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GENERATION OF COHERENT RADIATION
BY RELATIVISTIC ELECTRON BEAM IN ONDULATOR

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A b s t r a c t

The existence has been shown of the effect of automodulation of the electron beam passing through an ondulator which is due to the collective interaction via radiation fields. The conditions on the beam parameters have been obtained under which the radiative instability in question takes place. The possibility is discussed to construct a source of coherent radiation, based on this principle. The numerical examples are given for the sources of submillimeter and infrared range radiation.

1. In recent years there has appeared a tendency to construct the sources of coherent radiation in which electrons moving along a periodically curved trajectory are used. The so-called free-electron laser (FEL) belongs to such a type of devices. The relativistic electron beam in this laser is passed through an ondulator (a periodic transverse magnetic field) located in an open optic resonator /1/.

In this work we study a simpler situation when the electron beam passes through an ondulator and unlike FEL, a resonator is absent. We investigate the problem of radiative instability in the beam in an ondulator. With certain restrictions on the beam parameters the harmonics of density whose wavelength at a given energy resonates with ondulator period become unstable. Generally speaking, for the instability to be revealed, some initial level of density oscillations (or of the current) is required at the entrance of the ondulator. At any rate, the statistical density fluctuations can play a role of the initial excitation. At a sufficient length of the ondulator the resonant harmonics of density fluctuations become so larger during a pass that it is the modulated beam that radiates from a definite section of the ondulator. In this case, modulation of the beam density and its deceleration take place only due to the internal particle interaction via radiation fields. Such a scheme can be used as an independent source of coherent radiation, or as an amplifier if the current flows to the entrance of the ondulator which is previously modulated by means of some source. It should be noted that a similar source has the capability to retune the operating wavelength of radiation merely by variation of the beam energy.

2. Let us dwell in brief upon the theory of developing the longitudinal modulation of the electron beam in an ondulator. Consider first a continuous beam of the electrons moving with the same velocity $\vec{v}$. Let the beam pass through a helical ondulator[1] whose period is $\lambda_0 = 2\pi \lambda\!\!\!^{-}_0 = 2\pi \varkappa^{-1}$. The transverse magnetic field of the ondulator is written in the form $H(y) = H_x + iH_z = H_0 \exp(-i\varkappa y)$ where $H_0 = const$ and y is the coordinate along the ondulator axis. For the beam motion in this ondulator to be stable, an additional longitudinal magnetic field $H_\parallel$ should be introduced. If $H_\parallel$ is much larger than the intrinsic longitudinal field of the ondulator $H_\parallel^0 \approx H_0 \sigma / \lambda\!\!\!^{-}_0$ (σ is the transverse dimension of the beam) which occurs at transverse deviation from the ondulator center, the forced rotation will be the same for all electrons. For the sake of clearness, we restrict ourselves to the case when the rotation amplitude is independent of the quantity $H_\parallel$ $(\omega_\parallel = eH_\parallel/\gamma m \ll \varkappa)$. Then we have: $v_\perp = v_x + iv_z = u\exp(-i\varkappa y)$, $V \equiv \sqrt{v^2 - |v_\perp|^2} = const$, $u = eH_0\lambda\!\!\!^{-}_0/\gamma m \equiv K/\gamma$, $\gamma = 1/\sqrt{1-v^2}$. The radius of electron rotation in a plane (x,z) is $r_0 = \lambda\!\!\!^{-}_0 u$ (in the following we shall assume that $|1-V| \ll 1$ and, therefore, we shall take $V = 1$ everywhere possible).

Let us study the dynamics of beam modulation under the radiation field (in this case, we neglect the action of a Coulomb field). As the transverse motion of electrons is given by external fields, the radiation can result in changing of the longitudinal motion only. If one proceeds to canonically conjugated

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1) In this study we consider a helical onlulator wherein the beam radiation is circularly polarized. Note that the choice of such a specific ondulator is not principal and we use it only for simplicity.

variables $s = y - Vt$ and $\mathcal{P} = \mathcal{E} - \mathcal{E}_0$ ($\mathcal{E} - \mathcal{E}_0$ stands for energy deviation) and expands $\mathcal{P}$ over small deviation, one gets the following Hamiltonian $\mathcal{H}(\mathcal{P}, s, y)$ (it is convenient to use the longitudinal coordinate as a time) describing a relative motion of electrons under the radiation:

$$\mathcal{H} = \frac{\mathcal{P}^2}{2\gamma_\parallel^2 \mathcal{E}} - e\vec{v}_\perp \vec{A}, \tag{1}$$

where $\gamma_\parallel = 1/\sqrt{1-V^2}$, $\vec{A}$ is the vector potential of the radiation field. This potential is produced by the current density $e\vec{v}_\perp \cdot \rho(\vec{s}_\perp, s, y)$ which is expressed in the variables $\vec{s}_\perp$ and s, y. $\vec{s}_\perp$ is the transverse coordinate of electrons. The dependence of ρ on y characterizes a slow variation of the modulation amplitude which takes place on wavelengths highly exceeding the ondulator period.

Hamiltonian (1) may be transformed into the following form[2] by means of a formula for retarding potentials:

$$\mathcal{H} = \frac{\mathcal{P}^2}{2\gamma_\parallel^2 \mathcal{E}} - e^2|u|^2 Re \int_0^y dy' \int d\vec{s}_\perp' \frac{\rho(\vec{s}_\perp', s + y' - y + V|\vec{z} - \vec{z}'|, y')}{|\vec{z} - \vec{z}'|} e^{i\varkappa(y'-y)}, \tag{2}$$

where $|\vec{z} - \vec{z}'| = \sqrt{(y-y')^2 + (\vec{s}_\perp - \vec{s}_\perp')^2}$. The dependence of $\mathcal{H}$ on $\vec{s}_\perp$ is included parametrically ($\dot{\vec{s}}_\perp = 0$). As seen from (2), radiation resonates with the harmonic of beam density which is modulated in the longitudinal direction with the period $\lambda = \lambda_0(1-V)$.

An equation which describes the variation of particle densities in the ondulator may be derived by the kinetic equation for a function of particle distribution $f(\mathcal{P}, s, y)$:

$$\frac{\partial f}{\partial y} + \frac{\partial \mathcal{H}}{\partial \mathcal{P}} \frac{\partial f}{\partial s} - \frac{\partial \mathcal{H}}{\partial s} \frac{\partial f}{\partial \mathcal{P}} = 0$$

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2) We ignore the radiation propagating in the direction opposite to motion of the beam.

In a linear-over-modulation-amplitude approximation it is not difficult to derive from the latter equation the following simple equation for $\rho = \int f d\mathcal{P}$:

$$\frac{\partial^2\rho}{\partial y^2} = \frac{1}{\gamma_\parallel^2 \mathcal{E}}\left(\frac{\partial^2 \mathcal{H}}{\partial s^2}\right)\rho_0 \qquad (3)$$

where $\rho_0(\vec{s}_\perp)$ is the beam density at the entrance of the ondulator (at $y = 0$).

Represent ρ in the form

$$\rho = \rho_0 + a(\vec{s}_\perp, s, y)\exp(-iKs) + a^*(\vec{s}_\perp, s, y)\exp(iKs),$$

where the strong dependence of ρ on S ($K = \varkappa/(1-V)$) is obvious. After simple calculations, with the help of eq.(2) ($|\vec{z} - \vec{z}'| \simeq (y-y') + |\vec{s}_\perp - \vec{s}_\perp'|^2[2(y-y')]^{-1}$) we obtain from (3) the following equations for the amplitude of modulation of the continuous beam if the distribution in the transverse cross section is Gaussian ($\rho_\perp(\vec{s}_\perp) = \frac{1}{2\pi\sigma^2}\exp(-\vec{s}_\perp^{\,2}/2\sigma^2)$). In the limit of the wide beam we obtain the equation

$$\frac{\partial^2 a}{\partial y^2} = \left(\frac{\pi}{i\gamma\gamma_\parallel^2}\right)\dot{N} r_e |u|^2 K \rho_\perp \int_0^y a(\vec{s}_\perp, s+(1-v)(y'-y), y')\,dy'. \qquad (4)$$

In the limit of the narrow beam the equation has the form:

$$\frac{\partial^2 a}{\partial y^2} = \frac{\dot{N} r_e |u|^2 K^2}{2\gamma\gamma_\parallel^2}\int_0^y \frac{a(\vec{s}_\perp, s+(1-v)(y'-y), y')}{y-y'+iK\sigma^2}\,dy' \qquad (5)$$

where $e\dot{N} = eV\int \rho\, d\vec{s}_\perp$ is the beam current, $r_e = e^2/m$. The characteristic quantity σ^2 distinguishes these two limiting cases and is equal to $\sigma_{cr}^2 = (\gamma_\parallel \lambda^2/u)\sqrt{\dot{N} r_e/\gamma}$.

Let us solve eqs.(4) and (5) for the resonant amplitude of modulation. Let a be independent of S. The initial conditions for a in the case when all the electrons in the beam have the same initial velocity $\vec{v}$ will be the following:

$a_{y=0} = a_i$, $(da/dy)_{y=0} = 0$. For the wide beam we obtain $a(y) = \frac{a_i}{3}(\exp(\Lambda_1 y) + \exp(\Lambda_2 y) + \exp(\Lambda_3 y))$ where $\Lambda_3 = \Lambda_2 \exp i\frac{\pi}{3} = \Lambda_1 \exp i\frac{2\pi}{3} = i\left[2\pi\frac{\dot{N} r_e}{\gamma}|u|^2\varkappa\rho_\perp\right]^{1/3}$. Hence, the amplitude of modulation grows exponentially. The characteristic length on which the amplitude becomes e times larger (at $\exp|\Lambda|y \gg 1$) is[3]

$$\ell = \frac{2\gamma}{\sqrt{3}\,\sqrt[3]{2\pi \dot{N} r_e \mathcal{K}^2 \varkappa \rho_\perp}} \,, \qquad (6)$$

For the narrow beam the corresponding expression for $\ell = Re\,\Lambda^{-1}$ satisfies the relation obtained from eq.(5):

$$\Lambda^2 = \frac{\dot{N} r_e |u|^2 K^2}{2\gamma\gamma_\parallel^2}\left[\ln(iK\sigma^2\Lambda)^{-1} - 0.58 + \dots\right]. \qquad (7)$$

Equations (4) and (5) also enables one to find a characteristic width of the spectrum of density harmonics which can be unstable. A relative width of the resonance $\Delta K/K$ is approxximately equal to λ_0/ℓ .

3. Let us write down the restrictions on the beam and ondulator parameters at which the obtained results are valid.

Radiation fields will play the dominant role in the dynamics of density modulation if the periodical part of the projection of Coulomb field $\vec{E}_c$ onto the particle velocity is small compared to the radiation field projection: $\vec{E}_c\vec{v} \simeq E_c \ll |\vec{A}\dot{\vec{v}}_\perp| \approx |\vec{A}|u$.

3) Note that the instability increment may be also found by solution of the dispersion equation. This approach has been used in the just published paper /2/ whose authors study a particular case of instability resulting from the interaction between the infinitely wide electron beam ($\sigma \to \infty$) and the colliding weak laser wave ($\gamma u \ll 1$). In the case of $\sigma \to \infty$, the dispersion equation coincides with the well studied equation for a running wave lamp. The increment expression in Ref./2/ agrees (if the equivalent replacement of a laser wave by an ondulator is made) with the expression (6) of the present study.

Hence, we obtain the following conditions:

$$\mathcal{K}^2 \gg \min\left(\dot{N} r_e/\gamma\, ,\ \sqrt{\dot{N} r_e/\gamma}\ \frac{\gamma \lambda}{\sigma}\right), \qquad \sigma^2 \gg \gamma^2 \lambda^2/\mathcal{K}^2,$$

which should be fulfilled.

Transverse components of the Coulomb field leads to adiabatic extension of the beam. However, as the analysis shows, an influence of the extension effect on the development of radiative instability is eliminated, for example, by a longitudinal magnetic field magnetizing the transverse motion of electrons.

Evaluate now the restrictions imposed on both the energy $\Delta\mathcal{E}/\mathcal{E}$ and angular $\Delta\theta$ spread of the electron beam. Radiative instability will take place if the particle shifts due to the spread of longitudinal velocities can be neglected on the wavelengths of the order ℓ : $\ell \Delta V/V \lesssim \lambda$. Therefore, we obtain

$$\Delta\mathcal{E}/\mathcal{E} \lesssim \lambda_0/\ell \quad , \quad \Delta\theta \lesssim \sqrt{\lambda/\ell} .$$

Let us find the limiting value of the beam current, based on the fact that the ondulator field H_0 must be dominant compared to the radiation fields acting on the particles inside the beam. The condition $H_0 \gg |\vec{A}|(1-V)$ means that[4]

$$e\dot{N} \ll e\frac{\gamma}{r_e}\ \frac{\gamma_{\parallel}^2 \sigma^2}{\lambda_0^2} . \qquad (e/r_e = 1{,}6\cdot 10^4 A). \qquad (8)$$

4. Thus, the beam, after its passage of $L \simeq \ell \cdot \ln(\rho_0/\tilde{\rho}_i)$ in the ondulator ($\tilde{\rho}_i/\rho_0$ is the initial level of the resonant harmonic of density), will have the modulation amplitude close to the total one and will radiate in this state on a length of the order ℓ . The total power of coherent radiation will be about

$$W \approx \gamma m \dot{N} \lambda_0/\ell ,$$

4) When fulfilling the condition (8) the growth length always exceeds the ondulator period.

As wee see, the value of λ_0/ℓ determines a portion of the beam energy conversing into radiation.

In the case when the modulation amplitude is formed from the spectrum of density fluctuations, the spectral width of radiation line $\Delta\omega/\omega$ will be of the order of λ_0/ℓ , and radiation for both the wide and narrow beams will be concentrated within the angle $\hat{\theta}$ of the order $\sqrt{\lambda/\ell}$. If the initial condition is prepared with the help of an external monochromatic radiator[5], then $\Delta\omega/\omega \approx \lambda/\ell$. With the narrow beam the angular divergence of radiation is $\hat{\theta} \approx \sqrt{\lambda/\ell}$, with the wide beam $\hat{\theta} \approx \lambda/\sigma$.

5. It seems quite promising to apply the described above principle of automodulation of the beam for creating a source in a submillimeter range. Let us consider the example[6]. Let $\gamma = 3.5$, $\lambda_0 = 2$ cm, $H_0 = 1.7$ kG, $H_{\parallel} = 4$ kG. Then the radiation wavelength λ is 1 mm. For a 5 A beam current the characteristic beam area is $2\pi\sigma_{cr}^2 = 4$ cm^2. If the beam area is 0.5 cm^2, we get that the growth length ℓ is 50 cm. The energy

5) It is of interest to note that, as a modulating radiator, a source can be used, based on the considered principle wherein the spectrum width required is cut off with the monochromator. There is also the possibility to create a feed-back source in which a small fraction of the output radiation is applied again to the entrance of the ondulator after monochromatization. The total length of the ondulator is also reduced in such a scheme.

6) For the indicated parameters the corrections associated with both the longitudinal field and the Coulomb one have been taken into account.

and angular spreads of the beam should not exceed the following values: $\Delta\mathcal{E}/\mathcal{E} \lesssim 5\cdot10^{-3}$, $\Delta\theta \lesssim 2\cdot10^{-2}$. This beam[7)] will radiate a power of the order 50 kW in the ondulator. In the case when modulation developes from the fluctuation spectrum[8)]: $\Delta\omega/\omega \approx 5.10^{-3}$, $\hat{\theta} \approx 2\cdot10^{-2}$, whereas in the case of development from one harmonic of density the value of $\Delta\omega/\omega \approx 3\cdot10^{-4}$ is diminished.

Such a method of producing the coherent radiation enables one to use the pulsed electron sources as well. An increase of the duty factor (with the same power of a source of electrons) decreases the characteristic growth length and increases a fraction of the beam energy conversing into radiation. For example, the following parameters are chosen[9)]: $\gamma = 20$, $\lambda_0 = 2$ cm, $2\pi\sigma^2 =$

7) To produce the electron beam with these parameters, one can use the electrostatical method of acceleration. Currently the sources of such a kind are in progress in connection with the electron cooling problem of antiproton beams in storage rings /3/. The electrostatical acceleration method is especially convenient due to the capability to recuperate, in the simplest way, the energy of the applied electron beam.

8) When the modulation developes from the spectrum, the necessary length of the ondulator is approximately equal to 10 growth lengths of ℓ.

9) These parameters of the beam may be attained in the existing high-current accelerators with the pulse current duration of about 10^{-7} sec (see, e.g., /4/). Note that in our case the local energy spread in the beam is of importance and it can be less than the integral one.

$= 1$ cm^2, $e\dot{N} = 10^4$ A, $H_0 = 2.5$ kG, $H_{\parallel} = 10$ kG, $\lambda = 3.3\cdot10^{-3}$cm, $2\pi\sigma_{cr}^2 = 10^{-2}$ cm^2, $\Delta\mathcal{E}/\mathcal{E} \leq 2\cdot10^{-2}$, $\Delta\theta \lesssim 5\cdot10^{-3}$. The growth length ℓ is 15 cm. The beam will radiate at the angle $\hat{\theta} \approx 10^{-3}$ and the degree of non-monochromatization is $\Delta\omega/\omega \approx 4\cdot10^{-5}$ if the modulation occurs from one harmonic of density, or $\Delta\omega/\omega \approx 2\cdot10^{-2}$, $\hat{\theta} \approx 5\cdot10^{-3}$ if it occurs from the spectrum of fluctuations. Radiation power will be $\approx 10^9$ W.

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