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THE INFLUENCE OF DIFFERENT TYPES OF MODULATIONS ON A DECREASE OF THE STOCHASTICITY LIMIT IN BEAM-BEAM EFFECTS

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A b s t r a c t

The influence of various modulations in the beam-beam interactions on a decrease of the critical values of beam currents is investigated. The stochastic effects resulting from the parameter modulation in the simple models of "round" and "band" beams are considered. The results concerning the analytical and numerical analysis of the particle behavior of such models with application to the realistic conditions at colliding beam facilities are discussed as well.

INTRODUCTION

A maximum luminosity of colliding beam facilities is limited, as known, by the electro-magnetic interaction effects. An experimental study of this problem seems insufficient for the adequate understanding of all the reasons, which cause a decrease in luminosity of some facilities. This is explained by the fact that in the experimental conditions it is rather difficult, and often impossible at all, to separate one mechanism or another and to determine its role in the phenomenon under study. On the other hand, the theoretical investigation confronts with serious difficulties caused by the essentially nonlinear character of the force of a colliding beam. In view of this, the interest to computer simulation of the beam-beam effects is currently growing. This approach often makes it possible to reveal the character and the relative role of the effects under study with fairly simple models.

In the paper /1/ the modulation of the betatron motion by a synchrotron one was found to lead to a significant decrease of the limiting current because of the presence of a dispersion energy function at the interaction point. The goal of the present paper is to investigate the other possible types of modulation and to consider their joint effect.

The first section describes the basic model, presents some analytical estimates for the "round" and "band" beams and discusses the results obtained in /1/. The influence of the modulation of a betatron phase advance is studied in section 2. Section 3 is devoted to the modulation, the creation of which is due to the dependence of the β -function on the azimuthal coordinate. And, finally, the effects of various types of modulation are compared in section 4. Their mutual influence on the stochastic instability limit is discussed in section 4 as well.

1. Description of the model

We will consider the behaviour of a single particle multiply interacting with a constant bunch in the usual approximation (see, e.g., /1,2/). The colliding beam is assumed to be infinitely short along the longitudinal coordinate; in this case, the motion only along one transverse coordinate, disregarding the other coordinate, is considered. The radiation effects (noise and damping) are not taken into account. This means that only the strong effects, "evolving" during the time periods shorter than the damping time of betatron oscillations, are considered. Despite the evident limitations of such a very simplified model of the beam-beam interaction, its investigation seems to be useful, particularly in our case, for comparison of different types of modulation. This problem will be discussed in more detail in section 4. If the interaction points in a storage ring are located in a period of the magnetic system, then the motion in a single particle may be described by a comparatively simple mapping. Such a mapping connects the transverse coordinate and the momentum of a particle after some interaction with a colliding beam, with the values of the coordinate and momentum after the previous interaction. In the simplest case, if the nonlinear effects, which are due to the motion of a particle between the interaction points may be neglected, the mapping will consist of a linear rotation on the phase plane (x, p) and a nonlinear kick caused by the beam-beam interaction. If the derivative of the β-function over

the longitudinal coordinate S is a zero derivative at the interaction point ($\frac{d\beta}{dS} = 0$), then the mapping has the form:

$$x_{n+1} = x_n \cdot \cos\mu + \beta_0 p_n \sin\mu + \beta_0 f(x_n) \sin\mu \qquad (1.1)$$

$$p_{n+1} = -\frac{x_n}{\beta_0} \sin\mu + p_n \cdot \cos\mu + f(x_n) \cdot \cos\mu$$

Here x and $p = \frac{dx}{dS}$ are the transverse coordinate and momentum, β_0 - is the value of the β-function at the interaction points, μ - is the betatron phase shift between the interaction points ($\mu = \frac{2\pi\nu}{m_0}$, where ν is the betatron oscillation frequency over x and m_0 is the number of interaction points over a ring).

The quantity $f(x)$ in (1.1) is proportional to the relative change of the momentum Δp and is determined by the charge distribution $\rho(x)$ in the transverse cross section. In what follows, for the sake of simplicity, $f(x)$ will be referred to as a force. In Ref./1/ two limiting cases concerning the various transverse charge distributions were considered. In one of them the beam is assumed to be round in the transverse plane, with the Gaussian distribution $\rho(x) = \rho_0 \exp(-x^2/2\sigma^2)$, where by x is meant the distance from the beam centre. In this connection, if the initial momentum of a particle is directed along this direction (along the radius), the motion remains one-dimensional and can be described by the mapping (1.1) with the force

$$f_1(x) = -\frac{8\pi\xi}{\beta_0}\sigma^2 \frac{1-\exp(-x^2/2\sigma^2)}{x}; \quad \xi_{x,z} = \frac{N_0 r_e \beta_{x,z} \sigma_{z,x}}{2\pi\gamma\sigma_z\sigma_x(\sigma_z+\sigma_x)} \qquad (1.2)$$

where N_0 is the number of particles in the bunch, r_e is the classic electron radius, γ is the relativistic factor, $\sigma = \sigma_x = \sigma_z$ is the rms beam size. The parameter ξ is introduced by an usual way and characterizes a magnitude of the interaction.

On the contrary, the second case corresponds to the strongly stretched cross section in one of the transverse directions, for example along the storage ring radius. In the limiting case, such a beam may be considered to be a band one, and the motion in the field of this beam will be also one-dimensional. Then x should mean the transverse coordinate along the small size of the beam. In the existing devices this case corresponds, to an extent, to the beam elliptic in its transverse cross section with a large relation between the transverse sizes. The force of such a "band" beam with a Gaussian distribution $\rho(x)$ has the form:

$$f_2(x) = -\frac{4\pi\xi}{\beta_0}\int_0^x e^{-x^2/2\sigma^2}\,dx \qquad (1.3)$$

The computer simulation for the mapping (1.1) has been carried out by various authors (see the review /2/). The main problem is to find the critical value of ξ at which the stochastic instability leading to an appreciable increase of the transverse energy arises. As known, the mechanism of such instability is the interaction between nonlinear resonances /3-6/. The neighbouring resonances caused by the nonlinear perturbation, overlap and form a stochastic region in the phase space. It means that the particle can move apart from the centre of the beam. It should be noted that the idea of a critical value of ξ_c is conditional to a certain extent. In the computer simulation by ξ_c is often meant the value of ξ at which the particles with the most typical initial data (x_0, ρ_0) achieve, for a given number of iterations ($N \sim 10^4$-10^6), large

x_m (see, e.g. /1/). It is more convenient, in the other cases, to follow the relative increase of the transverse particle energy, as it was done in /7/. In view of this, a direct comparison of the values of ξ_c derived in this manner with the experimental values is conventional, too. The main advantage of this approach is the dependence of ξ_c on different parameters of the model under study rather than the absolute values of ξ_c. As the VEPP-2M data evidence, even such a simplified model as the mapping (1.1) reflects correctly some experimental dependences.

The computer simulation in /1/ showed a noticeable decrease of ξ_c when the modulation of the colliding beam centre was introduced into (1.1). This modulation occurs if the dispersion energy function ψ at the interaction point does not vanishes. In this case, the orbit of a particle whose energy differs from the equilibrium one by the quantity ΔE shifts from the orbit of the equilibrium particle by the quantity /8/:

$$\Delta x = \psi R \cdot \frac{\Delta E}{E} \qquad (1.4)$$

where R is the mean radius of the storage ring. Hence, the particle whose energy deviates from the energy of an equilibrium particle at the interaction point passes through the region of the beam with the lower density and the larger nonlinearity. The presence of synchrotron oscillations leads therefore to the modulation of the colliding beam force acting on the test particle. In the model under consideration this results in the change of x in the force expression (1.2; 1.3):

$$x \to x + A_s \cos(2\pi \nu_s / m_0) \qquad (1.5)$$

where A_s and ν_s are the amplitude and frequency of synchrotron oscillations and n is the number of a kick ("time"). The angular intersection of the beams at the interaction points leads to the modulation (1.5) as well /9/.

The decrease of ξ_c during the modulation (1.5) is accounted for by the appearance of additional, synchrobetatron resonances, which facilitate the overlap of the main resonances and thereby decrease the stochasticity limit. In the paper /2/ some analytical estimates of the conditions of overlapping the resonances for (1.1) with the force (1.2) have been obtained. The overlap criterion turned out to depend, in a complicated manner, on the parameters of the model, namely: the parameter ξ, transverse energy a, betatron frequency ν, and the resonance number n. The latter parameter is not independent, and it is determined by the values of ξ, ν and a. Due to synchrotron oscillations, the analytical estimates become more complex and give mainly the qualitative results. In such a situation the resonance structure analysis of a perturbation is very important. It enables one to make some conclusions on the relative force of the modulation (see, e.g., /10/).

Let us write the Hamiltonian corresponding to the mapping (1.1) in the "action-phase" variables ($x=\sqrt{2J\beta_0}\cos\psi$, $p=\sqrt{\frac{2J}{\beta_0}}\sin\psi$, see /2/):

$$H=J\nu_0+V(J,\psi,\theta)\,\delta_T(\theta)$$

Here ν_0 is the unperturbed betatron frequency, $\delta_T(\theta)$ is the periodic delta-function, dependent on the phase θ introduced instead of the azimuthal coordinate S : $\theta=\frac{2\pi S}{L}$, where L is the -interaction period. The external perturbation period $V(J,\psi,\theta)$ is given by the value of θ_0, which is equal to $\theta_0=\frac{2\pi}{m_0}$. The perturbation $V(J,\psi,\theta)$ is determined by the

force $f(x)$ and is as follows for the round beam (1.2) /2/:

$$V_1(J,\psi,\theta)=-\frac{4\pi\xi}{\beta_0}\,\sigma^2\int_0^1\frac{1-\exp(-u_s z)}{z}\,dz \tag{1.6}$$

If a modulation of the type (1.5) is available, the quantity u_s also depends on the phase θ, which plays a role of the dimensionless time:

$$u_s=\frac{(x+A_s\cos\varphi_s)^2}{2\sigma^2}\,;\quad \varphi_s=\nu_s\theta=\frac{2\pi n\nu_s}{m_0} \tag{1.7}$$

Similarly, for the band beam we have:

$$V_2(J,\psi,\theta)=-\frac{4\pi\xi}{\beta_0}\,\sigma^2\cdot\frac{1}{2}\int_0^1\frac{1-\exp(-u_s z)}{z^{3/2}}\,dz \tag{1.8}$$

The comparison of (1.6) and (1.8) shows that despite the significant difference in the force $f(x)$ the perturbations associated with the round and band beams have similar resonance structures. It becomes more noticeable if we proceed to the resonant Hamiltonian, which describes the behaviour of the system near a particular resonance. To do this, it is necessary, as usual, to expand the perturbation in Fourier series of ψ and θ and to keep the most essential resonant terms.

For the round beam we have /2/:

$$\begin{aligned}H_1^2=J\nu_0+\varepsilon\int_0^1\frac{dz}{z}\Big\{1-e^{-(a+h)z}I_0(az)\,I_0^2(rz)\,I_0(hz)\Big\}-\\ -2\varepsilon\int_0^1\frac{dz}{z}\,e^{-(a+h)z}\sum_{p,q,m,n=-\infty}^{\infty}I_n(az)\,I_p(rz)\,I_q(rz)\,I_m(hz)\cos\vartheta_{npqkm}\end{aligned} \tag{1.9}$$

Similarly, one can obtain a resonant Hamiltonian for the band beam:

$$\mathcal{H}_2^{\tau} = J\nu_0 + \frac{\varepsilon}{2}\int_0^1 \frac{dz}{z^{3/2}}\left\{1 - e^{-(a+h)z} I_0(az) I_0^2(\tau z) I_0(hz)\right\} -$$

$$- \varepsilon \int_0^1 \frac{dz}{z^{3/2}} e^{-(a+h)z} \sum_{p,q,k,m,n=-\infty}^{\infty} I_n(az) I_p(\tau z) I_q(\tau z) I_m(hz) \cos\vartheta_{npqkm} \qquad (1.10)$$

In (1.9-1.10) I_k stands for the modified Bessel function and ϑ_{npkqm} is a new unperturbed phase of oscillations near a particular nonlinear resonance. For the convenience, in the Hamiltonian (1.9-1.10) the phase-independent term, which determines the nonlinear betatron frequency shift $\Delta\nu$, is separated. Correspondingly, the sum does not include the term for which all p, q, k, m, n are zero simultaneously. The new perturbation parameter ε is related to ξ by the relation $\varepsilon = 2m_0\sigma^2\xi/\beta_0$, and the quantities a, τ and h are expressed via the amplitudes of betatron and synchrotron oscillations x_m and A_s according to formulas:

$$a = \left(\frac{x_m}{2\sigma}\right)^2; \quad h = \left(\frac{A_s}{2\sigma}\right)^2; \quad \tau = \sqrt{2ah} = \frac{A_s x_m}{2\sigma^2} \qquad (1.11)$$

Generally speaking, by σ it is necessary to mean the total rms size of the beam, which is a sum of its betatron (σ_β) and synchrotron (σ_s) sizes: $\sigma^2 = \sigma_\beta^2 + \sigma_s^2$. But, since $\sigma_s \ll \sigma_\beta$ as a rule, it is assumed that $\sigma \approx \sigma_\beta$.

The resonance phase ϑ_{npqkm} in (1.9-1.10) is determined by the relation:

$$\vartheta_{npqkm} = (2n+p+q)\Psi - (p-q+2m)\varphi_s - km_0\theta \qquad (1.12)$$

where the θ -derivatives equal: $\varphi_s' = \nu_s$; $\theta' = 1$; $\Psi' = \nu = \nu_0 + \Delta\nu$. In the last relation the betatron frequency shift $\Delta\nu$ depends on the transverse energy $2a$ and is obtained below. From (1.12) the resonance condition is easy to derive:

$$\nu = \nu_0 + \Delta\nu(a) = \frac{m_0 k + (p-q+2m)\cdot\nu_s}{2n+p+q} \qquad (1.13)$$

In the absence of synchrotron oscillations $p = q = m = 0$, therefore the distance between the neighbouring resonances n and $n+1$ equals $\Delta\nu_n = \frac{m_0}{2n(n+1)}$. If the synchrotron motion is taken into account, there arise the synchrotron resonances in the system (see (1.13)). For simplicity, we call them the side-band resonances, since they are not far, in frequency, from the main resonances with harmonics n ($\nu_s \ll 1$). Note that if the modulation does not occur but the constant beam shift takes place ($\nu_s = 0$; $A_s \neq 0$), additional resonances between the main ones arise, too. The condition $p+q=\pm 1$ corresponds to the nearest resonance of this kind, and the distance between it and the main resonance n equals $(\Delta\nu)_s = m_0/2n(2n+1)$; at $n \gg 1$ this distance is two times shorter than that between the main resonances. The amplitude of these additional resonances is essentially determined (see (1.9-1.10)) by the values $p=0, q=\pm 1$ and $p=\pm 1, q=0$ and depends on the quantity τ. In the case of a constant large shift when $\tau \gtrsim 1$ these resonances become comparable with the main ones ($p=q=0$), and the critical value of ξ_c decreases at least by a factor of 2.

Since ξ_c is determined by the resonance overlap criterion, which is inversely proportional to the distance between the resonances (in frequency), it is clear that the presence of the side-band resonances can decrease significantly the value of

ξ_c . Indeed, at $\nu_s \ll 1$ the distance to the nearest side-band resonance, as it follows from (1.13), is $(\Delta\nu)_{ns} \approx \nu_s/n \ll \nu$. The power of the side-band resonances is determined by the values p, q, m and decreases sharply with their growth (at $A_s \ll \sigma$). However, since the distance between them is very short, they can overlap at the values of ξ which are less than it is necessary for overlapping the main resonances. In this case, the overlap of these side-band resonances creates a slow diffusion. If the whole ν -overlap region seems to be of the order of a distance between the main resonances, this leads to the diffusion over main resonances, and hence to a significant increase in the transverse energy. A noticeable decrease of ξ_c under the modulation (1.7) in the model is explained in Ref./1/ by means of this mechanism.

The nonlinear frequency shift $\Delta\nu(a)$ is easy to find from the Hamiltonian (1.9-1.10). Since $A_s \ll \sigma$ and hence $h \ll z \ll 1$, then with $h \ll a$ the shift $\Delta\nu(a)$ is practically the same as in the absence of this modulation:

$$\Delta\nu_1(a) = \frac{m_0 \xi}{a} \left\{1 - e^{-a} I_0(a)\right\}$$

$$\Delta\nu_2(a) = \frac{m_0 \xi}{2a} \left\{1 - e^{-a} I_0(a)\right\} + \frac{m_0 \xi}{4\sqrt{a}} \int_0^a \frac{dz}{z^{3/2}} \left\{1 - e^{-z} I_0(z)\right\} \tag{1.14}$$

At $a \ll 1$ an usual linear shift is obtained from (1.14): $\Delta\nu \approx m_0 \xi$, while with increasing the transverse energy (at $a \gg 1$) $\Delta\nu$ tends to zero:

$$\Delta\nu_1 \approx \frac{m_0 \xi}{a}; \quad \Delta\nu_2 \approx \frac{2 m_0 \xi}{\sqrt{2\pi a}} \tag{1.15}$$

The energy dependence of the frequency is one of the specific features of the nonlinear motion, and the condition of

resonance overlap depends significantly on it (see /3-6/). As follows from estimates (1.14-1.15) for the round beam, the resonances of the higher harmonics n , in comparison with those for the bend beam, lie at the same distance from the centre of the phase plane. And, alternatively, the resonance with the same number n lies much closer to the centre of the round beam as compared to the band one. For further comparison, let us write out the estimates for the width of a nonlinear resonance /3-6/:

$$(\Delta J)_z = 4\sqrt{\frac{\varepsilon V_n}{\partial\nu/\partial J}}; \quad (\Delta\nu)_z = 4\sqrt{\frac{\partial\nu}{\partial J}\cdot \varepsilon V_n} \tag{1.16}$$

Here $(\Delta J)_z$ determines the region occupied by the resonance in the action J (and hence in energy a), and $(\Delta\nu)_z$ is the resonance width in frequency ν . εV_n stands for the perturbation taken at the resonance value J_z , which in turn is determined by the resonance condition (1.13). It follows from expressions (1.9-1.10) that ($A_s \ll \sigma$):

$$(\varepsilon V_n)_1 \approx 2\varepsilon \int_0^{a_1} \frac{dz}{z} e^{-z} I_{n_1}(z) = 2\varepsilon F_1(n_1, a_1)$$

$$(\varepsilon V_n)_2 \approx \varepsilon\sqrt{a_2} \int_0^{a_2} \frac{dz}{z^{3/2}} e^{-z} I_{n_2}(z) = \varepsilon\sqrt{a_2}\, F_2(n_2, a_2) \tag{1.17}$$

At $a \gg 1$ the integrals in (1.17) depend on a only via n_1 and n_2 .

The resonance overlap criterion is a condition under which the width of a resonance, in frequency $(\Delta\nu)_z$, becomes comparable with the distance between the nearest resonances. In the absence of modulation this distance is equal, as already mentioned, to $(\Delta\nu)_n \approx \frac{m_0}{2n^2}$. The numerical results /1/ have shown that in this case ξ_c is slightly different for the round and band beams. Nevertheless, one can obtain from estimates (1.16-1.17) that the modulation should affect stronger the band beam than the round one. It is connected with the fact that at a constant perturbation of F_1 and F_2 near the main resonance n, $(\Delta\nu)_z$ depends on a in different ways. Indeed, as computer simulation has shown, under the same conditions the quantity for the band beam with modulation (1.5) is smaller than for the round one. Particularly, the conclusion can be drawn that the elliptical beam with the biGaussian distribution over both transverse coordinates x and z ($\rho=\rho_0 \exp(-\frac{x^2}{2\sigma_x^2}-\frac{z^2}{2\sigma_z^2})$) will be less stable in the presence of modulation in comparison with the round beam. In addition, since the expressions for forces f_x and f_z from such a beam depend on the ratio σ_x/σ_z and since their limits are the expressions (1.2-1.3) for the round and bean beams (see /11/), one can also assume that the instability for the elliptical beam is stronger in the direction perpendicular to the larger size of the beam. This conclusion is in agreement with the preliminary data concerning a study of the influence of various modulations and noise in the elliptical beam model (with taking into account the coupling of transverse oscillations with respect to x and z) /12/.

2. Betatron phase modulation

The presence of synchrotron oscillations in colliding beam facilities leads to a weak modulation of the betatron phase shift $\Delta\mu$ between the interaction points. Such a modulation arises due to the dependence of a rotation period on a particle energy /7/. The azimuthal deviation of a particle with momentum $p_\parallel+\Delta p_\parallel$ from

the equilibrium one for time t is /8/:

$$\Delta\theta = \int_{t_0}^{t_0+t} \eta\Omega_0 \cdot \frac{\Delta p_\parallel}{p_\parallel}\, dt; \quad \eta = \frac{p_\parallel}{\Omega_0} \frac{d\Omega_0}{dp_\parallel} \qquad (2.1)$$

where $\Omega_0 = \frac{2\pi}{T}$ is the angular frequency of revolution. Taking into consideration the weak time-dependence of $\Delta p_\parallel$ and Ω_0 , in the relativistic case we obtain:

$$\Delta\mu = \mu(t)-\mu_0 = \frac{R\cdot\Delta\theta}{\beta_0} \approx -\alpha \frac{\Delta p_\parallel}{p_\parallel} \frac{\pi}{m_0} \cdot \frac{R}{\beta_0} = \Delta\mu_0 \sin(\nu_s\Omega_0 t+\delta) \qquad (2.2)$$

Here α is the momentum compaction factor.

$$\frac{\Delta p_\parallel}{p_\parallel} = \left(\frac{\Delta p}{p}\right)_0 \cdot \sin(\Omega_0\nu_s t+\delta).$$

A similar modulation occurs if there exists the chromatism in the ring, when $\partial\nu_{x,z}/(\partial p/p) \neq 0$:

$$\Delta\mu_{x,z} = \frac{2\pi^2}{m_0} \cdot \frac{\partial\nu_{x,z}}{(\partial p/p_\parallel)} \cdot \left(\frac{\Delta p}{p}\right)_0 \cdot \sin(\Omega_0\nu_s t+\delta) \qquad (2.3)$$

Just as the preceding form of modulation, chromatism arises when changing the particle energy between the interaction points. But since the chromatism is usually compensated by sextupoles, this modulation does not practically work.

With the magnetic field pulsations in the storage ring components a similar modulation arises, too. In this case, the betatron phase advance between the interaction points is also time-dependent. It is worth mentioning a particular case of betatron phase modulation due to the inaccurate azimuthal adjustment of the ring components. Here μ varies kick-likely; for two interaction points for example, the variation of μ is as follows:

$$\mu_1 = \mu_0 + \Delta\mu_0; \quad \mu_2 = \mu_0 - \Delta\mu_0; \quad \Delta\mu_0 \approx \frac{\Delta S}{\beta} \qquad (2.4)$$

where ΔS is the geometrical shift of the centre of the opposite gap from the interaction point, indices 1 and 2 refer to the first and second interaction points, respectively. This form of modulation is obtained from (2.3) at $\nu_s = 1$. As a rule, $\beta_z \ll \beta_x$, therefore the modulations (2.2) and (2.4) should manifest themselves more strongly in the case of z-motion. For numerical investigation concerning the influence of the betatron phase modulation in the round and band beam models (1.2-1.3) the quantity μ in the transition matrix (1.1) is considered to be dependent on the kick number according to the relation:

$$\mu = \mu_0 + \Delta\mu = \mu_0 + B\cos\left(\frac{2\pi\nu_s n}{m_0} + \delta\right); \quad \mu_0 = \frac{2\pi\nu_0}{m_0} \tag{2.5}$$

Just as in /7/, such a value of ξ_c is taken for ξ_c, at which the quantity W proportional to a maximum amplitude of the transverse motion:

$$W = \sqrt{\left(\frac{x}{\sigma}\right)^2 + \left(\frac{\rho\beta_0}{\sigma}\right)^2} \equiv \sqrt{X^2 + \mathcal{P}_x^2}$$

increases by $\Delta W = 2$. The total time (the number of iterations) for each experimental particle was $t_m = 10^5$, the value of W being averaged during each time period, $\Delta t = 1000$. It was the way in which the time dependence of W was found. This dependence characterizes either the stable motion ($W \approx const$), or the stochastic instability (the irregular growth of W with time). As the test particles, the particles with the initial zero momentum ($\mathcal{P}_0 = 0$) and the initial shift $X_0 = 2;4;6$ were chosen. Despite the conditional nature of the definition of ξ_c this approach makes it possible to investigate the conditions of appearing a strong stochasticity, depending upon the various parameters of the system (see, e.g. /1/). The results

are presented in Fig.1, where the derived dependence of ξ_c on the modulation amplitude B at a constant frequency $\nu_s \approx 0.008$, is shown. (Such a value of ν_s is close to the real one for the VEPP-2M facility.) For the existing storage rings the quantity is usually small. Nevertheless, even for small quantities B this modulation affects strongly the quantity ξ_c. The comparison of curves 1 and 2 shows that for the band beam ξ_c becomes ever smaller than for the round beam as B is increased, though at $B = 0$ the quantity ξ_c is about 1.5 times larger as compared to that for the band beam (see /1/).

Let us consider the resonance structure of a perturbation under such a type of modulation. With this aim, let us write the resonance Hamiltonian, describing the oscillations near one of the side-band resonances. Since the perturbation potential $V(I,\Psi)$ in this case is independent of θ and the period of the δ-function is modulated only (see (1.6)), the expression for H^z is much simpler than that for modulation (1.7) /2/:

$$H_1^z = I\nu_0 + \varepsilon\int_0^1 \frac{dv}{v}\left\{1 - e^{-av} I_0(av)\right\} - 2\varepsilon\int_0^1 \frac{dv}{v} e^{-av} I_n(av) J_m(b_k)\cos\vartheta_{nkm}$$

$$H_2^z = I\nu_0 + \varepsilon\int_0^1 \frac{dv}{v^{3/2}}\left\{1 - e^{-av} I_0(av)\right\} - 2\varepsilon\int_0^1 \frac{dv}{v^{3/2}} e^{-av} I_n(av) J_m(b_k)\cos\vartheta_{nkm} \tag{2.6}$$

Here $J_m(b_k)$ is the usual Bessel function of the m-th order with argument $b_k = \frac{Bm_0k}{\mu_0\nu_s}$ and ϑ_{nkm} is a new phase: $\vartheta_{nkm} = 2n\Psi - k\theta + m\varphi_s$. The resonance condition has the form:

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$$H_1^r = J\nu_0 + \varepsilon\int_0^1 \frac{dv}{v}\left\{1 - e^{-av} I_0(av)\right\} - 2\varepsilon\int_0^1 \frac{dv}{v} e^{-av} I_n(av) J_m(b_k) \cos\vartheta_{nkm}$$

$$H_2^r = J\nu_0 + \varepsilon\int_0^1 \frac{dv}{v^{3/2}}\left\{1 - e^{-av} I_0(av)\right\} - 2\varepsilon\int_0^1 \frac{dv}{v^{3/2}} e^{-av} I_n(av) J_m(b_k) \cos\vartheta_{nkm} \tag{2.6}$$

Here $J_m(b_k)$ is the usual Bessel function of the m-th order with argument $b_k = \frac{B m_0 k}{\mu_0 \nu_s}$ and ϑ_{nkm} is a new phase: $\vartheta_{nkm} = 2n\Psi - k\theta + m\varphi_s$. The resonance condition has the form:

$$\nu = \nu_0 + \Delta\nu(a) = \frac{km_0 + m\nu_s}{2n} \tag{2.7}$$

The comparison of (2.7) with (1.13) shows that the spectrum of the resonance frequencies under modulation μ (2.5) is far simpler than that in case of the modulation x (1.7). It also follows from (2.7) that for the phase modulation the distance between the side-band resonances, $(\Delta\nu)_{ns} = \frac{\nu_s}{2n}$, is shorter by a factor of 2 than that for (1.7). But the most significant distinction between this type of modulation and the previous one is the following. It is seen from (2.6) that at $b_k >> 1$ the amplitude of the resonance perturbation for a fixed value of n changes slightly with increasing m, up to $m = m^* \approx b_k$. Therefore, for all side-band resonances which are removed from the main resonance not more than $m^* \cdot \nu_s$, the condition of their overlap is independent of m. Since the distance between these resonances is very small, the critical value of ξ_c can be much lower than that of ξ necessary for the overlap of the main resonances n and $n+1$ and is determined by the quantity $J_m(b_k)$. Estimation of the total width of the overlapped resonances is dependent on m^* and equal approximately to

$$m^* \approx \frac{B m_0 K}{\mu_0 \nu_s} = \frac{B n m_0}{\pi \nu_s} \tag{2.8}$$

In the above estimation the ratio $m_0 k = 2\nu_0 n$ has been used (see (2.7)). If the maximum number of the side-band resonance m^* is known, one can find the total width (in frequency) occupied by all the side-band resonances:

$$(\Delta\nu)_{m^*} \approx 2m^* \cdot \frac{\nu_s}{2n} = \frac{B m_0}{\pi} \tag{2.9}$$

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It is seen from (2.9) that $(\Delta\nu)_{m^*}$ is determined by the depth of modulation and is independent of the synchrotron frequency ν_s.

As a result of this overlap, the stochastic region with a weak diffusion arises. If the width $(\Delta\nu)_{m^*}$ becomes comparable with the distance between the main resonances, this leads to a stronger diffusion of the main resonances in the system. Estimation of the critical value of B_c, at which this effect occurs, yields:

$$B_c \simeq \frac{(\Delta\nu)_{m^*}}{\Delta\nu_n} \simeq \frac{\pi}{2n(n+1)} \qquad (2.10)$$

At $B > B_c$ the side-band resonances overlap all the interval between the main resonances n and $n+1$, and in this case the quantity ξ_c should significantly decrease.

Let us compare the derived estimate (2.10) with the results of numerical simulation. For simplicity, the case of a round beam is taken. The value of n is then determined by the relation (1.14) for $\Delta\nu_1(a)$:

$$\frac{m_0 k}{2n} = \nu_0 + \frac{m_0 \xi}{a}\left\{1 - e^{-a} I_0(a)\right\} \qquad (2.11)$$

where $a = \left(\frac{x_m}{2\sigma}\right)^2$. We are interested, for example, in the overlap of additional resonances in the region $x = 5\sigma$, i.e. for $a = 6.25$. In our case ($\nu_0 = 3.08$, $m_0 = 2$) at $\xi = 0.07$ this gives $n = 11$. And the value of B_c, according to (2.10) turns out to be equal to $B_c \simeq 0.014$ (cf. Fig.1). At $\xi = 0.1$ we have $n = 9$, i.e. at $B \simeq 0.01$ the number of m^* equals: $m^* = 7$, while on a half the interval between the resonances $n = 9$ and $n = 10$ there are $N_m = \frac{1}{\nu_s(n+1)} \simeq 12$ side-band resonances. In other words, the total, effective overlap of all the interval does

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not occur. This corresponds to some growth of ξ_c in Fig.1 at $B = 0.01$. It follows also from (2.8-2.10) that the total overlap arises first at the larger displacements of x and then it expands the domain of smaller x as B (or ξ) is increased.

Thus, the comparison shows that rather rough estimates (2.8-2.10) provide not only the correct qualitative meaning of the effect of overlapping the side-band (synchrobetatron) resonances but are in accordance with the numerical data.

It is worth stressing that even if the total overlap of the main resonances does not occur and only a certain layer is formed wherein the stochasticity gives only a limited change of energy along the coordinate x, an unlimited diffusion over z becomes possible, when the coupling to the z-coordinate motion is introduced. This is connected with the fact that the diffusion in this case appears along the formed stochastic layer (for details see /17,13,14/). Although such a diffusion is much weaker as compared to that considered above across the layer (along x), nevertheless it may be strong at the values of ξ below the threshold of appearing the strong stochasticity.

Numerical data show that despite the relatively small values of the modulation amplitude B, the critical value of ξ_c drops significantly. The modulation, which is due to the difference in the betatron phase shift in various ring periods between the interaction points because of the inequalities of these periods (see 2.4), also leads to a significant decrease of ξ_c (Fig.2). This may impose fairly rigid requirements for the azimuthal adjustment of the ring elements, for example for operation with the coupling resonance with equal and small β-functions at ~~the interaction point. In this case, the transverse motion is~~

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close to the one-dimensional, and the results of the previous analysis are valid.

3. Modulation of the perturbation amplitude

In particular, such a modulation is due to the azimuthal dependence of a β-function in the gap wherein the beam-beam interaction occurs. This dependence is given by a formula

$$\beta=\beta_0+\frac{\ell^2}{\beta_0} \tag{3.1}$$

where β_0 is the minimum value of the β-function at the centre of the gap, ℓ is the azimuthal deviation. For a strong-focusing device with a small β-function at the interaction point, a magnitude of the β-function within the interaction region can differ considerably from β_0.

The presence of synchrotron oscillations leads to changing the β-dependent value of ξ for a particle whose phase differs from the equilibrium one because of the modulation of the interaction point position.

$$\xi=\xi_0\left(1+\frac{\ell^2}{\beta_0^2}\right)^{1/2} \tag{3.2}$$

Formula (3.2) holds provided that the β-function is much larger in the second direction.

Since for the round beam $\xi_{x,z}\sim\frac{\beta_{x,z}}{\sigma^2}$ and $\sigma\sim\sqrt{\beta}$, it is easy to see that under the condition $\beta_x=\beta_z$ the dependence of the longitudinal coordinate does not hold, and hence the modulation (3.2) is absent. In view of this, for numerical simulation the band beam model (1.3) has been used, in which the parameter ξ varies according to the relation:

$$\xi=\xi_0\left\{1+A_0\cos^2\left(\frac{2\pi\nu_s n}{m_0}+\delta\right)\right\}^{1/2} \tag{3.3}$$

where $A_0 = \left(\frac{S_0}{\beta_0}\right)^2$; S_0 is the amplitude of oscillations of a particle with the non-equilibrium energy over the azimuth at the interaction point.

The results of the numerical simulation (Fig.3) shows a significant dependence of ξ_c on the modulation amplitude A_0. Taking into account the modulation (3.3) results in appearing the side-band resonances near the separated resonance with the number n. If $A_0 << 1$, one can assume that $\xi \approx \xi_0 (1 + \frac{A_0}{4} \cos \nu_s^* \theta)$; $\nu_s^* = 2\nu_s$. This means that in the first approximation there are only two side-band resonances on each side of ν_n, and the resonance condition takes the form (cf. (1.13) and (2.7)):

$$\nu = \frac{m_0 k \pm 2\nu_s^*}{2n} \qquad (3.4)$$

The amplitude of such a resonance perturbation εV_n will be proportional to $A_0/4$.

At the first glance, such a kind of modulation appears to be not dangerous because of a small number of side-band resonances. However, if one takes into consideration the following approximation (with respect to ε, see (1.6)), then the situation sharply changes. Indeed, the betatron phase equation is of the form: $\dot{\psi} = \nu = \nu_0 + \Delta\nu(a)$ (in the first approximation for $n >> 1$ and far from the integer resonance see /2/). Since the quantity ξ in $\Delta\nu$ is modulated now, this implies the modulation of a betatron phase:

$$\dot{\psi} \approx \nu_0 + (\Delta\nu)_0 \cdot \left[\frac{A_0}{4} \cos \nu_s^* \theta + 1\right] \qquad (3.5)$$

where $(\Delta\nu)_0$ is determined by (1.14) with $\xi = \xi_0$, i.e. we have the modulation described in the previous section. The relation (3.5) can be written in the form similar to (2.5):

$$\mu \approx \mu_0 + \Delta\mu_0 \left(\frac{A_0}{4} \cos \nu_s^* \theta + 1\right)$$
$$\Delta\mu_0 = \frac{2\pi \cdot (\Delta\nu)_0}{m_0} \qquad (3.6)$$

Hence, the parameter A_0 can be expressed through B. Then the critical value A_c at which the side-band resonances overlap the whole frequency interval between the main resonances n and $n+1$ can be estimated:

$$A_c = \frac{1}{2} B_c \cdot \frac{4}{\Delta\mu_0} = \frac{m_0^2}{4n(n+1)(\Delta\nu)_0} \qquad (3.7)$$

In the above relation the estimate (2.10) is used and the fact that the distance between the side-band resonances is two times longer than in the case considered in section 2 is taken into account. Let us compare the estimate (3.7) with numerical data (Fig.3). To do this, just as in section 2, we shall consider the condition of overlapping the main resonances in the range of $x_m = 5$ displacements ($a = 6.25$). From the expression for the frequency shift $(\Delta\nu)_2$ (the case of a band beam) one can find numerically: $(\Delta\nu)_2 \approx m_0 \xi \cdot 0.32$. The value of n is found from the resonance condition (2.7) as usual. For the values of ξ_c and A_0 taken from Fig.3, the ratio of A_c/A_0 (derived according to (3.7)) gives: $A_c/A_0 \approx 0.75 \div 2.7$. The results obtained indicates that the criterion (3.7) gives an overestimated value for A_c. This is quite possible since the dependence (3.6) does not correspond to (3.3), especially at not too small values of A_0. In other words, the estimate (3.7) does not take into account completely all side-band resonances. Nevertheless, the relation (3.7) yields in order of magnitude a correct value for A_c.

A similar modulation occurs when the beams intersect at an angle in the interaction point. This situation always takes place if the beams in the interaction have the same sign as well as it is the case for the beams with opposite signs if they store in different rings. Experimental results obtained at DORIS and DCI facilities show that such a modulation is most likely to contribute considerably to decreasing ξ_c (see, e.g./15,16/). The effects which arise upon the modulation of ξ because of the angular intersection of the beams are numerically and analytically studied in /9/.

4. A joint effect of various modulations

Of interest is to consider the result of a simultaneous effect of a few modulations, since it is the situation that occurs in a real experiment. The results of the numerical simulation with two modulations (2.5) and (3.3) are presented in Fig.4 (curve 1). It is seen that on the initial section ($B \leq 0.003$) the quantity ξ_c is mainly determined by the modulation of a parameter ξ (3.3). Indeed, for the chosen value of the amplitude of this modulation ($A_0 \approx 0.1$) the value of ξ_c is equal to 0.17, as follows from Fig.3. At $B \gtrsim 0.01$ curve 1 coincides with curve 4 obtained at $A_0 = 0$. This indicates that both these kinds of modulation act separately in the sense that ξ_c is determined by a stronger modulation. A certain joint effect presents only within a small intermediate range of values, $0.003 \leq B \leq 0.01$. Approximately the same occurs for a somewhat larger value of A_0, $A_0 = 0.25$ (cf. curves 3 and 4).

The result obtained may be qualitatively explained, starting from the analysis in sections 2 and 3. As has been seen, both modulations are similar, and the main mechanism is the

overlap of additional resonances caused by the phase modulation. The resonance structure in this case is nearly the same, and a simple addition of the amplitudes of resonance harmonics takes place.

For the purpose of a quantitative analysis by means of the analytical estimates derived in sections 2 and 3 we make use of the relation between A_0 and B (3.7). For $x_m = 5$ from (1.14) we have $(\Delta\nu)_2 = m_0 \xi \cdot 0.32$. Therefore, for $A_0 = 0.1$ and $\xi = 0.17$ (curve 1) we obtain $B \approx 0.004$ and for $A_0 = 0.25$ (curve 3) the value of B equals 0.006. It is seen that the found values of B are in good agreement with the intermediate region wherein both modulations gives a comparable effect.

Fig.4 (curve 2) also presents the result of the joint effect of the displacement modulation x (1.7) and the betatron phase modulation (2.5). As follows from the figure, the sum of these modulations leads to a significant decrease of ξ_c as compared to the case of one modulation (2.5).

As an example, let us consider a point corresponding to $B = 0.01$. A modulation of the type (1.7), according to the data of /1/, decreases ξ_c at $B = 0$, $A_s = 0.46$ from $\xi_c = 0.2$ to $\xi_c = 0.1$, i.e. by a factor of 2. The modulation (2.5) at $B = 0.005$ decreases ξ_c from 0.28 to 0.19, i.e. approximately by a factor of 1.5; while the joint effect of these modulations lowers ξ_c to the level of $\xi_c \approx 0.05$, i.e. approximately by a factor of 6. When changing the quantity B this relation changes as well. Hence, in comparison with the case considered above, both these types of modulation decreases the stochasticity threshold independently. This effect becomes clear if one reminds the difference in the resonance structure of a perturbation (see sections 1 and 2). As has been shown, the phase modulation leads to the appearance of side-band resonances near each resonance with number n (see (2.7)). However, with the presence of the modulation of x each of these additional resonances splits in turn according to (1.9-1.10). Of course, this strongly facilitates the overlap of all additional resonances.

5. Concluding remarks

As the experiments carried out at the VEPP-2M facility show, even such a simple one-dimensional model in which the motion along the one transverse coordinate is considered without the connection with the other, is in agreement with the experimental data if determining is the motion along the coordinate on which the beam's phase volume is larger. By the determining direction we understood such a direction in which the stochasticity threshold is achieved somewhat earlier (ξ_{xc} or ξ_{zc} , where x and z are the radial and vertical coordinates). The lifetime of the beam in this case is due to migration of the particles up to large amplitudes just in this direction. Special measurements show that at the storage ring VEPP-2M the radial direction is turned out the determining one. Apparently, it is associated with the presence of a large dispersion function ψ_x which causes the modulation of motion over x and thereby leads to a significant decrease of the critical value of ξ_{xc}. In the conventional operation regime of VEPP-2M the β_x-function at the interaction point is large enough ($\beta_x \simeq 40$ cm). In view of this, the modulation considered in section 3 proves to be insignificant as compared to that caused by the presence of the ψ_x-function (see section 1). Similarly, the influence of a betatron phase (section 2) whose amplitude is inversely proportional to

the β-function at the interaction point (see (2.2)) may be also regarded as a weak one.

It should mention that, nevertheless, below the stochasticity limit the beam size can increase in another direction (along z). This means that a decrease in luminosity and in lifetime is determined, generally speaking, by different effects.

In the case when the vertical motion is determining (e.g. the vertical frequency shift $\Delta\nu_z$ or ξ_z), the one-dimensional approximation, as experimental data show, appears to be insignificant. To study the influence of modulations in this case, both the two-dimensionalness of the beam and the coupling of transverse oscillations must be taken into account.

Also, note that the amplitude of the modulations considered in sections 2 and 3 increases as the beam energy is increased. It is quite possible that just this fact accounts for the absence of that ξ_c does not increase at energies higher than 2 GeV /18/, that has been observed at SPEAR.

Finally, the author wishes to thank A.N.Skrinsky and G.M.Tumaikin for fruitful discussions. The author is also grateful to L.F.Khailo for her assistance in computation.

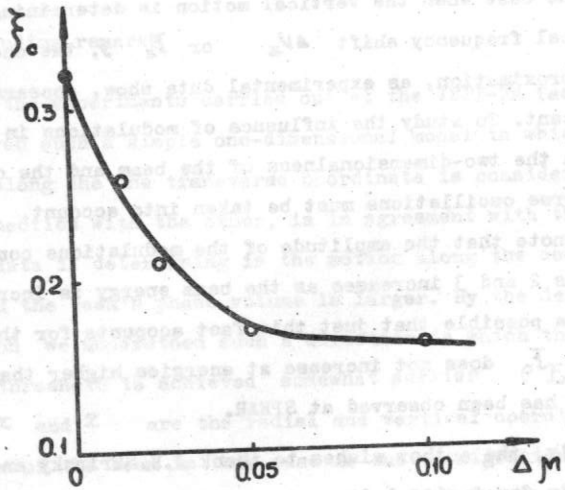

Fig.2. The critical value ξ_c dependence on $\Delta\mu$ (2.4) due to an inaccurate adjustment of the ring elements over the azimuth.

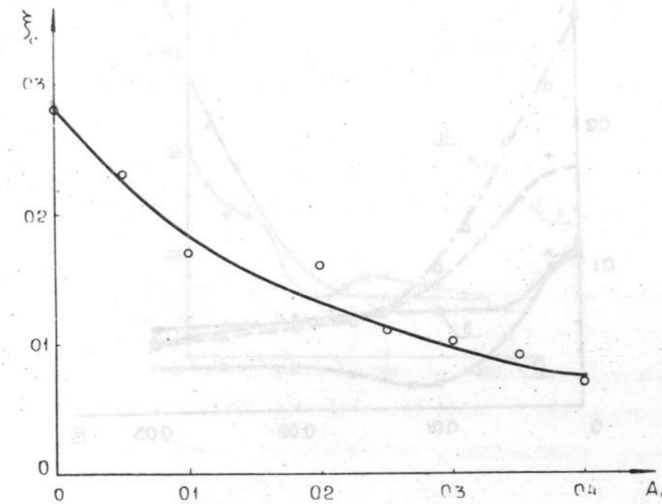

Fig.3. The critical value ξ_c dependence on the amplitude A_0 of the modulation ξ at the interaction point (3.3).

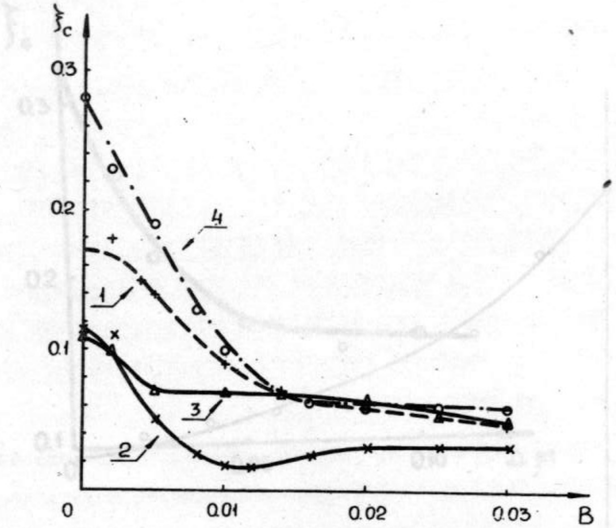

Fig.4. The joint effect of various modulations on ξ_c .
The curve 1 is the modulations (2.5) and (3.3) for A_o = 0.1 (see section 4), the curve 3 is the same for A_o = 0.25; curve 4 is taken from Fig.1 (curve 1) for comparison with curves 1 and 3 (A_o = 0). The curve 2 represents the joint effect of modulations (1.7) and (2.5) at A_S = 0.4 σ.

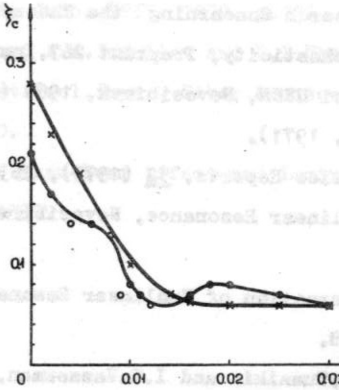

Fig.1. The critical value ξ_c dependence on the (2.5) betatron phase modulation amplitude. The curve 1 represents the band beam (1.3); the curve 2 does so the round beam (1.2).

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