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A b s t r a c t

It is shown that the inverse scattering transform method (the IST Method) and the equations integrable by this method can be formulated as a theory of pencils of principal and associated vector G-bundles of zero curvature. It is given a differential-geometric treatment of some properties of integrable equations.

I. Introduction

At present considerable attention is focused on the IST Method and the equations integrable by this method (see reviews /1-3/). As known, the starting point of the IST Method is to put correspondence a basic non-linear partial differen - tial equation to the linear matrix problem (the case of two independent variables x_1, x_2):

$$\frac{\partial \psi}{\partial x^1} = A_1(\lambda, \{u\})\psi, \qquad (1.1)$$
$$\frac{\partial \psi}{\partial x^2} = A_2(\lambda, \{u\})\psi,$$

where the elements of the matrices A_1 and A_2 are parametriz-ed by the functions $u^1(x_1, x_2), \ldots, u^\alpha(x_1, x_2)$ and λ is a parame - ter. The consistency condition of the system (1.1) is equiva - lent to the basical non-linear partial differential equation for the functions $u^1, u^2, \ldots, u^\alpha$.

Specific properties of integrable equations (solitons, infinite sets of conservation laws) are due to the existence of the representation (1.1) and reflect special geometric properties of these equations.

As it was noted by many authors, the consistency condi - tion of the system (1.1) may be interpreted as an equality to zero of the curvature of some space. A differential-geometric formulation of this observation was given in the papers /4-6/. The authors of the papers mentioned pointed out on the rela - tionship between the linear problem (1.1) and the structure equations of fibre bundles. But the only concrete case with the matrices A_1 and A_2 of 2 x 2 dimentionality and, corres - pondingly, with the structure group $SL(2, R)$ was analysed. The role of a structure group was not revealed. Moreover, the questions on symmetry groups, Backlund transformations, soli - ton solutions and the others were left open.

In the present paper it is suggested a general approach based on the treatment of the IST Method as a theory of the pencils of principal and associated vector G-bundles. The case of an arbitrary structure Lie group is considered. These results are an extension and generalization of those obtained in the papers /4-6/. It is shown that the given non-linear completely integrable differential equation is connected with some pencil of principal G-bundles and with the infinite set of the pencils of associated vector G-bundles of zero curvature. Therefore, the infinite number of linear spectral problems of the form (1.1) with matrices A_1 and A_2 of various dimensionality corresponds to the basic partial differential equation. It is demonstrated that a fundamental characteristic of the linear spectral problem is the structure group G rather than the dimensionality of the matrices A_1 and A_2. It is shown that soliton solutions correspond to degeneration of the basis in a fibre bundle. The structure of symmetry groups and Backlund transformations are discussed.

Our paper is organized as follows: Section 2 gives an interpretation of the IST Method as a theory of the pencils of vector G-bundles; in Section 3 some properties of integrable equations: specific features of soliton solutions, symmetry groups and Backlund transformations, are considered.

2. The IST Method as a theory of the pencils of vector bundles

"Embedding" of the IST Method to the theory of the pencils of vector bundles is due to interpretation of the linear spectral problem (1.1) as equations governing the motion of a basis in a certain pencil of G-bundles.

Partial differential equations integrable by the IST Method turn out to be connected with two types of bundles: principal G-bundles and associated vector G-bundles, to be more precise, the pencils of these bundles.

1. A pencil of principal G-bundles is the infinite family $\{(P,G,M,\pi);\lambda\}$ of principal G-bundles. The multicomponent parameter $\lambda=(\lambda_1,\lambda_2,\ldots)$ enumerates the terms of this family*,

* Until now, the spectral problems with one parameter λ have been only considered what corresponds to a one-parametric family of bundles.

(P,G,M,π) is the standard notation /7,8/ of the principal G-bundle with the structure group G, M is the base of a bundle, P is the bundle space, π is the projection: $P \to M$. The structure group G is the arbitrary linear Lie group. As shown, each fibre π^{-1} of the bundle is homeomorphic to the group G /7,8/. In our case, the base space M is built in a special manner: it is a family of all N-dimensional manifolds embedded in the infinite-dimensional manifold M_∞. Namely, let $z=\{z_1, z_2,\ldots\}$ be local coordinates of the manifold M_∞. Enumerate them in a way convenient for further purposes: $x_i \equiv z_i\ (i=1,2,\ldots,N)$; $u^\beta \equiv z_{N+\beta}\ (\beta=1,2,\ldots,\alpha)$. The remaining coordinates are denoted by $u^\beta_{i_1\ldots i_\ell}$ ($\ell = 1,2,\ldots,\infty$). Embedding of the manifold M to the infinite-dimensional manifold M_∞ is given by the infinite set of equations (here and below, d denotes an exterior differentiation):

$$
\begin{aligned}
&du^\beta - u^\beta_i\, dx_i = 0,\\
&du^\beta_{i_1} - u^\beta_{i_1 i_2}\, dx_{i_2} = 0, \qquad\qquad (2.1)\\
&\cdots\cdots\cdots\cdots\cdots\\
&du^\beta_{i_1\ldots i_\ell} - u^\beta_{i_1\ldots i_\ell i_{\ell+1}}\, dx_{i_{\ell+1}} = 0,
\end{aligned}
$$

where $i_\ell = 1,\ldots,N$; $\beta = 1,\ldots,\alpha$; $\ell = 1,2,\ldots,\infty$. Roughly speaking, $x=\{x_1,\ldots,x_N\}$ are independent variables, $u^1, u^2,\ldots,u^\alpha$ are dependent variables, and $u^\beta_{i_1\ldots i_\ell}$ are partial derivatives of the dependent variables over independent ones of various orders. A very important for a further analysis is the fact that the base space M is an embedded manifold. The set of all independent and dependent variables (together with derivatives) will be denoted, for short, by z $(z \overset{df}{=} (x_i, u^\beta, u^\beta_{i_1\ldots i_\ell}))$.

For each elements of the principal G-bundles pencil $\{(P,G,M,\pi);\lambda\}$ (i.e. at a given parameter λ) the connection form $\omega(\lambda)$ may be introduced in a standard way /7,8/. As a result, we have a pencil of connections $\{\omega(\lambda)\}$ for a pencil of principal G-bundles. Fixation of a pencil of principal G-bundles consists in fixing a type of the dependence of the pencil of connections $\{\omega(\lambda)\}$ on a parameter λ : different pencils correspond to a concrete type of the dependence of

$w(\lambda)$ on this parameter. Further, a pencil of curvature forms is introduced in an obvious way:

$$\Omega(\lambda) = d w(\lambda) + \tfrac{1}{2} w(\lambda) \wedge w(\lambda). \qquad (2.2)$$

Proceed now to a pencil of principal G-bundles with a zero curvature. It can be easily seen that the condition

$$d w(\lambda) + \tfrac{1}{2} w(\lambda) \wedge w(\lambda) = 0 \qquad (2.3)$$

is equivalent to a certain partial differential equation. Let $w^a\ (a=1,\dots,m)$ be a complete set of left-invariant one-forms of the m-parametric group G. Expanding the forms $w(\lambda)$ with respect to $w^a(\lambda)$, we obtain:

$$d w^a(\lambda) = f^{abc}\, w^b(\lambda) \wedge w^c(\lambda), \qquad (2.4)$$

where f^{abc} are the structure constants of the group G. Then, consider a space Γ of all cross sections of the pencil of principal G-bundles. If we introduce the local coordinates $\{z\}$ and take into account (2.1), we get:

$$w^a(\lambda) = w_i^a(z,\lambda)\, dx_i \qquad (2.5)$$

Finally, from eq. (2.4) we find:

$$\mathcal{D}_i w_j^a(z,\lambda) - \mathcal{D}_j w_i^a(z,\lambda) + f^{abc} w_i^b(z,\lambda) w_j^c(z,\lambda) = 0 \qquad (2.6)$$

where

$$\mathcal{D}_i = \frac{\partial}{\partial x^i} + u^\beta \frac{\partial}{\partial u_i^\beta} + \dots + u^\beta_{i_1 \dots i e} \frac{\partial}{\partial u^\beta_{i_1 \dots i e i}} + \dots$$

is the total derivative with respect to variables x_i.

Equations (2.6) should be satisfied for the whole pencil, i.e. for any value of the parameter λ. This means that eq. (2.6) is satisfied in virtue of some system of equations

$$F(z) = 0. \qquad (2.7)$$

The system (2.7) is a system of partial differential equations in partial derivatives with independent variables $\{x_i\}$. In principle, eqs. (2.7) can include all the z-coordinates, i.e. the derivatives of $u^\beta(x)$ of any order. Local equations usually investigated by the IST Method correspond to the case of a finite number of derivatives.

Thus, we see that the differential equations (2.6) and (2.7) can be put correspondence to some pencil of principal G-bundles of zero curvature. Since a principal G-bundle of zero curvature is a local group G_x, one may say that a partial differential equations corresponds to a pencil of local group G_x with structure equations (2.4). Whether a concrete partial differential equation can correspond to a pencil of local groups, i.e. whether there exist one-forms $w^a(z,\lambda)$ satisfying eqs. (2.4) - this requires a special study in each particular case. Unfortunately, a general criterion is not yet known.

2. The linear spectral problem of the IST Method is, however, associated with a pencil of associated vector G-bundles of zero curvature rather than with a pencil of principal G-bundles.

Let Q be a space of the arbitrary linear representation of the group G. Consider the vector G-bundle $(Q(P), G, M, \pi)$ /7,8/ associated with the principal G-bundle described above. Remind that the fibre $\pi^{-1}(z)$ of the vector G-bundle is a space Q of the representation of the group G. Introduce a pencil $\{(Q(P), G, M, \pi); \lambda\}$ of vector G-bundles associated with the pencil $\{(P, G, M, \pi); \lambda\}$ of principal zero-curvature G-bundles and denote the basis in a bundle space via $F(\lambda)$. With a given λ we have, as known /7,8/,

$$d F(\lambda) = \tilde{w}(\lambda) F(\lambda) \qquad (2.8)$$

where $\tilde{w}(\lambda)$ is the Q-valued connection form of the vector G-bundle. The zero-curvature condition has the form /7,8/:

$$d \tilde{w}(\lambda) + \tfrac{1}{2} \tilde{w}(\lambda) \wedge \tilde{w}(\lambda) = 0. \qquad (2.9)$$

Consider a cross section space $\tilde{\Gamma}$. If eq. (2.1) is taken into account, then in local coordinates eqs. (2.8) and (2.9) take the form

$$\mathcal{D}_i F(z,\lambda) = \tilde{\omega}_i(z,\lambda) F(z,\lambda) \qquad (2.10)$$

and

$$\mathcal{D}_i \tilde{\omega}_j(z,\lambda) - \mathcal{D}_j \tilde{\omega}_i(z,\lambda) + [\tilde{\omega}_i(z,\lambda), \tilde{\omega}_j(z,\lambda)] = 0 \qquad (2.11)$$

where the Q-valued functions are determined by the relation

$$\tilde{\omega}(z,\lambda) = \tilde{\omega}_i(z,\lambda)\, dx_i \qquad (2.12)$$

If Q is the n-dimensional space, then $\tilde{\omega}_i(z,\lambda)$ and $F(z,\lambda)$ are the n x n matrices.

Equation (2.10) is the linear spectral problem of the IST Method for the partial differential equation (2.7) written in the form (2.11). In a general case the linear equations (2.10) are matrix equations of arbitrary dimensionality.

There is a simple relationship between connections $\tilde{\omega}(z,\lambda)$ and connections $\omega(z,\lambda)$ of the pencil of principal G-bundles. It is obvious that the arbitrary Q-values function $\tilde{\omega}_i(z,\lambda)$ can be represented in the form

$$\tilde{\omega}_i(z,\lambda) = T^a \omega^a_i(z,\lambda) \qquad (2.13)$$

where $\omega^a_i(z,\lambda)$ are the left-invariant numerical functions on the group G and the matrices T^a is a matrix representation of the generators of the G-group algebra ($[T^a, T^e] = f^{abc}T^c$). Just the formula (2.13) determines a correspondence between the connection $\tilde{\omega}(\lambda)$ of the pencil of vector G-bundles and the connection $\omega(\lambda)$ of the pencil of principal G-bundles. Recall that Q is a space of the arbitrary representation of the group G . Formula (2.13) reflects the obvious fact: the con - nection in a principal G-bundle induces a definite connection in an arbitrary associated G-bundle.

If one takes into account the relation (2.13), then eq. (2.11) reduces to eq. (2.6) and eqs. (2.10) are written as

follows:

$$\mathcal{D}_i F(z,\lambda) = T^a \omega^a_i(z,\lambda) F(z,\lambda) \qquad (2.14)$$

Hence, if the partial differential equation $\mathcal{F}(z)=0$ can be represented as (2.6) (or (2.11)), then it can be connected with some pencil of vector G-bundles and, as a consequence, with the linear spectral problem (2.14). Moreover, from (2.6) and (2.14) it follows that in this case the partial differen- tial equation corresponds to the infinite set of the pencils of vector G-bundles, which corresponds, in turn, to the infi- nite set of the group's G representations. Hence, a diffe - rential equation corresponds to the infinite set of linear spectral problems (2.14) with the matrices A_1, A_2 of various dimensionality*.

All these connections $\tilde{\omega} = T^a \omega^a$ in the pencils of vector G-bundles are induced, as we see, by the single connec- tion $\omega = \{\omega^a, a = 1,\dots,m\}$ of the pencil of principal G-bundles. Therefore, the fundamental characteristic of the spectral problem of the type (1.1) is the structure group G rather than the dimensionality of matrices A_1 and A_2 .

Thus, a decisive importance for applicability of the IST Method to a concrete partial differential equation is of the existence of the one-forms ω^a satisfying eqs. (2.4) on the manifold of solutions.

As an example, let us consider a sine-Gordon equation

$$\frac{\partial^2 u}{\partial x_1^2} - \frac{\partial^2 u}{\partial x_2^2} + m^2 \sin u = 0$$

This equation corresponds to the pencil of principal G-bundles with the structure group SU (2) and to the infinite set of the type (2.10) spectral problem with the connection components equal to

$$\tilde{\omega}_i = i\frac{1}{2}\epsilon_{ik}\frac{\partial u}{\partial x^k}T^1 + i\cos\frac{u}{2}\lambda_i T^2 + i\sin\frac{u}{2}\epsilon_{ik}\lambda_k T^3 \qquad (2.15)$$

where i,k = 1,2; ϵ_{ik} is the antisymmetric tensor, $\lambda = (\lambda_1, \lambda_2)$ is the two-component parameter satisfying the condition $\lambda_i \lambda_i = m^2$, The matrices T^1, T^2, T^3 form the representations of the generators of the groups SU (2) of arbitrary dimensionality. Confining ourselves to the two-dimensional representations $T^1=\frac{1}{2}\sigma_1$, $T^2=\frac{1}{2}\sigma_2$, $T^3=\frac{1}{2}\sigma_3$ of the group SU (2) (where $\sigma_1, \sigma_2, \sigma_3$ are Pauli matrices), we obtain the usual linear spectral problem for the sine-Gordon equation, which is written in the relativistic-invariant form /9,10/.

The infinite set of linear problems can be written in an analogous way for any equation which the IST Method in applicable to. Much more clearly the fundamentalness of a structure group appears for the equations whose structure group is of the range higher or equal to 2 (the three-waves model /11/, the σ -model /12/, the chiral fields /13/). The notion of a structure group turns out to be very effective for a study of the reduction problem (for this problem see /13/) and also for establishing of a gauge equivalence of various equations, namely: at least, some part of the general chiral fields reduction problem has a strickly group character and it consists in reducing the structure group G with respect to all its possible subgroups. In the establishing of a gauge equivalence it is essential that transformations of coordinates of the cross sections of the pencil of principal G-bundles must conserve a structure group. These questions will be studied in more detail in a separate paper.

III. Differential-geometric treatment of some properties of integrable equations

Discuss now some general properties of integrable equations for which there exists a linear spectral problem of the type (2.14).

1. We shall consider the equation (2.14) as an equation of the moving basis in a space of the n x n matrices. Matrices $F(z,\lambda)$ form a basis in each fibre $\pi^{-1}(z)$ or, what is equivalent, they are fundamental matrices-solutions of the linear system (2.14): each column of the matrix $F(z,\lambda)$ is one of n linear independent solutions of the set of equations (2.14). Assume

that within the limit $|x|\to\infty$ $\omega_i(z,\lambda)\to\omega_i^0(\lambda)$. Then, there are non-trivial asymptotics $F(z,\lambda)\to F^0(x,\lambda)=\exp(\omega_i^0(\lambda)x_i)$ at $|x|\to\infty$. Of course, one can take various complete sets of solutions and all they are connected with each other. Restrict oneself to the case of two independent variables x_1, x_2 . Determine the matrices-solutions F^+ and F^- as follows:

$$F^+(z,\lambda)\to F^0(x,\lambda) \quad \text{at} \quad x_2\to+\infty$$

$$F^-(z,\lambda)\to F^0(x,\lambda) \quad \text{at} \quad x_2\to-\infty$$

These two bases are connected by the linear relation $F^+=F^-S$. The matrix S is called the transition matrix /1-3/.

It is easy to see that the diagonal elements S_{pp} $(p=1,\dots,n)$ of this matrix are independent of the coordinates x_1, x_2 and, therefore, they are invariant characteristics of the problem. Since $S_{pp}(\lambda)$ are the functionals of dependent variables, $S_{pp}(\lambda)$ are the generating functionals of the integrals of motion of the basic partial differential equation. As known, to find an explicit formof the infinite sets of the integrals of motion, it is necessary to expand $S_{pp}(\lambda)$ in a power series of λ . As a rule, the integrals of motion appearing after expansion in the neighbourhood of an arbitrary point λ_0 will be nonlocal. The series of local integrals appear in the expansion in the neighbourhoods of the singular points of the pencils of connections $\tilde{\omega}(z,\lambda)$.

Besides the bases F^+ and F^-, there are many other bases. Most interesting of them are a bases $F_p^{+,-}$ constructed in a following way: the matrix $F_p^{+,-}$ is obtained from the matrix F^-, by replacing its p -th column by the p -th one of the matrix F^+ . For the bases $F_p^{+,-}$ the following relations are valid:

$$\frac{\det F_p^{+,-}(z,\lambda)}{\det F^-(z,\lambda)} = S_{pp}(\lambda) \qquad (p=1,\dots,n) \tag{3.1}$$

It is clear that some solution of a basic partial differential equation corresponds to each concrete cross section of a pencil of G-bundles (to each concrete matrix-basis $F(z,\lambda)$). The equations integrable by the IST Method usually have two different types of solutions: the solutions of continuous spectrum and the solutions of soliton type. Most clearly the distinction between these types of solutions appear in the bases $F_p^{+,-}(z,\lambda)$. For the continuous spectrum solutions, $S_{pp}(\lambda)\neq 0$ and, hence, the matrices $F_p^{+,-}$ are nondegenerate. Meanwhile, the soliton solutions correspond to the zeros of the diagonal elements of the transition matrix. Then

$$\det F_p^{+,-}(z,\lambda=\xi)=0 \qquad (3.2)$$

Thus, soliton solutions correspond to the degenerate bases $F_p^{+,-}$ in the constructed above pencil of vector G-bundles.

At $n\geq 3$ various types of soliton solutions are possible. This is due to that what diagonal elements of the transition matrix have zeros (see, for example, Ref. 11). Different variants of a degeneration of the bases $F_p^{+,-}$ correspond to different types of soliton solutions.

2. As known, partial differential equations integrable by the IST Method possess the infinite sets of the integrals of motion /1-3/ and, correspondingly, the infinite symmetry groups /14/. Among the groups admissible by an integrable equation there are infinite Lie groups acting transitively on the integral manifold $\sum$ of this equation. The minimal of such groups is called a dynamical group /15/. The dynamical group contains the group of Backlund transformations as a subgroup. All these groups admit a natural interpretation within the framework of the pencils of vector G-bundles.

Let us consider equation $\mathcal{F}(z)$ containing the finite number of variables z (i.e. a local equation). Any solution of this equation is a certain cross section of the pencil of vector G-bundles. Therefore, the transformations which transform one solution of the partial differential equation to another one, are the transformations of one cross section to another one. In our case, these are transformations which the following equations remain invariant:

$$\mathcal{D}_i F(z,\lambda)=\tilde{\omega}_i(z,\lambda)\,F(z,\lambda), \qquad (3.3)$$

$$\Omega_{ij}=\mathcal{D}_i\tilde{\omega}_j(z,\lambda)-\mathcal{D}_j\tilde{\omega}_i(z,\lambda)+[\tilde{\omega}_i(z,\lambda),\tilde{\omega}_j(z,\lambda)]=0 \qquad (3.4)$$

These transformations are the pencil of the well known /7,8/ transformations from one local coordinates in a bundle to the others:

$$F(z,\lambda)\to F'(z,\lambda)=g(z,\lambda)\,F(z,\lambda), \qquad (3.5)$$

$$\tilde{\omega}_i(z,\lambda)\to\tilde{\omega}'_i(z,\lambda)=g^{-1}(z,\lambda)\tilde{\omega}_i(z,\lambda)g(z,\lambda)+g^{-1}(z,\lambda)\mathcal{D}_i g(z,\lambda) \qquad (3.6)$$

where $g(z,\lambda)$ is the Q-valued function of the z-coordinates and the parameter λ *. It is here important that $g(z,\lambda)$ is not the arbitrary function of the parameter λ. It is necessary that transformations (3.4) conserve the form of dependence of the connection $\tilde{\omega}(z,\lambda)$ on the parameter λ, i.e. that $\tilde{\omega}'_i(z,\lambda)=\tilde{\omega}_i(z',\lambda)$ This imposes strong limitations on the possible dependence of $g(z,\lambda)$ on λ.

It is clear that a major problem is just the enumeration of all possible $g(z,\lambda)$; if $g(z,\lambda)$ are known, we can restore all the solutions of the system (3.3)-(3.4) starting from one solution. The IST Method and its various variants reduce mainly to a construction of admissible $g(z,\lambda)$.

From formulae (3.5) and (3.6) an essential difference between the Backlund transformations of continuous spectrum and the soliton Backlund transformations becomes obvious. Consider the transformation (3.5) for bases $F_p^{+,-}$. For the continuous spectrum, $\det F_p^{+,-}\neq 0$, $\det F_p'^{+,-}\neq 0$, and hence, the transformation matrices $g(z,\lambda)$ are non-degenerate. In the case of Backlund transformations which add one soliton to the solution, $\det F_p'^{+,-}=0$. Therefore, the matrix $g(z,\xi)$ corresponding to this transformation is degenerate ($\det g(z,\xi)=0$), and the formula (3.6) is absent. An analogue of transformation (3.6) may be only written for some non-degenerate submatrix $\tilde{g}$ of the matrix g. This reduction leads to Backlund transformations usually considered in lite-

* Transformation (3.5), (3.6) coincide in their form with gauge transformations in the Yang-Mills theory.

rature.

Let us draw our attention to the fact that transformations (3.5) and (3.6) are not the most general transformations admissible by the basic partial differential equation. As known, integrable equations possess the Lie-Backlund symmetry /14,16/ groups for which

$$z \to z' = f(z, a)$$

and the functions f depend on all the variables z. In the transformations (3.5) and (3.6) the matrix $g(z,\lambda)$ can only depend on the variables z which enter the basic equation and, therefore, Lie-Backlund transformations are not contained in transformations (3.5) and (3.6).

The way out is the following. It is necessary to consider not only eqs. (3.3) and (3.4) but all possible their differential consequencies either. Consider the system

$$\begin{aligned} &\mathcal{D}_i F(z,\lambda) = \tilde{\omega}_i(z,\lambda) F(z,\lambda), \\ &\mathcal{D}_k \mathcal{D}_i F(z,\lambda) = \mathcal{D}_k \tilde{\omega}_i(z,\lambda) F(z,\lambda), \\ &\ldots\ldots\ldots\ldots \\ &\mathcal{D}_j \cdots \mathcal{D}_k \mathcal{D}_i F(z,\lambda) = \mathcal{D}_j \cdots \mathcal{D}_k \tilde{\omega}_i(z,\lambda) F(z,\lambda), \\ &\ldots\ldots\ldots\ldots \end{aligned} \qquad (3.7)$$

It is easy to see that the integrability condition of the infinite system (3.7) is the equation (3.4) with all its differential consequencies:

$$\begin{aligned} &\Omega_{ij} = 0 \\ &\mathcal{D}_k \Omega_{ij} = 0 \\ &\ldots\ldots \\ &\mathcal{D}_\ell \cdots \mathcal{D}_k \Omega_{ij} = 0 \\ &\ldots\ldots \end{aligned} \qquad (3.8)$$

A symmetry group of the infinite system (3.7)-(3.8) is the group admissible by the basic partial differential equation. The minimal group acting transitively on the solutions of the systems (3.7) and (3.8) is the dynamical group of the basic differential equation. It contains the infinite-dimensional symmetry group and the infinite-dimensional group of Backlund transformations as subgroups.

IV. Conclusion

We see that the IST Method can be formulated in a natural way within the framework of differential geometry as a theory of pencils of principal and associated vector G-bundles with zero curvature. Emphasize that we have considered non-arbitrary bundles but those with a linear structure group. Just such bundles lead to linear spectral problems.

The more general type bundles are apparently associated with the theory of prolongation structures /17/. A study of the structure of such bundles should help to understand much better the procedure of prolongation /17/.

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