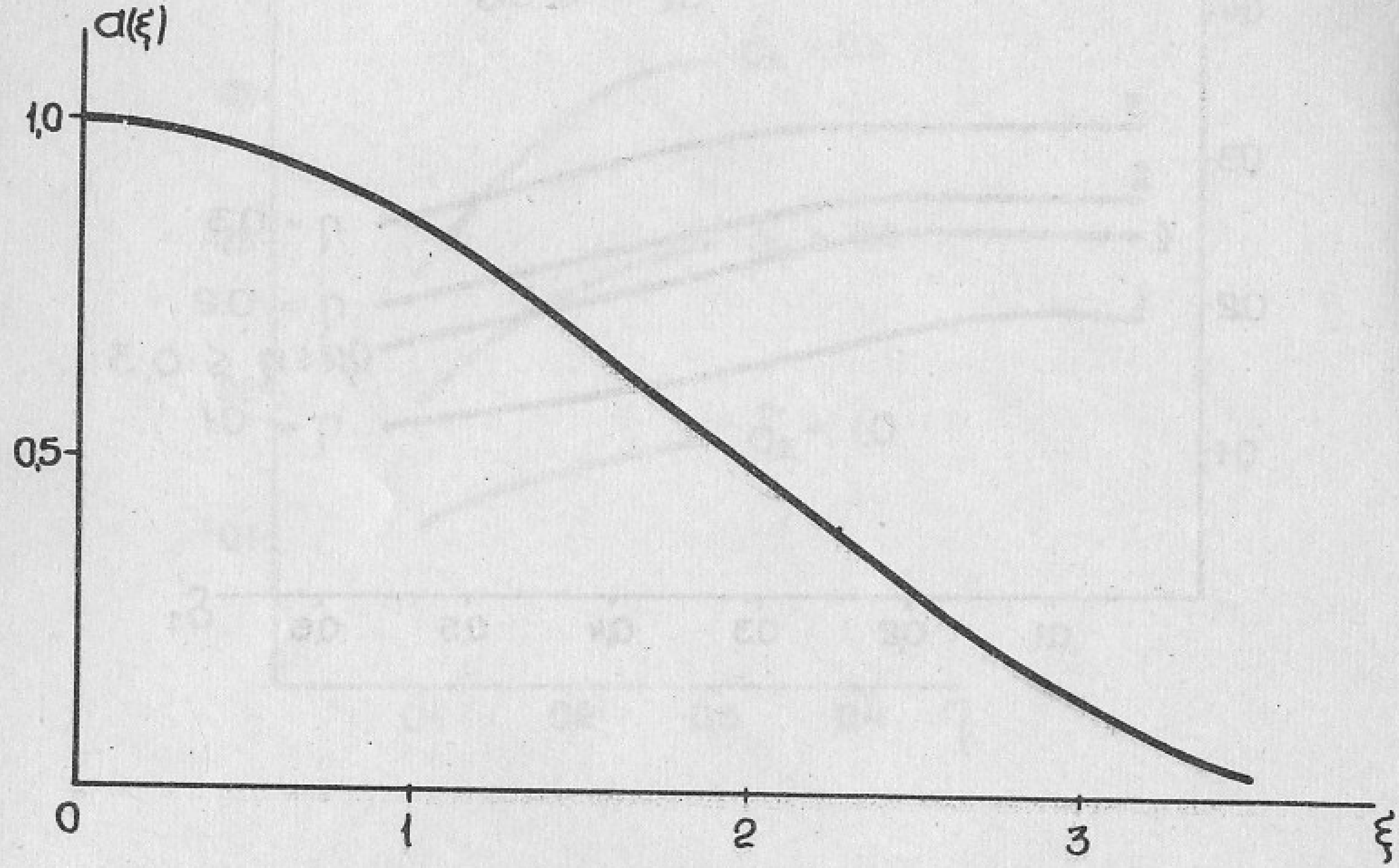

Fig.3 The function $a(\xi)$ which takes into account the difference between the cross-sections of the electron-electron scattering and Born one.

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