СИБИРСКОЕ ОТДЕЛЕНИЕ АН СССР
ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ

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P-ODD NUCLEAR FORCES AS A SOURCE OF PARITY NON-CONSERVATION IN ATOMS

ПРЕПРИНТ 80-136

Новосибирск

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A b s t r a c t

We consider the electromagnetic P-odd interaction of electron with nucleus caused by parity non-conserving nuclear forces. New information concerning these forces can be obtained already now in experiments on optical activity of heavy atoms and diatomic molecules. In the case of deuteron the P-odd vector-potential is expressed via the parameters of parity non-conservation in np scattering.

1. The discovery of the neutral current weak interaction made in Novosibirsk [1] by observation of optical activity of atomic bismuth vapour is only the first positive result in the study of weak interaction structure by the atomic spectroscopy methods. The subsequent increase in the accuracy of this experiment [2] makes qualitatively new results here sufficiently realistic. We mean in particular the measurement in atoms [3] and molecules [4] of the constant characterizing the weak interaction of electron with nucleus spin. The natural scale of this effect is Z^{-1} of that measured already since only the valent nucleon contributes to it, but not all nucleons of the nucleus. One could expect that the effects of such an order of magnitude can be measured already now. However, within the Weinberg-Salam model at the value of the mixing parameter $\sin^2\theta = 0.23$ that follows from the existing experimental data, the constant of the interaction discussed is very small numerically.

In the present paper we wish to note that the effects of parity violation in atoms, dependent on the nucleus spin, can be caused not only by neutral currents. They are induced also by P-odd nuclear forces. Due to these forces, the electromagnetic vector-potential of nucleus acquires an additional term of wrong parity acting on electron. The contribution of this mechanism can exceed appreciably that of neutral currents. Therefore, the experimental searches for the effects discussed are real even at present. They can give valuable information about parity violation in nucleus.

Specific for μ - mesoatoms effects caused by P-odd nuclear forces were considered previously in the ref. [5]. Parity violation in the electron-nucleus interaction induced by P-odd nuclear forces was discussed from the general point of view in the recent work [6].

2. The origin of the effect discussed can be understood most easily starting from the fact that parity violation in nucleus leads to helical spin structure in it [7,8] and toroidal currents [7]. The resulting contribution to the vector-potential

of the nucleus leads to the P-odd interaction between electron and nucleus, of interest to us.

We shall show that this electromagnetic interaction is necessarily of contact type. This result is contained in fact in the refs.[9-14]. According to communication by P.Sandars, it was known also to N.Ramsay. Nevertheless, for the sake of completeness of the exposition it seems to us worth-while to present here the proof of this fact.

The matrix element j_μ of the electromagnetic current operator between the states of a given spin I and momenta k and k' can be expanded in four independent vectors of the problem:

$$p_\mu = (k'+k)_\mu , \quad q_\mu = (k'-k)_\mu , \quad s_\mu , \qquad (1)$$

$$r_\mu = i\varepsilon_{\mu\nu\chi\lambda} p_\nu q_\chi s_\lambda$$

Here s_μ is four-dimensional spin operator defined, e.g., for the state with the momentum k and satisfying therefore the condition $k_\mu s_\mu = 0$:

$$s_0 = \frac{\vec{I}\cdot\vec{k}}{m} , \quad \vec{s} = \vec{I} + \frac{\vec{k}(\vec{k}\vec{I})}{m(m+k_0)}$$

Of course, in this case $k'_\mu s_\mu \neq 0$. Taking into account the conservation law for electromagnetic current $q_\mu j_\mu = 0$, this matrix element can be written as follows [10]:

$$j_\mu = \langle I,k'|p_\mu F_1 + r_\mu F_2 + [q^2 s_\mu - q_\mu (qs)]\, F_3|I,k\rangle \qquad (2)$$

The invariant functions F_i depend on the scalars that can be constructed from the vectors (1). Since $(pq) = 0$ and $(sk) = 0$, there are only two such invariants. As arguments of F_i it is convenient to choose the hermitian operator $\tau = iq_\mu s_\mu$ and the usual variable $t = q^2$. The substitution of an operator function F_i into the expression (2) should be accompanied, to retain the hermiticity condition, by symmetrization in non-commuting operators. In the expansion of $F_i(t,\tau)$

in powers of τ

$$F_i(t,\tau) = \sum_{n=0}^{N_i} f_{in}(t)\, \tau^n \qquad (3)$$

the maximal power N_i is evidently fixed by the spin I:

$$N_1 = 2I, \quad N_2 = N_3 = 2I - 1$$

Therefore, the total number of electromagnetic form-factors $f_{in}(t)$ constitutes 6I + 1.

The operator $\tau = is_\mu q_\mu$ changes sign not only under reflexion, but under inversion of time as well. Hence, the T-invariance condition restricts the summation in (3) to the even powers of τ . But then all the functions $F_i(t,\tau)$ are P-even, and parity violation in the electromagnetic current (2) is caused by the structure $q^2 s_\mu - q_\mu (sq)$ only. The term proportional to q_μ corresponds to gradient of a scalar function in the vector-potential of the particle. This contribution is eliminated by a gauge transformation. And the factor q^2 in the remaining term in the current cancels out the propagator $1/q^2$ of the particle field. Therefore, the P-odd electromagnetic interaction is indeed necessarily contact one. Note, that the term with $n = 0$ in the expansion (3) for the function $F_3(t,\tau)$ corresponds to the well-known anapole moment, considered for the first time in the ref. [7].

As to the T-odd part of the electromagnetic current, here those and only those terms that conserve P-invariance are of contact type. Finally, the part of current that has wrong C-parity is contact one [13].

The total number of invariant form-factors f_{in}, conserving (+) and violating (-) P-, T- and C-invariance is found at a given spin I without any difficulty and is presented in the table. If in one column of the table two values are indicated, the upper one refers to integer spin and the lower one to half-integer spin. This classification of electromagnetic form-factors was for the first time carried out in the ref. [12] by means of transition into annihilation channel.

P	+	−	+	−
T	+	+	−	−
C	+	−	−	+
		I	I	
	2I+1	I+$^1/2$	I-$^1/2$	2I

In the conclusion of this section note that in the current matrix elements non-diagonal in the internal state the condition of T-invariance does not forbid non-contact P-odd terms. A testimony of it is the circular polarization of radiation when parity is violated. The physical origin of such a difference is quite obvious. The above mentioned toroidal currents [7] create the field only inside a system which does not possess definite space parity. That is why the diagonal matrix element of the interaction is contact one. And only when the system is rebuilt, i.e., in the case of non-diagonal matrix element, this field goes out.

3. Pass now to the direct calculation of the parity violating electromagnetic field of nucleus. The nucleus is naturally assumed to be non-relativistic, so that the P-odd part of j_μ looks as follows:

$$j_0(\vec{q}) = 0\,, \qquad \vec{j}(\vec{q}) = [\vec{q}^{\,2}\vec{I} - \vec{q}(\vec{q}\,\vec{I})]\,F \qquad (4)$$

The first of these equalities has a simple physical meaning: P-odd interaction does not influence charge distribution in a system. And from the second one it follows by the way that

$$\int d\vec{r}\,\vec{j}(\vec{r}) = \vec{j}(\vec{q}=0) = 0 \qquad (5)$$

Since typical atomic momenta are much smaller than nuclear ones, the dependence of F on $\vec{q}^{\,2}$ and $\vec{q}\,\vec{I}$ can be neglected. In other words, the nucleus will be assumed to be point-like. In this case the vector-potential can be presented as:

$$\vec{A}(\vec{r}) = \int \frac{d\vec{r}\,'\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} = [-(\vec{b}\vec{\nabla})\vec{\nabla} + \vec{b}\Delta]\,\frac{1}{r} \qquad (6)$$

To find the constant vector $\vec{b}$ it is sufficient to integrate the equality (6) over $\vec{r}$ taking into account (5). We come than to the following expression for the vector-potential:

$$\vec{A}(\vec{r}) = -\pi\delta(\vec{r}) \int d\vec{r}\,' r'^2 \vec{j}(\vec{r}\,') \qquad (7)$$

The term $-\vec{\nabla}(\vec{b}\vec{\nabla})\frac{1}{r}$ that can be eliminated by a gauge transformation is omitted here. Therefore, neglecting the dependence of F on $\vec{q}$ we consider only the lowest of P-odd multipoles - anapole moment of nucleus

$$\vec{a} = -\pi \int d\vec{r}\; r^2 \vec{j}(\vec{r}) \qquad (8)$$

Its calculation constitutes in general extremely complicated problem. Here we shall restrict to the simplest approximation of shell model: one nucleon above completely closed shells. Anapole moment is due to the mixing of one-particle states of opposite parity with the same total angular momentum. The state arising in such a way we shall write down as

$$|0\rangle + \sum_k \frac{|k\rangle\langle k|H_W|0\rangle}{E_0 - E_k} = \qquad (9)$$

$$= R_0(r)\Omega_{Ilm}(\vec{n}) + i\sum_k \eta_k R_k(r)\Omega_{Il'm}(\vec{n})$$

where R_0, R_k are radial wave functions, Ω_{Ilm} are spherical spinors, $l' = 2I - 1$, $\vec{n} = \vec{r}/r$. The mixing coefficient

$$i\eta_k = \frac{\langle k|H_w|0\rangle}{E_0 - E_k}$$

at the usual definition of spherical function is purely imaginary due to T-invariance of the weak interaction operator H_w .

We start from the calculation of the contribution to the effect from the term in the current that is connected with spin magnetic moment of nucleon. Using the identity $\Omega_{Il'm} = -(\vec{\sigma}\vec{n})\,\Omega_{Ilm}$, we shall present the axial part of spin current

$$\vec{j}^s(\vec{r}) = \frac{e\mu}{2m}\vec{\nabla}\times\phi^+\vec{\sigma}\phi \qquad (11)$$

that is of interest to us, as

$$\frac{e\mu}{m}\,\vec{\nabla}\times\sum_k \eta_k R_0 R_k\,\Omega^+_{Ilm_2}\,\vec{n}\times\vec{\sigma}\,\Omega_{Ilm_1} \qquad (12)$$

Here e and m are the charge and mass of proton, μ is magnetic moment of nucleon. Substituting (12) into (8) and integrating by part, we get without any difficulty

$$\vec{a}^s = 2\pi\frac{\varkappa\vec{I}}{I(I+1)}\,\frac{e\mu}{m}\sum_k \eta_k r_{0k} \qquad (13)$$

$$r_{0k} = \int_0^\infty dr\, r^3 R_0 R_k\,, \qquad \varkappa = (-1)^{I+1/2-1}\,(I+1/2)$$

We pass now to the calculation of the contribution connected with the orbital motion of a proton (for a neutron this contribution is of course absent). It is convenient to present the integral $\int d\vec{r}\, r^2\vec{j}(\vec{r})$ by means of the substitution $\vec{r} = i[H,\vec{r}]$ as

$$\frac{ie}{2}\langle\, r^2[H,\vec{r}] + [H,\vec{r}]\, r^2\rangle \qquad (14)$$

where the expectation value is taken over the disturbed state $|0\rangle + i\sum_k \eta_k|k\rangle$. We split the total non-relativistic Hamiltonian of the valent nucleon into three terms:

$$H = H_r + H_l + H_w \qquad (15)$$

Here

$$H_r = -\frac{1}{2m}\,\frac{1}{r}\,\partial_r^2\, r + U(r)\,, \qquad H_l = \frac{\vec{l}^2}{2mr^2}$$

and weak interaction Hamiltonian H_w looks as

$$H_w = \frac{G}{\sqrt{2}}\,\frac{1}{2m}\,[\vec{\sigma}\vec{p}f(r) + f(r)\vec{\sigma}\vec{p}] \qquad (16)$$

The matrix element

$$i\,\frac{e}{2}\,\langle r^2[H_r,r]\vec{n} + [H_r,r]r^2\vec{n} + 2r^3[H_l,\vec{n}] + 2r^2[H_w,\vec{r}]\rangle$$

arising at the substitution of (15) into (14) is transformed by means of the identity

$$r^2[H_r,r] + [H_r,r]r^2 = \frac{2}{3}[H_r,r^3]$$

as follows:

$$ie\,\langle\,\frac{1}{3}\,[H - H_l - H_w, r^2\vec{r}] + r^2[H_l,\vec{r}] + r^2[H_w,\vec{r}]\rangle =$$

$$= \frac{2}{3}\,ie\,\langle\,\frac{1}{2m}\,[\vec{l}^2,\vec{r}] + r^2[H_w,\vec{r}] - \vec{r}(\vec{r}[H_w,\vec{r}])\rangle \qquad (17)$$

The expectation value of the commutator $[\vec{l}^2,\vec{r}]$ over the state $|0\rangle + i\sum_k \eta_k|k\rangle$ is calculated easily. It is equal to

$$-i\,\frac{l(l+1) - l'(l'+1)}{I(I+1)}\,\vec{I}\sum_k \eta_k r_{0k} = -2i\cdot\frac{\varkappa\vec{I}}{I(I+1)}\sum_k \eta_k r_{0k} \qquad (18)$$

The terms in (17) that contain the operator of the so-called contact current

$$\vec{j}^c = ie\,[H_w,\vec{r}] = \frac{G}{\sqrt{2}}\,\frac{e}{m}\,\vec{\sigma}f(r) \qquad (19)$$

are reduced to the form

$$- \frac{2}{3} \frac{G}{\sqrt{2}} \frac{e}{m} \frac{\varkappa \vec{I}}{I(I+1)} \langle 0|r^2 f(r)|0\rangle \qquad (20)$$

Finally, we got the following result for the orbital contribution to the anapole:

$$\vec{a}^{\,or} = - \frac{2\pi}{3} \frac{e}{m} \frac{\varkappa \vec{I}}{I(I+1)} \left\{ \sum_k \eta_k r_{0k} - \frac{G}{\sqrt{2}} \langle 0|r^2 f(r)|0\rangle \right\} \qquad (21)$$

The direct calculation, not grounded from the very beginning on the contact nature of the interaction, turns out incomparably more cumbersome, but leads of cource to the same results for the vector-potentials $\vec{A}^s$ and $\vec{A}^{or}$

Arising in this way P-odd Hamiltonian of electron-nucleus interaction is equal to

$$H^{em} = e\vec{\alpha}(\vec{A}^s + \vec{A}^{or}) =$$

$$= \delta(\vec{r}) \frac{2\pi\alpha}{m} \frac{\varkappa \vec{I}\vec{\alpha}}{I(I+1)} \left\{ (\mu - \frac{q}{3}) \sum_k \eta_k r_{0k} + \frac{q}{3} \frac{G}{\sqrt{2}} \langle 0|r^2 f(r)|0\rangle \right\} \qquad (22)$$

where $\vec{\alpha}$ are the Dirac matrices for electron and q is the charge of the external nucleon in the units of e. We present for comparison the weak interaction Hamiltonian, due to neutral currents, nucleon axial and electron vector ones:

$$H^{nc} = - \delta(\vec{r}) \frac{G}{\sqrt{2}} K_2 \frac{(\varkappa - 1/2)\vec{I}\vec{\alpha}}{I(I+1)} \qquad (23)$$

Here K_2 is a dimensionless constant of this interaction. In the Weinberg-Salam model

$$K_{2p} = -K_{2n} = - \frac{1}{2} (1 - 4\sin^2\theta) \cdot 1.25 \approx -0.05 \qquad (24)$$

The Hamiltonian (22) can be rewritten in analogous way

$$H^{em} = \delta(\vec{r}) \frac{G}{\sqrt{2}} K \frac{\varkappa \vec{I}\cdot\vec{\alpha}}{I(I+1)} \qquad (25)$$

To estimate the dimensionless constant K we shall retain in it the term with μ only, taking into account that $\mu_n \gg 1/3$

$$K = 2\sqrt{2}\, \pi\alpha\mu m \cdot 10^5 \sum_k \eta_k r_{0k} \qquad (26)$$

Assuming that $r_{0k} \sim A^{1/3}/m_\pi$ and $\eta_k \sim 10^{-7} - 10^{-6}$ we find

$$K \sim 0.1 - 1 \qquad (27)$$

Therefore, this constant can turn out much larger than the constant K_2 in the Weinberg-Salam model.

We do not take into account here the contribution to the nuclear anapole moment from the proper anapole moment of nucleon caused by weak interaction radiative corrections. This contribution constitutes about 10^{-2}.

Atomic calculations with the electromagnetic Hamiltonian (25) do not differ in any way from those with the Hamiltonian (23) carried out in the ref.[3] 1). For the transition in bismuth with the wave-length λ = 648 nm the difference in the degree of circular polarization of radiation for different hyperfine components of the line reaches 4% at K = 1. It is sufficiently close to the accuracy 14% attained already now in this experiment [2]. The unique possibility of measurement of anapole moments of various nuclei could be given in principle by study of optical activity of diatomic molecules gas which is caused just by the interactions (22) and (23).

Therefore, atomic and molecular experiments are possible sources of new information about parity violation in nuclei.

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1) We note only that an accurate account of nucleus finite size leads in atomic problem to the corrections that do not exceed $Z^2\alpha^2/2$.

4. Separately we would like to dwell on the calculation of the deuteron anapole moment since in the approximation of zero-range nuclear forces it can be expressed through the parameters of parity violation in np scattering.

In this approximation the deuteron wave-function with the account of P-odd effects was found in elegant paper [15]:

$$\psi(\vec{r}) = \sqrt{\frac{\varkappa}{2\pi}}\left\{1+[c(\vec{\sigma}_p+\vec{\sigma}_n)+\lambda_t(\vec{\sigma}_p-\vec{\sigma}_n)](-i\vec{\nabla})\right\}\frac{e^{-\varkappa r}}{r}\chi \qquad (28)$$

Here $\vec{r}=\vec{r}_p-\vec{r}_n$, $\varkappa=\sqrt{m\varepsilon_d}$, ε_d is the deuteron binding energy, χ is the two-nucleon triplet spin function. The constant c determines the P-odd part of np scattering amplitude that changes the isotopic spin T of the system, but conserves the usual spin S. The constant λ_t corresponds, on the contrary, to the conservation of T and change of S.

The current densities of proton and neutron are calculated according to usual formulae by means of the wave-function (28). Substituting the expressions obtained in this way into the formula (8), we come to the following result for the deuteron anapole moment:

$$\vec{a} = \frac{\pi e}{m}\vec{I}\left\{c(\mu_p-\mu_n-\frac{1}{3})+\lambda_t(\mu_p+\mu_n)\right\} \qquad (29)$$

The contribution of the contact current to the anapole, as well as structural electromagnetic vertex, can be neglected in the zero-range approximation. Indeed, since the range of weak interaction does not of course exceed the nuclear forces range r_0, all such terms should be proportional to the small factor $|\psi(r_0)|^2 r_0^3 \sim \varkappa r_0$. The absence of the mentioned smallness in the formula (29) is caused by the fact that the corresponding vector-potential is described in the language of perturbation theory by pole diagrams with small energy denominators. Therefore, the result (29) is in fact a peculiar low-energy theorem. Unfortunately, the real accuracy of the used approximation is not high. The parameter $\varkappa r_0$ in deuteron is known to be about $^1/3$.

It follows from the analysis presented in the section 2 that deuteron, even in general case (i.e., without the assumption that it is point-like), has only one P-odd form-factor. We take the opportunity to note that two P-odd structures in the deuteron electromagnetic vertex written down in the ref.[6] (see also [16]) can be reduced one to another by means of the identity

$$\xi^*_\mu\xi_\nu - \xi^*_\nu\xi_\mu = -\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon_{\alpha\beta\chi\lambda}\,\xi^*_\chi\xi_\lambda$$

We shall write the P-odd electromagnetic interaction of electron with deuteron as

$$H^{em} = \delta(\vec{r})\frac{G}{\sqrt{2}}K\vec{I}\vec{\alpha} \qquad (30)$$

(of course, in the case of deuterium it is quite sufficient to use non-relativistic approximation for the operator $\vec{\alpha}$). Here even at the optimistic estimate of the np scattering parameters $c\sim\lambda\sim 10^{-6}m^{-1}$ the dimensionless constant $K\sim 10^{-2}$. Nevertheless, the anapole contribution to the P-odd effects, dependent on the nuclear spin, can turn out essential in the atom of deuterium. The point is that in the Weinberg-Salam model the axial hadronic current is an isovector and hence in the isoscalar deuteron it is absent. Therefore, the discussed effects in deuterium are caused, besides the calculated deuteron anapole moment, by the isoscalar part of radiative corrections only.

Acknowledgements

The authors are sincerely grateful to L.M.Barkov, A.I.Vainshtein, M.S.Zolotorev and O.P.Sushkov for valuable discussions.

R e f e r e n c e s

1. L.M.Barkov, M.S.Zolotorev, Pis'ma Zh. Eksp. Teor. Fiz. /JETP Lett./ 26, 379, 1978; 27, 544, 1978.
2. L.M.Barkov, M.S.Zolotorev, Phys. Lett 85B, 308, 1979.
3. V.N.Novikov, O.P.Sushkov, V.V.Flambaum, I.B.Khriplovich, Zh. Eksp. Teor. Fiz. 73, 802, 1977 /Sov. Phys. - JETP, 46, 420, 1977/.
4. O.P.Sushkov, V.V.Flambaum, Zh. Eksp. Teor. Fiz. 75, 1208, 1978.
5. D.P.Grechukhin, A.A.Soldatov, Zh. Eksp. Teor. Fiz. 73, 31, 1977.
6. E.M.Henley, W.-J.P.Hwang, G.N.Epstein, preprint RLO-1388-797, University of Washington, 1979. Phys.Lett.,88B,349,1979.
7. Ya.B.Zel'dovich, Zh. Eksp. Teor. Fiz. /JETP /33, 1531, 1957.
8. L.M.Barkov, I.B.Khriplovich, M.S.Zolotorev, Comments Atom. Mol. Phys. 8, 79, 1979.
9. L.Durand III, P.C. De Celles, R.B.Marr, Phys. Rev., 126, 1883, 1962.
10. A.A.Cheshkov, Yu.M.Shirokov, Zh. Eksp. Teor. Fiz. /JETP/ 44, 1982, 1963.
11. A.A.Cheshkov, Zh. Eksp. Teor. Fiz. /JETP/ 50, 144, 1966.
12. I.Yu.Kobzarev, L.B.Okun', M.V.Terent'ev. Pis'ma Zh. Eksp. Teor. Fiz. /JETP Lett./ 2, 466, 1965.
13. A.D.Dolgov, Pis'ma Zh. Eksp. Teor. Fiz. /JETP Lett./ 2, 494, 1965.
14. V.M.Dubovik, A.A.Cheshkov, Zh. Eksp. Teor. Fiz. /JETP/ 51, 1369, 1966.
15. G.S.Danilov, Phys. Lett. 18, 40, 1965.
16. W.-J.P.Hwang, E.M.Henley, preprint RLO-1388-804, University of Washington, 1979.

Работа поступила - I7 апреля I980 г.

Ответственный за выпуск - С.Г.Попов
Подписано к печати 28.IУ-I980 г. МН 07II7
Усл. 0,9 печ.л., 0,8 учетно-изд.л.
Тираж 250 экз. Бесплатно
Заказ № I36.

Отпечатано на ротапринте ИЯФ СО АН СССР