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A b s t r a c t

The nonlinear stage of amplification of a signal in a free electron laser has been considered. The general formulation of the problem has been obtained. Two types of the input signal have been analysed in detail: 1) "white noise" and 2) two-monochromatic waves. For these cases the values of the output signal and the evolution of spectral curves have been obtained.

Recently the problem concerning the action of a free electron laser (FEL) at high gains $G \gtrsim 1$ /1-3/ is being under discussion. The interest to this problem is stimulated, in particular, by the experiments on amplification and generation of the coherent electromagnetic radiation at a passage of the relativistic electron beam through a periodical magnetic field /"magnetic lattice") /4-7/. Theoretical analysis of the action of the FEL at $G \ll 1$ (that corresponds to the experimental conditions /4,5/) has been carried out in the authors' paper /8/ (see also Refs. /2,9/). The action of the FEL at $G \gtrsim 1$ which is described in Ref /1/ is a direct generalization of the approach used in Ref. /8/.

As it follows Ref. /1/, in the case when the relativistic electron beam passes through the magnetic lattice of sufficient length, in the region of resonance frequencies the field strength of an electromagnetic wave ("signal") fastly increases, that is due to the particle bunching in the beam, the mechanism of which is, in some sense, identical to that which takes place in the traveling wave lamp. This increase continues up to a certain universal level at which the growth of the phase oscillations in the bunched electron beam ceases the growth of the signal. This nonlinear stage of development of the signal was studied /1/ for the case of exciting the monochromatic electromagnetic wave. If the signal is excited, for instance, from the spectrum of fluctuations, the wave interaction becomes significant in the nonlinear regime. The present paper is devoted to the evolution of the spectrum and to the growth of the signal when at the nonlinear stage the interaction of the waves with each other is significant.

Let us represent the field of the magnetic lattice (ondulator) as

$$A_0^\mu = \frac{mc^2\Omega_0}{2e\nu_0}\left(\epsilon^\mu e^{i\nu_0\frac{z}{c}} + \epsilon^{*\mu} e^{-i\nu_0\frac{z}{c}}\right) \quad (1)$$

where $\epsilon^\mu, \epsilon^{*\mu}$ are the helicity unit vectors, $\epsilon^\mu = n_1^\mu - i n_2^\mu$, $n_k^\mu = g_k^\mu$, g_k^μ are the components of the metric tensor,

$\epsilon^2 = \epsilon^{*2} = 0, \ \epsilon\epsilon^* = -2, \ \Omega_0 = \frac{eH_0}{mc}$, H_0 is the lattice field. The vector-potential of the wave is represented in the form

$$A^\mu_\omega = \frac{mc^2}{2e}\left(\epsilon^\mu a_\omega + \epsilon^{*\mu} a^*_\omega\right), \quad (2)$$

the fact that the vector-potential A^μ has only the transverse components means that the field of the spatial charge is neglected. It is true if the wavelength of Langmuir oscillations is long as compared to the characteristic growth length of the electromagnetic wave ℓ_g (see Ref. /1/).

From the equation of particle motion, in the superposition of the wave and lattice fields, it follows that

$$\frac{du^0}{d\tau} = \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{e\vec{A}}{mc^2}\right)^2, \quad \frac{du^z}{d\tau} = -\frac{c}{2}\frac{\partial}{\partial z}\left(\frac{e\vec{A}}{mc^2}\right)^2 \quad (3)$$

where $\vec{A} = \vec{A}_0 + \vec{A}_\omega$, τ is a proper time and u^μ are the components of particle four-dimensional velocity.

In the superposition of the fields (1) and (2) the transverse part of the generalized momentum $\vec{P}_\perp$ is conserved, so that one may put $\vec{P}_\perp = 0$ ($\vec{p}_\perp = \frac{e}{c}\vec{A}_\perp$). Taking account of this, the Maxwell equation for the potential of the electromagnetic wave with relativistic accuracy has the following form:*

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right) a_\omega = -\frac{\omega_p^2 \Omega_0}{c^2 \nu_0 \gamma_0}\, \bar{f}(z)\, e^{i\nu_0 z/c} \quad (4)$$

where $\bar{f}(z) = \int f(z, u^z)\, du^z$, f is the electron distribution function, $\omega_p^2 = 4\pi e^2 n/m$, n is the electron density in the beam, $\gamma_0 = u^0(0)$. Let us come now to the variables

$$\tilde{T} = t - z/c, \quad z = z/c \quad (5)$$

Then the kinetic equation for the distribution function is of the form (with accepted accuracy)

* Here and below all of the quantities are expanded systematically in powers of $1/\gamma$ (γ is the Lorentz factor).

$$\frac{\partial f}{\partial z} + \frac{1+\frac{\Omega_0^2}{\nu_0^2}}{2(u^z)^2}\frac{\partial f}{\partial \tilde{T}} + \frac{1}{2\gamma_0}\frac{\Omega_0}{\nu_0}\left(\frac{\partial a_\omega}{\partial \tilde{T}} e^{-i\nu_0 z} + \frac{\partial a^*_\omega}{\partial \tilde{T}} e^{i\nu_0 z}\right)\frac{\partial f}{\partial u^z} = 0 \quad (6)$$

For these variables, the Maxwell equation (4) takes the form

$$\frac{\partial^2}{\partial z\, \partial \tilde{T}} a_\omega = -\frac{\omega_p^2}{2\nu_0\gamma_0} e^{i\nu_0 z}\, \bar{f}(z) \quad (7)$$

Let us proceed to dimensionless variables (cf. Ref. /1/):

$$\nu_R = \frac{2\nu_0\gamma^2}{1+\Omega_0^2/\nu_0^2}, \quad a = \frac{\omega_p^2}{2\nu_0\Omega_0}, \quad \Gamma = \frac{\Omega_0}{\gamma_0\nu_0}a^{1/3}, \quad s = \frac{\Omega_0 z}{\gamma_0}a^{1/3},$$

$$y = \frac{a^{-2/3}}{\Omega_0}\frac{\partial a_\omega}{\partial \tilde{T}}, \quad u^z = \gamma_0\left(1 - \frac{\Gamma\sigma}{2}\right), \quad T = \nu_R \tilde{T} \quad (8)$$

Let us note that, as shown in Ref. /1/, the parameter $\Gamma \simeq \frac{\Delta\nu}{\nu}$, $\Delta\nu$ being the frequency interval in which the monochromatic signal is amplified, for relativistic particles $\Gamma \ll 1$. Moreover, as it is shown in Ref. /1/, $\frac{u(z)-u(0)}{u(0)} \sim \Gamma$. This means that σ can be of the order of unity. Taking this into account, one may write out equations (6) and (7) as

$$\frac{\partial f}{\partial s} + \left(\sigma + \frac{1}{\Gamma}\right)\frac{\partial f}{\partial T} - \left(y e^{-i\frac{s}{\Gamma}} + y^* e^{i\frac{s}{\Gamma}}\right)\frac{\partial f}{\partial \sigma} = 0$$

$$\frac{\partial y}{\partial s} = -e^{i\frac{s}{\Gamma}}\, \bar{f}(s) \quad (9)$$

Let us perform the Fourier transform

$$y = \frac{1}{2\pi}\int y_\omega e^{i\omega T}\, d\omega, \quad f = \frac{1}{2\pi}\int f_\nu e^{i\nu T}\, d\nu \quad (10)$$

and introduce the new functions

$$F_\nu = f_\nu e^{\frac{is\nu}{\Gamma}}, \quad z_\nu = y_\nu e^{i\frac{s}{\Gamma}(\nu-1)} \quad (11)$$

For these functions we get from (9) the following set of equations:

$$\frac{\partial F_\nu}{\partial s} + i\nu\sigma F_\nu - \int \frac{d\omega}{2\pi}\left[z_\omega \frac{\partial F_{\nu-\omega}}{\partial\sigma} + z_\omega^* \frac{\partial F_{\nu+\omega}}{\partial\sigma}\right] = 0$$
$$\frac{\partial z_\nu}{\partial s} - \frac{i}{\Gamma}(\nu-1) z_\nu = -\overline{F_\nu(s)} \qquad (12)$$

From the second equation of the set (12) it is seen that the signal with $\nu \sim 1$ is excited and $\Delta\nu \sim \Gamma$, while from the first equation it follows that F_ν do not vanish in the region of higher harmonics for the frequency interval of $\sim \Gamma$ only. For this reason, it is convenient to introduce the quantity $F_n^\omega = F_{\omega-n}$ near the n-th harmonic. After the inverse Fourier transform we find

$$\frac{\partial F_n}{\partial s} + in\sigma F_n - z\frac{\partial F_{n-1}}{\partial\sigma} - z^* \frac{\partial F_{n+1}}{\partial\sigma} = 0$$
$$\frac{\partial z}{\partial s} - \frac{1}{\Gamma}\frac{\partial z}{\partial T} = -\overline{F_1(s)} \qquad (13)$$

Coming to $F_\alpha = \sum F_n e^{in\alpha}$ we have

$$\frac{\partial F_\alpha}{\partial s} + \sigma\frac{\partial F_\alpha}{\partial\alpha} - (ze^{i\alpha} + z^* e^{-i\alpha})\frac{\partial F_\alpha}{\partial\sigma} = 0$$
$$\frac{\partial z}{\partial s} - \frac{1}{\Gamma}\frac{\partial z}{\partial T} = -\overline{F_1(z)} = -\int_0^{2\pi}\frac{d\alpha}{2\pi} e^{-i\alpha}\int_{-\infty}^{+\infty} d\sigma\, F_\alpha(\sigma) \qquad (14)$$

The first from Eqs. (14) can be regarded as the kinetic equation for the function F_α where α is the "coordinate", σ the "momentum", s the "time", and the time T enters as a parameter via the dependence $z(T)$. For a mono-energy electron beam the boundary condition is $F_\alpha(s=0) = \delta(\sigma)$. The integral $\int_{-\infty}^{+\infty} d\sigma\, F_\alpha(\sigma)$ is the density of particles at the point α. From the said above it follows that the second equation of the system (14) may be written as follows:

$$\frac{\partial z}{\partial s} - \frac{\partial z}{\partial T_1} = -\int_0^{2\pi} \frac{d\alpha_0}{2\pi} e^{-i\alpha(s,\alpha_0)}, \qquad (15)$$
$$T_1 = \Gamma T$$

where $\alpha(s, \alpha_0)$ satisfies the equations of motion*

$$\frac{d\alpha}{ds} = \sigma, \quad \frac{d\sigma}{ds} = -(ze^{i\alpha} + z^* e^{-i\alpha}) \qquad (16)$$

with boundary conditions

$$\alpha(0, \alpha_0) = \alpha_0, \quad \sigma(0) = 0$$

The set of equations (15) and (16) does not contain any parameters and their solution yields the universal behaviour of the physical process under study. In some sense, this system is analogous to the system, which was considered in Ref. /1/, but possesses the very important distinction: the spectrum of signal is not fixed here. Note, that this system allows the solution in the form of a monochromatic wave. Substituting $z = e^{i\nu T_1} B(s)$ and making use of $e^{i\alpha(s,\alpha_0)}$ - periodocity, it's easy to verify the equivalency of the equations derived and equations considered in Ref. /1/.

A further analysis of the systems (15) and (16) is numerical. The evolution of an electromagnetic signal has been studied in two cases: 1) the input signal is a set of a large number of harmonics with random phases ("white noise") and 2) the two-harmonics signal is supplied to the entrance of the system. In the second case one can observe the mutual influence and growth of the number of harmonics which are due to the nonlinear interaction. The evolution of the spectral distribution of the signal against the path 's' is shown in Fig. 1 for s = 7,8,9. The spectral energy distribution is given:

$$\int_0^T y^2(t)\frac{dt}{T} = \int \frac{d\omega}{2\pi} I_\omega$$

when $|y(0)|$ = 0.01 at the frequency interval $|\omega| < 3$ (in Γ units). It is seen that the signal excitation appears at the

* The transition from T to $T_1 = \Gamma T$ is equivalent to the transition in the spectral representation from the frequency ω to $\omega_1 = \omega/\Gamma$.

same width as in the case of monochromatic wave excitation. The average (full) intensity of the output signal against 's' is given in Fig. 2. Comparing Figs. 2 and 2 /1/ we see that the modulation depth of the output signal whose appearance is due to the development of phase oscillations in the beam of bunched electrons is much less for the case of "white noise" compared to the case of a monochromatic wave, that is explained by the non-uniform growth of the signal on different harmonics. The evolution of the spectrum when two harmonics with $\omega = -1$ and $\omega = 0$ are supplied to the entrance is given in Fig. 3. One can follow the energy transfer between various harmonics. The average intensity which is slightly differ from that in Fig. 2 is given in Fig. 3 as well.

All of these results have been obtained for the mono-energetic beam of electrons which reach the magnetic lattice at a given angle. The presence of the energy and angular spreads will lead to partial "washing out" of the picture. Therefore, the inequalities (21) in Ref. /1/ should be satisfied.

In conclusion, it should be mentioned that the physical mechanism of generation of the electromagnetic radiation in a free electron laser, which is discussed in the presented paper, is analogous, to a certain degree, to the mechanism in the case of developing the beam instability in a plasma (see, for example, Ref. /10/).

FIGURE CAPTIONS

Fig. 1. The spectral distribution of the output signal at different values of s : $s = 7(1)$, $s = 8(2)$, $s = 9(3)$; the input signal has constant spectral distribution in the interval $|\omega| < 3$ with random phases ("white noise"), $\sqrt{\overline{y^2(c)}} = 0.01$.

Fig. 2. Average intensity of the output signal as a function of s for white noise (see caption for Fig. 1).

Fig. 3. Evolution of the output signal when the input signal consists of two harmonics $\omega = 0$ and $\omega = -1$ at $\sqrt{I(0)} = 0.01$. The frequencies of the harmonics are indicated, symbol T denotes the summary signal.

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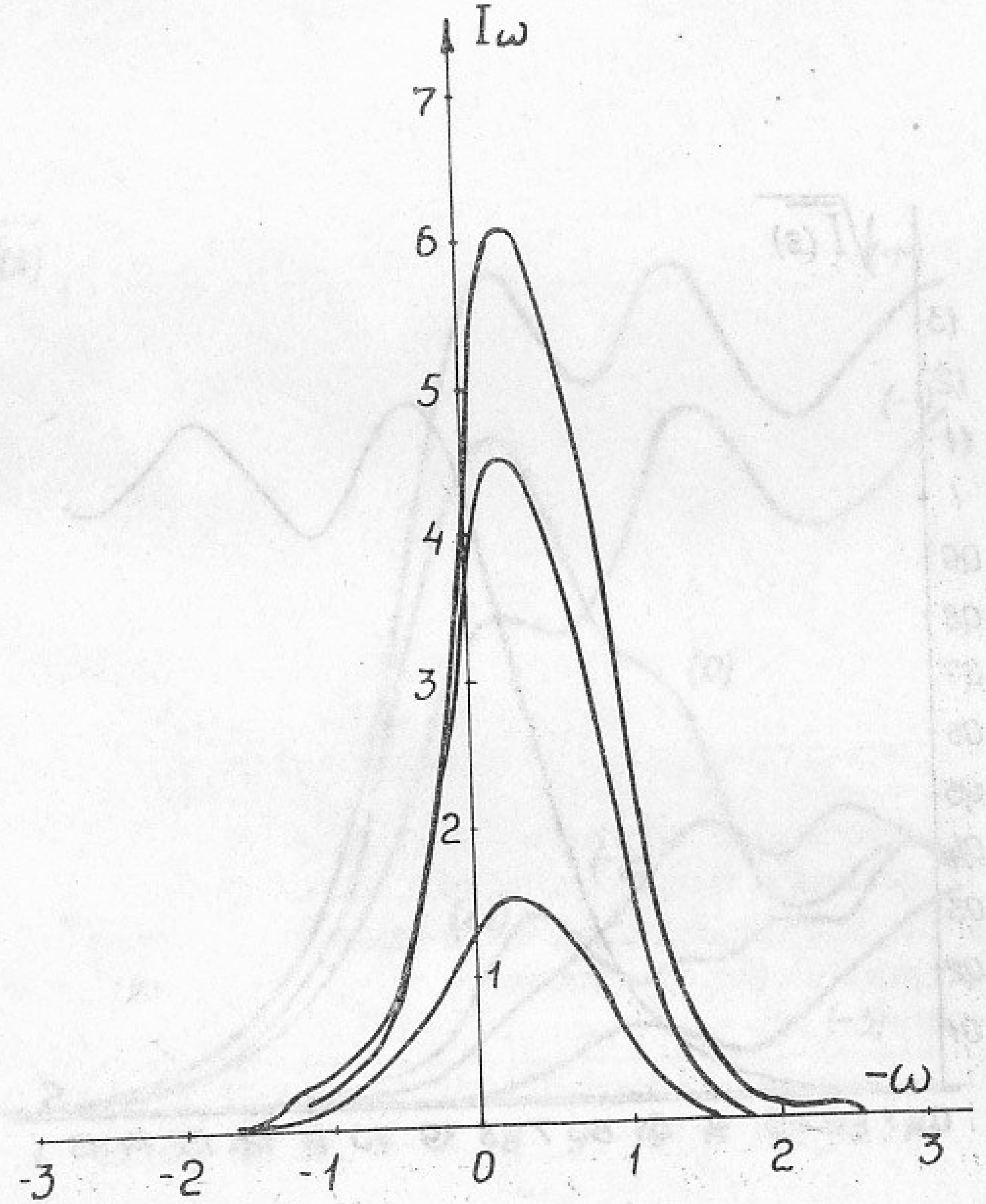
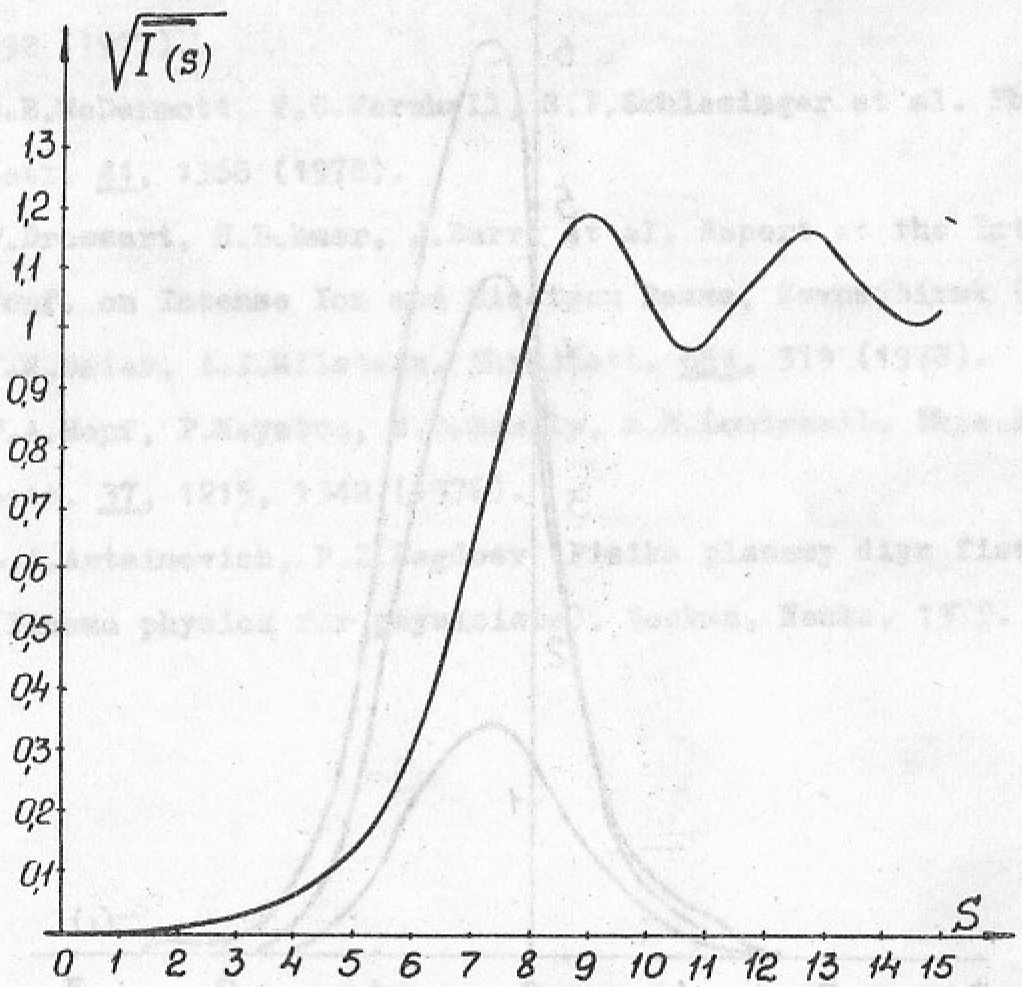
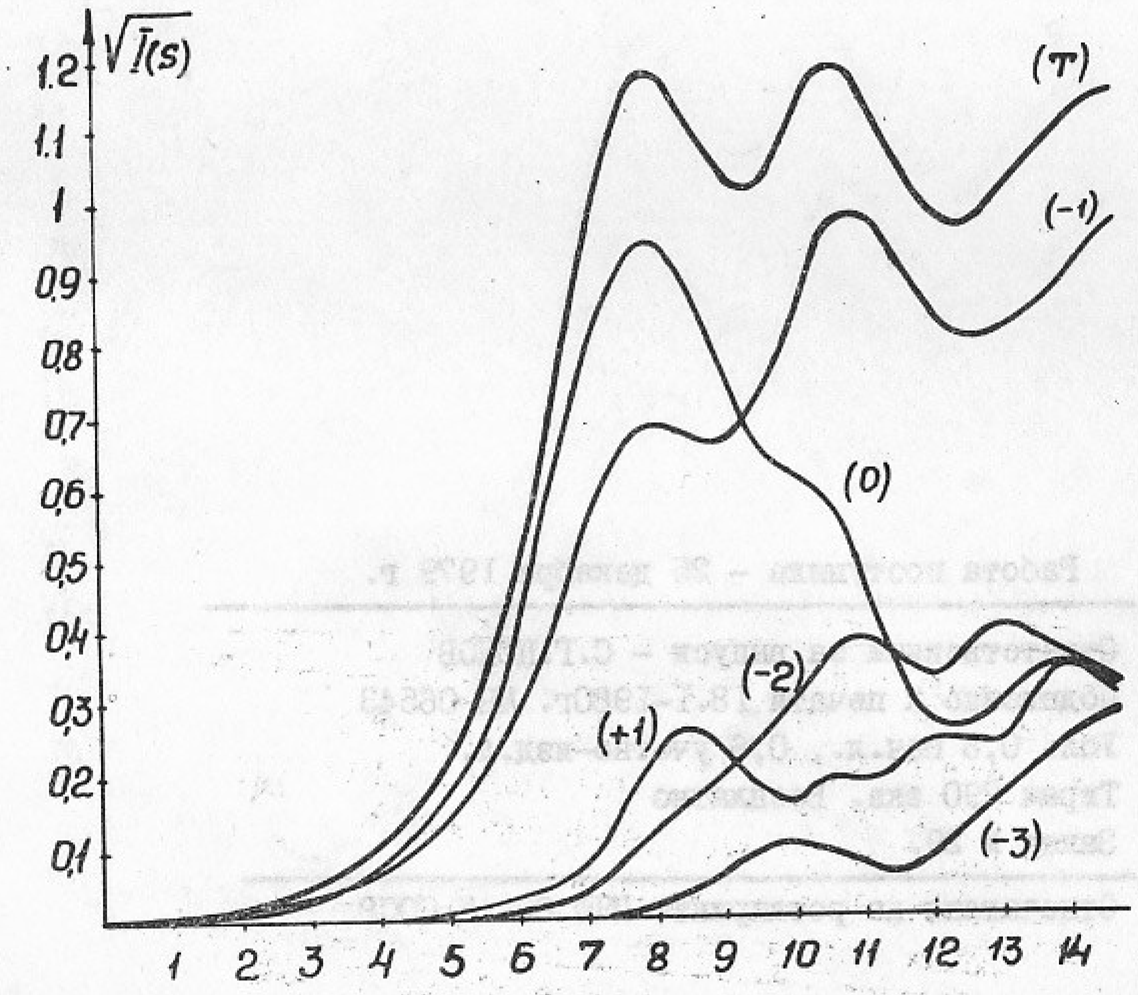


Fig. 1.



$\sqrt{\overline{I}(s)}$
1,3
1,2
1,1
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
S
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Fig. 2.



$\sqrt{\overline{I}(s)}$
1.2
1.1
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
(T)
(-1)
(0)
(-2)
(+1)
(-3)
S
1 2 3 4 5 6 7 8 9 10 11 12 13 14

Fig. 3.

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