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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
СО АН СССР

Yu.I.Skovpen'

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ПРЕПРИНТ 81 - 32

Новосибирск

TO THE QUESTION ON THE $\Sigma^+ \to p\gamma$ DECAY AMPLITUDE

Yu.I.Skovpen'

Institute of Nuclear Physics
630090, Novosibirsk 90, USSR

A b s t r a c t

The contribution of barion resonances to the $\Sigma^+ \to p\gamma$ decay amplitude is calculated in the framework of the bag model. The amplitude obtained is not consistent with its experimental value.

1. Introduction

The weak radiative decays of hyperons with $\Delta S = 1$ are of considerable interest since they can provide information on the nonleptonic weak interactions. By now the rate of $\Xi^0 \to \Lambda\gamma$ and $\Sigma^+ \to p\gamma$ decays as well as the asymmetry parameter for $\Sigma^+ \to p\gamma$, which characterizes the correlation between the proton momentum and the spin of the Σ^+, have been experimentally measured. A lot of theoretical papers are devoted to the calculation of the asymmetry parameter and rate of the $\Sigma^+ \to p\gamma$ decay (see the cited references in /1/). It is known from the experiment /1/ that

$$\frac{\Gamma(\Sigma^+ \to p\gamma)}{\Gamma(\Sigma^+ \to p\pi^0)} = (2.26 \pm 0.26)\cdot 10^{-3}$$

$$\alpha(\Sigma^+ \to p\gamma) = -0.70^{+0.31}_{-0.27} \tag{1}$$

As for the decay rate of $\Sigma^+ \to p\gamma$, it is apparently a well established quantity but the asymmetry parameter has been measured within poor accuracy and varies from experiment to experiment.
The most general form of the Hamiltonian, which describes the $\Sigma^+ \to p\gamma$ decay, is as follows:

$$H = \frac{i}{2} e \bar{U}_p (A + C\gamma_5) \sigma_{\mu\nu} F_{\mu\nu} U_{\Sigma^+} \tag{2}$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor. The decay rate Γ and the asymmetry parameter α are expressed via A and C as follows:

$$\Gamma = \frac{e^2 \omega^3}{\pi} (A^2 + C^2)$$

$$\frac{d\Gamma}{d\Omega} \sim [1 + \alpha(\vec{s}\hat{p})] \tag{3}$$

$$\alpha = 2\,\frac{Re(A^* C)}{A^2 + C^2}$$

where ω is the photon energy, $\vec{s}$ is the spin of Σ^+, $\hat{p}$ is the unit vector along the momentum direction of the proton.

The calculation based on the single-quark transition $s \to d\gamma$ (Fig. 1) /2/ gives rise to the $\Sigma^+ \to p\gamma$ amplitude which is one order of magnitude less than the experimental one. The contribution of barion resonances to the amplitude decay under consideration is shown in Figs. 2 and 3. The cross is used to denote the weak interaction inside a hadron. As shown in the quark diagrams (Figs. 4,5,6), it is due to the weak two-quark interaction and to the interaction of the quark with the gluon field of a hadron. To calculate their contribution to the effective Hamiltonian (2), it is necessary to make use of a model of hadrons because these diagrams refer to long distances. In Ref. /3/ the contribution of baryon resonances to the P-odd amplitude has been calculated in the framework of the bag model. The result obtained in /3/ is in good agreement with the experiment.

Note that the problem with the matching between the calculated value of the $\Sigma^+ \to p\gamma$ decay amplitude and its experimental value is a very serious one. Indeed, one of the attempts to introduce the right-handed currents was motivated by the aim to obtain the $\Sigma^+ \to p\gamma$ decay amplitude coinciding with the experimental one. In the present paper the P - even and P - odd amplitudes determined by graphs 4 and 5 are calculated within the bag model. The value obtained is of the same order of magnitude as the single-quark amplitude. The bag's parameters have been taken the same as in /3/. The value of the P - odd amplitude is distinguished from that obtained in Ref. /3/ by one order of magnitude.

2. The contribution of baryon resonances

Before calculating the contribution of baryon resonances to the $\Sigma^+ \to p\gamma$ decay amplitude a bag model has to be briefly described.

The quarks in the bag are described by the Dirac equation

$$(-i\gamma\partial + m_\alpha)\psi_\alpha = 0 \qquad (4)$$

Confinement of the quarks is provided by the boundary conditions

$$in\gamma\psi = \psi$$
$$\Sigma\, n\,\partial(\bar{\psi}\psi) = 2B \qquad (5)$$

where n is the unit vector normal to the bag's surface, B is the bag's constant. The bag is assumed to be spherically-shaped. The solution of equation (4) has the form:

$$\psi_{jlm} = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} \sqrt{E+m}\; j_{|\varkappa+\frac{1}{2}|-\frac{1}{2}}(\beta r)\,\Omega_{jlm} \\ -\sqrt{E-m}\; j_{|\varkappa-\frac{1}{2}|-\frac{1}{2}}(\beta r)\,\Omega_{jl'm} \end{pmatrix} \qquad (6)$$

where $\varkappa = (-1)^{j-l+\frac{1}{2}}(j+\frac{1}{2})$, j is the total moment, l is the orbital moment, $l' = 2j - l$, Ω_{jlm} is the spherical spinor, $j_\varkappa(\beta r)$ is the Bessel function, $\beta = \sqrt{E^2-m^2}$, E and m are the energy and mass of a quark, respectively. If the quarks have $j = \frac{1}{2}$, then from the normalization condition

$$\int_0^R \psi^+\psi\, d^3r = 1 \qquad (7)$$

one finds

$$N^{-2} = \frac{R}{E+\varkappa m}\, j_0^2 \left[2ER(ER+\varkappa) + mR\right] \qquad (8)$$

where R is the bag radius. Conditions (5) in this case reduce to the following equations:

$$tg(\beta R) = \frac{\varkappa\beta R}{ER - \varkappa m R + \varkappa}$$
$$\sum_i \frac{(E_i R + \varkappa)\beta_i^2 R^2}{2E_i R(E_i R + \varkappa_i) + m_i R} = 2\pi R^4 B \qquad (9)$$

where $\sum_i$ is the sum over all the quarks in a hadron.

The effective Hamiltonian of the weak two-quark interaction (ΔS = 1) with taking into account the strong interactions is of the form /4/:

$$H_W = \sqrt{2}\, G \sin\theta_c \cos\theta_c \left[C_+ O_+ + C_- O_-\right]$$

$$O_\pm = \bar s_L \gamma_\mu u_L \bar u_L \gamma_\mu d_L \pm \bar s_L \gamma_\mu d_L \bar u_L \gamma_\mu u_L$$

$$C_+ = x^{-\frac{6}{25}}, \quad C_- = x^{\frac{12}{25}} \tag{10}$$

$$x = 1 + \frac{25}{3}\frac{1}{4\pi}\ln\frac{m_W^2}{\mu^2}$$

where θ_c is the Cabibbo angle, m_W is the mass of the W - boson, $\mu \sim m_\pi \div m_p$ is the infrared parameter. In the Hamiltonian (10) the strong interactions are taken into account under the assumption that the $SU(4)$ symmetry is valid up to μ. The more precise calculation, when the intermediate scale m_c (m_c is the mass of the c - quark) /5/ is introduced, gives rise to appearing the operators with right-handed currents. However, they appear with small coefficients /5/, and in diagrams 4 and 5 they are not enhanced in comparison with the operators without right-handed currents. In this connection, we make use of the effective Hamiltonian (10) in our calculation.

Let us proceed now to a calculation of the $\Sigma^+ \to p\gamma$ decay amplitude. The main contribution to the P - even amplitude is determined by the intermediate states Σ^+ and p. The resulting formula for A is as follows:

$$A = \sqrt{2}\, G \sin\theta_c \cos\theta_c\, x^{\frac{12}{25}}\, I_{qq,qs} \times$$

$$\times\left\{\frac{10}{9}\left[\frac{\mu_{qq}}{E_{\Sigma^+}-E_p} + \frac{\mu'_{qq}}{E_p - E_{\Sigma^+}}\right] + \left[\frac{\mu_{qq}}{E_{\Sigma^+}-E_p} + \frac{\frac{2}{3}\mu'_{qq} + \frac{1}{3}\mu'_{ss}}{E_p - E_{\Sigma^+}}\right]\right\} \tag{11}$$

The integral determining the weak P-even two-quark interaction has the form:

$$I_{ij,kl} = \int \frac{N_i N_j N_k N_l}{(4\pi)^2}\, d^3 r \times$$

$$\times\left[\sqrt{\varepsilon_i + m_i}\sqrt{\varepsilon_j + m_j}\, j_0(\beta_i r) j_0(\beta_j r) + \sqrt{\varepsilon_i - m_i}\sqrt{\varepsilon_j - m_j}\, j_1(\beta_i r) j_1(\beta_j r)\right] \tag{12}$$

$$\times\left[\sqrt{\varepsilon_k + m_k}\sqrt{\varepsilon_l + m_l}\, j_0(\beta_k r) j_0(\beta_l r) + \sqrt{\varepsilon_k - m_k}\sqrt{\varepsilon_l - m_l}\, j_1(\beta_k r) j_1(\beta_l r)\right]$$

where i, j, k, l are the quarks in the $S_{1/2}$ state. An expression for the magnetic dipole transition is given by the following formula:

$$\mu_{ij} = \frac{1}{\omega}\int \frac{N_i N_j}{4\pi}\, d^3 r\, j_1(\omega r) \times$$

$$\times\left[\sqrt{\varepsilon_i + m_i}\sqrt{\varepsilon_j - m_j}\, j_0(\beta_i r) j_1(\beta_j r) + \sqrt{\varepsilon_i - m_i}\sqrt{\varepsilon_j + m_j}\, j_0(\beta_j r) j_1(\beta_i r)\right] \tag{13}$$

Indices q and s in (11) stands for the u-, d- and s-quarks, respectively; μ and μ' are the magnetic quark moments for the $N-N^*$ and $\Sigma-\Sigma^*$ transitions, respectively. In calculating (11) with the bag parameters taken from the paper /6/, one finds:

$$A = 0.3 \cdot 10^{-8}\,\frac{1}{GeV} \tag{14}$$

The contributions from the other resonances may be neglected.

The contribution to the P-odd amplitude is due to the $\frac{1}{2}^-$ states. From experiment there are known nonstrange resonances, S'_{11} (1535), S''_{11} (1700), S'_{31} (1650), and one strange resonance S''_{11} (1750). The bag model predicts /7/ three nonstrange states, N_a (1327), N_b (1275), Δ (1362), and three strange states, Σ_1 (1445), Σ_a (1517), Σ_b (1473). The contribution of the $\frac{1}{2}^-$ resonances to a decay amplitude depends strongly on that what numbers are employed. It is clear that the experimental numbers are more preferable, then $E_{\Sigma^+} - E_{N^*} \approx -0.440$ GeV with an accuracy of about 20% and $E_p - E_{\Sigma^*} \approx -0.810$ GeV. Neglecting the difference in energies of the N^* resonances and of the Σ^*, we have:

$$\frac{\langle\Sigma^+|H_W|N^*\rangle\langle N^*|H^\gamma|P\rangle}{E_{\Sigma^+}-E_{N^*}} + \frac{\langle\Sigma^+|H^\gamma|\Sigma^*\rangle\langle\Sigma^*|H_W|P\rangle}{E_p - E_{\Sigma^*}} = \tag{15}$$

$$= \frac{\langle\Sigma^+|H_W H^\gamma|P\rangle}{E_{\Sigma^+}-E_{N^*}} + \frac{\langle\Sigma^+|H^\gamma H_W|P\rangle}{E_p - E_{\Sigma^*}}$$

that significantly simplifies calculations (H^γ is the Hamiltonian of the γ - quant radiation). With the use of (15) one finds the P-odd amplitude which is due to the $\frac{1}{2}^-$ states:

$$C = -\sqrt{2}\, G \sin\theta_c \cos\theta_c\, x^{\frac{12}{25}} \times$$

$$\times \left[\frac{\frac{1}{3} I'_{qq,qs} d'_{ss} + \frac{2}{3} I'_{qq,sq} d'_{qq}}{E_p - E_{\Sigma^*}} - \frac{I'_{qs,qq} d_{qq}}{E_{\Sigma^*} - E_{N^*}} \right] \quad (16)$$

If i, j, k are the quarks in the $S_{\frac{1}{2}}$ state and l in the $P_{\frac{1}{2}}$ state, then $I_{ij,kl}$ - the integral characterizing their P-odd interaction is derived from $I_{ij,kl}$ (12) by the substitution $m_l \to -m_l$. N_l is replaced for the relevant value of the normalization coefficient for the $P_{\frac{1}{2}}$ state. The formula for an electric dipole transition is of the form

$$d_{ij} = \frac{1}{\omega} \int \frac{N_i N_j}{4\pi} d^3 r \Big[\Big(\sqrt{\varepsilon_j - m_j} \sqrt{\varepsilon_i + m_i}\, j_0(\beta_i r) j_0(\beta_j r) - \quad (17)$$

$$- \frac{1}{3} \sqrt{\varepsilon_j + m_j} \sqrt{\varepsilon_i - m_i}\, j_1(\beta_i r) j_1(\beta_j r) \Big) j_0(\omega r) -$$

$$- \frac{2}{3} \sqrt{\varepsilon_j + m_j} \sqrt{\varepsilon_i - m_i}\, j_1(\beta_i r) j_1(\beta_j r)\, j_2(\omega r) \Big]$$

where indices i, j refer to the quarks in the $S_{\frac{1}{2}}$ and $P_{\frac{1}{2}}$ states, respectively; d and d' are the dipole electric moments for the $N - N^*$ and $\Sigma - \Sigma^*$ transitions, respectively. In calculating (16) one finds:

$$C = -0.5 \cdot 10^{-8} \frac{1}{GeV} \quad (18)$$

The contribution from the higher resonances may be neglected. As has been mentioned in Introduction, the value of the P-odd amplitude obtained in the present paper is distinguished from the results in Ref. /3/. This disagreement is due to not only to the use of different energies for the N^* and Σ^* states. If we neglect the difference in energies of the strange and of the non-strange resonances, then it obviously follows from (2) and (15) that the contributions of N^* and Σ^* to the P-odd amplitude have opposite signs. In our paper the contributions of N^* and Σ^* have the opposite signs but they are the same signs in /3/.

In addition, the contributions obtained in /3/ and our paper do not coincide in their magnitude.

3. Conclusion

The quark-gluon interaction with $\Delta S = 1$ due to diagram 7 calculated in /5/. It is seen from estimation of graph 6 that its contribution may be neglected. The magnitude of diagram 8 is proportional to $\ln \frac{m_q^2}{m_{\pi,K}^2}$, i.e. goes to infinity when $m_K = m_\pi = 0$. In practice, this logarithm is small. Although the contributions from various graphs 8 to the amplitude are about $10^{-7} \cdot \frac{1}{GeV}$, the strong compensations take place.

The author is indebted to I.B.Khriplovich who has stimulated the presented work and to O.P.Sushkov for discussions.

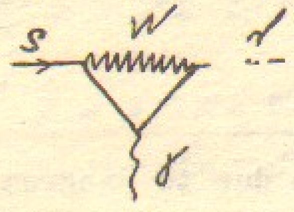

Fig. 1

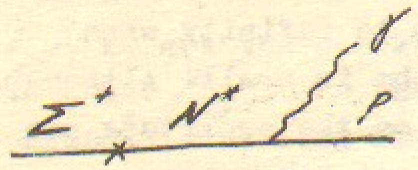

Fig. 2

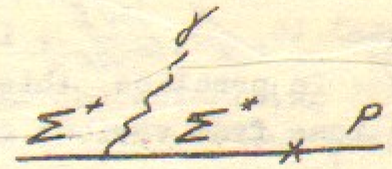

Fig. 3

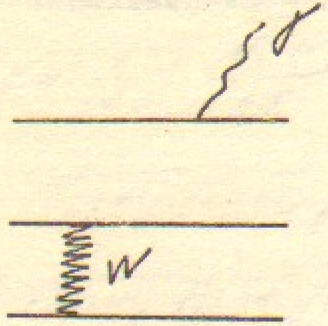

Fig. 4

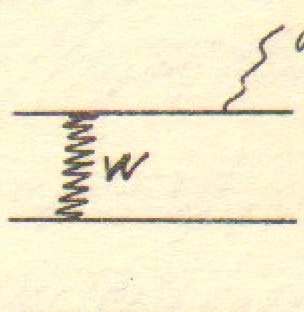

Fig. 5

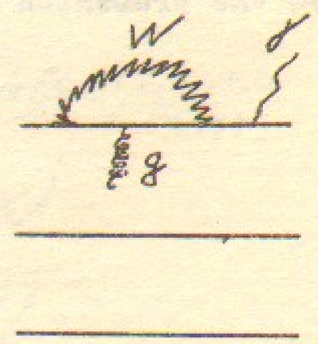

Fig. 6

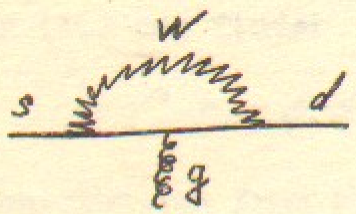

Fig. 7

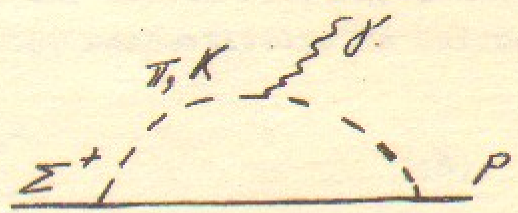

Fig. 8

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