68

ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
СО АН СССР

Ya.S.Derbenev and A.N.Skrinsky

ON HIGH ENERGY ELECTRON COOLING

ПРЕПРИНТ ИЯФ 79 - 87

Новосибирск

ON HIGH ENERGY ELECTRON COOLING

Ya.S.Derbenev and A.N.Skrinsky

Institute of Nuclear Physics of Siberian Division
of the USSR Academy of Sciences, Novosibirsk

1. Electron cooling as a method of producing intense monochromatic beams of heavy particles in storage rings can be used in a broad field of experiments on elementary particle physics, nuclear physics, etc. /1/. By now, this method has successfully been tested in two laboratories /1,2/. High energy electron cooling is mainly intended as a method to ensure high luminosity of colliding proton-antiproton beams /3/ if we already can obtain the required number of antiprotons.

This paper is a short review of our conceptions about possibilities of heavy particle beams cooling at high energies by the beam of ultrarelativistic electrons coasted in the storage ring. It contains preliminary considerations reported at Workshop at Madison /4,5/ and also the results of the studies carried out for recent time.

Before proceeding to the subject of this paper let us say a few words in general on the methods which enable one to cool the beams of fast charged particles, or to keep them cold enough despite any heating diffusion processes.

2. Synchrotron radiation damping - currently, the most familiar - is of extreme usefulness for electron and positron beam cooling, especially today for high luminosity e^+e^- colliding beam experiments. Radiation cooling will be very useful for the next (or even after-the-next) generation of $p\bar{p}$ colliding beam

facilities at energies more than 5 TeV, assuming the bending magnetic field would be about 100 KGs or more.

3. Ionization losses should be useful for cooling muon beams (of course, the average energy losses should be compensated from the external energy source in the same way as for the synchrotron radiation case). Ionization cooling becomes effective for relativistic particles ($\gamma \geq 2$), because only in this case there is no antidamping due to decreasing ionization losses with increasing particle energy.

This cooling is useful only for muons, because for all relativistic hadrons the cross-section of "strong interaction death" is too large and for $e^{\pm}$ the radiative losses are bigger than ionization losses.

To have better equilibrium emittance it is necessary to place the light material target in a region with very low beta-value.

Ionization cooling makes it possible to obtain intense, high energy low emittance muon beams by accelerating the cooled muons. With the use of a special high-field storage ring it would be possible to obtain very intense and narrow $\nu_e, \tilde{\nu}_e, \nu_\mu, \tilde{\nu}_\mu$- beams, and even high-luminosity muon colliding beams /6/.

4. Stochastic cooling is most effective when it is necessary to damp large amplitudes of betatron oscillation and large momentum spread in proton (antiproton) beams; this is especially important for the initial stage of antiproton storing. But the rate of stochastic cooling goes down with increasing linear density of the stored beams (special disadvantage for bunched beams) or for beams of small emittance and small momentum spread. The good feature of stochastic cooling is that its rate does not decrease for higher energies.

5. Electron cooling gives high cooling rate in the case of medium and, especially, low emittance proton (antiproton) beams in the medium and low energy range. The use of electron cooling for higher energies is the subject of our discussions today. Note, cooling <u>intense</u> proton beams need experimental study.

6. While interaction, cooling helps to confine the beam emittances and consequently the beam dimensions to their minimum admissible level at the collision points. This enables one to maintain the maximum attainable density and the maximum attainable collision luminosity. The cooling time, naturally, should be in any case much smaller than antiproton life-time due to strong interaction with colliding particles:

$$\tau_c << N_{\bar{p}}/L_{\Sigma}\sigma_T \approx 10^5 \div 10^6 \text{ sec.}$$

Some effects of diffusional character lead, in the interaction regime, to beams tend to expand. Among these effects are: the multiple scattering on the nuclei of residual gas and on the colliding beam particles (pair collisions); intra-beam scattering which leads to self-heating of a beam at high energies; an influence of the different noises of the magnetic field and RF systems; accumulating effects of non-linearities of the guiding fields and, finally, the coherent electromagnetic field effect of the colliding beam. Particularly, the existence of this latter effect is in practice the most significant and, therefore, let us specially dwell upon this effect, at least schematically.

If the transverse dimensions turn out to be too small for a beam which contains (for the sake of simplicity) only one bunch with a number of particles N , the colliding particle during one collision can acquire a scattering angle larger than the admissible angle which is determined by the admittance of the storage ring. While decreasing the colliding bunch density only the multiturn effects become destructive. In this case, the colliding beam field can be considered as a lens which varies the frequency of the particle betatron oscillations. An influence of this lens can be characterized by the frequency shift $\Delta\nu$. If $\Delta\nu$ is so high that the frequency is nearing the linear machine resonances, the particle oscillation amplitude increases very rapidly (in a few turns). With further decrease in bunch density, $\Delta\nu$ also decreases and, at the value $\Delta\nu \approx 0.1$, the influence of the colliding beam field in "good" operating points leads to quasidiffusional behavior of the amplitude of betatron oscillations under the effect of strongly non-linear field of the bunch. In this case, the diffusion rate rapidly decreases with decrease of betatron frequency shift and at $\Delta\nu \lesssim 10^{-4}$ the diffusion induced by collisional effects becomes practically negligible (even at the largest life-times of the beams, supposing that all the other sources of the diffusion have been ultimately suppressed). In the range $\Delta\nu = 10^{-1} \div 10^{-4}$ the diffusion rate not only depends on the given value $\Delta\nu$ but also depends on the presence of modulation of this shift by synchrotron oscillations as well as noises of magnetic field and also on a number of other factors. The beam "erosion" caused by the diffusion can be suppressed with friction. For e^+ and e^- at high energies the very strong friction is the radiation friction. Under these conditions one can achieve the following values: $\Delta\nu_{max} = (3 \div 5)\cdot 10^{-2}$. At these values one can overcome the diffusion by strong radiation friction. For $p\bar{p}$ colliding beams at energies of hundreds of GeV, as mentioned above, the only kind

of friction for which one can hope today is electron cooling by circulating electron beam of the same average velocity. Though the cooling rate will be a few orders of magnitude lower than the radiation cooling rate attained. Therefore, it is hardly possible to hope for machine operation with the frequency shifts higher than, say, $\Delta\nu_{max} = 1\cdot 10^{-2}$.

At a given $\Delta\nu_{max}$ a relation occurs between maximum achievable summary luminosity L_Σ of the installation and the number of particles in the weaker beam $N_{\bar{p}}$. Since, in the first approximation, L_Σ does not depend on the number of particles in a strong beam, then we will take $N_p = N_{\bar{p}} = N$. In this case, the relation mentioned above in the beams which are symmetrical over r- and z-directions ($\Delta r = \Delta z$, $\beta_z = \beta_x = \beta_o$ at the collsion points) has the form:

$$L_\Sigma = \frac{\gamma N \Delta\nu_{max}}{r_p \beta_o} f_o \quad , \tag{1}$$

where r_p is the classical radius of the proton, γ is the relativistic factor for the beams at the experiment energy, f_o is the rotation frequency.

If the beams are separated into several bunches, the total ultimate luminosity over all the collision points, at a given total number of particles N, remains the same. It is determined by the above formula if the bunches would collide only in the useful and similar (over β_o-value) points and the collisions would not occur at all parasitic points. The latter can be achieved in the one-track storage ring with the help of the transverse quasiresonant electrostatic fields sufficiently high (as it is planned for VEPP-4), but in the case of the two-track storage ring it can be achieved by the appropriate selection of the intersection geometry.

The minimum emittance of the beams which corresponds to the given above formula for the ultimate luminosity will depend on the number of bunches n_b in the colliding beams. It is determined by the formulae:

$$\varepsilon_{min} = \frac{r_p N}{4\pi \Delta\nu_{max} \gamma n_b}$$

$$\varepsilon_{min} = \frac{r_p^2 L_\Sigma \beta_0}{4\pi (\Delta\nu_{max})^2 f_0 n_b \gamma^2} . \qquad (2)$$

Of course, this emittance should be less than the admittance of the storage ring (taking into account the losses in useful aperture if one needs to separate orbits at parasitic points of collisions):

$$\varepsilon_{min} < \varepsilon_{adm} .$$

This condition gives the limit for achievable L_Σ. Note, that operation, using $N_p \gg N_{\bar p}$, does not allow one neither to increase the luminosity considerably nor to decrease the emittance of the antiproton beam compared to the value given above and consequently, it does not ease the problem of antiproton cooling. This regime is rather a partial replacement if high energy electron cooling fails.

7. Let us shift now to the discussion of electron cooling directly in the colliding regime at energies of hundreds of GeV. Let us assume initially that the transverse electron temperature is rather small (longitudinal temperature is small due to relativistic character at $\Delta p_\perp$ and $\Delta p_\parallel$ comparable in the laboratory system), so that in the cooling region the angular spread and electron beam dimension are smaller than those of the beam under cooling. In this case, the cooling rate will be maximum and will be

determined by the transverse velocities of antiprotons (or protons) /1/:

$$\tilde{\tau}_c^{-1} = 20 \frac{J_e^{max}}{e} \cdot \eta_c L_c \frac{r_e r_p}{\gamma^5 S_{\bar p} \theta_{\bar p}} , \qquad (3)$$

where r_e, e are the classical radius and the charge of the electron; $L_c \simeq 10$ is the Coloumb logarithm of antiproton-electron collisions; J_e^{max} is the peak electron current at the moments of cooling; η_c is the fraction of the antiproton orbit occupied by the cooling region; $S_{\bar p}$ and $\theta_{\bar p}$ are the cross-section and the characteristic angles of the antiproton beam on the cooling region.

As to $\theta_{\bar p}$, one should have in mind that not only the particles giving the main contribution to the luminosity (which corresponds to the emittance ε_{min}) should be cooled but the particles which have the angles by factor $K = 2 \div 3$ bigger, should also be cooled, since otherwise the luminosity decreasing time will be too small due to the irreversible diffusion pumping of particles into the "tails" of the distribution function. Under these conditions:

$$\theta_p = K \left(\frac{\varepsilon_{min}}{\beta_p} \right)^{1/2}$$

$$S_p = 2\pi K^2 \varepsilon_{min} \beta_p \qquad (4)$$

$$\tilde{\tau}_c^{-1} = 3 \frac{J_e^{max}}{e} \eta_c L_c \frac{r_e r_p \beta_p^{1/2}}{(\gamma K \sqrt{\varepsilon_{min}})^5} ,$$

where β_p is the value of beta-function in the cooling region. Consequently, using expressions for ε_{min} one gets:

$$\tilde{\tau}_c^{-1} = 10^3 \frac{\bar{J}_e}{e} \cdot \frac{\Pi_e}{\ell_b} \cdot \frac{r_e L_c}{r_p^{3/2}} \cdot \frac{\eta_c (\Delta\nu_{max})^{5/2} \beta_p^{1/2} n_b^{5/2}}{\gamma^{5/2} N^{5/2} K^5} \qquad (5)$$

or

$$\tilde{\tau}_c^{-1} = 10^3 \frac{\bar{J}_e}{e} \cdot \frac{\Pi_e}{\ell_b} \cdot \frac{r_e L_c}{r_p^4} \cdot \frac{\eta_c (\Delta\nu_{max})^5 \beta_p^{1/2} f_0^{5/2} n_b^{5/2}}{L_\Sigma^{5/2} \beta_0^{5/2} K^5} \tag{6}$$

(note, that in the latter formula the dependence on γ disappears). It is assumed here that $J_e^{max} = \bar{J}_e \Pi_e / \ell_b$, where $\bar{J}_e$ is average electron current in the electron storage ring, Π_e is the circumference of this storage ring which should be by integer times less or should be equal to the distance between the antiproton (proton) bunches; it is assumed that the electron beam is accumulated into one bunch with the same length ℓ_b, or less, as that for the bunches in the main storage ring.*) From formulae (5) and (6) it is clear that the maximum cooling decrement decreases quite rapidly with an increase in the antiproton energy and current (or luminosity).

8. One should also take into account that possibilities for increasing the energy and current of antiproton beam are confined with the effect of selfheating due to multiple internal scattering (see also pp.10,11). This effect appears at γ -values higher than its some crutial value which in smooth approximation coincides with the betatron frequency (or transition energy)

*) Note, that in order to cool protons and antiprotons in one track collider in a time, it is necessary to have two independent electron storage rings withelectron velocities in the cooling sections parallel to the beam under cooling. The interaction of electron bunch with antiparallel beam should be excluded by correponding phase shift to prevent very dangeous beam-beam effects, especially on electrons having two throusands times lower momenta.

/ 7, 8/:

$$\gamma_{cr} = \nu \ .$$

The more accurate calculation /9/ shows that the value γ_{cr} can be increased due to nullifying the dispersion function Ψ in the straight sections and also due to its decreasing in bending magnets (see formula (8)). But it is hardly feasible to increase γ_{cr} in the heavy particle storage ring up to the values of hundreds and more.

The ratio of the self-heating rate to cooling power does not depend on the emittance and their comparison leads to the simple condition of the cooling effect existence:

$$N < N_{cr} \simeq 2 N_e \eta_c \frac{M}{m} \left(\frac{\gamma_{cr}}{\gamma}\right)^2 , \qquad \text{at } \gamma > \gamma_{cr} \tag{7}$$

9. For colliding beams, estimations carried out by the latter formulae show that large electron currents are required in order to obtain cooling times on the order of 10^2-10^3 sec at the luminosity of 10^{30} cm^{-2} s^{-1} (the pulse currents should be on the level of tens or even hundreds amperes in the many bunches regime of the collider).

Achievement of such currents in the storage rings at electron energies of hundreds of MeV and higher is not a problem now. But difficulties appear due to necessity of having a low effective temperature of electrons in order that the angles in the electron beam should be lower than those in the proton (antiproton) beam and should not decrease the cooling rate. Having in mind, that electron beam cross-section should not be bigger than that of antiproton beam, we obtain the restriction on electron beam emittance:

$$\varepsilon_e \leq \varepsilon_p \ .$$

Only in this case the cooling rate would not decrease additionnally.

In order to maintain small enough spreads in electron beam, asufficiently strong effective friction is required for electrons. For this purpose, it is natural to use radiation friction. This can be done either directly in the main storage rings or with the transportation of electron bunches after their heating into the special deeply and fast cooling storage ring and also by injection into the main electron storage ring of just cooled portion of particles.

A few processes can lead to the heating of electron beams: heating with the beams under cooling themselves; appearance of electron coherent instabilities; self-heating due to the intrabeam electron collisions and also the influence of the synchrotron radiation quantum fluctuations.

The first effect is the simplest and it can be easily handled. For this purpose, it is sufficient that the time of effective cooling of the electrons is sufficiently small:

$$\tilde{\tau}_{rad} < \tilde{\tau}_c \cdot \frac{m}{M} \cdot \frac{N_e}{N_{\tilde{p}}} \cdot \left(\frac{\theta_e}{\theta_{\tilde{p}}}\right)^2$$

(this formula is derived from the simple thermodynamic considerations "heat flow balance"). Even if one takes into account that θ_e should be lower than $\theta_{\tilde{p}}$, for the parameters required, we obtain simple requirements for $\tilde{\tau}_{rad}$.

The problem of coherent instability is much more complicated and has many forms. Operational experience of electron storage rings, though, gives the confident hope that one may obtain the required parameters. So, at VEPP-2 storage ring the peak electron currents one can manage to obtain are 20-40 A with moderate

temperature even at an energy as low as 100 MeV.

10. In order to overcome the effect of internal scattering which in the relativistic region nearly innevitably leads to the beam self-heating, both the ultimate powerful damping and selection of a very special structure of the electron storage ring are required. In particular, it requires the ultimate radial focusing in the bending sections and zero values for the dispersion functions and their derivatives at the long straight sections (cooling sections, for example) (elimination of dangeous influence of quantum fluctuations of radiation requires similar ways but looks like it is easier). This problem seems to be solvable.

Preliminary calculations for the storage ring of such a type show that self-heating disappears under the condition:

$$\gamma < \gamma_{cr} \simeq \frac{\pi}{\ln(1/\eta_\psi)} \left[\langle \beta_e^{1/2} \rangle \langle \beta_e^{-1/2} \rangle\right]^{1/2} \cdot \frac{1}{\langle \chi \rangle}, \qquad (8)$$

where $\beta_e \simeq \beta_{ex} \simeq \beta_{ez}$ are β -functions,

$$\chi = \left[\frac{\psi^2}{\beta^2} + \left(\frac{d\psi}{dS}\right)^2\right]^{1/2}, \qquad \langle \chi \rangle \simeq \chi_{max} \cdot \eta_\psi,$$

η_ψ is the fraction of the orbit, where ψ or its derivative $d\psi/dS$ are different from zero. The brackets $\langle \dots \rangle$ mean averaging over the orbit of electrons. As it is seen from formulae (8), the crutial energy can additionally be made higher due to strong modulation of β -function on the sections where $\psi \equiv 0$.

At $\gamma_{cr} > \gamma$ the transverse temperature of electrons will be determined by quantum fluctuations of energy. In this case, the bunch can be longer due to its thermalization by internal scattering. The compensation for this enlargement will require stronger RF-voltage.

11. In the case when the condition $\gamma_{cr} > \gamma$ is too difficult to achieve technically and the self-heating is unavoidable, the equilibrium emittance and energy spread for the electron beam will be the following:

$$\varepsilon_e = \left[\frac{N_e L_c r_e^2 c \tau_{rad}}{2 \ell_b \gamma^5}\left(\frac{\gamma}{\gamma_{cr}}\right)^2 \langle \beta_e^{1/2} \rangle\right]^{2/5}, \quad (\gamma_{cr} < \gamma) \qquad (9)$$

$$\left(\frac{\Delta\gamma_e}{\gamma}\right)^2 = \frac{1}{4}\gamma_{cr}^2 \left(\langle \beta_e^{-1/2} \rangle / \langle \beta_e^{1/2} \rangle\right) \cdot \varepsilon_e, \quad (\gamma_{cr} < \gamma) \qquad (10)$$

(we assume here that $\varepsilon_{z\bar{e}} = \varepsilon_{x\bar{e}} = \varepsilon_e$ which can be achieved due to resonance coupling for X and Z oscillations; in this case the self-heating attenuates because of decrease in the beam density). From (9) and (10) follows that the longitudinal temperature is approximately by $(\gamma/\gamma_{cr})^2$ times less than the transverse temperature. At given parameters of the electron storage ring the maximum decrement of cooling appears to be independent of electron current:

$$(\tau_c^{-1})_{max} \simeq \frac{4 N_e r_e^2 c L_c}{\gamma^5 \ell_b \varepsilon_e^{5/2}} \beta_c^{1/2} \cdot \eta_c \frac{m}{M} \simeq 2\eta_c \frac{m}{M}\left(\frac{\gamma_{cr}}{\gamma}\right)^2 \frac{\beta_c^{1/2}}{\langle \beta_e^{1/2} \rangle} \tau_{rad}^{-1} \qquad (11)$$

$$(\theta_p < \theta_e, \quad \delta_p < \delta_e, \quad \gamma_{cr} < \gamma)$$

At energies much higher than the crutial value ($\gamma \gg \gamma_{cr}$), cooling power achieveable appear to be too small if no special means are used.

12. In the region $\gamma \gg \gamma_{cr}$ one can use the mean which enables one to weaken strongly the effect of high transverse electron temperature appeared at self-heating on the cooling rate. It is achieved by introducing the strong focusing of electron beam on the cooling section so that the β-function becomes manyfold less than the cooling section length: $\beta_c \ll \Pi_c$

(whence $\beta_c \ll \beta_p$). Similar effects was already used in the case of "direct" electron beam (at $T_{e\parallel} \ll T_{e\perp}$) with the help of longitudinal magnetic field "freezing" the transverse motion of electrons in the proton-electron collisions /9-11/. The circulating beam of high energy can be magnetized with the system of standard quadrupoles which is of preference by technical reasons. When introducing the strong focusing the beam cross-section decreases but the transverse temperature is correspondingly increases and the contribution of collisions with the "microscopic" impact parameters decreases. But in this case, contribution of the proton-electron interaction at distances of the order of transverse sizes of the beams increases sharply since this interaction goes during a large number of oscillations of transverse velocities of electrons (possibly even along the whole length of cooling) and its efficiency is already not restricted by the transverse temperature. One can imagine the proton interacting with the electron disk with the diameter of the order of oscillation amplitude and having the electron mass and charge. In this case, the proton exchange with the heat energies only with the longitudinal degree of freedom of these objects. The cooling rate here is determined by the beam dimensions and the spread of relative velocities (in moving frame):

$$(\Delta\vec{u})^2 = \Delta v_{e\parallel}^2 + \gamma^2 \theta_p^2 c^2 \qquad (12)$$

(assuming $c\gamma\theta_p \gg \Delta v_{p\parallel}$).

If the longitudinal velocity spread in an electron beam and its size are so small that

$$\gamma\theta_p \gg \Delta v_{e\parallel}/c, \quad \sigma_p \gg \sigma_e \quad (\text{but } \theta_p \ll \theta_e)$$

the cooling decrement is determined again by the proton emittance with the new (smaller) value of the Coloumb logarithm:

$$\tau_c^{-1} \simeq \frac{4 N_e z_e^2 c}{\gamma^5 \ell_b \varepsilon_p^{5/2}} \sqrt{\beta_p} \cdot \ln(\rho_{max}/\rho_{min}) \cdot \frac{m}{M} \eta_c \qquad (13)$$

wherė the minimum impact parameter is equal to the electron beam size: $\rho_{min} = \sigma_e$ and the parameter ρ_{max} is defined of maximum interaction time:

$$\rho_{max} = (\gamma\theta_p c)\cdot(\Pi_c/\gamma c) = \theta_p \Pi_c$$

it is assumed that

$$\beta_p \geq \Pi_c, \quad \rho_{max} \gg \rho_{min}.$$

The parameters $\Delta v_{e\parallel}$ and σ_e increase with an increase of electron current. It may seem that this increase can be compensated in the wide range by decreasing parameters γ_{cr} and β_c. But one should take into account that the longitudinal velocity spread is not only determined by the energy spread and also by the spread of amplitudes and angular oscillations of the electron velocity:

$$\overline{\Delta v_\parallel^2} = \overline{\left(\frac{\Delta\gamma}{\gamma}\right)^2} + \frac{1}{4}\overline{(\gamma^2\theta^2)^2} \qquad (14)$$

($\Delta v_\parallel = \Delta v - \frac{1}{2}\theta^2$ in the laboratory frame). At the energy values $\gamma = 10^2$-10^3 this contribution becomes substantial, so one can only minimize the cooling time over the parameters γ_{cr} and β_c. Optimum conditions correspond to the approximate equalities:

$$\frac{\Delta\gamma_e}{\gamma} = \frac{1}{2}\gamma^2\theta_e^2 = \gamma\sigma_e/\Pi_c \quad . \qquad (15)$$

An existence of the optimum over the parameter β_c is connected with that at relative velocities sufficiently small the region of efficient interaction during collisions becomes smaller than the diameter of electron beam.

Under the conditions $\gamma\theta_p \leq (\Delta v_{e\parallel}/c)$ and $\sigma_p \leq \sigma_e$ the cooling decrement is equal to:

$$\tilde{\tau}_c^{-1} = 4\eta_c \frac{N_e z_e z_p c}{\gamma^2 \sigma_e^2 (\Delta v_{e\parallel}/c)^3 \ell_b} \quad . \qquad (16)$$

With the help of the relations (9) and (10) one can find out the optimum values for the parameters γ_{cr} and β_c as the functions of N_e and τ_{rad}:

$$\left(\frac{\Delta\gamma_e}{\gamma}\right)^{1/2}_{opt} \simeq \left(\frac{\gamma_{cr}}{\gamma}\right)_{opt} \simeq \left(\frac{\beta_c}{\Pi_c}\right)_{opt} \simeq \left(\frac{N_e z_e^2 c\, \tau_{rad}}{\ell_b \Pi_c^2} L_c\right)^{1/9}; \qquad (17)$$

from the formula (16) follows:

$$(\tilde{\tau}_c^{-1})_{opt} = \eta_c \frac{m}{M}\left(\frac{\gamma}{\gamma_{cr}}\right)_{opt} \cdot \tilde{\tau}_{rad}^{-1} \quad . \qquad (18)$$

Apparently, in practice one can always ensure the condition:

$$\gamma\theta_p \lesssim (\Delta v_{e\parallel}/c)_{opt}$$

by making β_p sufficiently large on the cooling section. Then the cooling rate will be determined by the largest size of two beams. Taking into account that $\beta_p \geq \Pi_c$ one can differ the following cases:

1) $\gamma^2 \frac{\varepsilon_p}{\Pi_c} > \left(\frac{\Delta v_{e\parallel}}{c}\right)^2_{opt}$, then

$$(\tilde{\tau}_c^{-1})_{opt} \simeq \eta_c \frac{4 N_e z_e z_p c}{\gamma^5 \ell_b \varepsilon_p^{5/2}} (\sqrt{\beta_p})_{opt}; \quad \gamma^{-2}(\beta_p)_{opt} = \varepsilon_p \Big/ \left(\frac{\Delta v_{e\parallel}}{c}\right)^2_{opt} \qquad (19)$$

(compare with the formula (13)).

2) $\gamma^2 \frac{\varepsilon_p}{\Pi_c} < \left(\frac{\Delta v_{e\parallel}}{c}\right)^2_{opt}$, in this case the cooling decrement is determined by the formula (18).

With the proton emittance given, the cooling rate increases in the case (1) with the increase in number of electrons and decreases in the case (2). The decrease is connected with the rapid increase in the spread of the longitudinal electron velocities which is due to the increase of electron emittance $\left(\Delta v_{e\parallel}/c \simeq \frac{1}{2}\gamma^2\theta_e^2\right)$. The maximum is achieved when

$$\left(\frac{\gamma_{cr}}{\gamma}\right)_{opt} \simeq \left(\gamma^2 \frac{\mathcal{E}_p}{\Pi_c}\right)^{1/4} ,$$

in this case

$$\tilde{\tau}_c^{-1} = \left(\tilde{\tau}_c^{-1}\right)_{max} \simeq \eta_c \frac{m}{M}\left(\frac{\Pi_c}{\gamma^2 \mathcal{E}_p}\right)^{1/4} \cdot \tilde{\tau}_{rad}^{-1} \quad . \qquad (20)$$

13. At sufficiently low longitudinal temperature in the dense magnetized beam one more factor can become essential, namely, a shielding by the electrons remained of the interaction of the proton with electron circle (the shielding like the Debay shielding in plasma). This effect becomes essential in the case when Π_c is larger than the length of Langmuir longitudinal oscillations:

$$\omega_{pl} \cdot \frac{\Pi_c}{\gamma c} >> 1 ,$$

where

$$\omega_{pl} = \left(\frac{4 N_e e^2}{\gamma m \ell_b \sigma_e^2}\right)^{1/2} .$$

In this case, the shielding radius can become less than the transverse size of electron beam:

$$r_{sh} \geq \left(\Delta v_{e\parallel}^2 + \gamma^2\theta_p^2\right)^{1/2} / \omega_{pl}$$

and maximum cooling rate is decreased by shielding by no more than the factor $\omega_{pl}\Pi_c/\gamma c$. Estimations for the real conditions show that this effect should be taken into account under optimization of the cooling beam parameters.

The results presented here seem to give us a basis for the optimism in the question of suppressing the detrimental effect of the electron beam self-heating. The results, though, are preliminary and for the final conclusions additional thourough studies are required.

The authors are very grateful to Drs. N.S.Dikansky, I.A.Koop, I.N.Meshkov, V.V.Parkhomchuk, G.M.Tumaikin, with whom discussions have helped to form these considerations.

R e f e r e n c e s

1. G.I.Budker, A.N.Skrinsky. Usp.Fiz.Nauk 124, N°4 (1978).
2. M.Bell et al. CERN-EP/79-96, 3 Sept. 1979 (submitted to Phys.Lett.).
3. C.Rubbia. Proc. of Intern. Conf. on Relativistic Electron Cooling. Madison University, Visconsin, USA, 2-3 November 1978.
4. A.N.Skrinsky, ibid.
5. Ya.S.Derbenev, ibid.
6. E.Perevedentsev, A.Skrinsky. Proc. of the 6-th National Conf. on Charge Particle Acceleratorc, Dubna, 1978, v.2 p.272.
7. A.Pivinsky, Proc. of the 10-th Intern. Conf. on High Energy Accelerators, Stanford, Calif. 1974, p.405.
8. Ya.S.Derbenev. Proc. of the 6-th National Conf. on Charge Particle Accelerators, OIYaI,Dubna, 1978, vol.I, p.119.
9. Ya.S.Derbenev. Abstract of the thesis. Novosibirsk, 1978.
10. Ya.S.Derbenev, A.N.Skrinsky. Particle Accelerators, vol.8, p.235 (1978).
11. N.S.Dikansky et al. Proc. of 6-th National Conf. on Charge Particle Accelerators. OIYaI, Dubna, 1978, vol.I, p.99.

Работа поступила I8 сентября I979г.

Ответственный за выпуск С.Г.ПОПОВ
Подписано к печати 24.09.I979г. МН 03045
Усл. 0,7 печ.л., 0,6 учетно-изд.л.
Тираж 200 экз. Бесплатно
Заказ № 87

Отпечатано на ротапринте ИЯФ СО АН СССР