ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
СО АН СССР

54

G.L.Kotkin, E.A.Kuraev and V.G.Serbo

RADIATIVE CORRECTIONS IN ELASTIC ELECTRON-PROTON SCATTERING IN THE RANGE OF MOMENTUM TRANSFERRED $m_e^2 \ll -t \ll m_p^2$

ПРЕПРИНТ ИЯФ 79-72

Новосибирск

ABSTRACT

Radiative corrections in elastic electron-proton scattering are calculated for such a set up when only the scattered electron is registered. The accuracy of calculation is $\sim$ 0,1%.

АННОТАЦИЯ

Вычислены радиационные поправки к сечению реакции в постановке, когда регистрируется только рассеянный электрон. Точность расчета $\sim$ 0,1%.

1. INTRODUCTION

The accuracy in elastic electron-proton scattering experiments was essentially increased during the last few years. In some experiments [1, 2] the cross section $d\sigma$ was measured with accuracy exceeding 1%. However, electromagnetic form factors of proton are extracted from Rosenbluth cross section $d\sigma_0$, with amplitude M_0 shown in Fig. 1 (wavy line corresponds to photon, double one - to proton). The measured cross section $d\sigma$ is distinguished from the Rosenbluth one $d\sigma_0$ due to electromagnetic radiative corrections (RC): $d\sigma=(1+\delta)\cdot d\sigma_0$. The magnitude of RC δ depends on the set up of the experiment. In this paper we shall consider such one when only the energy and scattering angle of electron are registered, and recoil proton and photons are not registered.

In such a set up one usually uses the formulae of Schwinger [3], Maximon [4] and Tsai [5] (see also review [6]). However, the accuracy of these formulae is unsufficient for the high-precised experiments [1, 2] because in these papers either the recoil effects of proton (including its bremsstrahlung) as in Refs. [3, 4] had not been taken into account, or the terms whose contribution into δ has the magnitude of α = 1/137 order, as in Ref. [5], had been neglected.

Meanwhile, the model independent calculation of such terms is possible if momentum transferred to proton is small compared to proton mass

$$m^2 \ll -t \ll M^2\,, \quad m \equiv m_e\,, \quad M \equiv m_p\,. \qquad (1)$$

Indeed, uncertainty due to strong interaction effects arises mainly from the amplitude M_5 with two-photon exchange whose contribution into δ is smaller or of the order of the magnitude

$$\alpha \frac{t}{M^2} . \qquad (2)$$

In this paper, initiated by experiment [2], we obtained formulae for δ with the accuracy exceeding the one in Refs. [3-6]. With this purpose in the first order of α (the amplitudes M_{1-6}) we have taken into account all the recoil effects neglecting the terms of order of (2) only. Unlike [3-5] we did not assume $\Delta\mathcal{E}$ (inaccuracy of electron energy) to be very small and also calculated the terms proportional to $\Delta\mathcal{E}/\mathcal{E}$ and $\Delta\mathcal{E}/M$. It may be important when scattering angle is close to 180°. Besides, in the second order of α we took into account the main contributions containing the products of four or three large logarithms $\ln(-t/M^2)$ and one or two logarithms of $\ln(\Delta\mathcal{E}/\mathcal{E})$ kind, neglecting the terms in δ of the kind

$$\frac{\alpha^2}{\pi^2}\cdot\left(\ln\frac{-t}{m^2}\right)^2 \qquad (3)$$

(for example, M_{11}), so far as in experimental conditions of Ref. [2] type, $-t \sim \mathcal{E}^2 \sim (110 \text{ MeV})^2$, quantities (2) and (3) are of the same order $\sim 5\cdot 10^{-4}$. Below in Section 2 we give our main results. In Section 3 the comparison with previously obtained results is given and the generalization of our formulae for the elastic electron-nuclei scattering is discussed (the "peak" approximation for this case is given in Appendix 4). Sections 3-4 and Appendices 1-3,5 are devoted to the details of calculations.

The main results of the paper are presented in formulae (4-6), (9), (22).

2. MAIN RESULTS

1. Basic notations we use in the paper:

m and M - mass of electron and proton; $\mathcal{E}$ and θ -energy and scattering angle of electron (rest frame of initial proton everywhere is implied); $r = (2\mathcal{E}/M)\sin^2(\theta/2)$ - recoil parameter; $\mathcal{E}_0' = \mathcal{E}/(1+r)$ -energy of elastically scattered electron; $\mathcal{E}'$ - energy of inelastic (with the emission of photon) scattered electron; $\nu = \Delta\mathcal{E}/\mathcal{E}_0'$ - relative distance from elastic peak; $t = -4\mathcal{E}\mathcal{E}_0'\sin^2\frac{\theta}{2}$ - square of transferred momenta at elastic scattering;

$$L = \ln\frac{-t}{m^2}, \quad \ell = \ln\frac{1}{\sin^2(\theta/2)},$$

$$\varphi(x) = -\int_0^x \ln(1-y)\frac{dy}{y}, \quad \varphi(1) = \frac{\pi^2}{6};$$

G_E and G_M - electric and magnetic form factors of proton; $d\sigma_0$ - Rosenbluth cross section:

$$d\sigma_0 = \frac{Z^2\alpha^2\cos^2\frac{\theta}{2}}{4\mathcal{E}^2\sin^4\frac{\theta}{2}}\cdot(C+D)\cdot\frac{dO_e}{1+r},$$

$$C = -\frac{t}{2M^2}G_M^2\, tg^2\frac{\theta}{2}, \quad D = \frac{G_E^2 - tG_M^2/4M^2}{1-t/4M^2},$$

dO_e - solid angle element of scattered electron. For convenience we have also intriduced Z -charge of proton ($Z = 1$). Throughout the article we consider electron-proton scattering;

formulae for positron-proton scattering may be obtained by means of formal replacement $Z \to -Z$.

2. Let us consider such an experiment when the scattered in solid angle element dO_e electron is registered, its energy ε' lying in the interval

$$\varepsilon'_0 - \Delta\varepsilon \leqslant \varepsilon' \leqslant \varepsilon'_0, \quad (\nu = \Delta\varepsilon/\varepsilon'_0 \ll 1)$$

(the so-called "elastic peak"). In this case $d\sigma(\nu) = (1+\delta)\cdot d\sigma_0$, where RC δ is

$$\delta = \delta_{e\gamma} + \delta_{interf} + \delta_{p\gamma} + \delta_{\mu} + \delta_{h} \tag{4}$$

Here, $\delta_{e\gamma}$ corresponds conditionally to electron bremsstrahlung:

$$\delta_{e\gamma} = (1+\beta)e^{\delta_{soft}} - 1 + \frac{2\alpha}{\pi}\left[\frac{1}{2}(L-1)\cdot \ln(1+r) - a\nu + b\nu^2\right],$$

$$\beta = \frac{2\alpha}{\pi}\left[\frac{13}{12}(L-1) - \frac{17}{36} - f(\theta)\right],$$

$$\delta_{soft} = -\frac{2\alpha}{\pi}(L-1)\ln\frac{1}{\nu},$$

$$f(\theta) = \frac{\pi^2}{12} - \frac{1}{2}\Phi\left(\cos^2\frac{\theta}{2}\right) = \frac{1}{2}\ln\sin^2\frac{\theta}{2}\cdot\ln\cos^2\frac{\theta}{2} + \frac{1}{2}\Phi\left(\sin^2\frac{\theta}{2}\right), \tag{5}$$

$$a = \frac{1}{C+D}\left[D - \frac{\varepsilon D}{2M}\left(L - 3 - l\cdot tg^2\frac{\theta}{2}\right) + C\cdot L\right],$$

$$b = \frac{1}{4(C+D)}\left[2D\cdot\left(L - 3 + \frac{1+l\cdot\cos\theta}{1+\cos\theta}\right) + C\left(L - 1 + l + \frac{2}{1-\cos\theta}\right)\right]$$

Quantity δ_{interf} corresponds to the interference of emission from electron and proton, i.e. to the interference of amplitudes M_1 and M_2, M_5 and M_0:

$$\delta_{interf} = \frac{Z\alpha\pi\sin\frac{\theta}{2}}{1+\sin\frac{\theta}{2}} - \frac{4Z\alpha}{\pi}\left[\ln\frac{1}{\nu} + 1 - \frac{l}{4}\left(1 + \frac{l}{1+\cos\theta}\right)\right]\cdot\ln(1+r). \tag{6}$$

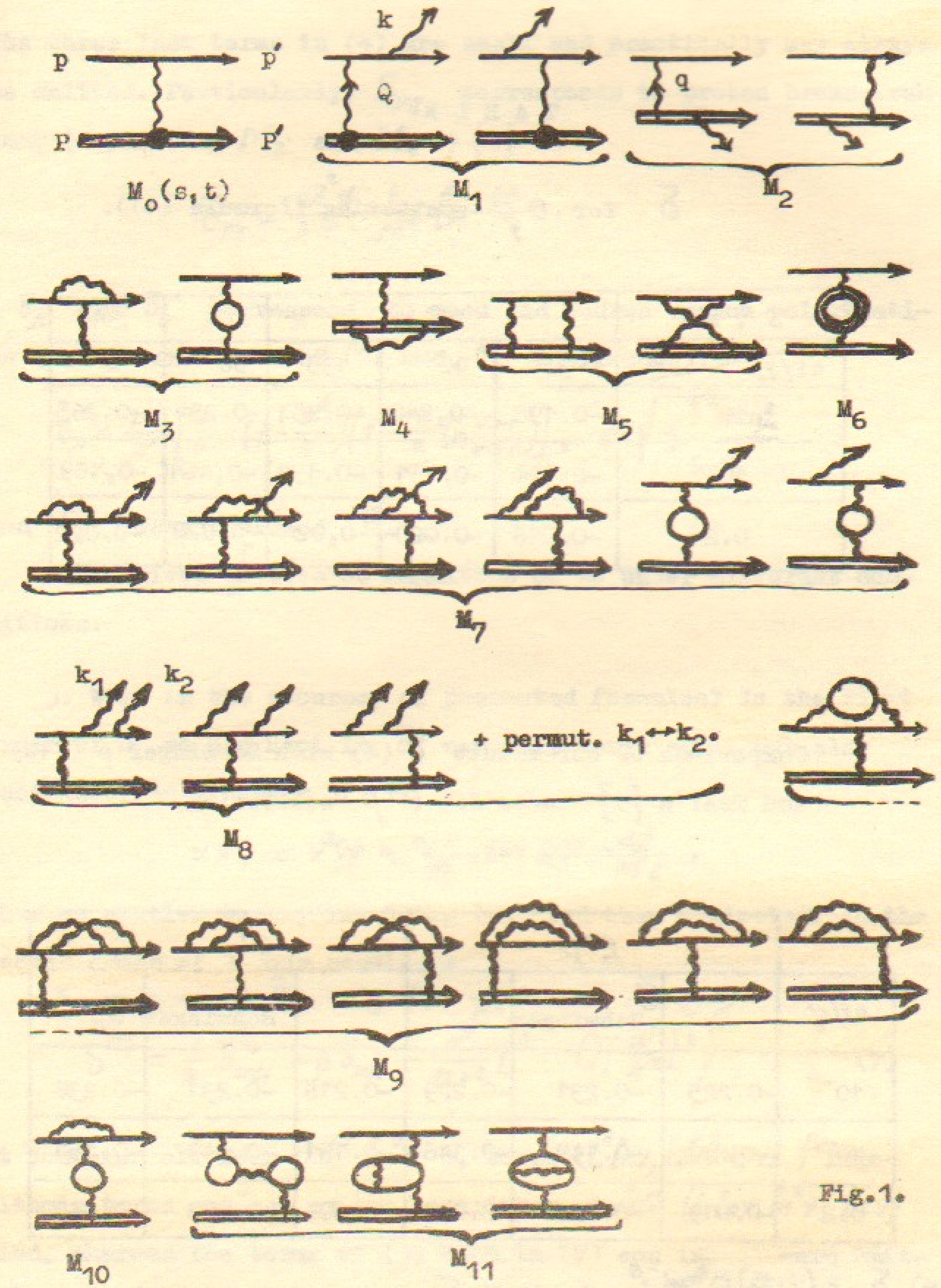

Fig.1.

TABLE

δ for e^-p-scattering (formula (4)).

ε, MeV	100			200	500
$\Delta\varepsilon/\varepsilon_0'$ \ θ	30°	90°	135°	60°	30°
10^{-3}	-0.198	-0,240	-0.251	-0.254	-0.265
10^{-2}	-0.124	-0.151	-0.158	-0,161	-0,169
0.2	-0.018	-0.020	-0,024	-0,022	-0.022

Comparison of our result δ (4) with Schwinger's (11b) and Tsai's [5] ones for $e^\mp p$-scattering,

ε = 100 MeV, $\theta = 60^\circ$

	e^-p			e^+p		
$\Delta\varepsilon/\varepsilon_0'$	δ	$\tilde{\delta}_{Schwinger}$	δ_T ж)	δ	$\tilde{\delta}_{Schwinger}$	δ_T ж)
10^{-3}	-0.225	-0.231	-0.229	-0.216	-0.231	-0.236
10^{-2}	-0,142	-0.149	-0.146	-0.131	-0.149	-0.151
0.2	-0.019	-0.028	-0.024	-0.006	-0.028	-0.043

ж) $\delta_T = (1+\beta)e^{\delta_{Tsai}-\beta} - 1$ where δ_{Tsai} from the paper by Mo and Tsai [5], formula (II.6).

The three last terms in (4) are small and practically may always be omitted. Particularly, $\delta_{p\gamma}$ corresponds to proton bremsstrahlung (amplitudes M_2 and M_4) [7]

$$\delta_{p\gamma} = \frac{2Z^2\alpha}{3\pi}\cdot\frac{t}{M^2}\ln\frac{M}{\varepsilon\nu},$$

δ_μ and δ_h correspond to muon and hadron vacuum polarization (the interference of M_6 and M_0 amplitudes):

$$\delta_\mu = \frac{2\alpha}{\pi}\left[\frac{1}{9} - \left(1-\frac{v^2}{3}\right)\left(1-\frac{v}{2}\ln\frac{v+1}{v-1}\right)\right], \quad v = \sqrt{1-\frac{4m_\mu^2}{t}},$$

and δ_h doesn't exceed δ_μ.

Table gives an idea on magnitude of RC under different conditions.

3. What is the accuracy of presented formulae? In the first order of α we had kept in δ not only terms $\sim\alpha$, but also contributions of order of

$$\alpha\nu,\ \alpha\nu^2,\ \alpha\frac{\Delta\varepsilon}{M},\ \alpha r \sim \frac{\alpha t}{M\varepsilon},$$

whereas smaller terms, involving (2), had been neglected. In the second order of our result is

$$\delta^{(2)} = \frac{1}{2}\delta^2_{soft} + \beta\cdot\delta_{soft} - \frac{\alpha^2}{18\pi^2}L^3 + O\left(\frac{\alpha^2}{\pi^2}L^2\right). \qquad (7)$$

It contains all four ($L^2\cdot\ln^2\nu$), three ($L^2\ln\nu, L\ln^2\nu, L^3$) logarithmic terms and one or two logarithms of $\ln\nu$ ($L\ln\nu, \ln^2\nu, \ln\nu$) kind, whereas the terms of (3) kind in (7) and in were omitted. We omitted in (4) the last term in (7) $-\frac{\alpha^2}{18\pi^2}L^3$ because it did not exceed the neglected ones (3) in the whole region (1): $L/18 < 1$.

4. For typical in experiment [2] conditions

$$\varepsilon = 111\ MeV, \quad \theta = 58{,}3^{o}, \tag{8}$$

$\nu = 0.01$ RC and its contributors are $\delta = -0.144$; $\delta_{e\gamma} = -0.149$; $\delta_{interf} = 0.005$, $\delta_{p\gamma} = -1.3 \cdot 10^{-4}$; $\delta_{\mu} = 3\ 10^{-4}$; $\delta_{soft} = -0.207$; $\beta = 0.045$. Uncertainty of quantity δ in these conditions if of $5 \cdot 10^{-4}$ order.

5. Let us consider now such a set up when only scattered in solid angle element dO_e electron is registered and its energy ε' is situated at small distance $\Delta\varepsilon = \varepsilon_0' - \varepsilon' \equiv \nu \cdot \varepsilon_0'$ from elastic peak (the so-called "radiative tail"). The corresponding expression for the cross section is[*)]

$$d\sigma = d\sigma_0 \cdot \frac{2\alpha}{\pi} \frac{d\varepsilon'}{\Delta\varepsilon}\Big[L - 1 - a\nu + 2b\nu^2 + 2Z\ln(1+r) - \frac{Z^2 t}{3M^2} + \\ + (\delta_{soft} + \beta)(L-1)\Big]. \tag{9}$$

The accuracy of this formula is $\lesssim 1\%$ for $\nu \lesssim 0.2$ and decreases for larger ν. More exactly: when inferring (9) we omitted the terms of relative order of

$$\nu^3 L, \quad \nu L \frac{\varepsilon}{M}, \quad \frac{t}{M^2}\nu, \quad \alpha\nu L.$$

6. In experiments [2] the ratio

---

*) It can be obtained from the cross section $(1+\delta)\, d\sigma_0$ (4) by differentiation over ν.

$$R = \frac{\Delta\varepsilon \cdot d\sigma}{d\varepsilon' dO_e} \Big/ \frac{d\sigma(\nu)}{dO_e} \equiv \frac{d\ln[d\sigma(\nu)/dO_e]}{d\ln\nu}$$

was measured. From (9), (4) one can easily obtain that the quantity R is approximately constant under conditions (8) and $\nu = 0.01 \div 0.2$ in accordance with the results of experiment [2]. Indeed, if we keep in R/R_0 the terms $\sim\alpha$ and omitted $\delta_{p\gamma}$, δ_{μ} and δ_h, we have

$$\frac{R}{R_0} = 1 + \frac{1}{L-1}\big[-a\nu + 2b\nu^2 + 2Z\ln(1+r)\big] - \frac{2\alpha}{\pi}\Big[\frac{1}{2}(L-1)\cdot\ln(1+r) - \\ - a\nu + b\nu^2\Big] - \delta_{interf}; \qquad R_0 = \frac{2\alpha}{\pi}\cdot(L-1). \tag{10}$$

The magnitude of R/R_0 varies from 1,005 to 1,016 for wide range of ν variation from 0.01 to 0.2.

3. COMPARISON WITH RESULTS OF OTHER WORKS. DISCUSSION.

1. First, RC (4) was calculated by Schwinger [3] in case of infinite large mass of proton. For practical calculation one uses the following formula (see reviews [4–6]):

$$\delta_{Schwinger} = e^{\delta_s + \beta_s} - 1; \qquad \delta_s = -\frac{2\alpha}{\pi}(L_s - 1)\cdot\ln\frac{\varepsilon}{\Delta\varepsilon}, \\ \beta_s = \frac{2\alpha}{\pi}\Big[\frac{13}{12}(L_s - 1) - \frac{17}{36} - f(\theta)\Big], \quad L_s = \ln\Big(\frac{4\varepsilon^2}{m^2}\sin^2\frac{\theta}{2}\Big). \tag{11a}$$

In the limit $M \to \infty$, $\Delta\varepsilon/\varepsilon \to 0$ our result (5) differs from that in the first order of α by the term

$$\delta_{MF} = \frac{Z\alpha\pi \sin\frac{\theta}{2}}{1+\sin\frac{\theta}{2}} . \tag{12}$$

This term, well-known for the case of scattering of electron on the external field (McKinley, Feshbach [8]), arises due to M_S and M_0 interference. At $\theta = 60^\circ$ it equals $\delta_{MF} = 0.008$. In the second order of α the Schwinger's result

$$\delta^{(2)}_{Schwinger} = \frac{1}{2}\delta_s^2 + \beta_s \cdot \delta_s + \frac{1}{2}\beta_s^2 \tag{13a}$$

differs, strictly speaking, from ours (7). Sometimes, instead of (11a) one uses the formula

$$\tilde{\delta}_{Schwinger} = (1+\beta_s)e^{\delta_s} - 1. \tag{11b}$$

In this case the result in the second order

$$\tilde{\delta}^{(2)}_{Schwinger} = \frac{1}{2}\delta_s^2 + \beta_s \cdot \delta_s \tag{13b}$$

also differs from (7). Nevertheless, in region (1) difference between (7) and (13a,b) is beyond our accuracy.

2. The Maximon result [4, 6] will coincide with (11b) if one replaces in it L_s for $L = L_s - \ln(1+r)$. That added to $\tilde{\delta}_{Schwinger}$ the terms $\sim \alpha r \ln\nu$ and corresponds to partial account of proton recoil influence on electron emission. But in (5) and in δ_{interf} (6) there are some other terms of the same order and the terms $\sim \alpha r L$ which may even be larger; all

these terms were not taken into account by Maximon.

3. In Tsai's paper [5] the recoil of proton and its emission was taken into account, i.e. the terms of $\alpha r \ln\nu$ and $\alpha r L$ order, but the terms of order α (for example, δ_{MF} (12)) were omitted. In conditions (8) these omitted terms have the same order $\alpha r \ln\nu \sim \alpha r L \sim \alpha \sim 1/100$, so the Tsai's result has the same accuracy as the Schwinger's one (11). The difference of (4) from the Tsai's result [5] is mainly connected with $M_{1,2,5}$:

$$\delta - \delta^{Tsai} \approx -\frac{2\alpha}{\pi}\left[a\nu - b\nu^2 + f(\theta)\right] + \frac{Z\alpha\pi \sin(\theta/2)}{1+\sin(\theta/2)} .$$

For the electrons scattering these two terms essentially cancel each other, so $\delta - \delta^{Tsai} < 10^{-2}$. For the positrons scattering $(Z \to -Z)$ this difference may reach 0.02 (see for details Section 5 and Appendix 2).

4. Formulae (4), (9) may be applied for the scattering of electrons on spinless nuclei. In this case Z and M denote the charge and mass of nuclei, $C = 0$, $D = F^2(t)$, where $F(t)$ is the form factor of nucleus. However, the range of validity and the order of omitted terms are changed. The main uncertainty, as before, is connected with the amplitude M_5, but the order of omitted terms is determined by radius of nucleus R $(\sim Z\alpha t R^2)$ or by the distance to the nearest excitation energy level of nucleus. But this uncertainty may be excluded by means of comparison of the experimental cross sections for the electron and for positron scattering on the same nuclei.

4. CALCULATION OF REAL PHOTONS EMISSION

1. When the distance from the elastic peak $\Delta\varepsilon = \nu\cdot\varepsilon_0'$ is sufficiently small, the classical approximation to RC is good:

$$|M_1+M_2|^2 = -4\pi\alpha\,(b-ZB)^2\cdot|M_0|^2, \tag{14}$$

$$b=\frac{p'}{kp'}-\frac{p}{kp}, \quad B=\frac{\mathcal{P}'}{k\mathcal{P}'}-\frac{\mathcal{P}}{k\mathcal{P}}.$$

The corresponding RC has the form

$$\delta_1+\delta^R_{interf}+\delta_2 = -\frac{\alpha}{\pi}\int\left(b^2-2Zb\cdot B+Z^2B^2\right)\frac{d^3k}{4\pi\omega},$$

where $\omega=\sqrt{\vec{k}^2+\lambda^2}$ is the energy of photon, λ is its "mass" introduced for the removal of infrared divergence. In region (1) the electron is ultrarelativistic while the proton is non-relativistic, so the probability of photon emission by proton is very small compared to the electron one:

$$B\approx\left(-\frac{\vec{q}\vec{k}}{M\omega^2},\frac{\vec{q}}{M\omega}\right), \quad \left|\frac{M_1}{M_2}\right|^2=Z^2\frac{B^2}{b^2}\sim Z^2\frac{t}{M^2}. \tag{15}$$

Photon energy ω weakly depends on the direction of its emission for the fixed p' (see (23)), so one may put $\omega_{max}=\Delta\varepsilon$ by calculation of δ_2 and δ^R_{interf}. Averaging over photon emission angles

$$\langle b\cdot B\rangle\equiv\int b\cdot B\,\frac{d o_\gamma}{4\pi}=\frac{2r}{\omega^2};\ \langle B^2\rangle=-\frac{\vec{q}^2}{M^2\omega^2}\left(1-\frac{\vec{k}^2}{3\omega^2}\right),$$

we have

$$\delta^R_{interf}+\delta_2=\frac{2\alpha}{\pi}\int_0^{\Delta\varepsilon}\frac{\vec{k}^2d|\vec{k}|}{\omega}\cdot\left\langle 2Zb\cdot B-Z^2B^2\right\rangle=$$

$$=-\frac{4Z\alpha}{\pi}r\left[\left(1-\frac{Z\varepsilon}{3M}\right)\ln\frac{2\Delta\varepsilon}{\lambda}-1+\frac{5Z\varepsilon}{18M}\right]. \tag{16}$$

2. The dependence of ω on emission angle is to be taken into account by calculation of δ_1. However, one can show (see Appendix 1) that the relation of (16) type

$$\delta_1=-\frac{\alpha}{\pi}\int_0^{\omega_{max}}\langle b^2\rangle\frac{\vec{k}^2d|\vec{k}|}{\omega} \tag{17a}$$

is valid but in this case

$$\omega_{max}=\varepsilon\cdot\nu \tag{17b}$$

Averaging b^2 over photon emission angles we have

$$\langle -b^2\rangle=\int_0^1\frac{2pp'\,dx}{p_x^2\vec{k}^2+(\lambda\varepsilon_x)^2}-\frac{m^2}{(m\vec{k})^2+(\lambda\varepsilon)^2}-\frac{m^2}{(m\vec{k})^2+(\lambda\varepsilon_0')^2}, \tag{17c}$$

$$p_x^2=-q^2x(1-x),\quad \varepsilon_x=x\varepsilon+(1-x)\varepsilon_0'.$$

Calculation of (17c) (see Appendix 2) leads to the expression

$$\delta_1=\frac{2\alpha}{\pi}\left[(L-1)\ln\frac{m\,\omega_{max}}{\lambda\sqrt{\varepsilon\varepsilon_0'}}+\frac{1}{4}L^2-\frac{\pi^2}{12}-f(\theta)\right]+O\left(\frac{\alpha}{\pi}\cdot\frac{t}{M^2}\right) \tag{18}$$

where $f(\theta)$ had been defined in (5). This formula differs from the one obtained by Tsai [5] on the term $-\frac{2\alpha}{\pi}f(\theta)$ (see Appendix 2 for discussion). Let us note that in the limit $M\to\infty$ formulae (17), (18) coincide with the Schwinger result (see, for example, [7], § 117).

3. Let us consider now the correcting terms to approximate expression (14) (proportional to ν, ν^2 and $\nu\frac{\varepsilon}{M}\approx\frac{\Delta\varepsilon}{M}$) which is essential in the region of the radiative tail. These corrections have to be done to $|M_1|^2$ only due to estimation (15). After averaging over the initial and summing over the final spin states we have

$$|M_1|^2 = -\frac{(4\pi\alpha)^3 Z^2}{Q^4}\left[g^{\mu\nu}Q^2 G_M^2(Q^2) + 4g^{\mu}g^{\nu}D(Q^2)\right]\cdot\frac{1}{2}\, Sp\left[\ell^{\mu\alpha}(\hat p+m)\bar\ell^{\nu\alpha}(\hat p'+m)\right],$$

$$\ell^{\mu\nu} = \gamma^\mu b^\nu + \gamma^\mu\frac{\hat k}{2kp}\gamma^\nu + \gamma^\nu\frac{\hat k}{2kp'}\gamma^\mu, \qquad (19)$$

$$Q^2 \equiv (p-p'-k)^2 = q^2-2qk.$$

Expression $|M_1|^2$ in terms of invariants is given in Appendix 3. Here we give more convenient dimensionless expression*)

$$T = \frac{q^4\omega^2|M_1|^2}{(4\pi\alpha)^3 Z^2\, 16M^2\varepsilon\varepsilon'(1+\cos\theta)}.$$

Expressing it through dimensionless parameters $x=\frac{\omega}{\varepsilon}$, $y=\frac{\varepsilon'}{\varepsilon}$ (energy fractions of photon and scattered electron) and emission angles of photon and electron we have

$$T = \left[C(Q^2)\cdot(c_0+xc_1+x^2c_2) + D(Q^2)\cdot(d_0+xd_1+x^2d_2)\right]\cdot\left[1+\frac{x\tau}{y(1-\cos\theta)}\right]^{-2},$$

*) T is normalized in such a way that $T\to -\frac{1}{2}(\omega b)^2(C+D)$ if $k\to 0$.

$$x=\frac{\omega}{\varepsilon};\; y=\frac{\varepsilon'}{\varepsilon};\; \varkappa=\frac{kp}{\omega\varepsilon}=1-\frac{\vec n\vec p}{\varepsilon},\; \varkappa'=\frac{kp'}{\omega\varepsilon'}=1-\frac{\vec n\vec p'}{\varepsilon'};\; \vec n=\frac{\vec k}{\omega};\; \tau=\varkappa-y\varkappa';$$

$$Q^2=-2y\varepsilon^2(1-\cos\theta)\left[1+\frac{x\tau}{y\cdot(1-\cos\theta)}\right];$$

$$c_0=d_0=-\frac{1}{2}(\omega b)^2=\frac{(1-\cos\theta)}{\varkappa\varkappa'}-\frac{m^2/\varepsilon^2}{2\varkappa^2}-\frac{m^2/\varepsilon^2}{2(y\varkappa')^2};$$

$$d_1=\frac{1}{y(1+\cos\theta)}\left[-\tau d_0+(1+y)\left(\frac{1}{\varkappa'}-\frac{1}{\varkappa}\right)+\frac{(1-y)(1-\cos\theta)}{\varkappa\varkappa'}+\frac{m^2}{\varepsilon^2}\left(\frac{y}{\varkappa^2}-\frac{1}{(y\varkappa')^2}\right)\right]; \qquad (20)$$

$$d_2=\frac{1}{2(1+\cos\theta)}\left(\frac{2-\varkappa'}{\varkappa}+\frac{2-\varkappa}{y^2\varkappa'}\right);$$

$$c_1=\frac{2\tau c_0}{y(1-\cos\theta)}-\frac{\tau}{y\varkappa\varkappa'}-\frac{m^2}{2\varepsilon^2(1-\cos\theta)}\left(\frac{\varkappa'}{\varkappa^2}-\frac{\varkappa}{y^3\varkappa'^2}\right);$$

$$c_2=\frac{1}{2(1-\cos\theta)}\left(\frac{\varkappa'}{\varkappa}+\frac{\varkappa}{y^2\varkappa'}\right).$$

It is easy to see that $|M_1|^2$ and T are invariants under the exchange $p\leftrightarrow -p'$, $k\to -k$. So, under the exchange $\varkappa\leftrightarrow\varkappa'$, $y\to 1/y$, $\tau\to -\tau/y$ we have

$$c_n\to(-y)^n c_n,\quad d_n\to(-y)^n d_n. \qquad (21)$$

As a result the cross section corresponding to amplitudes M_1+M_2 has the form

$$d\sigma=\frac{(Z\alpha)^2\cos^2\frac{\theta}{2}}{4\varepsilon^2\sin^4\frac{\theta}{2}}\cdot\frac{do_e}{1+r}\left[T+T'\cdot(C+D)\right]\frac{2\alpha}{\pi}\frac{d\varepsilon'}{\varepsilon_0'-\varepsilon'}\frac{do_\gamma}{4\pi}, \qquad (22)$$

where T was defined in (20) and

$$T' = \frac{1}{2}\omega^2\left(2Zb\cdot B - Z^2B^2\right) =$$

$$= \frac{Z\varepsilon}{M+\varepsilon\tau}\left(\frac{1-\cos\theta-\tau}{\varkappa'} + \frac{y(1-\cos\theta)+\tau}{\varkappa}\right) +$$

$$+ \frac{Z^2\varepsilon^2}{2(M+\varepsilon\tau)^2}\left[2y(1-\cos\theta)+2(1-y)\tau-\tau^2\right].$$

Here the fraction of photon energy is (see A1)

$$x = \frac{\omega}{\varepsilon} = \frac{\nu}{1+\tau\frac{\varepsilon}{M}}. \tag{23}$$

Specific energy dependence of this cross section $\frac{d\varepsilon'}{\varepsilon_0'-\varepsilon'}$ corresponds to the well-known infrared behaviour $\frac{d\omega}{\omega}$.

4. To integrate cross section (22) on photon emission angles it is convenient to expand the integrand using small difference of Q^2 from t and of x from ν:

$$Q^2 = t(1-\nu)(1+x\eta),\quad \eta = \frac{\tau}{y(1-\cos\theta)}\ ;\quad x \approx \nu\left(1-\tau\frac{\varepsilon}{M}\right).$$

Besides, we use $G_{E,M}(Q^2) = G_{E,M}(t) + O(\nu t/M^2)$. The expression T (20) expansion has the form

$$T = C\cdot\left[c_0(1-\nu) + \nu(1-\nu-\varepsilon\tau/M)(c_1-\eta c_0) + \nu^2(c_2+\eta^2c_0)\right] +$$

$$+ D\cdot\left[d_0 + \nu(1-\varepsilon\tau/M)(d_1-2\eta d_0) + \nu^2(d_2+3\eta^2d_0)\right].$$

The further integration over dO_γ is now trivial. The main contribution is connected with d_0 averaging

$$\langle d_0\rangle = \langle c_0\rangle = \ln\left(\frac{-q^2}{m^2}\right) - 1 \approx L - 1 - \nu - \frac{\nu^2}{2}.$$

It is useful to note that $\langle d_1 - 2\eta d_0\rangle$, $\langle c_1 - \eta c_0\rangle$ are proportional to small parameter $1-y \approx r+\nu$, which can be seen from symmetry relations (21). As a result we have $\langle T\rangle = L-1-a\nu+2b\nu^2$, where a and b were defined in (5).

So, cross section (22) averaged over photon emission angle is

$$d\sigma = d\sigma_0\cdot\frac{2\alpha}{\pi}\cdot\frac{d\varepsilon'}{\varepsilon_0'-\varepsilon'}\left[L-1-a\nu+2b\nu^2+Zr(1+y)-\frac{Z^2t}{3M^2}\right]. \tag{24}$$

Appendix 4 is devoted to the so-called "peak approximation"; it is more rude than (21), (24) but works for the case of hard photon.

All the corrections in question are connected with the term $-a\nu+2b\nu^2$. It's energy $d\varepsilon'$ integration gives the additional contribution to δ_1 (18):

$$-\frac{2\alpha}{\pi}\int_0^\nu\left(a\nu-2b\nu^2\right)\frac{d\nu}{\nu} = -\frac{2\alpha}{\pi}\left(a\nu-b\nu^2\right). \tag{25}$$

5. By calculation of RC to cross section (24) it is sufficient to take into account the emission of photon from electron only and the recoil effects may be neglected, i.e. we may consider $M_{7,8}$ diagrams only. Connected with M_7 RC can be extracted from Fomin paper [9]:

$$d\sigma_7 = 2(\Gamma+\Pi)\cdot d\sigma_1\ ;$$

$$\Gamma = \frac{\alpha}{\pi}\left[(L-1)\ln\frac{\lambda}{m} - \frac{1}{4}L^2 + \frac{3}{4}L - 1 + \frac{\pi^2}{12}\right]; \tag{26}$$

$$\Pi = \frac{\alpha}{\pi}\left(\frac{1}{3}L - \frac{5}{9}\right).$$

To calculate RC to elastic peak one needs to integrate (26) over photon energy up to $\omega_{max} = \Delta\varepsilon$:

$$d\sigma_7 = \delta_\gamma\, 2(\Gamma + \Pi)\, d\sigma_0 , \tag{27}$$

where δ_γ coincides with δ_1 (18) if we neglect the recoil effect in δ_1 , i.e. $\varepsilon_0' \to \varepsilon$.

The contribution of M_8 in RC to the elastic peak is

$$\delta_8 = \frac{1}{2}\frac{\alpha^2}{\pi^2}\int b_1^2 b_2^2 \frac{d^3k_1}{4\pi\omega_1}\frac{d^3k_2}{4\pi\omega_2} ,$$

$$b_i = \frac{p'}{k_i p'} - \frac{p}{k_i p} , \quad i = 1,2,$$

and the upper boundary of photon energy is determined from inequality $\omega_1 + \omega_2 \leq \Delta\varepsilon$. Coefficient $\frac{1}{2}$ is connected with the identity of photons. After averaging over photon angles we have (compared with (17))

$$\delta_8 = \frac{1}{2}\delta_\gamma^2 + \frac{\alpha^2}{\pi^2}(L-1)\cdot I ,$$

$$I = \int_0^{\Delta\varepsilon}\left(\int_0^1 \frac{2pp'\,dx}{p_x^2 \vec{k}^2 + \lambda^2\varepsilon^2} - \frac{2m^2}{m^2\vec{k}^2 + \lambda^2\varepsilon^2}\right) \ln\left(1 - \frac{\omega}{\Delta\varepsilon}\right)\frac{\vec{k}^2 d|\vec{k}|}{\omega} . \tag{28}$$

One can put $\lambda = 0$ in I and after this one gets $I = -\frac{\pi^2}{3}(L-1)$. Differentiating δ_8 over $\Delta\varepsilon$, we obtain the M_8 contribution to RC in the region of the radiative tail

$$d\sigma_8 = \delta_\gamma \frac{d\delta_\gamma}{d\varepsilon'} d\varepsilon' d\sigma_0 = \delta_\gamma \cdot d\sigma_1 . \tag{29}$$

Adding (26), (29), we obtain the total RC to cross section (24):

$$d\sigma_7 + d\sigma_8 = (\beta + \delta_{soft})\cdot d\sigma_1 . \tag{30}$$

The sum of cross sections (24) and (30) leads to (9). In the same way the sum of (27) and (28) gives RC to the elastic peak from $M_{7,8}$:

$$d\sigma_7 + d\sigma_8 = d\sigma_0\cdot\left[\delta_\gamma\left(\frac{1}{2}\delta_\gamma + 2\Gamma + 2\Pi\right) + O\left(\frac{\alpha^2}{\pi^2}L^2\right)\right] . \tag{31}$$

5. CALCULATION OF RC DUE TO VIRTUAL PHOTONS

1. RC from $M_{3,9,10}$ diagrams are well known:

$$|M_0 + M_3|^2 = (1 + \Gamma + \Pi)\cdot|M_0|^2 ;$$

$$2Re(M_0^* M_{10}) = 2\Gamma\Pi\cdot|M_0|^2 ;$$

$$2Re(M_0^* M_9) = \delta_9\cdot|M_0|^2 ;$$

where Γ and Π were defined in (26), and δ_9 was calculated in [10]:

$$\delta_9 = \Gamma^2 - \frac{\alpha^2}{18\pi^2}L^3 + O\left(\frac{\alpha^2}{\pi^2}L^2\right) .$$

Together with (18), (25) and (31) these corrections lead to δ_{el} (5), (7).

2. When the recoil effects are neglected the δ_5 contribution of diagram M_5 is well known (see (12)). We will now take into account the recoil effects whose magnitude is of αr order, but will neglect the strong interaction effects whose magnitude is of $\alpha t/M^2$ order. In this approximation we can consi-

der the proton as the pointlike one. Moreover, we can consider it as a spinless particle in the same approximation (We have done more complicated calculation for spinor proton by using the results of paper [11], naturally, it gives the same re-

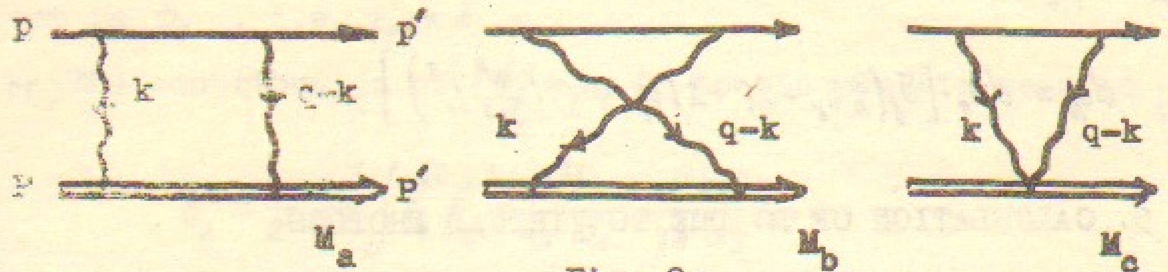

Fig. 2.

sults). So we may consider M_5 as a sum of diagrams in Fig.2. It is easy to see that $M_c \sim m^2$ and, therefore, is irrelevant. Two remaining diagrams give

$$\delta_5 = \frac{2\,Re\,(M_a+M_b)M_0^*}{|M_0|^2} = Re\,\frac{iZ\alpha t}{8\pi^3 R}\int \frac{S^{\mu\nu\sigma}}{V_1 V_2 V_3}\, Sp\left[(\hat{p}'+m)\ell^{\mu\nu}(\hat{p}+m)\gamma^\sigma\right] d^4k\,,$$

$$R = \frac{1}{4}Sp\left[(\hat{\mathcal{P}}+\hat{\mathcal{P}}')(\hat{p}+m)(\hat{\mathcal{P}}+\hat{\mathcal{P}}')(\hat{p}'+m)\right] = 4M^2\varepsilon\varepsilon_0'\,(1+\cos\theta)\,,$$

$$S^{\mu\nu\sigma} = (2\mathcal{P}+k)^\mu(2\mathcal{P}'+k-q)^\nu(\mathcal{P}+\mathcal{P}')^\sigma\,,$$

$$V_1 = k^2-\lambda^2\,,\quad V_2 = (k-q)^2-\lambda^2\,,\quad V_3 = 2k\mathcal{P}+k^2\,,$$

$$\ell^{\mu\nu} = \left(\frac{2p'^\mu}{V_4}+\frac{2p^\mu}{\tilde{V}_4}\right)\gamma^\nu + \gamma^\mu\frac{\hat{k}}{V_4}\gamma^\nu - \gamma^\nu\frac{\hat{k}}{\tilde{V}_4}\gamma^\mu\,,$$

$$V_4 = 2kp'+k^2\,,\quad \tilde{V}_4 = -2kp+k^2.$$

Since, due to gauge invariance $k^\mu\ell^{\mu\nu} = \ell^{\mu\nu}\cdot(q-k)^\nu = 0$ we change $S^{\mu\nu\sigma}$ for $8\,\mathcal{P}^\mu\mathcal{P}'^\nu\mathcal{P}^\sigma$ and have

$$\delta_5 = -\frac{Z\alpha}{\pi}\,t\left[g(u,t)-g(s,t)\right],\quad s=(p+\mathcal{P})^2,\ u=(p-\mathcal{P}')^2,$$

$$g(u,t) = 2(p'\mathcal{P})\,J + \frac{4}{R}\,A^\mu I^\mu\,,$$

$$A^\mu = \frac{1}{4}Sp\left[\hat{\mathcal{P}}\gamma^\mu\hat{\mathcal{P}}'(\hat{p}+m)\hat{\mathcal{P}}(\hat{p}'+m)\right], \tag{32}$$

$$J = Re\,\frac{-i}{\pi^2}\int\frac{d^4k}{V_1V_2V_3V_4}\,,\quad I^\mu = Re\,\frac{-i}{\pi^2}\int\frac{k^\mu d^4k}{V_1V_2V_3V_4}\,.$$

The calculation of A^μ gives

$$A^\mu = a\cdot(p+p')^\mu + c\cdot(\mathcal{P}+\mathcal{P}')^\mu\,,$$

$$2a = -2(p'\mathcal{P})(\mathcal{P}'\mathcal{P}) - \frac{1}{2}tM^2 = -2\varepsilon_0'M^2(M+\varepsilon-\varepsilon_0') + M^3(\varepsilon-\varepsilon_0')\,,$$

$$2c = 2(p'\mathcal{P})(p\mathcal{P}+p'\mathcal{P}) + \frac{1}{2}tM^2 = 2\varepsilon_0'M^2(\varepsilon+\varepsilon_0') - M^3(\varepsilon-\varepsilon_0')\,,$$

and

$$A^\mu I^\mu = (a+c)\left(\frac{t}{2}J - J_1\right) + a\,J_3 + c\,J_4\,,$$

where

$$J_i = Re\,\frac{-i}{\pi^2}\int\frac{V_i}{V_1V_2V_3V_4}\,d^4k \tag{33}$$

The asymptotical expressions for J, J_i are presented in Appendix 5. As a result of rather cumbersome calculations one obtains

$$\delta_5 = \delta_{MF} + \frac{2Z\alpha}{\pi}\left[-\ln(1+r)\ln\frac{-t}{\lambda^2} + \frac{r\,\ell^2}{2(1+\cos\theta)} - \frac{1}{2}\,r\,\ell\right] \tag{34}$$

where the first term was defined in (12).

The sum of δ_S and δ^R_{interf} (16) (using (24)) leads to δ_{interf} (6). It is easy to see that the "infrared term" ($\frac{4Z\alpha}{\pi} \ln v \cdot \ln(1+r)$) from (6) coincides with the corresponding one in the Tsai result [5], while the δ_{MF} term is absent in Ref. [5].

We are very much grateful to V.F.Dmitriev, I.F.Ginzburg, V.S.Fadin and A.I.Vainshtein for useful discussions. We express our gratitude to D.M.Nikolenko, S.G.Popov, D.K.Toporkov and B.B.Voytsekhovsky for having discussed the possibility of the experiments, and N.I.Popova for helping to calculate the Table.

APPENDIX 1.

Let us introduce 4-vector $h = \mathcal{P} + p - p'$. From the law of momenta conservation we have $h = \mathcal{P}' + k$ and $(h-k)^2 = h^2 - 2hk + \lambda^2 = M^2$ or

$$\omega = \frac{h^2 - M^2 + \lambda^2}{2(h_0 - \vec{h}\vec{n})}, \quad \vec{n} = \frac{\vec{k}}{\omega}, \tag{A1}$$

i.e. the lab. system energy of photon depends on the angle between the vectors $\vec{q}$ and $\vec{n}$. Let us consider, following Tsai [5], the special frame of reference where vector h has only the zero component $\tilde{h}_0 = \sqrt{h^2}$ (below all the quantities in this frame of reference we mark using the tilde sign). In this frame the photon energy $\omega = (h^2 - M^2 + \lambda^2)/2\sqrt{h^2}$ does not depend on angles of its emission and changes from λ to $\omega_{max} = \varepsilon \cdot v[1 + \mathcal{O}(v)]$ when ε' varies from ε'_0 up to $\varepsilon'_0 - \Delta\varepsilon$. The integration of in this frame is rather simple and gives ($\tilde{k} \equiv \sqrt{\tilde{\omega}^2 - \lambda^2}$)

$$\delta_1 = \frac{\alpha}{\pi}\int_0^{\omega_{max}} \left[\int_0^1 \frac{2pp' \, dy}{p_y^2 \tilde{k}^2 + \lambda^2 \varepsilon_y^2} - \frac{m^2}{m^2\tilde{k}^2 + (\lambda\tilde{\varepsilon}')^2} - \frac{m^2}{m^2\tilde{k}^2 + (\lambda\varepsilon)^2} \right] \frac{\tilde{k}^2 d\tilde{k}}{\tilde{\omega}} \tag{A2}$$

$$p_y^2 = [yp + (1-y)p']^2 = -q^2 y(1-y) + m^2, \quad \varepsilon_y = y\tilde{\varepsilon} + (1-y)\tilde{\varepsilon}'.$$

Taking into account that $\tilde{\varepsilon} = ph/\sqrt{h^2} = \varepsilon'_0[1 + \mathcal{O}(v)]$, $\tilde{\varepsilon}' = p'h/\sqrt{h^2} = \varepsilon[1 + \mathcal{O}(v)]$ and making substitution $y = 1 - x$ we obtain expression (17).

APPENDIX 2.

It is easy to see the origin of difference between our result (18) and the Tsai's [5] one. For this aim we perform the exact tranformation in (17a)

$$\frac{\vec{k}^2 d|\vec{k}|}{\omega} = \frac{1}{2} d\vec{k}^2 - \frac{1}{2}\left(1 - \sqrt{\frac{\vec{k}^2}{\vec{k}^2 + \lambda^2}}\right) d\vec{k}^2 \tag{A3}$$

The first term in the right-hand side of (A3) leads after simple integration to the Tsai's result δ_1^{Tsai}. The difference $\delta_1 - \delta_1^{Tsai} = -\frac{\alpha}{\pi} I$ is defined by the integral

$$I = -\frac{1}{2}\int_0^{\omega_{max}} \langle b^2 \rangle \left(1 - \sqrt{\frac{\vec{k}^2}{\vec{k}^2 + \lambda^2}}\right) d\vec{k}^2 \tag{A4}$$

Making the substitution $|\vec{k}| = \lambda z$ and replacing ω_{max}/λ for infinity (due to fast convergence of integral: dz/z^3) we get after z -integration

$$I = \int_0^1 \frac{2pp'}{p_x^2} \varphi(a) dx, \quad a = \frac{p_x^2}{\varepsilon_x^2},$$

$$\varphi(a) = \left(\frac{1}{\sqrt{1-a}} - 1\right) \ln\frac{2}{\sqrt{a}} - \frac{1}{\sqrt{1-a}} \ln\frac{2}{1 + \sqrt{1-a}}.$$

When the limit $-q^2 \gg m^2$ is considered we have

$$\frac{2pp'}{p_x^2} \to \frac{1}{x(1-x)}\ , \quad a \to \frac{2x(1-x)(1-\cos\theta)}{\left[x\sqrt{\varepsilon_0'/\varepsilon} + (1-x)\sqrt{\varepsilon/\varepsilon_0'}\right]^2}\ .$$

From symmetry (A4) under the exchange $p \leftrightarrow p'$ one can deduce that the expansion of I on powers of small parameters $(\varepsilon-\varepsilon_0')/(\varepsilon+\varepsilon_0') \approx r/2$ contains only even powers of this parameter, so

$$I = \int_0^1 \frac{\varphi(a_0)}{x(1-x)}\,dx + O\left(\frac{q^2}{M^2}\right), \quad a_0 = 2x(1-x)(1-\cos\theta).$$

This result leads to

$$\delta_1 - \delta_1^{Tsai} = -\frac{2\alpha}{\pi} f(\theta) + O\left(\frac{\alpha}{\pi}\frac{t}{M^2}\right)$$

where $f(\theta)$ was defined in (5).

APPENDIX 3.

We present here the exact result for $|M_1|^2$ (vector $N = P/M$, $N^2 = 1$):

$$|M_1|^2 = -\frac{(4\pi\alpha)^3 Z^2}{Q^4}\left[8D(Q^2)M^2 S_1 + 2Q^2 G_M^2(Q^2) S_2\right]$$

$$S_1 = b^2\left(2Np\cdot Np' + \tfrac{1}{2}q^2 - qk\right) - 2bN\cdot(p+p')N +$$
$$+ 2kN\left[(bp-1)\frac{Np'}{kp} + (bp'-1)\frac{Np}{kp'}\right] + \frac{kp'}{kp} + \frac{kp}{kp'} + m^2\left[b^2 + \frac{2(kN)^2}{kp\cdot kp'}\right],$$

$$S_2 = b^2(q^2 - 4qk) - 2b(p+p') + 2(bp+1)\frac{kp'}{kp} + 2(bp'+1)\frac{kp}{kp'} + \frac{2kp'}{kp} + \frac{2kp}{kp'} + 2m^2b^2.$$

when $M \to \infty$ trace S_1 coincides with the corresponding trace for scattering of electron on external field (see [7], § 91). Terms in $S_{1,2}$ which are proportional to m^2 were omitted in (20) because they are of m^2/ε^2 order compared to the first ones.

APPENDIX 4.

For the rude estimation of RC one uses the so-called "peak approximation" [5, 12]. In this approximation one takes into account the emission of photons only in small cones along the direction of the initial and the scattered electrons and neglects the interference between these cones, so the accuracy of this approximation is logarithmic (the terms of $1/L$ order are neglected). The corresponding (21) cross section has the form

$$d\sigma = d\sigma_0(t_1)\,dw_1 + d\sigma_0(t_2)\,dw_2$$

where the first term corresponds to photon emission by initial electron and the second one - to the scattered electron:

$$d\sigma_0(t_2) = \frac{\alpha^2\cos^2(\theta/2)}{4\varepsilon^2\sin^4(\theta/2)}\cdot\frac{D(t_2) + C(t_2)}{1 + 2(\varepsilon/M)\sin^2(\theta/2)}\ ,$$

$$t_2 = \frac{4\varepsilon^2\sin^2(\theta/2)}{1 + 2(\varepsilon/M)\sin^2(\theta/2)}\ ,$$

$$dw_2 = \frac{\alpha}{\pi}\left[\frac{2 - 2x_2 + x_2^2}{x_2\ kp'} - \frac{m^2(1-x_2)}{(kp')^2}\right]\frac{d^3k}{4\pi\omega}\ , \quad x_2 = \frac{\omega}{\varepsilon'}$$

and $d\sigma_0(t_1)$, dw_1 can be obtained by replacement

$$\varepsilon \to \varepsilon - \omega\ , \quad p' \to p\ , \quad x_2 \to x_1 = \frac{\omega}{\varepsilon}\ .$$

The integration of this expression over the photon emission angles up to $\theta_\gamma \sim \theta$ leads to

$$dW_1 = \frac{\alpha}{\pi}\,\frac{d\varepsilon'}{\varepsilon_0'-\varepsilon'}\cdot\frac{\varepsilon - r\varepsilon_0'}{\varepsilon - r\varepsilon'}\left[\left(1 - x_1 + \frac{1}{2}x_1^2\right)L_1 - 1 + x_1\right],$$

$$dW_2 = \frac{\alpha}{\pi}\,\frac{d\varepsilon'}{\varepsilon_0'-\varepsilon'}\left[\left(1 - x_2 + \frac{1}{2}x_2^2\right)L_2 - 1 + x_2\right],$$

$$x_1 = \frac{\nu(1+r)}{1+\nu}\,,\quad x_2 = \frac{\nu}{1-\nu}\,,\quad L_1 = \ln\left(\frac{4\varepsilon^2}{m^2}\sin^2\frac{\theta}{2}\right),\quad L_2 = \ln\left(\frac{4\varepsilon'^2}{m^2}\sin^2\frac{\theta}{2}\right).$$

For the scattering of electrons on spinless nuclei it is necessary to replace $D(t)+C(t)$ in $d\sigma_0(t)$ for $Z^2F^2(t)$ where $F(t)$ is form factor of nuclei.

APPENDIX 5.

We present here the asymptotical expressions for the integrals $\mathcal{J}$ and $\mathcal{J}_i$ (32), (33) for $m \ll \varepsilon \ll M$:

$$\mathcal{J} = \mathcal{J}(u,t) = -\frac{1}{t}\,\frac{2}{M^2-u}\ln\left|\frac{M^2-u}{Mm}\right|\ln\frac{-t}{\lambda^2}\,,$$

$$\mathcal{J}_1(u) = \mathcal{J}_2(u) = -\frac{1}{2(M^2-u)}\left[2\ln\left|\frac{M^2-u}{Mm}\right|\ln\frac{M^2}{\lambda^2} + \ln^2\left|\frac{M^2-u}{M^2}\right| - \right.$$

$$\left. - \frac{1}{2}\ln^2\frac{m^2}{M^2} - \Phi\left(\frac{-u}{M^2-u}\right)\right],$$

$$\mathcal{J}_3 = -\frac{\pi^2}{2\sqrt{-t}\,M} - \frac{1}{2M^2}(L-2)\,,$$

$$\mathcal{J}_4 = \frac{1}{2t}\left(L^2 + \frac{4\pi^2}{3}\right).$$

The function $g(s,t)$ can be expressed in terms of $\tilde{\mathcal{J}}, \tilde{\mathcal{J}}_i$ which can be obtained from $\mathcal{J}, \mathcal{J}_i$ by the change of $V_4 \to \tilde{V}_4$ in the integrand: $\tilde{\mathcal{J}} = \mathcal{J}(s,t)$, $\tilde{\mathcal{J}}_1 = \tilde{\mathcal{J}}_2 = \mathcal{J}_1(s) - \frac{\pi^2}{2(s-M^2)}$, $\tilde{\mathcal{J}}_{3,4} = \mathcal{J}_{3,4}$.

REFERENCES

1. F.Borkowski et al. Z.Physik, <u>A275</u>, 29, 1975, W.Schütz, Z.Physik, <u>A273</u>, 69, 1975;
   Ю.К.Акимов и др. ЖЭТФ, <u>35</u>, 65I, 1972.
2. Б.Б.Войцеховский и др. Письма в ЖЭТФ, <u>29</u>, I05, I979.
3. J.Schwinger, Phys.Rev. <u>76</u>, 790, 1949.
4. L.C.Maximon, Rev.Mod.Phys., <u>41</u>, 193, 1969.
5. Y.S.Tsai, Phys.Rev., <u>122</u>, 1898, 1961.
   L.W.Mo, Y.S.Tsai, Rev.Mod.Phys., <u>41</u>, 205, 1969.
6. Hans Breuer.
   Nuclear Data Tables, <u>9</u>, 169-233 (1971).
7. В.Б.Берестецкий, Е.М.Лифшиц, Л.П.Питаевский, Релятивистская квантовая теория, Москва, I968.
8. McKinley, Feshbach, Phys.Rev., <u>74</u>, 1759, 1948.
9. П.Фомин, ЖЭТФ, <u>35</u>, 707, I958.
   А.И.Ахиезер, В.Б.Берестецкий, Квантовая электродинамика, § 40, Москва, I969.
10. Barbieri et al. Il.Nuovo Cimento, <u>11A</u>, 824, 1972.
11. Э.А.Кураев, Г.В.Меледин, Препринт ИЯФ 76-9I, I976.
12. V.N.Baier, V.S.Fadin, V.A.Khose, Nucl.Phys., <u>B65</u>, 381, 1973.