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EXCLUSIVE DECAYS OF HEAVY MESONS

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A B S T R A C T

The method of calculation of the quarkonium decays is described. The corresponding selection rules are obtained. It is shown that using of the π -meson wave function described in the previous paper /4/ allows consistent with experimental data predictions for the widths of the following decays:

$\chi_0(3415)\to\pi^+\pi^-$, $\chi_2(3555)\to\pi^+\pi^-$, $\psi(3100)\to\pi^+\pi^-$, $\psi(3685)\to\pi^+\pi^-$.

The analogous predictions for the Υ -family are presented.

I. Introduction.

The general method of finding the asymptotic behaviour of exclusive processes in QCD has been presented in /1/. The goal of this paper is to apply the methods of papers /1/ to the exclusive decays of the mesons containing the heavy quarks. Two main types of processes belong to this class: a) strong decays of the $\bar{Q}Q$-mesons, for instance $\chi_0 \to \pi^+\pi^-$ (where Q is a heavy quark); b) weak decays of the $\bar{q}Q$-mesons, for instance $D^0(1865) \to K^-\pi^+$ (where q is a light quark).

The main idea of using QCD for the description of the exclusive amplitudes with the large momentum transfer is to separate in the amplitude contributions of the interactions at small and large distances. The small distance contributions can be exactly calculated in terms of the perturbation theory. They determine the main dependence of the amplitude on the asymptotic variables. The large distance contributions can be expressed in terms of the slowly varying hadronic wave functions and enter as coefficients in the asymptotic amplitude.

It is important that the wave function of a given hadron enters into the asymptotic expressions and is the same for the different processes in which this hadron is involved. Therefore, one can verify the possibility of description of various processes with π -mesons with the help of the same π -meson wave function.

In our previous paper /4/ using a method of sum rules a number of first moments of the π -meson wave function has been obtained. Using these results, we have described qualitative behaviour of the π -meson wave function and proposed the model wave function $\varphi(\xi, \mu_0^2)$ which fulfil the sum rules.

In this paper we show that using the model wave function gives the $\pi^+\pi^-$ -decay widths of $\chi_0(3415), \chi_2(3555), \psi(3100)$ and $\psi'(3685)$ which agree with the experimental data.

The same wave function also allows to obtain the satisfactory description of the charmed particle nonleptonic decays: $D^0 \to K^-\pi^+, \bar K^0\pi^0; D^+ \to \bar K^0\pi^+, \pi^0\pi^+$ etc. These results will be described in the next paper /5/.

In Sect.II the general properties of the heavy meson decay amplitudes are described and the corresponding selection rules are obtained. In Sect. III the decays of the charmonium into pions are described. Some properties of the hadronic wave functions are briefly presented in the Appendix (for more details see /2/).

II. General properties and selection rules.

In this Section we describe the formal theory of the heavy quarkonium ($M_q \to \infty$) decays and derive the corresponding selection rules.

The decay of the C-even quarkonium into two light mesons is described by the Fig.1 diagram /2,3/. (Dashed lines show gluons, solid lines correspond to quarks). This diagram (and all other analogous diagrams) must be understood in the following way /1/. The internal quark and gluon lines having the large virtuality ($\sim M_q^2$) correspond to the usual perturbation theory propagators. The external quark and gluon lines having small virtuality remain operators and the matrix element of the type of $\langle 0|\psi_a\psi_b|\text{Meson}\rangle$ (i.e. the hadron wave function) should be taken for each pair of the quark operators. (The definition and properties of the meson wave functions are given in Appendix). The wave function depenus on the quark virtuality through the logarithmic corrections. So the wave function should be taken at the characteristic virtuality point for a given process.

Let us consider as an example the $\pi^+\pi^-$ decay mode of the $^3P_0$ quarkonium level. Using the properties of the wave functions (see Appendix) we obtain:

$$M(^3P_0 \to \pi^+\pi^-) = (4\pi\alpha_s)^2 \frac{64}{27} \varphi_V(0) \frac{f_\pi^2}{4M_q^2} I_0 + O(\mu^2/M_q^4), \tag{1}$$

$$I_0 = \int_{-1}^{1} \frac{d\xi_1\, \varphi_\pi(\xi_1)}{1-\xi_1^2} \int_{-1}^{1} \frac{d\xi_2\, \varphi_\pi(\xi_2)}{1-\xi_2^2} \frac{1}{1-\xi_1\xi_2}\left[1+\frac{1}{4}\frac{(\xi_1-\xi_2)^2}{1-\xi_1\xi_2}\right].$$

Here M_q is a heavy quark mass, μ is the characteristic mass scale for the light hadrons ($\mu \ll M_q$), α_s and the wave functions $\varphi_\pi(\xi)$ are taken at the characteristic virtuality points (for more details see Sect. III). The decay probability is:

$$W(^3P_0 \to \pi^+\pi^-) = \frac{1}{32\pi M_q} |M(^3P_0 \to \pi^+\pi^-)|^2 \tag{2}$$

The total width is found from the diagram shown in Fig.2 and has a form:

$$W_{tot}(^3P_0) = (4\pi\alpha_s)^2 |\varphi_V(0)|^2 \frac{1}{4\pi M_q} \tag{3}$$

Therefore, the branching ratio is independent of $\varphi_V(0)$ *:

$$Br(^3P_0 \to \pi^+\pi^-) = \frac{W(^3P_0 \to \pi^+\pi^-)}{W_{tot}(^3P_0)} = \left| \pi \tilde{\alpha}_s \frac{16\sqrt{2}}{27} \frac{f_\pi^2}{M_Q^2} I_0 \right|^2 . \qquad (4)$$

If the π -meson wave function is known, we can predict the definite value of the branching ratio with the help of (4).

Similarly one can found the probabilities of other types of the quarkonium decays: the decay of the C-odd quarkonium either into two mesons (Fig.3) or into the baryon-antibaryon pair (Fig.4); the decay of the C-even quarkonium either into two gluonium states (Fig.5) or into the baryon-antibaryon pair (Fig.6), etc.

One can see from (4) that the branching ratios for two meson decays decrease as $\sim M_Q^{-4}$ if M_Q increases. Such a behaviour is already evident from the dimensional considerations. The two particle meson wave function has the dimension "μ", so that the branching ratio decreases as $\sim M_Q^{-4}$. The three particle baryon wave function has the dimension "μ^2" and the branching ratio for the $\bar{B}B$-decay decreases as $\sim M_Q^{-8}$.

Let us consider now the selection rules. The existence of the selection rule in our case means that the definite decay modes have additional suppression ($\lesssim \mu/M_Q$) in comparison with the dimensional counting. It is convenient to use the quantum number (naturalness) $\sigma = (-1)^S \cdot P$, where S is the me-

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* The values of α_s in expressions (1) and (3) can be slightly different. That is why we have introduced in formula (4) the effective coupling constant (see Sect.III).

son spin and P is its parity. The "natural particles" have $\sigma = +1 (\rho, f, {}^3S_1, {}^3P_0, {}^3P_2)$ while for "unnatural particles" $\sigma = -1 (\pi, A_1, {}^1S_0, {}^3P_1, {}^1P_1)$. So we can formulate the main selection rule as follows: <u>the naturalness in the heavy quarkonium decays into two light mesons is conserved.</u>* That means that the initial state naturalness is equal to the product of the final mesons naturalnesses. The decay amplitudes depend on M_Q as $\lesssim M_Q^{-5}$ if the selection rule is violated. (Let us note that in the decay ${}^3P_0 \to \pi^+\pi^-$ considered above the naturalness is conserved).

For explaining the origin of the selection rule it is useful to remind some properties of the meson wave functions /1/.

If the quark pair constituting a given meson has the nonzero projection of the orbital momentum on the direction of the meson momentum, so that $L_z \neq 0$, then the contribution of such a state into the decay amplitude will have the additional suppression $\sim (P_\perp/M_Q)^{L_z}$, where $P_\perp$ is the characteristic transverse quark momentum inside the meson. So the main contribution in decay amplitude comes from components of the meson wave function with the meson helicity equal to the sum of the quarks helicities (i.e. $L_z = 0$). For a meson with helicity $\lambda \geq 2$ the angular momentum projection $L_z \geq 1$ and the corresponding quarkonium decay amplitude is obviously suppressed.** For the meson with $\lambda = 0$ (antiparallel quark spins) the po-

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* For simplicity we restrict ourselves here to the S and P-wave quarkonium levels.

** For the gluonium states $L_z \geq 1$ at $\lambda \geq 3$.

larization tensor has a form: $M^{J}\mathcal{E}^{\lambda=0}_{\mu_1\dots\mu_J}(p)\simeq p_{\mu_1}\dots p_{\mu_J}$ at $p\to\infty$. Therefore, its wave function has the same properties as that of a spinless meson (see Appendix). The main contribution to the decay amplitude comes from the following component of the meson wave function

$$\langle 0|\bar\psi(z)\gamma_\mu\psi(-z)|J,\lambda=0,\sigma=+1,p\rangle \to p_\mu\left(M^{J}\mathcal{E}^{\lambda=0}_{\mu_1\dots\mu_J}z^{\mu_1}\dots z^{\mu_J}\right)\varphi_1(zp)+$$

$$+M^{J}\mathcal{E}^{\lambda=0}_{\mu\mu_2\dots\mu_J}z^{\mu_2}\dots z^{\mu_J}\varphi_2(zp)\to \tag{5}$$

$$\to p_\mu\left[(zp)^{J}\varphi_1(zp)+(zp)^{J-1}\varphi_2(zp)\right]\equiv p_\mu\varphi_J(zp)$$

(for the particles with $\sigma=-1$, $\bar\psi\gamma_\mu\psi\to\bar\psi\gamma_\mu\gamma_5\psi$).

For a meson with $|\lambda|=1$ (parallel quark spins) the polarization tensor has a form: $M^{J-1}\mathcal{E}^{|\lambda|=1}_{\mu_1\dots\mu_J}(p)=\{\mathcal{E}^{|\lambda|=1}_{\mu_1}\cdot p_{\mu_2}\dots p_{\mu_J}+perm\}$ so its wave function has the same properties as that of a vector meson with $|\lambda|=1$ (see Appendix). The main contribution to the corresponding decay amplitude give the following component of the meson wave function:

$$\langle 0|\bar\psi(z)\sigma_{\mu\nu}\psi(-z)|J,\lambda=\pm1,\sigma=+1,p\rangle\to\left(p_\mu\epsilon^{\lambda=\pm1}_\nu-p_\nu\epsilon^{\lambda=\pm1}_\mu\right)\varphi_J(zp) \tag{6}$$

(for the particles with $\sigma=-1$, $\sigma_{\mu\nu}\to\sigma_{\mu\nu}\gamma_5$).

For the following considerations it is important that the leading component of the wave functions of the mesons with $\lambda=0$ and $|\lambda|=1$ (5),(6) have opposite chiral parities (i.e. the parities with respect to γ_5 -transformation).

Now it is easy to show that the selection rule is just that mentioned above. If the initial and final states naturalnesses have opposite signs the decay amplitude includes the unit an-

tisymmetric tensor $\epsilon_{\mu\nu\lambda\sigma}$. What quantities can the tensor indices be contracted with? We have two meson momenta p_1 and p_2, two light meson polarization tensors and one quarkonium polarization tensor. If mesons have zero helicities ($\lambda_1=\lambda_2=0$), their polarization tensors reduce to their momenta and the decay amplitude equals zero. If $\lambda_1=0$ and $|\lambda_2|=1$ the leading contribution comes from the wave functions (5),(6) and decay amplitude equals zero due to the chirality conservation (the trace over the spinor indices equals zero in the diagrams shown in Figs. 1,3 in the massless quark limit; the nonzero light quark mass gives the correction $\sim m/M_q$).

Let us consider now the decay into two mesons with the helicities $\lambda_1=\lambda_2=1$.

a) Natural C-even quarkonium, Fig.1 ($^3P_0$, $^3P_2$). The corresponding decay amplitude has a form: $M\sim\rho_{\mu\nu}\langle\hat p_2\hat\epsilon_2^{\,1}\gamma_\mu\hat p_1\hat\epsilon_1^{\,1}\gamma_\nu\gamma_5\rangle$ where $\rho_{\mu\nu}$ is the amplitude of the quarkonium decay into two virtual gluons. Since the meson momenta p_1, p_2 are orthogonal to ϵ_1, ϵ_2 and $p_1^2\sim p_2^2\sim 0$, the parts of amplitude $\rho_{\mu\nu}\sim g_{\mu\nu}$, $\rho_{\mu\nu}\sim p_{1\mu}p_{2\nu}$ and $\rho_{\mu\nu}\sim p_{2\mu}p_{1\nu}$ give zero contribution. Therefore, the leading term in the decay amplitude of the $^3P_0$ state is equal to zero. There is the additional term $\rho_{\mu\nu}\sim$ $\sim\epsilon^{\lambda=0}_{\mu\nu}\sim(\epsilon^1_\mu\epsilon^{-1}_\nu+\epsilon^1_\nu\epsilon^{-1}_\mu)$ in the case of the $^3P_2$ -state decay. Its contribution is equal to zero as well, because the trace $\langle\dots\rangle$ is antisymmetric under the permutation $\mu\leftrightarrow\nu$

b) Unnatural C-even quarkonium, Fig.1 ($^1S_0$, $^3P_1$). The corresponding decay amplitude has a form: $M\sim\rho_{\mu\nu}\langle\hat p_2\hat\epsilon_2^{\,1}\gamma_\mu\hat p_1\hat\epsilon_1^{\,1}\gamma_\nu\rangle$ As the projection of the total angular momentum is conserved, $\lambda=S_z=\lambda_1-\lambda_2=0$. The polarization vector of the quarkonium with $\lambda=0$ is: $\epsilon^{\lambda=0}_\mu\sim(p_1-p_2)_\mu$. So for the $^1S_0$ and

$^3P_1$ states we have: $\rho_{\mu\nu} \sim \epsilon_{\mu\nu\lambda\sigma} p_1^{\lambda} p_2^{\sigma}$. The trace $\langle \ldots \rangle$ in this case is symmetric under permutation $\mu \leftrightarrow \nu$: $\langle \ldots \rangle \sim$ $\sim (\epsilon_2^{\mu} \epsilon_1^{\nu} + \epsilon_2^{\nu} \epsilon_1^{\mu})$. So, the leading contribution to the decay amplitude is equal to zero.

c) Similar considerations give for the C-odd quarkonium ($^3S_1$, $^1P_1$) the result $M = 0$ (see Fig.3).

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Only the $^3P_2$ -state can decay into two mesons with $\lambda_1 - \lambda_2 = \pm 2$. The total amplitude has the form: $M \sim \rho_{\mu\nu} \langle \hat{p}_2 \hat{\epsilon}_2^{\lambda 1} \gamma_\mu \hat{p}_1 \hat{\epsilon}_1^{\lambda 1} \gamma_\nu \gamma_5 \rangle$. The amplitude $\rho_{\mu\nu}$ has the form: $\rho_{\mu\nu} \sim \epsilon_{\mu\nu}^{\lambda = \pm 2} \sim \epsilon_\mu^{\pm 1} \epsilon_\nu^{\pm 1}$, while the trase $\langle \ldots \rangle$ is antisymmetric in $\mu \leftrightarrow \nu$ So, $M = 0$.

We have shown, therefore, that the above stated selection rule is fulfi-led for all S- and P-wave quarkonium decays.

It is surprising at first sight that the $^1S_0$ -state decay amplitudes into two massless gluons (Fig.2) or two nearly massless ρ -mesons with $|\lambda_{1,2}| = 1$ (Fig.1) have different properties. The last amplitude is suppressed while the first is not. The reason is that the gluon wave function is $\epsilon_\mu^{|\lambda| = 1}$, while the expression $[p_\mu, \epsilon_\nu^{|\lambda| = 1}]$ plays the role of the ρ-meson wave function there.

We give below the examples of the allowed and suppressed decays.

a) $^1S_0 \not\to \rho\rho, A_1A_1, B(1235)B, \ldots$ $\quad ^1S_0 \to \rho B, \delta(980)\pi, A_1A_2, \ldots$

b) $^3S_1 \not\to \rho\pi, \rho A_1, \delta B, \ldots$ $\quad ^3S_1 \to \pi B, \rho A_2, \delta\rho, \ldots$

c) $^3P_1 \not\to \rho\rho, \pi A_1, A_1A_1, \ldots$ $\quad ^3P_1 \to \rho B, \pi A_2, \delta A_1, \ldots$

d) $^3P_0 \not\to \pi A_2, \rho B, A_1A_2, \ldots$ $\quad ^3P_0 \to \pi\pi, \rho\rho, \pi A_1, \ldots$

e) $^3P_2 \not\to \pi A_2, \rho B, \delta A_1, \ldots$ $\quad ^3P_2 \to \pi\pi, \rho\rho, \pi A_1, \ldots$

f) $^1P_1 \not\to \rho\delta, \rho A_2, A_1B, \ldots$ $\quad ^1P_1 \to \pi\rho, BA_2, \rho A_1, \ldots$

Additional selection rules can exist in each given case besides the above stated one. For instance, the decay of $^3P_0$ state (or $^3P_2$ -state with $\lambda = 0$) into two mesons with the same naturalness and with $|\lambda_{1,2}| = 1$ is suppressed.

Now consider shortly the role of the resonances in the 3-particle quarkonium decays. The diagrams in Fig.7 describe the contribution to the 3-particle decay of the kinematical region where $(p_i p_j) \sim M_q^2$, $i,j = 1,2,3$ (p_i are the light meson momenta). This contribution to the decay amplitude is $\sim O(M_q^{-4})$ and gives the branching ratio $\sim (\mu/M_q)^6$. Let us remind that the two particle branching ratio is $\sim (\mu/M_q)^4$. It is clear now that the kinematical region where the mass of the $(M_i M_j)$ pair is small will give the dominant contribution to the 3-particle decay amplitude at $M_q \to \infty$. In other words, the 3-particle decay proceeds mainly in such a way, that at first stage one final meson and one resonance are produced. Then this resonance decays into two final mesons, Fig.8. Evidently, these considerations are applicable also to the decays into more than three particles.

In conclusion of this Section we want to stress that all the above results are exact at the formal limit $M_q \to \infty$ only. However, the real value of the C-quark mass is not very large. Therefore, the values of other quantities, in particular the meson wave function width, can play a significant role. For example, the contribution of the diagram in Fig.3 into the $\psi(3100) \to \rho\pi$ decay is suppressed at $M_c \to \infty$, while there is no suppression in the case $\psi(3100) \to B(1235)\pi$ decay. This suppression is not large really ($\sim m_\rho/E_\rho \sim m_\rho/M_c \sim 1/2$ in the amplitude). Therefore, if the ρ -meson wave function width is larger than that of B-meson, then the integral analogous I_o is larger for the ρ -meson case and so the $\psi \to \rho\pi$ decay rate can exceed that of $\psi \to B\pi$ *.

We expect, nevertheles, that the selection rules will already work well for the Υ -family ($M_b \simeq 4.7\,Gev.$).

III. Charmonium decays.

In this Section we apply the general method to the description of the charmonium decays. The main difficulty is of course due to a small C-quark mass ($M_c \simeq 1.5\,Gev.$). It seems at first sight that the charmonium mass is large enough ($\bar{M}^2 \simeq 10$ Gev^2). However, the thing is that the large light meson momenta divide into the smaller quark momenta and it is the latter that determine the propagator virtualities at diagrams in Figs.1,4,7,... For instance, the mean gluon virtuality in Fig.1 diagram is: $\bar{q}_L^2 \simeq \bar{x} \cdot \bar{y} \cdot M^2$. Here M is the charmonium mass, X and Y are the longitudinal momentum fractions carried by two light quarks. If the probability to find the quark with small momentum fraction ($x, y << 1$) is not very small, then $\bar{q}_L^2 <<$ M^2. If $\bar{q}_L^2$ is too small, then the operator expansion is useless. The characteristic values of X,Y are determined by the form of the meson wave function.

Our main assumption is that we can restrict ourselves to the leading terms only and to neglect the power corrections

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* Besides, the diagram in Fig.9 corresponding to the 3-particle light meson wave function can play the role in the $\Psi(3100)$ decays. Although the contribution of this diagram includes power suppression, it is the Born diagram however, while the diagram in Fig.3 has suppression due to the loop ($\sim \frac{\alpha_s}{\pi} \simeq 0.1$).

when applying the scheme of Sect.II to the description of the charmonium decays into $\pi(K)$ -mesons. Within this assumption we show that using the model π -meson wave function from the paper /4/ leads to the good description of the charmonium decays.

1. The branching ratio for the decay $\chi_0(3415) \to \pi^+\pi^-$ is given by the formula (4), where now $M_\psi = M_c \simeq 1.5\,Gev.$ For the case of the $\chi_2(3555) \to \pi^+\pi^-$ decay the analogous formulae are:

$$W(\chi_2 \to \pi^+\pi^-) = \frac{1}{240\pi M_c} \left| (4\pi\alpha_s)^2 \frac{32}{27} \frac{f_\pi^2}{4M_c^2} \chi_V(0) I_2 \right|^2 \quad (7a)$$

$$W_{tot}(\chi_2) = \frac{1}{45\pi M_c} (4\pi\alpha_s)^2 |\chi_V(0)|^2 \quad (7b)$$

$$Br(\chi_2 \to \pi^+\pi^-) = \left| \pi\tilde{\alpha}_s \frac{8\sqrt{3}}{27} \frac{f_\pi^2}{M_c^2} I_2 \right|^2 \quad (7c)$$

$$I_2 = \int_{-1}^{1} \frac{d\xi_1 \varphi_\pi(\xi_1)}{1-\xi_1^2} \int_{-1}^{1} \frac{d\xi_2 \varphi_\pi(\xi_2)}{1-\xi_2^2} \frac{1}{1-\xi_1 \cdot \xi_2} \left[1 - \frac{1}{2} \frac{(\xi_1-\xi_2)^2}{1-\xi_1 \xi_2} \right]^2 \quad (8)$$

Everywhere below we use the following strong coupling constant:

$$\alpha_s(\mu^2) = \frac{4\pi}{b} \frac{1}{\ln \mu^2/\Lambda^2}, \quad b = 9, \quad \Lambda \simeq 100\,Mev. \quad (9)$$

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* (continued from page 12) The above formulated selection rule is not applicable to the contributions like those of the diagram in Fig.9.

Let us point also that the rormulae (1),(3),(4),(7) are written down in a somewhat conditional form because the arguments of α_s are not indicated there. (That is why we have introduced the effective value $\tilde{\alpha}_s$ in the formulae (4),(7c)). We take α_s at the point: $\alpha_s = \alpha_s(M^2/4 \simeq 3\,GeV^2) \simeq 0.24$ in the expressions (3),(7b) for the total widths. This point corresponds to the case when each of two gluons in Fig.2 carries one half of the total energy. In the formulae (1),(7a) we take α_s' at the point corresponding to the gluon virtualities (Fig.1): $\alpha_s'\left(q_1^2 = \frac{1-\xi_1}{2}\cdot\frac{1-\xi_2}{2}M^2\right)$ and $\alpha_s'\left(q_2^2 = \frac{1+\xi_1}{2}\cdot\frac{1+\xi_2}{2}M^2\right)$. Then we extract them from the integrals over ξ_1 and ξ_2 replacing $q_{1,2}^2$ by its characteristic values $\bar{q}_{1,2}^2$, the later being determined by the wave function form.

In order to show to what extent the wave function form influences the values of the branching ratios $\chi_{0,2} \to \pi^+\pi^-$, we take the narrow wave function: $\varphi_n(\xi) \simeq \delta(\xi)$. This wave function describe the case when each of two quarks carries one half of the π -meson momentum. For this wave function: $\alpha_s(\bar{q}_1^2 = M^2/4) = \alpha_s(\bar{q}_2^2 = M^2/4) \simeq 0.24$, $\tilde{\alpha}_s \simeq 0.24$ and the integrals I_0 in (1) and I_2 in (8) are equal: $I_0 = I_2 = 1$. We have now from the formulae (4),(7), $f_\pi \simeq 133\,MeV$:

$$Br(\chi_0 \to \pi^+\pi^-) \simeq 2.5\cdot 10^{-3}\%, \quad Br(\chi_2 \to \pi^+\pi^-) \simeq 10^{-3}\%.$$

The experimental numbers are:

$$Br(\chi_0 \to \pi^+\pi^-) = (1 \pm 0.3)\% \ /6/$$
$$Br(\chi_2 \to \pi^+\pi^-) = (0.15 \pm 0.07)\% \ /6/ \qquad (10)$$
$$Br(\chi_2 \to \pi^+\pi^-) = (0.21 \pm 0.11)\% \ /7/.$$

Therefore, any narrow wave function, i.e. the wave function which gives $I_0 \simeq I_2 \simeq O(1)$, evidently contradicts to the experimental data.*

We use now the model wave function proposed in the paper /4/:

$$\varphi(\xi, \mu_0^2 \simeq (500\,MeV)^2) = \frac{15}{4}\xi^2(1-\xi^2). \qquad (11)$$

The function (11) has maxima at the points $\xi \simeq \pm 0.7$. So the characteristic gluon virtualities in Fig.1 are: $\bar{q}_1^2 \simeq (500\,MeV)^2$, $\bar{q}_2^2 \simeq 9\,GeV^2$ and correspondingly $\alpha_s(\bar{q}_1^2) \simeq 0.43$ and $\alpha_s'(\bar{q}_2^2) \simeq 0.20$, $\tilde{\alpha}_s = 0.43\cdot 0.20/0.24 \simeq 0.36$.

As for the π -meson wave functions virtualities μ^2, they are determined mainly by the propagator with the smallest virtuality in Fig.1: $1/\mu^2 \simeq 1/\bar{q}_1^2 + 1/\bar{q}_2^2 + 1/\bar{\sigma}^2 \simeq 1/\bar{q}_1^2$, i.e. $\mu^2 \simeq (500\,MeV)^2$. So we can substitute into I_0 (1) and I_2 (8) just the wave function (11). As a result we have ($M_c \simeq 1.5\,GeV$, $f_\pi = 133\,MeV$)

$$Br(\chi_0 \to \pi^+\pi^-) \simeq 1.1\%, \quad Br(\chi_2 \to \pi^+\pi^-) \simeq 0.24\% \qquad (12)$$

(In this case: $I_0 \simeq 14$, $I_2 \simeq 11$!). These numbers agree with the experimental data (10).

2. The $\psi(3100) \to \pi^+\pi^-$, $\psi'(3865) \to \pi^+\pi^-$ decay probabilities can be expressed through the π -meson wave function as well. These decays are forbidden in the exact isotopic symmetry limit and

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* The analogous calculation with the asymptotic wave function $\varphi(\xi, \mu^2 \to \infty) \equiv \varphi_\infty(\xi) = \frac{3}{4}(1-\xi^2)$ gives:

$$Br(\chi_0 \to \pi^+\pi^-) \simeq 3\cdot 10^{-2}\%, \quad Br(\chi_2 \to \pi^+\pi^-) \simeq 0.7\cdot 10^{-2}\%$$

are due to the electromagnetic interaction.* There is a number of contributions to the decays $\psi(\psi') \to \pi^+\pi^-$,Figs.10-13. The diagram in Fig.11 includes a small factor $\sim \frac{\alpha_s}{\pi} \sim 0.1$ due to the loop,as compared with that in Fig.10. The diagram in Fig.13 takes into account the isotopic symmetry breaking in the meson wave function,i.e. the appearance of the antisymmetric in "ξ" part of the wave function (see Appendix). It is small also for the same reason. It is impossible at present to make a reliable estimate of the contribution of the diagram in Fig.12 which corresponds to the 3-particle component of the pion wave function. We can note only that this contribution includes a power suppression $\sim (\mu/M_c)^2$ in the amplitude,where μ is some characteristic mass for the 3-particle pion wave function. It may be also that the 3-particle pion wave function is narrow,while the 2-particle one is wide (see (11)). Then the 3-particle wave function contribution will have an additional suppression. In any case,according to our main assumption,the power corrections like those of Fig.12 digram contribution would not play a significant role.

Therefore,the decays $\psi, \psi' \to \pi^+\pi^-$ are determined mainly by the Fig.10 diagram contribution and can be expressed in terms of the pion electromagnetic form factor:

$$Br_e(\psi, \psi' \to \pi^+\pi^-) \equiv \frac{W(\psi, \psi' \to \pi^+\pi^-)}{W(\psi, \psi' \to e^+e^-)} = \frac{1}{4} F_\pi^2(M^2_{\psi,\psi'}) \qquad (13)$$

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* The contribution due to $m_d - m_u \neq 0$ into the decay amplitudes is small in comparison with that of the diagram in Fig.10.

$$M^2 F_\pi(M^2) \to \frac{32}{9} \pi \alpha_s f_\pi^2 |I|^2, \quad I = \int_{-1}^{1} \frac{d\xi}{1-\xi^2} \varphi_\pi(\xi, \mu^2). \qquad (14)$$

The experimental data agree with the ρ -meson contribution: $M^2 F_\pi(M^2) \simeq m_\rho^2 \simeq 0.6\ GeV^2$ and are

$$M_\psi^2 F_\pi(M_\psi^2 = 9.6\ GeV^2) = (0.7 \pm 0.3)\ GeV^2;\ M_{\psi'}^2 F_\pi(M_{\psi'}^2 \simeq 13.9\ GeV^2) = (0.7 \pm 0.4)\ GeV^2 \qquad (15)$$

Substituting into (14) the wave function $\varphi(\xi) = \delta(\xi)$ one obtains (in this case $\alpha_s = \alpha_s(k^2 = \frac{1-\xi_1}{2} \cdot \frac{1-\xi_2}{2} M_\psi^2) = \alpha_s(\frac{1}{4} M_\psi^2) \simeq 0.2$, $I = 1$):

$$M_\psi^2 F_\pi(M_\psi^2) \simeq 0.04\ GeV^2 \qquad (16)$$

that contradicts to the experimental data.*

The characteristic virtuality $\bar{k}^2$ in Fig.10 for the wave function (11) is: $\bar{k}^2 \simeq (500\ MeV)^2$ and so in (14) $\alpha_s = \alpha_s(500\ MeV) \simeq 0.43$. Using this value α_s and the wave function (11) one has:

$$M_\psi^2 F_\pi(M_\psi^2) \simeq M_{\psi'}^2 F_\pi(M_{\psi'}^2) \simeq 0.53\ GeV^2 \qquad (17)$$

that agrees with (15). (In this case $I \simeq 2.5$).

3. Let us consider shortly the properties of the K-meson wave function and the role of SU(3)-symmetry breaking. The K-meson wave function is even in "ξ" and coincides with the π -meson one in the exact SU(3)-symmetry limit. For the true K-meson wave function one has:

$$\int_{-1}^{1} d\xi\, \tilde{\varphi}_K(\xi, \mu^2) = f_K \simeq 1.25 f_\pi \simeq 165\ MeV \qquad (18)$$

---

* Using of the wave function $\varphi_{as} = \frac{3}{4}(1-\xi^2)$ gives:

$$M_\psi^2 F_\pi(M_\psi^2) \simeq 0.1 \div 0.2\ GeV^2$$

and besides SU(3)-symmetry breaking leads to the appearance of the ξ -odd part in the wave function: $\varphi_K(\xi) = \varphi_K^+(\xi) + \varphi_K^-(\xi)$ *

In a number of papers the tendency appears to choose the K-meson wave function in the form: $\tilde{\varphi}_K(\xi) = (f_K/f_\pi)\tilde{\varphi}_\pi(\xi)$.

One would have for such a function

$$\frac{Br(\chi_0 \to K^+K^-)}{Br(\chi_0 \to \pi^+\pi^-)} = \left(\frac{f_K}{f_\pi}\right)^4 \simeq 2.5$$

that contradicts to the experimental data /6/. The reason is clear: the SU(3)-symmetry breaking changes not only the wave function normalization (i.e. $f_\pi \to f_K$) but its shape as well. So the integral I_0 in formula (1) has different values in the π and K-meson case.** The equality of the π and K-meson widths (19) shows that the effect of $f_K > f_\pi$ is compensated by the change of the wave function form.

The experimental data for the $\psi, \psi' \to K^+K^-$ decays are:

$$Br(\psi \to K^+K^-) = (2.8 \pm 0.8)10^{-4}, \quad Br(\psi \to \pi^+\pi^-) = (1.0 \pm 0.5)10^{-4} \tag{20}$$

$$Br(\psi' \to K^+K^-) = (1.3 \pm 0.8)10^{-5}, \quad Br(\psi' \to \pi^+\pi^-) = (0.7 \pm 0.5)10^{-5}.$$

---

* The even orders in the SU(3)-symmetry breaking Hamiltonian give contribution to the even part of the K-meson wave function $\varphi_K^+(\xi)$ only as well as the odd ones contribute in to the $\varphi_K^-(\xi)$.

** The contribution of the function $\varphi_K^-(\xi)$ into the integral I_0 has the same effect as $f_K > f_\pi$. The main change of the value I_0 is due to a change of the shape of the wave function $\varphi_K^+(\xi)$ (~~in the second order in the SU(3)-symmetry breaking~~).

Inspite of the fact that the experimental errors are large, it seems that there is indication to the difference between the π - and K-meson widths. We want to point out that this difference can have the natural explanation, so that (19),(20) do not contradict to each other. The reason is that $W(\psi,\psi' \to K^+K^-) \neq 0$ is due to the SU(3)-symmetry breaking while $W(\psi,\psi' \to \pi^+\pi^-) \neq 0$ is due to the isotopic symmetry breaking. In the last case, as was pointed out above, the main contribution comes from the diagram in Fig.10. It may be not the case for the decays $\psi, \psi' \to K^+K^-$. Let us give the rough estimate of the diagram in Fig.14 contribution in comparison with that of the diagram in Fig.10:

$$\frac{(Fig.14)}{(Fig.10)} \sim \frac{(4\pi\alpha_s)^2 \frac{\alpha_s}{\pi} \varphi_V(0) f_K^2 I^{+-}}{(4\pi\alpha)(4\pi\alpha_s)\varphi_V(0) f_K^2 I^{++}} \simeq \frac{\alpha_s^2 I^{+-}}{\pi\alpha I^{++}} \sim 5\frac{I^{+-}}{I^{++}} \simeq 0.5 \div 1.$$

Here: $\alpha = 1/137$, $\alpha_s \simeq 0.3$, I^{++} and I^{+-} are the corresponding integrals of the SU(3)-even and the SU(3)-odd wave functions $\varphi_K^+(\xi)$, $\varphi_K^-(\xi)$ correspondingly. To obtain the better estimate one needs to know the wave function $\varphi_K^-(\xi)$.*

4. In conclusion of this Section we give the predictions for the $\pi^+\pi^-$ -decay widths of the Υ -family. Because $M_b^2 \simeq 10 M_c^2 \simeq 22.5\ GeV^2$ all the characteristic scales increase by a factor of $\simeq 10$. The wave function (11) should be replaced by the following one:

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* The diagram in Fig.9 can be also significant. See the foot-note on the page 12.

$$\varphi(\xi, \mu^2 = 25\,GeV^2) = \frac{15}{4}(1-\xi^2)\left\{\left(\frac{\alpha_s(25\,GeV^2)}{\alpha_s(0.25\,GeV^2)}\right)^{0.617}\left(\xi^2 - \frac{1}{5}\right) + \frac{1}{5}\right\} = \tag{21}$$

$$\simeq \frac{15}{4}(1-\xi^2)(0.72\,\xi^2 + 0.06).$$

The effective coupling constant $\tilde{\alpha}_s$ is now:

$$\tilde{\alpha}_s = 0.25 \cdot 0.16 / 0.19 \simeq 0.21 .$$

As a result:

$$\frac{W(^3P_0(\bar{b}b) \to \pi^+\pi^-)}{W(^3P_0(\bar{b}b) \to hadr)} \simeq 1.8 \cdot 10^{-3}\,\% ; \quad \frac{W(^3P_2(\bar{b}b) \to \pi^+\pi^-)}{W(^3P_2(\bar{b}b) \to hadr)} \simeq 0.43 \cdot 10^{-3}\,\% . \tag{22}$$

(For the wave function (21): $I_0 \simeq 9.7$, $I_2 \simeq 8$). Instead of (17) we have now:

$$M_\Upsilon^2 F_\pi(M_\Upsilon^2) \simeq 0.24\ GeV^2 \tag{23}$$

$$Br\left(\frac{\Upsilon \to \pi^+\pi^-}{\Upsilon \to e^+e^-}\right) \simeq 1.8 \cdot 10^{-6} \tag{24}$$

It is seen from (22)-(24) that the two-particle widths are very small.

IV. Conclusions.

We have described above the properties of the heavy meson exclusive decays. In the previous paper /4/ the properties of the π -meson wave function were elucidated, so here the main attention was put on to π -meson decay modes.

The main result of this paper together with the paper /4/ is as follows. The π -meson wave function (11) satisfying the sum rules /4/ gives at the same time good description

of the charmonium decays. That shows that the whole approach is selfconsistent.

The main characteristic property of the wave function (11) is that it is "wide" /4/. Just this property of the wave function leads to the good agreement of the theoretical calculations and the experimental data. (The narrow wave function $\varphi_\pi(\xi) \simeq \delta(\xi)$ gives the results smaller than the experimental results by a factor $\sim 10^{-2}$).

It will be shown in the next paper /5/ that as soon as we take into account the property of the π -meson wave function described above, the importance of the gluon effects in the charmed meson decays ($D^0 \to K^-\pi^+$, $D^0 \to \bar{K}^0\pi^0$ etc.) becomes clear.

We are indebted to A.I.Vainshtein and E.V.Shuryak for useful discussions.

Appendix.

Bellow the definition and some properties of the hadron wave functions used in the text are given.

a) K-meson.

The "two-particle" wave function is defined as follows:

$$\Phi_{\alpha\beta}(z,p) = \langle 0 | \bar{S}_\beta(z) \exp\left\{ ig \int_{-z}^{z} d\sigma_\mu B_\mu(\sigma) \right\} U_\alpha(-z) | K^+(p) \rangle \equiv \tag{A1}$$

$$\equiv \langle 0 | \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \bar{S}_\beta(0) (z_\mu \overleftrightarrow{D}_\mu)^n U_\alpha(0) | K^+(p) \rangle .$$

Here: α, β are the spinor indices, S and U are the quark field operators, B_μ is the gluon field operator, $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu$, $\overrightarrow{D}_\mu = \overrightarrow{\partial}_\mu - ig B_\mu$, $\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + ig B_\mu$, the sum over the color indices is implied in (A1). (In what follows we do not write out explicitly the gluonic exponent in the wave functions). It is convenient to represent $\tilde{\Phi}_{\alpha\beta}$ in the form:

$$4\tilde{\Phi}(z,p) = -i\hat{p}\gamma_5\varphi_A - \hat{z}\gamma_5\tilde{\varphi}_2 - i\gamma_5\tilde{\varphi}_p - \sigma_{\mu\nu}\gamma_5 p^\mu z^\nu \tilde{\varphi}_T,$$

$$\langle 0|\bar{s}(z)\gamma_\mu\gamma_5 u(-z)|K^+(p)\rangle = ip_\mu \tilde{\varphi}_A(zp,z^2) + z_\mu(zp)\tilde{\varphi}_2(zp,z^2),$$

$$\langle 0|\bar{s}(z)\gamma_5 u(-z)|K^+(p)\rangle = -i\tilde{\varphi}_p(zp,z^2), \tag{A2}$$

$$\langle 0|\bar{s}(z)\sigma_{\mu\nu}\gamma_5 u(-z)|K^+(p)\rangle = (z_\mu p_\nu - z_\nu p_\mu)\tilde{\varphi}_T(zp,z^2),$$

$$\langle 0|\bar{s}(z)\, u(-z)|K^+(p)\rangle = \langle 0|\bar{s}(z)\gamma_\mu u(-z)|K^+(p)\rangle = 0,$$

$$\sigma_{\mu\nu} = \tfrac{1}{2}[\gamma_\mu,\gamma_\nu].$$

The leading contribution is given by the function $\varphi(zp,z^2) \simeq \varphi(zp,0)$ defined by the matrix element of the lowest twist bilocal operator $\bar{s}(z)\gamma_\mu\gamma_5 u(-z)$ at the light cone $z^2 \simeq 0$, because in all the cases considered in the text $p_\mu \sim M$, $z_\mu \sim 1/M$, $z^2 \sim 1/M^2$, $zp \sim 1$, $M \to \infty$. The other components of the K-meson wave function give the power corrections $\lesssim O(\mu/M)$. In the exact SU(3)-symmetry limit the wave functions $\tilde{\varphi}_i$ are symmetric under the replacement $(zp) \to -(zp)$. When SU(3)-symmetry breaking effects are taken into account, the wave functions $\tilde{\varphi}_i$ acquire an antisymmetric in zp part. The function $\tilde{\varphi}(\xi)$ defined as

$$\tilde{\varphi}_A(zp) = \int_{-1}^{1} d\xi\, e^{i\xi(zp)}\tilde{\varphi}_A(\xi) \tag{A3}$$

has a simple meaning. In the reference frame in which the K-meson momentum $P_z \to \infty$, two quarks have the longitudinal momenta xP_z and $(1-x)P_z$. The quantity ξ is: $x-(1-x) = 2x-1$, $0 \le x \le 1$. Therefore, the wave function $\tilde{\varphi}(\xi)$ describes the longitudinal momentum distribution for the quarks.

The wave function $\tilde{\varphi}_A(\xi)$ overall normalization is determined by the matrix element $\langle 0|\bar{s}(0)\gamma_\mu\gamma_5 u(0)|K^+\rangle = ip_\mu f_K$ which is known from the weak decay $K \to \mu\bar{\nu}$: $\tilde{\varphi}_A(zp=0) = \int_{-1}^{1} d\xi\, \tilde{\varphi}_A(\xi) = f_K$. (For the π -meson the analogous constant is $f_\pi \simeq 133\,MeV$). It is not difficult to find out the normalization of the wave functions $\tilde{\varphi}_p(zp)$, $\tilde{\varphi}_T(zp)$ using the QCD equations of motion:

$$\varphi_p(0) = 3\varphi_T(0) \simeq f_K\left(\frac{m_K^2}{m_u+m_s}\right) \simeq f_K(1.8\,GeV). \tag{A4}$$

Here m_u and m_s are the current quark masses:

$$m_s \simeq 150\,MeV,\quad m_u \simeq 4\,MeV,\quad m_d \simeq 7\,MeV.$$

We use in the text the dimensionless wave function $\varphi_A(\xi)$

$$\varphi_A(zp=0) = \int_{-1}^{1} d\xi\, \varphi_A(\xi,\mu^2) = 1. \tag{A5}$$

The π - and K-meson wave functions have the same general properties, the only difference is that the π -meson wave function $\varphi_\pi(\xi)$ is a symmetric function in the exact isotopic symmetry limit.

b) ρ -meson.

The two-particle wave function has the form:

$$\langle 0|\bar{d}(z)\gamma_\mu u(-z)|\rho^+(p)\rangle = \epsilon_\mu m_\rho f_\rho \varphi_{1V}(zp,z^2) + ip_\mu(\epsilon z) m_\rho f_{2V}(zp)\varphi_{2V} +$$
$$+ z_\mu(\epsilon z)\, m_\rho^2 f_{3V}\varphi_{3V},$$

$$\langle 0|\bar{d}(z)\gamma_\mu\gamma_5 u(-z)|\rho\rangle = \epsilon_{\mu\nu\lambda\sigma} p^\nu z^\lambda \epsilon^\sigma m_\rho^2 f_A \varphi_A \tag{A6}$$

$$\langle 0|\bar{d}(z)\, u(-z)|\rho\rangle = (\epsilon z)\, m_\rho^2 f_S \varphi_S$$

$$\langle 0|\bar d(z)\sigma_{\mu\nu} u(-z)|\rho\rangle = (p_\mu \epsilon_\nu - p_\nu \epsilon_\mu) f_T\, \varphi_T(zp, z^2) +$$

$$+ i(z_\mu \epsilon_\nu - z_\nu \epsilon_\mu)\, m_\rho^2 f_{2T}\cdot(zp)\cdot\varphi_{2T} + (p_\mu z_\nu - p_\nu z_\mu)\cdot(\epsilon z)\cdot m_\rho^2 f_{3T}\varphi_{3T} \quad \text{(A6)}$$

$$\langle 0|\bar d(z)\gamma_5 u(-z)|\rho\rangle = 0 .$$

Here ϵ_μ is the ρ -meson polarization vector, m_ρ is its mass. All functions $\tilde\varphi_i$ in (A6) are symmetric under the replacement $zp \to -zp$ in the exact isotopic symmetry limit. For the case of the ρ -meson helicity $\lambda = 0$ the main contribution is given by the functions φ_{1V} and φ_{2V} and for the $|\lambda| = 1$ case that of $\tilde\varphi_T(zp)$. (Let us remind that at $p \to \infty$, $m_\rho \epsilon_\mu^{\lambda=0}(p) \sim p_\mu(1 + O(m/p))$, $m_\rho \epsilon_\mu^{\lambda=\pm 1} \sim m_\rho)$. The constant f_ρ which is analogous to f_π and f_K is known from the $\rho^0 \to e^+e^-$ decay:

$$\tilde\varphi_{1V}(zp=0) = \int_{-1}^{1} d\xi\, \tilde\varphi_{1V}(\xi) = f_\rho \simeq 1.5 f_\pi \simeq 200\, MeV. \quad \text{(A7)}$$

The wave function of any meson with the helicity $\lambda = 0$ is analogous to this of the K-meson and the wave function of any meson with the helicity $|\lambda| = 1$ is analogous to this of the ρ -meson. Taking into account the $|\lambda| \geq 2$ two quark wave function contributions gives the power corrections only /1/.

c) Heavy meson wave function.

The $^3P_0$ -state wave function has the form:

$$\Phi_{\alpha\beta}(z, p) = \langle 0|\bar C^a_\beta(z) C^a_\alpha(-z)|^3P_0\rangle . \quad \text{(A8)}$$

(The summation over the color is implied). The expansion of the expression (A8) into the power series in Z is at the same time the expansion in powers of the relative momentum of quarks. The relative momentum is small in the nonrelativistic case: $P_{rel} \sim v/c\, M_q \ll M_q$, so one can restrict himself to the first nontrivial order in Z in (A8).* In this approximation the $^3P_0$ -wave function components have the form:

$$\langle 0|\bar Q(z) Q(-z)|^3P_0\rangle \simeq i\varphi_S(0), \quad \text{(A9)}$$

$$\langle 0|\bar Q(z)\gamma_\mu Q(-z)|^3P_0\rangle = p_\mu (zp)\cdot\varphi_V(0) + z_\mu \varphi_{2V}(0)$$

$$\langle 0|\bar Q(z)\sigma_{\mu\nu} Q(-z)|^3P_0\rangle \simeq (p_\mu \epsilon_\nu - p_\nu \epsilon_\mu)\varphi_T(0).$$

Using the QCD equations of motion it is not difficult to connect four constants in (A9) with each other. As a result the $^3P_0$ -state wave function has the form:

$$\langle 0|\bar Q_\beta(z) Q_\alpha(-z)|^3P_0(p)\rangle \simeq \frac{1}{4}\Big\{ i\hat p(zp) - i4M_q^2\hat z - \quad \text{(A10)}$$

$$- 6M_q - 2iM_q \sigma_{\mu\nu} p_\mu z_\nu \Big\}_{\alpha\beta} \cdot \varphi_V(0) .$$

Analogously one can find the wave functions of other heavy mesons.

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* The wave function moments $\langle \xi^n \rangle$ are the coefficients in the expansion of the formula (A8) into Z series and are proportional to the matrix elements $\langle 0|\bar Q \overleftrightarrow{D}_{\mu_1} \ldots \overleftrightarrow{D}_{\mu_n} Q|^3P_0\rangle$. Because the relative momentum is small, the heavy meson wave function is narrow, its width is $\sim v/c \ll 1$ So:

$$\langle \xi^n \rangle = \int_{-1}^{1} d\xi\, \xi^n \varphi_{3P_0}(\xi) \sim \left(\frac{v}{c}\right)^n \int_{-1}^{1} d\xi\, \varphi_{3P_0}(\xi) .$$

i) ${}^3P_2$ -state:

$$\langle 0|\bar{Q}_\beta(z)Q_\alpha(-z)|{}^3P_2(p)\rangle \simeq \frac{1}{4}\left\{\hat{\rho}+\sigma_{\mu\nu}\rho_\mu p_\nu\right\}_{\alpha\beta}\cdot\chi_\nu(0), \tag{A11}$$

$$\rho_\mu = \epsilon_{\mu\nu} z_\nu ,$$

where $\epsilon_{\mu\nu}$ is the ${}^3P_2$ -meson polarization tensor.

ii) ${}^3S_1$ -state:

$$\langle 0|\bar{Q}_\beta(z)Q_\alpha(-z)|{}^3S_1\rangle \simeq \frac{1}{4}\left\{\hat{\epsilon}\,2M_q-\sigma_{\mu\nu}\epsilon_\mu p_\nu\right\}_{\alpha\beta}\varphi(0), \tag{A12}$$

where ϵ_μ is the ${}^3S_1$ -meson polarization vector.

iii) ${}^1S_0$ -state:

$$\langle 0|\bar{Q}_\beta(z)Q_\alpha(-z)|{}^1S_0\rangle \simeq \frac{1}{4}i(\hat{P}\gamma_5)_{\alpha\beta}\,\varphi_z(0)-\frac{i}{2}M_q(\gamma_5)_{\alpha\beta}\,\varphi_q(0). \tag{A13}$$

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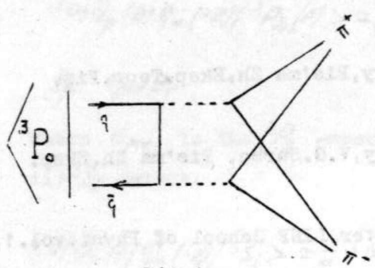


Fig.1

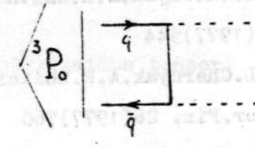
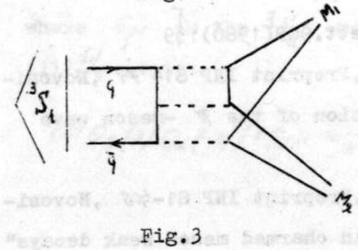


Fig.2



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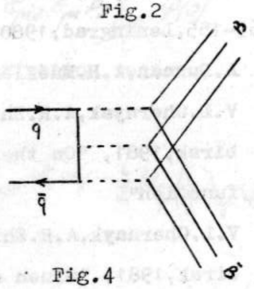

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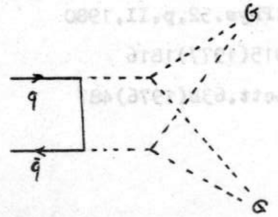

Fig.5

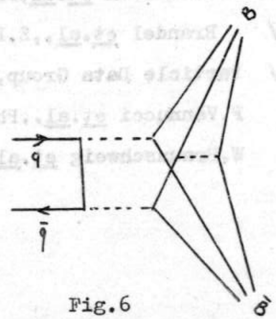

Fig.6

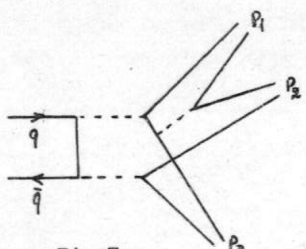

Fig.7a

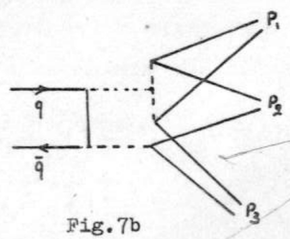

Fig.7b

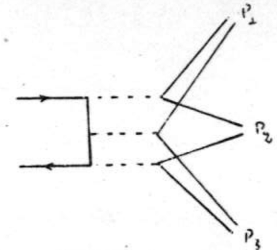

Fig.7c

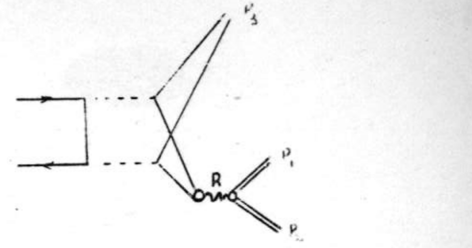

Fig.8

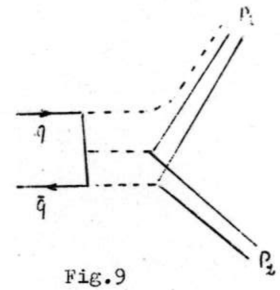

Fig.9

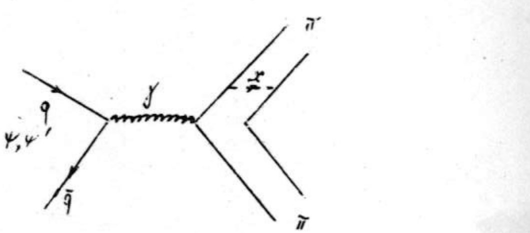

Fig.10

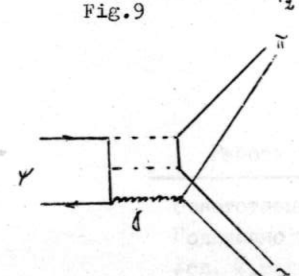

Fig.11

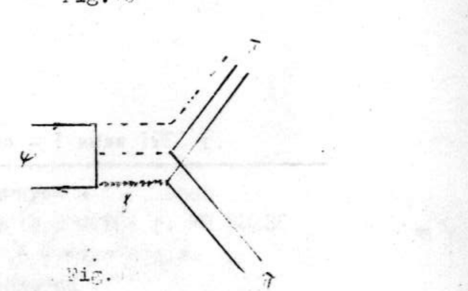

Fig.

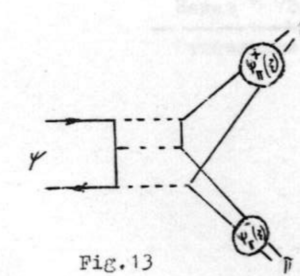

Fig.13

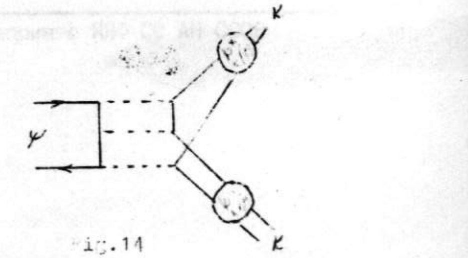

Fig.14