СИБИРСКОЕ ОТДЕЛЕНИЕ АН СССР
ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ



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# EFFECTS OF PARITY NONCONSERVATION IN THE COLLECTIVE MOTION IN A NUCLEUS



ПРЕПРИНТ 80-214

Новосибирск

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ABSTRACT

The mixing of the collective states of opposite parity due to a weak interaction is discussed. The existence of a regular mechanism of enhancement of the weak interaction is revealed. The value of the enhancement is proportional to a power of the collectivity of the states under discussion. For the giant resonanses $1^-$ and $1^+$ the coefficient of enhancement is of the order of $\sim A^{1/2}$.

## 1. INTRODUCTION

The mean scale of the weak interaction in nuclei is of the order of $\sim 10^{-7}$ compared to the scale of a nuclear shell structure, $\mathcal{E}_F/A^{1/3}$. But in many cases the real effects of parity violation (PV) in nuclei are greater then this simple estimate, up to one, two and sometimes three order of magnitude (Ref./1-3/). The various mechanisms of enhancement of the PV-effects in nuclei are discussed in Ref./4,5/. Here we would like to pay attention to the possibility of strengthening of the PV-effects due to the existing collective motion in nuclei. Since the probabilities of electromagnetic transitions from collective states are enhanced, one should expect the similar enhancement in the vector part of weak interaction in a nucleus. This mechanism has an obvious feature of universality through the Periodic Table, although the magnitude of enhancement may be not large.

For estimation of the PV-effects one has to calculate the mixing coefficient

$$\alpha_{mn} = \frac{\langle m|W|n\rangle}{E_n - E_m} \qquad (1)$$

between the states of opposite parity, where W is the weak interaction, $E_n$ and $|n\rangle$ are the energy and the state of a nucleus. In random-phase approximation (RPA) the matrix element of W has two kinds of terms. One of them represents a direct PV-interaction between the collective states, the other one corresponds to a PV-interaction of a single particle with a core. In other words, a collective state in RPA is a coherent superposition of particle-hole states, and the PV-effects can arise not only from direct PV-interaction between particle-hole pairs, but from PV-effects in single-particle states too. As it will be seen below, these terms have the same value as the direct interaction.

## 2. Basic Equations.

Let us discuss now the properties of collective states of a system with a Hamiltonian which is disturbed by a small two-particle interaction. For description of the collective phenomena in a nucleus we shall use the formalism of the Generalized Density Matrix (GDM) proposed by Belyaev and Zelevinsky, Ref/6/. In this formalism the dynamical equation for unperturbed density matrix $\hat{R}$ is

$$[\hat{R}, \hat{S}+\hat{H}] = 0, \tag{2}$$

where $\hat{S}$ is the generalized nuclear field including the independent particle energy $\hat{\mathcal{E}}$ and the self-consistent part $\tilde{S}$ dependent upon the effective two-particle interaction V(a,b)

$$\hat{S}(a) = \hat{\mathcal{E}}(a) + \hat{\tilde{S}}(a) = \hat{\mathcal{E}}(a) + Tr_{b}\{V(a,b)\hat{R}(b)\}, \tag{3}$$

$\hat{H}$ is the collective Hamiltonian in the space of considered collective states,

$$\hat{H} = Tr_{a}\{\mathcal{E}(a)\hat{R}(a)\} + \frac{1}{2}Tr_{ab}\{\hat{R}(a)V(a,b)R(b)\} = \tag{4}$$

$$= Tr\{\hat{S}\hat{R}\} - \frac{1}{2}Tr\{\hat{\tilde{S}}\cdot\hat{R}\}. \tag{4'}$$

The equation (2) is understood as the set of equations for matrix elements of $\hat{R}$, $\hat{S}$ and $\hat{H}$ inside the considered collective band. In such a sense the $\hat{R}$ and $\hat{S}$ introduced here are the operators in the extended space of the single-particle and collective variables.

Let us find the linear respons of a system to a small perturbation of the effective interaction

$$V(a,b) \longrightarrow V(a,b) + W(a,b).$$

The linearized equation for the correction $\delta\hat{R}$ to density matrix is

$$[\delta\hat{R}, \hat{S}+\hat{H}] + [\hat{R}, \delta\hat{S}+\delta\hat{H}] = 0, \tag{5}$$

where the corrections to nuclear field $\delta\hat{S}$, collective Hamiltonian $\delta\hat{H}$ and to $\delta\hat{R}$ are related, due to the consistensy conditions (3) and (4).



$$\delta\hat{S}(a) = \delta\hat{\tilde{S}}(a) = Tr_{b}\{V(a,b)\delta R_{b}\} + Tr_{b}\{W(a,b)\hat{R}(b)\}, \tag{6}$$

$$\delta\hat{H} = Tr\{\hat{S}\cdot\delta\hat{R}\} + \frac{1}{2}Tr_{ab}\{\hat{R}(a)W(a,b)\hat{R}(b)\}. \tag{7}$$

To solve the equations (5-7) we need to define more precisely the structure of the unperturbed system. Further, we shall discuss the situation where the collective motion may be described by a set of harmonic vibrations in RPA.

## 3.Random-Phase Approximation in GDM.

In the frame of GDM the unperturbed quantities $\hat{R}$, $\hat{S}$ and $\hat{H}$ can be expanded in the infinite series on powers of $\hat{A}_k^+$ and $\hat{A}_k$ which are the creation and annihilation operators of normal modes (see Ref./6/). RPA corresponds to the first order in the series of $\hat{R}$ and $\hat{S}$ and to the second order in $\hat{H}$.

$$\hat{R} = \bar{R} + \sum_k (R_k\hat{A}_k^+ + R_k^+\hat{A}_k) + \dots, \tag{8}$$

$$\hat{S} = \bar{S} + \sum_k (S_k\hat{A}_k^+ + S_k^+\hat{A}_k) + \dots, \tag{8'}$$

$$\hat{H} = \sum_k \omega_k\hat{A}_k^+\hat{A}_k + \dots . \tag{8''}$$

$\bar{S}$ and $\bar{R}$ are static parts which, diagonalized simultaneously, determine the single-particle spectrum $E_1$ and the occupation numbers $n_1$. At this order we get the basis $|1)$ of the independent particle model,

$$\bar{R}_{12} = n_1\cdot\delta_{12}, \quad \bar{S}_{12} = E_1\cdot\delta_{12}. \tag{9}$$

The states $|1)$ have a spinor structure which allows the effects of Cooper - pairing to be included automatically. The precise spinor structure of operators and single-particle states can be found in Ref./6/. The next terms in the series (8) correspond to RPA unharmonic corrections and may be omitted if we are not interested in the region of transitional nuclei.

The quantities $R_k$ satisfy the equation

$$[R_K, \bar{S}] + [\bar{R}, S_K] = \omega_K R_K \tag{10}$$

determining, together with the consistency condition,

$$S_K(a) = Tr_\delta \{V(a,\delta) R_K(\delta)\}, \tag{10'}$$

the frequancies $\omega_K$ of normal vibrations.
The normalization of $R_k$ is determined by Eqs.(7) and (8'').

For a further convenience let us introduce a complete set $\varphi_\alpha(1,2)$ of the solutions of RPA equation (10). This set will be used as a basis in the space of two-quasiparticle states. The equation for $\varphi_\alpha(1,2)$ is

$$\hat{L}\varphi_\alpha \equiv [\varphi_\alpha, \bar{S}] + [\bar{R}, Tr_\delta\{V(a,\delta)\varphi_\alpha(\delta)\}] = \omega_\alpha \varphi_\alpha, \tag{11}$$

and

$$\varphi_\alpha(\omega_\alpha) = \varphi_\alpha^+(-\omega_\alpha).$$

The RPA operator $\hat{L}$ is not Hermitian, and an equation for conjugate functions $\chi_\beta^{(1,2)}$ is

$$(L^+)^T\chi_\beta \equiv [\chi_\beta, \bar{S}] + Tr_\delta\{V(a,\delta)[\bar{R}, \chi_\beta(\delta)]\} = \omega_\beta \chi_\beta \tag{11'}$$

$(\hat{L})^T$ denotes a transposed operator.

The functions $\chi_\beta$ and $\varphi_\alpha$ form a biorthogonal set $\{\alpha\}$

$$Tr\{\chi_\beta^+ \varphi_\alpha\} = \delta_{\alpha\beta} \tag{12}$$

The set $\{\alpha\}$ can be divided into two classes. The first one, containing the physical modes $\varphi_\alpha$, describes the transitions through the Fermi surface only, with $\omega_\alpha > 0$ and $\varphi_\alpha = [\bar{R}, \chi_\alpha]$. The second class includes the modes $\varphi_{\kappa\kappa'}$ describing the "parallel" transitions on the same side of the Fermi surface, with $\omega_{\kappa\kappa'} = E_\kappa - E_{\kappa'}$ and



$$\chi_{\kappa\kappa'}(1,2) = \delta_{\kappa 1}\delta_{\kappa' 2} \tag{13}$$

These modes correspond to transitions of an odd particle.

Now, let us discuss the solution of the basic equations (5-7). In RPA, the correction to collective Hamiltonian $\delta\hat{H}$ can be written as follows

$$\delta\hat{H} = \sum_\alpha h_\alpha \hat{A}_\alpha^+ + h_\alpha^* \hat{A}_\alpha + \sum_{\alpha\beta}\left(\tfrac{1}{2} g_{\alpha\beta} \hat{A}_\alpha^+ \hat{A}_\beta^+ + \tfrac{1}{2} g_{\alpha\beta}^* \hat{A}_\alpha \hat{A}_\beta + h_{\alpha\beta} \hat{A}_\alpha^+ \hat{A}_\beta\right), \tag{14}$$

where $\hat{A}_\alpha^+$ is the phonon creation operator of the type $\alpha$ phonon. The coefficients $g_{\alpha\beta}$ and $h_{\alpha\beta}$ have the obvious symmetry properties,

$$g_{\alpha\beta} = g_{\beta\alpha}, \quad h_{\alpha\beta} = h_{\beta\alpha}^* \tag{14'}$$

The correction to density matrix $\delta\hat{R}$ and to self-consistent field $\delta\hat{S}$ should be found in the form

$$\begin{aligned} \delta\hat{R} &= \rho + \sum_\alpha (\rho_\alpha \hat{A}_\alpha^+ + \rho_\alpha^+ \hat{A}_\alpha), \\ \delta\hat{S} &= \sigma + \sum_\alpha (\sigma_\alpha \hat{A}_\alpha^+ + \sigma_\alpha^+ \hat{A}_\alpha), \end{aligned} \tag{15}$$

The $\sigma$ is the correction to single-particle shell model potential. The $\rho$ is the static distortion of the density matrix. The quantities $\rho_\alpha$ and $\sigma_\alpha$ determine the corrections to phonon amplitudes.

For $\sigma$ and $\sigma_\alpha$ we have, from the consistency condition (6)

$$\sigma(a) = Tr_\delta\{V(a,\delta)\rho(\delta)\} + Tr_\delta\{W(a,\delta)\bar{R}(\delta)\}, \tag{16}$$

$$\sigma_\alpha(a) = Tr_\delta\{V(a,\delta)\rho_\alpha(\delta)\} + Tr_\delta\{W(a,\delta)R_\alpha(\delta)\}. \tag{16'}$$

The similar conditions arise for the h and g coefficients of collective Hamiltonian $\delta\hat{H}$. For example, the coefficients $h_\alpha$ are (from (7))

$$h_\alpha = \mathrm{Tr}\{\bar{S}\rho\} + \mathrm{Tr}\{\sigma R_\alpha\} \, . \tag{17}$$

Inserting the expressions (14,15) into Eq.(5) we find the equations for $\rho$'s and $\sigma$'s ,

$$[\rho, \bar{S}] + [\bar{R}, \sigma] + \sum_\alpha (R_\alpha^+ h_\alpha - R_\alpha h_\alpha^*) = 0, \tag{18}$$

$$[\rho_\alpha, \bar{S}] + [\bar{R}, \sigma_\alpha] + [R_\alpha, \sigma] + [\rho, S_\alpha] - \omega_\alpha \rho_\alpha +$$
$$+ \sum_\beta (R_{\alpha\beta} h_\beta - 2\tilde{R}_{\alpha\beta} h_\beta^* + g_{\alpha\beta} R_\beta^+ - h_{\alpha\beta} R_\beta) = 0. \tag{19}$$

In Eq.(19) for $\rho_\alpha$ the second order density matrix $R^{(2)}$ has arisen

$$\hat{R}^{(2)} = \sum_{\alpha\beta} (\tilde{R}_{\alpha\beta} \hat{A}_\alpha^+ \hat{A}_\beta^+ + \tilde{R}_{\alpha\beta}^+ \hat{A}_\alpha \hat{A}_\beta + R_{\alpha\beta} \hat{A}_\alpha^+ \hat{A}_\beta) \, . \tag{20}$$

But, as we shall see, there is no need in the explicit expression for $\tilde{R}_{\alpha\beta}$ and $R_{\alpha\beta}$ for our purpose.

It is interesting to note, that the equations (18,19) and the conditions (16,17) do not determine the coefficients $h_\alpha$ and $h_{\alpha\beta}$ , $g_{\alpha\beta}$ uniquely. To demonstrate this, one should multiply Eq.(19) by $\bar{S}$ and take the Trace. Since $\mathrm{Tr}\{\bar{S}[\rho_\alpha \bar{S}]\} = 0$ and $\mathrm{Tr}\{\bar{S}[\bar{R}, \sigma_\alpha]\} = 0$ we find from (17)

$$\omega_\alpha h_\alpha = \omega_\alpha \mathrm{Tr}\{\bar{S}\rho_\alpha\} + \omega_\alpha \mathrm{Tr}\{\sigma R_\alpha\} = \sum_\beta \Big( h_\beta \mathrm{Tr}\{\bar{S} \cdot R_{\alpha\beta}\} -$$
$$- 2 h_\beta^* \mathrm{Tr}\{\bar{S}\tilde{R}_{\alpha\beta}\}\Big) + \mathrm{Tr}\{\bar{S}[R_\alpha, \sigma] + \bar{S}[\rho, S_\alpha]\} + \omega_\alpha \mathrm{Tr}\{\sigma R_\alpha\}.$$

Using the identity $\mathrm{Tr}\{A[B,C]\} = \mathrm{Tr}\{[A,B]C\}$ and Eqs.(10), (18), we find, after some transformations,

$$\omega_\alpha h_\alpha = \sum_\beta h_\beta \{\mathrm{Tr}\{R_\beta^+ S_\alpha\} + \mathrm{Tr}\{\bar{S} R_{\alpha\beta}\}\} - h_\beta^* \{\mathrm{Tr}\{R_\beta S_\alpha\} + 2\mathrm{Tr}\{\bar{S}\tilde{R}_{\alpha\beta}\}\}. \tag{21}$$

But, due to consistency conditions (4) for unperturbed collective Hamiltonian



$$\mathrm{Tr}\{R_\beta S_\alpha\} + 2\mathrm{Tr}\{\bar{S}\tilde{R}_{\alpha\beta}\} = 0,$$

$$\mathrm{Tr}\{R_\beta^+ S_\alpha\} + \mathrm{Tr}\{\bar{S} R_{\alpha\beta}\} = \omega_\alpha \delta_{\alpha\beta} \, . \tag{22}$$

The quantities $h_\alpha$ are therefore arbitrary. It can be shown in a similar way that the coefficients $g_{\alpha\beta}$ and $h_{\alpha\beta}$ (for $\alpha \neq \beta$ ) are as such arbitrary just as $h_\alpha$ are. The arbitrary character of these coefficients has a simple explanation. It reflects the invariance of the dynamical equation (2) under unitary transformations in the collective space. The coefficients of collective Hamiltonian depend obviously on the parameters of a unitary transformation. A definite set of $h_\alpha$ and $g_{\alpha\beta}$ , $h_{\alpha\beta}$ corresponds to some definite basis $\{\alpha\}$ in the collective space. For our purpose, a naturel basis is the basis of unperturbed states. In this case, the correction $\delta\hat{R}$ to the density matrix is equal to zero, becuase

$$\langle\beta| R_{12} |\alpha\rangle = \langle\beta| a_1 a_2^+ |\alpha\rangle,$$

by the definition and the states $|\alpha\rangle, |\beta\rangle$ are unperturbed ones. The equations of motion (18,19) become thus simpler

$$[\bar{R}, w] + \sum_\alpha (R_\alpha^+ h_\alpha - R_\alpha h_\alpha^*) = 0, \tag{23}$$

$$[\bar{R}, w_\alpha] + [R_\alpha, w] + \sum_\beta (\varphi_{\alpha\beta} R_\beta^+ - h_{\alpha\beta} R_\beta + R_{\alpha\beta} h_\beta - 2\tilde{R}_{\alpha\beta} h_\beta^*) = 0, \tag{24}$$

where $\quad w(a) = \mathrm{Tr}_b\{W(a,b)\bar{R}(b)\}\, ; \; w_\alpha(a) = \mathrm{Tr}_b\{W(a,b) R_\alpha(b)\}.$

From these equations we get immediately,

$$h_\alpha = - \mathrm{Tr}\{[\bar{R}, w]\chi_\alpha\},$$

or, using $\quad \varphi_\alpha = [\bar{R}, \chi_\alpha]$

$$h_\alpha = \mathrm{Tr}\{w \cdot \varphi_\alpha\} = \mathrm{Tr}_{ab}\{\varphi_\alpha(a) W(a,b) \bar{R}(b)\} \, . \tag{25}$$

For $g_{\alpha\beta}$ and $h_{\alpha\beta}$ we get

$$g_{\alpha\beta} = \mathrm{Tr}_{a,b}\{\varphi_\alpha^{*}(a)W(a,b)\varphi_\beta(b)\} - \mathrm{Tr}\{[\varphi_\alpha, w]\chi_\beta\} + \sum_\gamma \left(2h_\gamma^{*}\mathrm{Tr}\{\tilde{R}_{\alpha\gamma}\chi_\beta\} - h_\gamma \mathrm{Tr}\{R_{\alpha\gamma}\chi_\beta\}\right), \tag{26}$$

$$h_{\alpha\beta} = \mathrm{Tr}_{a,b}\{\varphi_\beta^{+}(a)W(a,b)\varphi_\alpha(b)\} + \mathrm{Tr}\{\chi_\beta^{+}[\varphi_\alpha, w]\} + \sum_\gamma \left(\mathrm{Tr}\{\chi_\beta^{+}R_{\alpha\gamma}\}h_\gamma - 2\mathrm{Tr}\{\chi_\beta^{+}\tilde{R}_{\alpha\gamma}\}h_\gamma^{*}\right). \tag{27}$$

These expressions give the solution of our problem.

## 4. The Model of Contact Interaction.

Now, let us estimate, with the help of (25-27), the order of the PV-effects in collective states. For this purpose we shall take the weak interaction W(a,b) in a simple contact form.

$$W(a,b) = \frac{G}{2\sqrt{2}\,m}\Big\{ (\hat{\vec{\sigma}}_a - \hat{\vec{\sigma}}_b)\cdot[(\vec{p}_a - \vec{p}_b), \delta(\vec{r}_a - \vec{r}_b)]_+ + (1+\mu_p-\mu_n)\cdot i(\hat{\vec{\sigma}}_a \times \hat{\vec{\sigma}}_b)\cdot$$
$$\cdot [(\hat{\vec{p}}_a - \hat{\vec{p}}_b), \delta(\vec{r}_a - \vec{r}_b)]_- \Big\} \hat{T}_{ab}^{(+)}, \tag{28}$$

where $\mu_p$ and $\mu_n$ are the magnetic moments of the proton and neutron,

$$\hat{T}_{ab}^{(+)} = \frac{1}{4}\left(\hat{\tau}_a^{+}\cdot\hat{\tau}_b^{-} + \hat{\tau}_a^{-}\cdot\hat{\tau}_b^{+}\right), \tag{29}$$

$\hat{\vec{\sigma}}$ and $\vec{\tau}$ are the spin and isospin Pauli matricies, and $[\hat{A},\hat{B}]_\pm = \hat{A}\cdot\hat{B} \pm \hat{B}\cdot\hat{A}$. We shall use the exchange part of W(a,b) only

$$W(a,b) = \frac{G}{2\sqrt{2}\,m}\Big\{ i(\hat{\vec{\sigma}}_a \times \hat{\vec{\sigma}}_b)\cdot[(\hat{\vec{p}}_a - \hat{\vec{p}}_b), \delta(\vec{r}_a - \vec{r}_b)]_- + (1+\mu_p-\mu_n)\cdot$$
$$\cdot(\hat{\vec{\sigma}}_a - \hat{\vec{\sigma}}_b)\cdot[(\hat{\vec{p}}_a - \hat{\vec{p}}_b), \delta(\vec{r}_a - \vec{r}_b)]_+ \Big\} \frac{1}{2}\left(1 - \hat{\tau}_a^3\,\hat{\tau}_b^3\right). \tag{30}$$

We shall discuss below the mixing of $1^-$ and $1^+$ giant resonances in an even nuclus. For simplicity, the resonance $1^-$ will be described in a "hydrodynamical" approximation following from RPA in the limit of large frequances and short-range interaction (Ref./7/).

Let us introduce, according to Ref./7/, the quantities $R_k^{(-)}(\vec{r}) = \sum_{1,2} R_k^{(-)}(1,2)\psi_1(\vec{r})\psi_2^{*}(\vec{r})$ and $S_k^{(-)}(r) = \mathrm{Tr}_{r'}\{V(\vec{r}-\vec{r}')R_k^{(-)}(\vec{r}')\} =$



$= G_0\tau^3\mathrm{Tr}\{\tau^3 R_k^{(-)}(r)\}$ where $\psi_i(\vec{r})$ is the wave function of a state $|1)$, and $V(\vec{r}_a - \vec{r}_b) = \vec{\tau}_a\vec{\tau}_b G_0\delta(\vec{r}_a - \vec{r}_b)$. For an isovector mode, $R_k^{(-)}(\vec{r}) = \tau^3 A_k(\vec{r})$ therefore,

$$S_k^{(-)}(\vec{r}) = 2\tau^3 G_0 A_k(\vec{r}) \equiv \tau^3 B_k(\vec{r}).$$

Expanding Eq.(10) on powers of $[\ ,\bar{S}]/\omega_-$, we find, for $B_k(\vec{r})$, the equation

$$\omega_-^2 B_k(\vec{r}) + C_s^2 \vec{\nabla}\left(n(\vec{r})\vec{\nabla}B_k(\vec{r})\right) = 0, \tag{31}$$

where $C_s^2 = \frac{2G_0\rho_0}{m}$ and $n(\vec{r}) = \rho(\vec{r})/\rho_0$, $\rho_0$ is the density in the centr of a nucleus.

Taking the density to be constant inside a nucleus and neglecting the diffusiveness of a nuclear surface, we obtain the equation

$$(\omega_-^2 + C_s^2\Delta)B_k(\vec{r}) = 0, \tag{32}$$

with the boundary condition

$$\left.\frac{dB_k(\vec{r})}{dr}\right|_{r=R} = 0. \tag{32'}$$

The solution of the Eq.(32), with the boundary condition (32') is

$$B_k(\vec{r}) = b\, j_1(\varkappa r)\frac{r_k}{r}, \tag{33}$$

where $\varkappa = \omega_-/c_s$ is determined by (32'), $\varkappa\cdot R = 2.08$.
The coefficient b is determined by the normalization condition (Ref./7/)

$$\frac{1}{m}\int d^3r\, \mathrm{Tr}\left\{\rho(\vec{r})\left(\vec{\nabla}S_k^{(-)+}(\vec{r})\cdot\vec{\nabla}S_{k'}^{(-)}(\vec{r})\right)\right\} = \frac{\omega_-^3}{2}\delta_{kk'} \tag{34}$$

and substituting $S_k^{(-)}(\vec{r})$ into Eq.(34), we get

$$b^2\frac{4\pi}{3}\frac{\rho_0 R}{m}\left(1 + \frac{\sin(2\varkappa R)}{2\varkappa R} - 2\frac{\sin^2(\varkappa R)}{\varkappa^2 R^2}\right) = \frac{\omega_-^3}{2}. \tag{35}$$

Using the value of $\varkappa R = 2.08$, we find

$$0.44\cdot\frac{8\pi}{3}\frac{\rho_0 R}{m}b^2=\omega_-^3 \tag{36}$$

The resonance $1^+$ will be discussed in the frame of the model with factorized interaction.

$$V(a,b)=(\vec\sigma_a\cdot\vec\sigma_b)\left[g_s+g_v(\vec\tau_a\cdot\vec\tau_b)\right]. \tag{37}$$

In this case, the main transitions are those between the components of a spin-orbit doublet. For simplicity we shall restrict ourselves to the case of a single doublet with a filled lower level. For the same reason, we shall discuss only the nuclei with $N = Z$, where the isoscalar and isovector resonances are not mixed.

The effective field for an isovector resonance is

$$S_\kappa^{(+)}=g_v\tau^3\sigma_\ell\,\alpha_{\kappa\ell}\ , \tag{38}$$

where

$$\alpha_{\kappa\ell}=Tr\{\sigma_\kappa\tau^3\varphi_\ell\}. \tag{39}$$

From (39) we obtain, taking into account that $\alpha_{\kappa\ell}=\alpha\cdot\delta_{\kappa\ell}$ for a spherical nucleus, the dispertion equation for the resonance frequancy $\omega_+$

$$1=2g_v\sum_{1,2}\frac{(n_1-n_2)\left|(\sigma^3)_{12}\right|^2}{E_1-E_2+\omega_+}. \tag{40}$$

For $R_k^{(+)}(1,2)$ we have an expression

$$R_k^{(+)}(1,2)=g_v\frac{(n_1-n_2)(\tau^3\sigma_\kappa)_{12}}{E_1-E_2+\omega_+}\,a, \tag{41}$$

$a$ is determined by the normalization condition

$$g_v^2a^2\sum_{1,2}\frac{(n_1-n_2)\left|(\tau^3\sigma^3)_{12}\right|^2}{(E_1-E_2+\omega_+)^2}=1. \tag{42}$$



Let us now estimate, using obtained expressions for $R_k^{(-)}$ and $R_k^{(+)}$, the coefficients $h_{kl}$ and $g_{kl}$ of a collective Hamiltonian. The coefficients $h_\alpha$ are equal to zero for the states $1^-$ and $1^+$, so the expressions (26,27) become much simpler

$$g_{\kappa\ell}=Tr_{ab}\{\varphi_\kappa^{(+)}(a)W(a,b)\varphi_\ell^{(-)}(b)\}-Tr\{[\varphi_\kappa^{(+)},w]\chi_\ell^{(-)}\}, \tag{43}$$

$$h_{\kappa\ell}=Tr_{ab}\{\varphi_\ell^{(-)+}(a)W(a,b)\varphi_\kappa^{(+)}(b)\}+Tr\{\chi_\ell^{(-)+}[\varphi_\kappa^{(+)},w]\}. \tag{43'}$$

It is enough to estimate $h_{kl}$ for our purpose, becuase both expressions (43,43') have a similar structure.

Let us rewrite $h_{kl}$ as follows:

$$h_{\kappa\ell}=Tr\{w_\ell^{(-)+}\varphi_\kappa^{(+)}\}+Tr\{[w,\chi_\ell^{(-)+}]\varphi_\kappa^{(+)}\}, \tag{44}$$

where $w_\ell^{(-)}(a)=Tr_b\{\varphi_\ell^{(-)}(b)W(a,b)\}$.

From the selection rules one can see that $w_\ell^{(-)}(a)$ is determined by the following part of the interaction:

$$\widetilde W(a,b)=\frac{G(1+\mu_p-\mu_n)}{4m\sqrt2}\vec\sigma_a\cdot\left[\vec P_b,\delta(\vec r_a-\vec r_b)\right]_+\tau_a^3\tau_b^3. \tag{45}$$

Then,

$$w_\ell^{(-)}(a)=\frac{G(1+\mu_p-\mu_n)}{4m\sqrt2}\sigma_a^\kappa\tau_a^3\int d^3r_b\,Tr\{\tau^3[P_b^\kappa,\delta(\vec r_a-\vec r_b)]_+\varphi_\ell^{(-)}(\vec r_b)\},$$

or

$$w_\ell^{(-)}=\frac{G(1+\mu_p-\mu_n)}{2\sqrt2}\sigma^\kappa\tau^3\hat j_\ell^\kappa(\vec r) \tag{46}$$

where

$$\hat j_\ell^\kappa(\vec r)=\frac{1}{2m}\int d^3r'\,Tr\{\tau^3[P'^\kappa,\delta(\vec r-\vec r')]_+\varphi_\ell^{(-)}(\vec r')\} \tag{47}$$

is the off-diagonal matrix element of current density, corresponding to the resonance $1^-$. In our approximation

$$\hat{j}^{\kappa}_{\ell}(\vec{r}) = -i\,\frac{\rho(\vec{r})}{m\omega_-}\,Tr\{\tau^3\nabla_\kappa S^{(-)}_{\ell}(\vec{r})\} \tag{48}$$

or, substituting $S^{(-)}_{k}(\vec{r})$

$$\hat{j}^{\kappa}_{\ell}(\vec{r}) = -i\,\frac{2\rho_0\varkappa b}{m\omega_-}\left(\frac{j_1(\varkappa r)}{\varkappa r}(\delta_{\kappa\ell}-\hat{n}_\kappa\hat{n}_\ell)+j_1'(\varkappa r)\hat{n}_\kappa\hat{n}_\ell\right). \tag{49}$$

For $w^{(-)}_{l}$ we have

$$w^{(-)}_{\ell} = -i\,\frac{G(1+\mu_p-\mu_n)}{\sqrt{2}\,m\omega_-}\varkappa b\rho_0\tau^3\left\{\frac{j_1(\varkappa r)}{\varkappa r}\sigma_\ell+\hat{n}_\ell(\vec{\sigma}\vec{n})\left(j_1'(\varkappa r)-\frac{j_1(\varkappa r)}{\varkappa r}\right)\right\} \tag{50}$$

In calculating $Tr\left(w^{(-)+}_{\ell}\varphi^{(+)}_{\kappa}\right)$, we shall use the fact of a slight dependence of radial wave functions over j inside a spin-orbit doublet, and the relation

$$\langle L-\tfrac{1}{2},LM'|n^{\ell}(\vec{\sigma}\vec{n})|L+\tfrac{1}{2},LM\rangle = \tfrac{1}{3}\langle L-\tfrac{1}{2},LM'|\sigma^{\ell}|L+\tfrac{1}{2},LM\rangle. \tag{51}$$

Taking $\varphi^{(+)}_{\kappa}$ from (41) and $w^{(-)}_{l}$ from (50) and using the dispertion equation (40), we find

$$Tr\{w^{(-)+}_{\ell}\varphi^{(+)}_{\kappa}\} = -i\,\frac{G(1+\mu_p-\mu_n)}{2\sqrt{2}\,m\omega_-}\rho_0\varkappa ba\,\langle L|\,j_1'(\varkappa r)+\frac{j_1(\varkappa r)}{\varkappa r}\,|L\rangle\,\delta_{\kappa\ell} \tag{52}$$

Now, let us find the second term in (44). The correction to a single-particle field is

$$w(a) = Tr_b\left(W_{ex}(a,b)\,\bar{R}(b)\right) = \frac{G(1+\mu_p-\mu_n)}{2\sqrt{2}\,m}\rho_0(\vec{\sigma}_a\vec{p}_a) \tag{53}$$

Since $\chi^{(-)}_{\ell}(\vec{r}) = \dfrac{S^{(-)}_{\ell}(\vec{r})}{\omega_-}$, we have

$$[w,\chi^{(-)+}_{\ell}] = \frac{G(1+\mu_p-\mu_n)}{2\sqrt{2}\,m\omega_-}\rho_0\left[(\vec{\sigma}\cdot\vec{p}),\,S^{(-)}_{\ell}(\vec{r})\right] =$$

$$= -i\,\frac{G(1+\mu_p-\mu_n)}{2\sqrt{2}\,m\omega_-}\rho_0\,(\vec{\sigma}\cdot\vec{\nabla})\,S^{(-)}_{\ell}(\vec{r})$$

Using (48), we obtain

$$[w,\chi^{(-)+}_{\ell}] = \frac{G(1+\mu_p-\mu_n)}{4\sqrt{2}}\tau^3\sigma^{\kappa}\hat{j}^{\kappa}_{\ell}(r) \tag{55}$$



Comparing (55) and (46), we see that in the case of a contact interaction the contribution to the mixing coefficient via distortion of a single-particle spectrum is only two times lesser then the contribution via the direct interaction of the collective states.

Now, we can estimate the value of a mixing coefficient

$$\alpha_{\kappa\ell} = \frac{h_{\kappa\ell}}{\omega_- - \omega_+} = \alpha\,\delta_{\kappa\ell}$$

With the expressions for a and b substituted into (52), we obtain

$$\alpha = -i\,\frac{G(1+\mu_p-\mu_n)}{\sqrt{2}\,(\omega_- - \omega_+)}\varkappa\left(\frac{1}{3.5\,\pi}\,\frac{\rho_0\omega_-}{mR}\,\frac{\Delta_{LS}}{\omega_+}\,\frac{L(L+1)}{2L+1}\right)^{1/2}\cdot \tag{56}$$
$$\cdot\langle L|\,j_1'(\varkappa r)+\frac{j_1(\varkappa r)}{\varkappa r}\,|L\rangle,$$

where $\Delta_{LS}$ is a spin-orbit splitting in a single-particle spectrum, R is a nuclear radius, L is an angular momentum of a splitted level. Taking into account $L \sim A^{1/3}$ and $\omega_-$, $\omega_+, \Delta_{LS} \sim A^{-1/3}$, we get a quantitative estimate

$$\alpha \sim (10^{-7} \div 10^{-6})\cdot A^{-1/6},$$

and the order of $h_{kl}$ is $\sim A^{-1/2}$.

The value of the mixing coefficient $\alpha$ is small, but for noncollective states $\alpha$ happens to be even lesser. For the noncollective states $h_{kl}$ is a matrix element of a perturbation over the particle-hole type states. This matrix element is of the order $\sim A^{-1}$, therefore the mixing of the collective states is enhanced by a factor of $\sim A^{1/2}$.

The auther is acknowledged to I.B. Khriplovitch, to V.V. Mazepus, to V.B.Telitsin and to V.G.Zelevinsky for numerous discussions of the questions concerned in the presented paper.

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Работа поступила - 28 октября 1980 г.

Ответственный за выпуск - С.Г.Попов
Подписано к печати I0.XI-I980 г. МН I35I5
Усл.0,9 печ.л., 0,6 учетно-изд.
Тираж I50 экз. Бесплатно
Заказ № 2I4.

Отпечатано на ротапринте ИЯФ СО АН СССР