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HADRON-NUCLEUS COLLISIONS.

II. COLLECTIVE MODELS.

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A b s t r a c t

Two kinds of collectivity: the interaction of projectile with collective states of target and the final state interactions of secondaries are considered in detail. The former one may be responsible for cumulative particle production, while the latter one influences strongly on the composition of secondaries, produced in central region. The crucial experiments are considered.

## 1. Introduction

In our previous paper [1] we have considered some most popular applications of cascade description for hadron-nucleus collisions. Let us remind, that the main advantage of this approach is the reduction of complex hadron-nucleus interaction process to a more simple sum of hadron-nucleon subcollisions. Its applicability implies essentially, that the characteristics of particle production process in each hN-subcollision are not affected by the presence of other subcollisions<sup>1)</sup>. In such a way they can be extracted from independent measurements, e.g. from the data on particle production off free nucleons. As a result, one has a number of parameter-free quantitative predictions, which can be easily tested by experiment.

Unfortunately, the multiparticle production process is most probably a rather soft one. At present accelerator energies the sizes of production volume exceed the internucleon distance in nuclei, and more than one nucleon can be involved in the same production process. By other words, the mutual influence of subcollisions may be very essential and one should expect for quite new collective phenomena to appear.

Up to now there is no clear answer for a question, whether any collectivity does really take place. The point is, that the most studied characteristics like the  $A$ -dependence of multiplicity of secondaries, the rapidity shift of their spectra toward the nucleus fragmentation region, etc. are as well explainable [3,4] within the pure cascade picture. Moreover, some attempts are known [5] to explain the production of cumulative hadrons by their multiple intranuclear rescatterings.

An additional difficulty is that the detailed characteristics of the collective processes in question are quite unknown, since the description of such soft hadronic interactions is still an unsolved problem. Any phenomenological input is only a qualitative idea, and in contrast to the cascade picture

<sup>1)</sup> Note, that the same approach underlies the hard scattering models (see, e.g. Ref. [2]), where the process of parton hard scattering occurs at distances being very small in comparison with the interparton ones. In these models the hadronization of scattered partons is assumed to be unaffected by the hadronization of other partons.

it allows only qualitative estimates rather than quantitative ones. In particular, the presence of any "noncollective background", being able to imitate the collectivity manifestation, makes the verification of collectivity quite unreliable.

In order to avoid this problem in our work we consider mainly those of collectivity manifestations, which either have no "background" at all, or the possible "background" has a qualitatively behaviour, being incompatible with data.

All the effects considered below suggest one of two possible underlying collective phenomena:

(i) the collision of projectile with the collective (quantum) state of intranuclear nucleons; below we refer it as the "initial state interactions" (sect.2), and

(ii) the mutual interactions of secondary hadrons, produced in different  $hN$ -subcollisions, the "final state interactions", resulting in formation of collective system (Sect. 3).

The influence of the initial state interactions must be most essential in the nucleus fragmentation region. Usually it is considered in connection with the "cumulative" hadron production. In the Frankfurt-Strikman model [6] the target collective state under consideration contains a fast nucleon (moving toward the projectile), while, in the flucton model [7] the corresponding target state is the local fluctuation of intranuclear density. In sects. 2.1-2 the predictions of both models for cumulative  $K, \bar{p}$ -production off deuterons are shown to be a clear test to choose between these models.

A very popular phenomenological idea known as "coherent tube model" (see, for a review, Ref. [8]) is considered in sect.2.3. It assumes, that the whole "tube" of nucleons, lying on the way of projectile, is a quantum state, which interact with projectile like a single "effective hadron". Both the grounds and applicability limits of this idea are not yet understood. We demonstrate, that this model, as well as its recent developments [9] have serious troubles in the whole kinematical region.

The final state interactions are expected to be most im-

portant in the central region, where the density of secondaries is maximal. There are known many theoretical attempts to describe them. The oldest one is presented by the Landau hydrodynamical model (LHM) [10], which assumes the interaction to be so strong that both the projectile and nucleons of "tube" appear to be involved in the common collective (hydrodynamical) process from the very beginning of collision. Recent studies [11-14] have shown, that LHM provides a reasonable description not only for the most general features<sup>2)</sup>, like the total multiplicity and rapidity distribution of secondaries, but for cumulative and high- $p_t$  hadron production. An interesting attempt to account for the recoil nucleons and leading particle effects has been developed in Ref. [15], where the hadronic cluster under consideration is in many respects very close to the hydrodynamical system, considered in LHM.

However, the most of the abovementioned features can be reproduced also within the pure cascade approach [3-5], and therefore the existence of collectivity requires for more reliable test. We expect that such a test comes from study of particle composition of secondaries. In sect. 3.1 we demonstrate that its experimentally observed A-dependence can hardly be compatible with any pure cascade approach. In sect. 3.2 we show, that the same data can be described by a simplest parameter-free version of LHM. At last, in sect. 3.3 we discuss briefly the possibility, that the process is not collective from the very beginning, as it is assumed by LHM, but the collectivity arises at the final stage of cascade process, assumed by conventional additive quark model [3,4].

The results of this paper are summarized in sect. 4, while in sect. 5<sup>we</sup> formulate the main results of the whole study including also our previous paper [1].

<sup>2)</sup> Let us note, that LHM is probably the only theoretical model, which had predicted the main features of  $hA$ -collisions long before the corresponding data became available.

## 2. Initial state interactions

When the projectile collides the nucleus, it meets some collective quantum states of interacting nucleons rather than a simple set of free ones. The role of such quantum fluctuations is most important in the nucleus fragmentation region, where they can be responsible, for instance, for the cumulative hadron production.

The simplest example of such fluctuations is the Fermi motion of nucleons, resulting in some Doppler widening of secondary particle spectra. However, its influence is rather small and can be neglected in comparison with other nuclear effects.

Another but likely more important phenomena are the multi-nucleon correlations at small distances ( $r \sim 1$  fm). There are known two extreme ways [6,7] to treat their influence on (cumulative) hadron production.

The model [6], proposed by Frankfurt and Strikman, considers such correlations as a high-momentum component of intranuclear motion of nucleons, being some generalization of Fermi motion. It assumes, that such correlated nucleons keep their individuality and their momentum distribution is governed by the short-range behaviour of internucleon potential. An additional assumption, concerning the production mechanism, is that each of such correlated nucleons interacts with projectile and contributes to the cumulative particle production independently, i.e. cascade approach is valid.

In contrast to this picture, the flucton model (FM) [7] assumes that the whole configuration of correlated nucleons interacts with projectile as a single "effective hadron". In order to calculate the particle production off such an "effective hadron" it assumes also the validity of quark counting rule [16].

Each of these two models describes well the  $p, \pi$ -production. In sect. 2.1-2 we demonstrate, that their predictions for cumulative  $K, \bar{p}$ -production off deuterons differ drastically and can be a good test for underlying mechanism.

A very popular idea is known as the "coherent tube model"

(CTM) [8]. In contrast to FM, which consider the rare intranuclear fluctuations, CTM implies that the whole "tube" of nucleons lying on the way of projectile interacts with it as a single "effective hadron". Despite the external attractivity of this idea, its grounds are quite unclear and may be wrong [11]<sup>3)</sup> Below (sect. 2.3) we demonstrate that <sup>it</sup> is unable to reproduce the current data on  $K, \bar{p}$ -production.

### 2.1 Intranuclear motion of nucleons (IMN)

This effect deforms the particle spectra mainly in the nucleus fragmentation region. In particular, it results in particle production in the so called "cumulative region", which is kinematically forbidden off free nucleons. Since IMN manifests itself completely in collision with lightest nucleus, we restrict our consideration by the deuteron case, where the rescattering and absorption effects are small and, probably, can be neglected.

The momentum distribution of IMN can be divided into two components [6]:  $p \lesssim p_F \sim 0.3$  GeV/c and  $p > p_F$ , which are the "usual" Fermi motion and high momentum component (HMC), respectively. The former component is the motion in the collective intranuclear potential. The latter one corresponds to the short-range interaction of nucleons at distances comparable with the radius of nucleon "core"  $r_0 \sim 0.5$  fm. Although the relative weight of this component is small (for deuteron  $w_{HMC} \sim 4-5$  per cent), its influence on the cumulative hadron production can be very important [6].

Let us consider the hadron-deuteron collision in antilab. system:  $D + h \rightarrow h' + X$ . In this case the cumulative hadron production corresponds to  $X_h > 0.5$ , where  $X_h \equiv P_{h'}/P_D$ . Assuming that only one of the deuteron nucleons takes part in the production of  $h'$ , while another nucleon is only a "spectator", one

<sup>3)</sup> Some arguments, given in Ref. [17] reflect only the growth (with energy) of particle formation length, therefore they are not sufficient basis for CTM. Moreover, these arguments, being supplemented by the parton dynamics, result in the qualitatively different quark-parton cascade model. At last, CTM looks very doubtful in high- $p_t$  and cumulative regions, where these arguments fail completely.

comes to the formulae [6]:

$$E \frac{dN}{d^3p}(X, p_{\perp}) \Big|_{D+h \rightarrow h} = \sum_{N=p,n} \int d^3k |\Psi(k)|^2 E \frac{dN}{d^3p} \left( \frac{X}{\alpha}, p_{\perp} - \frac{X}{\alpha} k_{\perp} \right) \Big|_{N+h \rightarrow h} \quad (2.1)$$

here  $\alpha = 1/2 (1 + K_{\parallel} / \sqrt{K^2 + M_N^2})$ ,  $\Psi(k)$  is the deuteron wave function and spectra  $E \frac{dN}{d^3p} \Big|_{N+h \rightarrow h}$  can be taken from independent direct measurements. Thus eq. (2.1) provides predictions with no free parameters.

In our estimates we use the simplest parametrization of data [18] on proton fragmentation  $pBe \rightarrow K^+, \bar{p}$ :

$$E \frac{dN}{d^3p} \Big|_{pBe \rightarrow h} = \exp [C_0 + C_1 \cdot \ln(1-X) - (p_{\perp}^2 + m_h^2)^{1/2} / T] \quad (2.2)$$

where  $T = 0.14$  GeV and fitted parameters  $C_0, C_1$  are shown in Table 1. We neglect completely the target dependence of proton fragmentation, assuming  $E \frac{dN}{d^3p} \Big|_{pp \rightarrow h} \approx E \frac{dN}{d^3p} \Big|_{pBe \rightarrow h}$ ; besides we assume for neutron fragmentation  $E \frac{dN}{d^3p} \Big|_{np \rightarrow h} = \alpha \cdot E \frac{dN}{d^3p} \Big|_{pp \rightarrow h}$ , where from the additivity of valence quarks one has for  $h = K^+$  and  $\alpha = 1$  for  $h = K^-, \bar{p}$ . Concerning the deuteron wave function we use the modified Hamada-Johnston one [19].

The estimated yield and slope parameter  $B$ , defined by

$$E \frac{dN}{d^3p} \propto \exp(-B \cdot p_{\perp}^2)$$

are plotted in Fig. 1. Dotted lines correspond the Fermi motion only (with cutoff in eq. (2.1) at  $|k| = 0.3$  GeV/c), while the dashed ones include the contribution of HMC too.

Although the relative weight of HMC is small ( $\sim 5$  per cent), its contribution dominates over the Fermi motion at  $X \geq 0.4-0.5$ . But the most interesting feature is, however, the considerable broadening of  $p_{\perp}$ -distribution of secondaries, produced in the cumulative region.

Its nature is very simple. In fact, taking a particle in the cumulative region one selects the events, when the collided nucleon has a large momentum  $K_{\parallel}$  towards the projectile. Since for HMC one has  $|\Psi(k)|^2 \propto 1/k^4 = (K_{\parallel}^2 + K_{\perp}^2)^{-2}$ , the

corresponding transverse component of  $\vec{K}$  is also large  $\langle k_{\perp} \rangle \sim K_{\parallel}$ . Thus the larger  $X$  is the larger  $K_{\parallel}$  and  $\langle k_{\perp} \rangle$  are, and the larger smearing in  $p_{\perp}$  should be expected for spectrum of produced particle.

Let us stress, that this result assumes essentially, that the spectator nucleon, keeping a balancing momenta  $-\vec{K}$  does not take a part in the production process, leaving the production region without interactions. Quite opposite possibility is considered below as a flucton model.

## 2.2 Flucton model

Let us assume now, that at distances  $r \sim r_c$  ( $r_c$  is the nucleon "core" radius) that the both nucleons of deuteron interact with projectile like a single "effective hadron", rather than two separated particles. In this way the production process involves the whole system. Supplementing this picture by the quark counting rules [16] to account for production of this "effective hadron" one comes to the so-called flucton model [7].

In deuteron case the flucton contribution is given by the formulae [7]:

$$E \frac{dN}{d^3p}(X, p_{\perp}) \Big|_{Dh \rightarrow h} \approx W_{fl} \cdot E \frac{dN}{d^3p}(X, p_{\perp}) \Big|_{Nh \rightarrow h} \cdot (1-X)^{2n} \quad (2.3)$$

where  $W_{fl}$  is the weight of flucton state, the damping factor  $(1-X)^{2n}$  [7] takes into account the existence of  $n$  extra spectator quarks as compared with  $Nh \rightarrow h'$  fragmentation. In  $K, \bar{p}$  production one has  $n = 3$ .

Since the value of  $W_{fl}$  is not well defined<sup>4)</sup> in our estimates (Fig. 1) we assume  $W_{fl} = 0.05$  and  $0.1$  (solid curves 1 and 2). For  $K, \bar{p}$  production flucton model predicts more copious yields than that could be due to HMC, but for  $K^+$  production the predicted yields are comparable.

<sup>4)</sup> Accordingly to [7]  $W_{fl} = 0.03-0.11$  from measurements of deuteron form-factor.

Another interesting feature arises, if the  $p_t$ -distribution of fragments of "effective hadron" is the same, as that in fragmentation of "usual" hadrons. In this case one expects no broadening in  $p_t$  at all. Thus the study of  $p_t$ -distribution provides a good test to <sup>make</sup> clear, what the mechanism (HMC or flucton) dominates.

A very interesting picture can be, if HMC and flucton effects coexist, corresponding to large and small internucleon distances. In this case one expects a nontrivial dependence of  $p_t$ -distribution on  $X$ : at some  $X$ , where HMC dominates, a broad maximum of  $\langle p_t^2 \rangle$  is expected. Moreover, such a behaviour is to depend heavily on the type of final particle, because the relative contributions of mechanisms depends on it.

At last, let us add two warnings. The first, our parametrization by eq. (2.2) is rather crude, especially for  $p_t$ -distribution; moreover, our additional assumption on proton and neutron fragmentation can be unsatisfactory too. Nevertheless, the qualitative picture is not sensitive to this details. In any case, the recalculation with the corresponding more accurate data has no problems. The second, we neglect completely the triple-reggeon mechanism, being essential at  $X \rightarrow 1$ . Its contribution violates strongly the quark counting rules in  $K^+$ -production, but for  $K^-, \bar{p}$  production its influence seems to be not so strong [20]. In any case, this question deserves an additional study.

### 2.3 Coherent tube model

This model is based on the idea, that the characteristic longitudinal length of "coherent" hadronic interaction growth with the projectile energy. If this length exceeds the nucleus diameter, the whole "tube" of nucleons, lying on the way of projectile interacts with it like a single "effective hadron" (having the tube quantum numbers). In the forward hemisphere, where the target quantum numbers are unessential, one has [8]:

$$f_{h+vN}(E_0, y, p_t) = f_{h+N}(E_0' = E_0 v, y + \ln v, p_t) \quad (2.4)$$

where  $f = E \frac{dN}{d^3p}$ , and  $v$  is the number of nucleons inside the tube.

In the tube fragmentation region eq. (2.4) fails, because one should take into account the quantum numbers of tube. In order to do this, it is very convenient to rewrite eq. (2.4) in antilab. system of reference:

$$f_{(vN)+h}(E_0; X, p) = f_{N+h}(E_0 v; X, p_{\perp}) \cdot \mathcal{D}(v, X) \quad (2.5)$$

If one neglects the tube quantum numbers,  $\mathcal{D}(v, X) = 1$ . In the picture, developed in Ref. [9] the estimates are obtained on the basis similar to that for the quark counting rules [16]<sup>5)</sup>:

$$\mathcal{D}(v, X) \sim (1-X)^{2n} \quad (2.6)$$

where  $n$  is the number of spectators, being additional to those in  $Nh$ -collision. If one treats the whole tube on the quark level [7] one has  $n = 3(v-1)$ . Another estimate [9]  $n = (v-1)$  arises, when one assumes that the only nucleon participates in the backward (cumulative) particle production, while the rest of tube nucleons are the spectators only.

Returning in the lab system and averaging eq. (2.5) over the tube length one obtains

$$E \frac{dN}{d^3p} \Big|_{hA, E_0} = \sum P_A(v) \cdot \mathcal{D}(v, \dots) \cdot f_{hN}(E_0 v, y + \ln v, p_{\perp}) \quad (2.7)$$

The results of our numerical estimates for  $\tilde{\alpha}$  defined as

$$\tilde{\alpha} = \ln \left( E \frac{dN}{d^3p} \Big|_{pW} / E \frac{dN}{d^3p} \Big|_{pAl} \right) / \ln(184/27) \quad (2.8)$$

are shown in Fig. 2. As input spectra we use the parametrizations, obtained in Ref. [1].

In forward cone CTM predicts  $\tilde{\alpha} \rightarrow 0$  ( $\tilde{\alpha} > 0$ ), since the corresponding input spectra scale (or increase) with increase of  $E_0$ , while the data [18] show  $\tilde{\alpha} < 0$ . In central region the CTM predictions:

<sup>5)</sup> For instance, compare it to eq. (2.3).

$$\tilde{\alpha}_K < \tilde{\alpha}_P$$

are also at variance with data [21,22]. At last, CTM underestimates considerably the  $A$ -dependence of  $K^+$ -spectra, but overestimates that of antiproton production.

The whole disagreement in central region becomes more dramatic, if one takes into account the intranuclear damping of secondaries (Fig. 2, dashed curves). Remind, that this damping results in considerable broadening of  $p_t$ -distribution of slow secondaries ( $y_{lab} \approx 1$ ), which can be easily revealed by corresponding measurements.

Recent measurements [24] of  $K^\pm$  spectra ( $pTa, E_0 = 400 \text{ GeV}, \theta_{lab} = 90^\circ$ ) have shown, that both  $p_t$ -distribution and  $+/-$  ratio are practically the same as those in pp-collisions. Corresponding predictions of CTM are shown in Figs. 3,4.

First of all, the predicted  $p_t$ -distributions appear to be considerably wider than the experimental ones, even if one does not take into account the intranuclear rescattering of secondaries. The  $K^+/K^-$  ratio is significantly overestimated (Fig. 4). When one takes the intranuclear interactions into account, the agreement for the ratio becomes better, but the corresponding  $p_t$ -distributions are incompatible with data (Fig. 3). Note, that the dip at  $p_t \sim 0$  seen in  $K^-$  spectrum (dashed line) reflects the fact, that slow  $K^-$  are completely absorbed by nucleus.

Thus we conclude, that CTM does not work, at least, in the central and nucleus fragmentation region, while in projectile fragmentation region more accurate data are needed for the final conclusion.

At last, let us note, that the data [24] can hardly be compatible with cascade models, including that considered in Refs. [3-7]. The momenta of kaons [24] are small:  $p \sim 0.3-0.6 \text{ GeV}/c$ , and in the frameworks of these models they are produced mainly inside the nucleus, therefore the absence of intranuclear interactions of them looks very surprising. Thus, the data [24] can be considered as an evidence for these kaons to be produced either outside the nucleus as result of decay of some long-lived hadronic system [10-12,15], or from the "hot spots"

on the nuclear surface [25]. In any case, this question deserves additional both theoretical and experimental study.

### 3. Final state interactions

Let us consider now the multiple strong interaction of secondaries at the final stage of hadroproduction process, when the density of secondaries can be enough to provide their kinetical "mixing". If the mixing is strong enough, the multiparticle production process is not a sum of independent ones in each of the intranuclear  $hN$ -subcollisions, but it goes via the formation and decay of some collective system.

The influence of collectivization must be <sup>most</sup> essential in the central region, where the density of secondaries is maximal. Correspondingly, all the effects, discussed below manifest themselves mainly in the central region.

The most outstanding one is the specific  $A$ -dependence of composition of secondaries. In sect. 3.1 some model-independent considerations demonstrate that the current data can hardly be compatible with the pure cascade picture of multiparticle production.

The more detailed consideration of collective phenomena is given in sect. 3.2, where simplified Landau hydrodynamical model [10] is considered as the case of "extreme" collectivity. This model assumes the collectivity to be quite essential from the very beginning of  $hA$ -collision. However, such characteristics, as the particle composition of secondaries and their  $p_t$ -distribution are rather insensitive to processes at the initial stage of collision. Therefore, one expects the corresponding manifestations of collectivity to be the same, even if the collective system in question is formed at final stages of (cascade) process. In sect. 3.3 we demonstrate that such a collectivization can take place, for instance, during the recombination stage of picture, considered in the additive quark model [3,4].



### 3.1. Model-independent considerations

Let us formulate the main difference between the cascade and collective pictures of particle production.

By the cascade process we mean the branching one, where each branch has individual kinematical constraints.

If it is not the case, and the particles, produced at different branches are strongly "mixed" kinematically by their mutual interactions, we have a collective process. In the extreme case the only remaining kinematical constraints are those for the collective system as a whole.

Consider now the  $A$ -dependence of production process. Increasing  $A$  in the cascade picture we have more branches, the mean energy of subcollision decrease, and kinematical restrictions become more essential. So, one expects that the heavy to light particle ratios must decrease with  $A$ . In contrast, in collective picture the mass and sizes of collective system grows with  $A$ , the kinematical restrictions become less essential and the same ratios must increase with  $A$ .

The current data [22,23,26] are rather scarce, but they show (Fig. 5), that<sup>6)</sup>

$$\Delta\alpha_{K/\pi} \equiv \alpha_K - \alpha_\pi > 0 \quad (3.1)$$

so the kaon multiplicity grows with  $A$  faster, than the pion one. Bearing in mind that the kaon to pion ratio grows with the collision energy too, one may conclude, that the kinematical restrictions are less essential for larger  $A$ , and collective picture is more preferable.

For antiprotons the data [22,23,26] have large errors and are not so conclusive, but some trend for  $\Delta\alpha_{\bar{p}/\pi^-} > 0$  is also seen<sup>7)</sup>. Note, that the data on  $\alpha_{\bar{p}}$  and  $\alpha_{\pi^-}$  are avail-

<sup>6)</sup> Where  $\alpha$  is defined by fit  $E \frac{dG}{d^3p} \propto A^\alpha$ .

<sup>7)</sup> At the first sight, one should expect also  $\alpha_{\bar{p}/K^-} > 0$ . However, the conditions for meson and baryon production are different even in the thermodynamical picture [10]: the chemical potential is zero for the mesons, while for baryons it has some nonzero (strictly speaking,  $A$ -dependent) value because of contribution of primary baryons.

able only at rather different  $y_{lab}$  and their comparison is not meaningful.

In the pure cascade picture, considered, e.g. in Refs. [3,4] one has no reason for excessive strange meson production, moreover, one could expect the suppression of their yields in comparison with the yields of more light pions.

At the first sight, the experimental  $A$ -dependence of particle composition

$$\Delta\alpha_{K^+/\pi^+} > \Delta\alpha_{K^-/\pi^-} \quad (3.2)$$

together with the observation, that [27]

$$\sigma_{K^+N} < \sigma_{K^-N} < \sigma_{\pi^\pm N} \quad (3.3)$$

suggest that the different  $A$ -dependence of particle spectra is the result of different intranuclear absorption of secondaries. In order to verify this guess, let us estimate the "original"  $A$ -dependence of (undamped) spectra, assuming it to be the same for kaon and pion production:  $\alpha_{K^+}^{(0)} \sim \alpha_{K^-}^{(0)} \sim \alpha_{\pi^\pm}^{(0)}$ . The intranuclear damping of particles is given by the factor

$$D_h \sim \exp[-n \cdot (\sigma_{in})_{hN} \cdot \ell] \quad (3.4)$$

where  $n = 0.17$  nucleon/fm<sup>3</sup> is the intranuclear density,  $\ell$  stands for the intranuclear path. Difference in absorption for pions and kaons results in

$$\Delta\alpha_{K/\pi} \sim \frac{d}{d \ln A} \left[ \ln \left( \frac{D_K}{D_\pi} \right) \right] \approx -n \cdot [(\sigma_{in})_{KN} - (\sigma_{in})_{\pi N}] \frac{d\ell}{d \ln A} \quad (3.5)$$

where  $\ell$  is assumed to be the same for kaons and pions. Let us accept for a crude estimate  $\ell \sim R(\text{nucleus}) \sim A^{1/3}$  (fm). Using [27]

$$\sigma_{K^+N} \sim 14 \text{ mb}, \quad \sigma_{K^-N} \sim 19 \text{ mb} \quad \text{and} \quad \sigma_{\pi^\pm N} \sim 22 \text{ mb},$$

one obtains

$$\Delta\alpha_{K^+/\pi^+} \sim 0.16, \quad \Delta\alpha_{K^-/\pi^-} \sim 0.06$$

which are in fair agreement with data (see, Fig. 5).

However, let us estimate the "original" values  $\alpha^{(0)}$  for undumped spectra:

$$\alpha_h^{(0)} = \alpha_h + (G_{in})_{hN} \cdot n \frac{d\ell}{d\ln A} \sim \alpha_h + \frac{1}{3} \cdot n \cdot R \cdot (G_{in})_{hN} \quad (3.6)$$

where  $\alpha_h$  stands for observed  $A$ -dependence. Numerically, one has  $\alpha_\pi^{(0)} \sim \alpha_K^{(0)} \sim 1.2-1.3$ . Such a value is too high in view of current cascade models [3,4]. In any case, it means that the intranuclear absorption of secondaries must be quite essential and neglect of it [3,4] makes meaningless any comparison with data.

A more fatal contradiction arises, if one assumes the same intranuclear path  $\ell$  in order to estimate the "original"  $A$ -dependence of antiproton spectra: the corresponding value  $\alpha_p^{(0)} \sim 1.8 > \alpha_{K,\pi}^{(0)}$ , which is again incompatible with the pure cascade picture.

Note, that our estimates provide the lowest limit, since we assume  $\alpha_K^{(0)} \sim \alpha_\pi^{(0)}$ , while in pure cascade picture one expects  $\alpha_K^{(0)} < \alpha_\pi^{(0)}$ . Moreover, the formation length for heavy particles should be shorter, than those for light ones, and for corresponding intranuclear paths one has, at least,  $\ell_K > \ell_\pi$ .

Thus the  $A$ -dependence of particle composition cannot be explained by the influence of intranuclear absorption of secondaries. The only open possibility to account for it in the pure cascade approach is that the mean elementary production process in  $hA$ -collision differs from the process in  $hN$ -collision (say, the relative contribution of hard processes grows with  $A$ , resulting in relative growth of heavy particle production). Unfortunately, there is no model for such a phenomenon. Instead of it, we consider below another possibility, assuming essentially the collectivization of secondaries (at least, at final stages of production process).

### 3.2 Landau's hydrodynamical model

The main assumption of this model is that the hadronic interaction in the multiparticle production process is so strong, that the classical hydrodynamical description is valid from the very beginning of collision. The whole process is

schematically divided into three stages [10]: (i) the compression stage (which results in formation of collective system), (ii) the hydrodynamical expansion and (iii) decay of hydrodynamical system into final hadrons.

Although the predictions of this model are [11-14] in a good agreement with current data, the question, whether this model is credible, is still open. The most unreliable assumption is that the hydrodynamics is applicable at earliest stages of collision [28,29]. Probably, it can be verified [14,30] by the detailed study of high- $p_t$  hadron production.

In this work we consider the features, reflecting the collectivity at the final stage of collision, which are rather insensitive to the specifics at earlier ones. Therefore in our estimates we restrict ourselves by a crude assumption, concerning the characteristics of initial state of hydrodynamical system.

We assume, that the initial state, of collective system, formed in nucleon-tube collision is similar to that produced in the nucleon-nucleon one (for details see Appendix):

$$\begin{aligned} T_i(E_0, \nu) &= T_i(E_0 \cdot \omega(\nu), 1) \\ S_i(E_0, \nu) &= S_i(E_0 \cdot \omega(\nu), 1) \cdot G(\nu) \\ l_i(E_0, \nu) &= l_i(E_0 \cdot \omega(\nu), 1) \cdot G(\nu) \end{aligned} \quad (3.7)$$

with

$$\omega(\nu) = \frac{2\nu}{1+\nu^2}, \quad G(\nu) = \left(\frac{1+\nu^2}{2}\right)^{1/2} \quad (3.8)$$

where  $T_i$ ,  $S_i$  and  $l_i$  are the initial temperature, total entropy and initial longitudinal size of hydrodynamical system, and  $\nu$  stands for the number of nucleons inside the tube.

The main feature of eqs. (3.7-8) is that they relate the characteristics of nucleon-tube collision with that the nucleon-nucleon one at lower energy  $E'_0 = E_0 \cdot \omega(\nu) \xrightarrow{\nu \gg 1} 2E_0/\nu$ , in contrast to the coherent tube model [8], which implies that  $E'_0 = E_0 \cdot \nu$ . In particular, hydrodynamical model predicts [12] the narrowing for the rapidity distribution of secondaries,

instead of broadening, predicted by GTM.

Another feature is that the initial state of hydrodynamical system is  $\mathcal{G}(v)$  times dilated in longitudinal direction as compared with the "similar" nucleon-nucleon collision. Strictly speaking, it means that the longitudinal rapidity distribution of secondaries must be even narrower than that in corresponding "similar" NN-collision, because of more essential transverse expansion. Obviously, in this case one should expect also the  $A$ -dependent broadening of transverse momentum distribution.

For simplest estimates let us neglect the transverse hydrodynamical expansion at all. In this way the relations (3.7-8) are valid up to the final stage of hydrodynamical expansion and for final particle spectra one has:

$$E \frac{dN}{d^3p} \Big|_{E_0, N\text{-tube}} = a(v) \cdot E \frac{dN}{d^3p} \Big|_{E'_0 = E_0 \cdot \omega(v), NN} \quad (3.9)$$

Since the pion multiplicity is proportional to the entropy of hydrodynamical system, one has  $a_\pi(v) = \mathcal{G}(v)$ . However, the multiplicity of secondary  $K, \bar{p}$  grows with entropy faster:  $a_{K, \bar{p}}(v) \cong [\mathcal{G}(v)]^\beta$ ,  $\beta > 1$  because of the influence [31,32] of (strangeness/baryon number) conservation laws. So, one expects  $\beta \sim 2$ , if the mean multiplicity of particle in question is small:  $\langle N \rangle \ll 1$ , and  $\beta \sim 1$  for  $\langle N \rangle \gg 1$ . This fact accounts [30,32] for the energy dependence of particle composition of secondaries, produced in the central region in hadron-hadron collisions. For example, parametrizing the energy dependence

$$E \frac{dN_{K, \bar{p}}}{d^3p} \propto \left( E \frac{dN_\pi}{d^3p} \right)^\beta$$

in the ISR energy interval one has [33]:

$$\beta_K \sim 1.6, \quad \beta_{\bar{p}} \sim 1.9 \quad (3.10)$$

Because of lack of such data at FNAL and IHEP energies we consider below the both extreme cases:  $\beta = 1$  and  $\beta = 2$ .

Averaging eq. (3.9) over the nucleus one has<sup>8)</sup>:

$$E \frac{dN}{d^3p} \Big|_{E_0, NA} = \sum_V p_A(v) \cdot \mathcal{G}(v)^\beta \cdot E \frac{dN}{d^3p} \Big|_{E'_0 = E_0 \cdot \omega(v), Y_{cm}(E_0/v, 1), NN} \quad (3.11)$$

The results of numerical estimates for  $\tilde{\alpha}$  (see eq. (2.8)) are plotted in Fig. 6. The data on pion production support  $\beta \sim 1$ , while the antiproton ones agree with  $\beta \sim 2$ . The kaons demonstrate intermediate case, which is reasonable too, since  $\langle N_K \rangle \sim 1$ .

Note, that our oversimplified version of Landau model ignores completely the forward-backward asymmetry in the hadron-tube collision and therefore its predictions outside the central region cannot be very reliable. Besides that our version neglects also the  $A$ -dependence of the inelasticity factor as well as the intranuclear interactions of secondaries. Let us consider the latter in more detail.

First of all, the intranuclear interactions of secondaries are absent at all, if the collective system decays outside the nucleus. Otherwise the intranuclear cascading becomes very essential; moreover, only forward part of "tube" takes part in the collective process of multiparticle production [12].

Let us estimate the corresponding "threshold" value of incident energy  $E_0$ . The "lifetime" of collective system can be characterized by the formation time of particles, produced in the central region [11,14]

$$\tau_f \sim \gamma_{c.s.} \cdot m_\pi^{-1} \approx \frac{1}{2} e^{Y_{c.s.}} \cdot m_\pi^{-1}$$

where  $\gamma_{c.s.}$  is the lab. Lorentz factor of the center of mass

<sup>8)</sup> Let us warn, that for the center of mass of hydrodynamical system the rapidity  $Y_{cm}(E_0, v) \neq Y_{cm}(E_0 \cdot \omega(v), 1)$ , but  $Y_{cm}(E_0, v) \cong Y_{cm}(E_0/v, 1)$ . This fact should be properly taken into account in the rapidity shifting of "effective" NN-spectra.

of hydrodynamical system, and  $Y_{c.s.}$  is the corresponding rapidity, which coincides in practice with the position of the maximum of spectra. So, if one has

$$\tau_f \approx 2R_{\text{nucleus}}$$

or, finally,

$$Y_{c.s.} \approx \frac{1}{3} \ln(4A) \quad (3.12)$$

the intranuclear interactions becomes unimportant. Taking  $A \sim 10^2$  one obtains  $Y_{c.s.} \approx 2$ , which corresponds to  $E_0 \approx 50 - 100$  GeV. This estimate agrees with the observations [12,34], that the ratio  $R = \langle n_{ch} \rangle_{pA} / \langle n_{ch} \rangle_{pp}$  reaches its asymptotic value at  $E_0 \sim 10^2$  GeV. In particular, it means that the data, used in Fig. 6 can be influenced by the intranuclear absorption, and the above analysis should be accepted with caution. Obviously, the analysis at higher  $E_0$  is badly needed.

Up to now we neglect completely the differences in spatial hydrodynamical expansion between the systems, produced in  $hA$  and  $hN$  collisions. However, the system, produced in  $hA$  collision is  $G(v)$  times dilated in the longitudinal direction (see eqs. (3.7)), and the transverse hydrodynamical expansion must be more essential.

It was pointed out in Refs. [30,35], that the transverse collective velocity distribution is formed mainly at latest stage of hydrodynamical expansion and, therefore, it is rather insensitive to earlier stages of collision. However, at latest stage of expansion the specific (unknown) conditions at the matter-vacuum boundary (the vacuum pressure, etc.) are quite essential [35] and the straightforward calculation [36], neglecting them, is unreliable. Moreover, the comparison [30,35] of experimental  $p_t$ -distributions with the thermal ones has shown, that transverse hydrodynamical expansion is practically absent in proton-proton collision.

Let us now estimate the upper limit for transverse expansion from the A-dependence of  $p_t$ -distributions. Accordingly to [30,35,36] we assume that transverse collective rapidity grows linearly along the radius of system.

It was shown in Ref. [30], that its maximum value, reached at side boundary of system is proportional to density of secondaries  $\eta_{\text{max}} \propto \left(\frac{dN}{dy}\right)$ , produced in central region. Assuming

$$\eta_{\text{max}} = \eta_0 \cdot A^{\langle \tilde{\alpha} \rangle}$$

where  $\langle \tilde{\alpha} \rangle$  characterizes the local A-dependence of  $\left(\frac{dN}{dy}\right)$  for secondaries, and  $\eta_0$  is a free parameter, we convolute transverse collective rapidity distribution with the thermal spectra of particles at temperature  $T = 0.13$  GeV. The results, parametrized as

$$\frac{1}{\langle N \rangle} \frac{dN}{dp_t^2} \propto A^{\delta \alpha(p_t)}$$

are plotted in Fig. 7. In the experimentally studied [21,22] rapidity region  $y_{\text{lab}} \sim 2-2.5$   $\langle \tilde{\alpha} \rangle \sim 0.18$  and one has for upper limit of  $\eta_0$  <sup>9)</sup>

$$\eta_0 \leq 0.05 - 0.1 \quad (3.13)$$

which is again considerably smaller, than that could be expected from the straightforward calculations [36]. Note, that  $\delta \alpha$  grows rapidly with  $\langle \tilde{\alpha} \rangle$ : small increase from 0.18 to 0.25 results in considerable changes of  $\delta \alpha$  (Fig. 7).

### 3.3 Additive quark model (AQM)

The multiparticle production process, assuming in the additive quark model [3,4] can be divided into two quite different subsequent stages:

- (i) the intranuclear quark-parton cascading, resulting in production of new ("constituent") quarks, and
- (ii) The recombination stage, when the quarks recombine into secondary hadrons.

<sup>9)</sup> Let us stress, that the competing contribution in  $\delta \alpha$  comes from intranuclear interactions of secondaries, which results also in broadening of  $p_t$ -spectra, therefore, the estimate (3.13) should be considered only as an upper limit.

While the former stage can be a real cascade process, having no considerable mutual interaction between the partons (and quarks), produced in different branches of cascade, the latter stage can be essentially a collective one. Let us summarize briefly some reasons for this.

First of all, the final state interactions at the recombination stage are definitely very strong, since no free quark is seen.

The multiplicity of recombining quarks is very high: in typical collision with heavy nucleus one has  $\langle n_{ck} \rangle \sim 20$ , which corresponds to the quark multiplicity  $\langle n_q \rangle \sim 50-60$ <sup>10)</sup>.

At last, the simplest combinatorial considerations show, that the probability for two or three quarks to be in a colourless state is very small:  $(\frac{1}{3})^2$  and  $(\frac{1}{3})^3$ , respectively. So, the multiple mutual soft interactions between the final hadrons are needed to make them colourless.

All this arguments, that the recombination stage corresponds to multiple interactions of secondaries, and collectivization at this stage is very likely.

If the collectivization is strong enough to reach the local thermal equilibrium, all the predictions, concerning the particle composition and  $p_t$ -distribution of secondaries should be quite similar to those expected in Landau model, since they depends mainly on the latest stage of evolution of collective system. In particular, careful experimental studies of particle composition and  $p_t$ -distributions ( $\lesssim 1$  GeV/c) allow us to verify the local thermal equilibrium at latest stages of the process [30], while at earlier ones it can be verified by study of high- $p_t$  hadron and nonresonant low mass dilepton production [30, 38, 39].

Let us turn now to more specific features of AQM. There are two possible types of collectivity in this model. The first one is the collectivization of secondary quarks, produced by the same constituent quark  $q$  of projectile hadron in diffe-

<sup>10)</sup> This number can be somehow diminished, if one assumes the main multiplicity coming from decay of resonances [37].

rent  $hN$ -subcollisions. The second one is the collectivization of quarks, produced by different constituent quarks of projectile.

If one assumes no collectivity at all, and takes the quasieikonal dynamics [3] for  $qA$ -interaction, one can reproduce [1] the current data on  $K, \bar{p}$  production in central region, if only the intranuclear absorption of secondaries is completely neglected. However, in this way the data on pion production are crucially overestimated (see, Fig. 9)<sup>11)</sup>. As we have seen in sect. 3.1, such a defect is inherent to any "pure" cascade model. Taking into account the intranuclear absorption, one spoils [1] fatally the agreement on  $K, \bar{p}$  production. Thus the collectivity seems to be inevitable in order to account for the current data on particle composition.

Consider now the possible ways to study each type of collectivity. In order to study the collectivity of the first type one should study the particle composition in the events, when only one of projectile constituent quarks takes part in the multiparticle production process, while the others are only spectators. If the particle composition is again incompatible with the expectations of pure cascade picture, the collectivity is proved. Such of the events can be probably chosen, if one fixes in final state a very fast ( $X > 1/2$  or  $2/3$  for incident meson or baryon, respectively) hadron with the quantum numbers of projectile.

The collectivity of the second type can be found in experimental studies of the Anisovich relation [40]<sup>12)</sup>

$$R_h \equiv E \frac{d^5}{d^3p} (BA \rightarrow h+X) / E \frac{d^5}{d^3p} (MA \rightarrow h+X) = 3/2 \quad (3.14)$$

where  $B$  and  $M$  stand for primary baryon and meson, respectively;  $h$  is secondary hadron, produced in the central region. This relation implies that each of quarks of projectile contributes into particle production independently. If the collectivity takes place, one expects  $R_h > 3/2$ . The current data [41]

<sup>11)</sup> This fact have been omitted in Ref. [1].

<sup>12)</sup> Note, that the intranuclear absorption effects cancel each other in this ratio.

are taken mainly at rather low incident energy ( $E_0 \sim 10^2 \text{ GeV}$ ) for unidentified secondary hadrons (mainly, pions); only pseudorapidity distributions are available. These data do not contradict to eq. (3.13). However, the pion production is not a good test for collectivity, since it is not kinematically suppressed (note, that  $m_\pi \ll \langle p_t^{(\pi)} \rangle$ ). More sensitive to the kinematics of process is the heavy particle ( $K, \bar{p}, \dots$ ) production and the corresponding violation of eq. (3.13) must be more essential.

#### 4. Summary

In this paper we have considered two kinds of collective phenomena, which can take place in hadron-nucleus collisions: the interactions of projectile with the collective states of target (the "initial state interactions", sect. 2) and the "final state interactions" of secondaries (sect. 3).

The most trivial example of the former kind of collectivity is the well-known Fermi-motion of nucleons. Its nontrivial "high momentum component" have been used in Refs. [6] to explain the cumulative  $p, \pi$  production. In sect. 2.1 this model have been used in order to estimate the cumulative  $K, \bar{p}$  production. The most important feature being inherent to this model is found to be a considerable broadening of  $p_t$ -distribution for cumulative  $K, \bar{p}$ .

Another kind of target collective states is the "fluctons" [7], being the local fluctuations of nuclear matter density, which interact with projectile like a single "effective hadron". For the same deuteron target the flucton model predicts (sect. 2.2) more copious yield of cumulative  $K, \bar{p}$  and no broadening of their  $p_t$ -distribution. Thus we conclude, that the experimental study of cumulative  $K, \bar{p}$  production off deuterons can provide a clear test to choose between these models.

The idea, that the whole "tube" of nucleons interacts with the projectile as a single "effective hadron" (the coherent tube model [8,9]) have been verified in sect. 2.3. It have been found to be in conflict with data in the whole kinematical region.

The final state interactions have been considered at a more qualitative level. First of all, in sect. 3.1 we demonstrate that the current data on  $A$ -dependence of  $K/\pi$  ratio cannot be reproduced within the framework of pure cascade model, i.e. that assuming no collectivity.

In sect. 3.2 we show that the predictions of Landau hydrodynamical model, assuming the extreme collectivity of the process, are in reasonable agreement with the same data. Besides that, from the analysis of the  $A$ -dependence of low  $p_t$ -distribution we obtain the upper limit for transverse hydrodynamical expansion of collective system. The result agrees with our earlier observation [35], that such expansion is quite unessential in  $pp$ -collisions.

Another possibility, that collective system is formed as a result of final state interactions during the recombination stage of additive quark picture, is discussed in sect. 3.3. If the collectivization is strong enough, the predictions, concerning the particle composition and  $p_t$ -distribution of secondaries are expected to be very close to those, predicted by the conventional Landau hydrodynamical model. The most important test for such a model is the verification of the Anisovich relations [40] for the centrally produced heavy particles.

#### 5. Conclusions

This paper completes our study, which was begun in Ref. [1]. Our main intention is to clear up, what the hypothesis, underlying the most popular current models are supported or ruled out by data. In particular, the question is studied, whether the production process is a pure cascade or collectivity takes place.

The problem is, that this question is open up to now. Remind, that the most of features of current data are in practice equally well reproduced within both cascade and collective models, i.e. they are not sensitive to the specifics of dynamics of  $hA$ -collision.

In our work we demonstrate the advantages, arising in

study of heavy particle production. First of all, corresponding yields are more sensitive to the kinematics of the process; moreover, the "background" coming from the low-energy cascading can be quantitatively estimated.

Our main result, concerning the cascade models is that the most successful one is the additive quark model. It describes reasonably the particle production in the projectile fragmentation region. Besides that, it can easily incorporate the interactions of projectile with possible collective (quantum) states of nuclear target (the intranuclear motion of nucleons, the "fluctons", etc.). In this way one has a real possibility of unified description of both forward and backward (cumulative) particle production.

However, a new problem arises, if one consider the particle composition of secondaries, produced in central region. In particular, the specific dependence of  $K/\pi$  ratio appears to be incompatible with any "pure" cascade model. The only possible explanation is seen, that the mutual interactions of secondaries are so strong, that the final particles are produced via formation and decay of collective system.

A possible composite picture can be proposed, which assumes, that the quark additivity of projectile manifests itself mainly in the projectile fragmentation region (excluding the region  $X \rightarrow 1$ , where the triple-reggeon contribution can dominate). The specifics of intranuclear collective (quantum) states dominates in the nucleus fragmentation region, while the particle production in central region is governed by the evolution of collective (thermodynamical) system, resulting from the final state interactions of secondaries.

Let us stress, that the current available data are too scarce for final conclusions, especially in the central and nucleus fragmentation regions. Any information on the particle composition and  $p_t$  - distributions is very desirable. A very interesting information on the space-time development can be obtained in studies of low and intermediate mass ( $M \approx 2-3 \text{ GeV}c^2$ ) nonresonant dilepton production (see, e.g. analogous studies [30, ] for hadron-hadron collisions). A very valuable information can be obtained [14,30,42] also from high- $p_t$  hadron pro-

duction.

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#### Appendix Initial state of hydrodynamical system

In this paper we are interested in those features of multiparticle production, which depend mainly on the final stage of hydrodynamical expansion. Since they are rather insensitive to the detailed story [10,13,14] of compression stage, we restrict ourselves by the crude estimate for the state of hydrodynamical system, formed to the end of compression stage.

For simplicity sake let us assume, that at this "initial" moment [11]

(i) hadronic matter is rest in the proton-tube CM reference frame, and its density (or temperature) is constant over the volume of system;

(ii) initial volume is proportional to the sum of the Lorentz-contracted volumes of the primary proton and tube.

Because of (i) the properties of the "initial state" in the proton-tube collision are similar to those in the proton-proton one. Assumption (ii) is a straightforward generalization of the corresponding one in the original Landau model [10] for hadron-hadron collisions.

Such an "initial state" is characterized by two parameters: the initial temperature  $T_i(E_0, v)$  and longitudinal size  $l_i(E_0, v)$  (the transverse size is assumed to be a constant  $\sim m_K^{-1}$ ). The former parameter is an intensive quantity, while the latter parameter is an extensive one. Thus the similarity relations contain two "scale factors"  $\omega, \phi$ :

$$T_i(E_0, v) = T_i(E_0 \cdot \omega(v), 1) \quad (\text{A.1})$$

$$l_i(E_0, v) = \phi(v) \cdot l_i(E_0 \cdot \omega(v), 1) \quad (\text{A.2})$$

In particular, for the total entropy (which characterizes the total multiplicity of secondaries) one expects:

$$S_i(E_0, v) = G(v) \cdot S_i(E_0 \cdot \omega(v), 1)$$

Let us define  $\omega(v), G(v)$ . The CM energy of system is

$$E_{CM} \simeq (2mE_0 v)^{1/2}$$

( $m$  is nucleon mass), and the CM Lorentz factor for tube is

$$\gamma_v^{(CM)} \simeq E_0/E_{CM} = (E_0/2mv)^{1/2}$$

since in CM the total momentum is equal to zero, the corresponding CM Lorentz factor for incoming proton is

$$\gamma_1^{(CM)} \simeq v \cdot \gamma_v$$

and for the initial longitudinal size one has

$$l_i(E_0, v) \propto \frac{1}{\gamma_1^{(CM)}} + \frac{1}{\gamma_v^{(CM)}} = \left(\frac{2m}{E_0}\right)^{1/2} \cdot \frac{(1+v^2)}{v^{1/2}} \quad (\text{A.3})$$

For the initial density of energy one has

$$\mathcal{E}(E_0, v) \propto E_{CM}/l_i(E_0, v) \propto E_0 \frac{v}{1+v^2}$$

whence it follows

$$\mathcal{E}(E_0, v) = \mathcal{E}(E_0 \cdot \omega(v), 1) \quad (\text{A.4})$$

$$\omega(v) = \frac{2v}{1+v^2}$$

which is the same for any intensive characteristics. From eqs. (A.2-4) one has

$$G = \left(\frac{1+v^2}{2}\right)^{1/2} \quad (\text{A.5})$$

Finally, let us compare this estimate with the straightforward calculations [10], which consider the shock waves at the compression stage of collision. For the total multiplicity one has:

$$N(E_0, v) = G(v) \cdot N(E_0 \cdot \omega(v), 1)$$

Since in Landau model [10]  $N \propto E^{1/4}$ , one has

$$N(E_0, v) \propto \left(\frac{1+v^2}{2}\right)^{1/2} \cdot \left(\frac{2v}{1+v^2}\right)^{1/4}$$

Using  $v = A^{1/3}$  and parametrizing  $N(E_0, v) \propto A^\alpha$ , one has

$$\alpha \simeq 0.20$$

in a good agreement with  $\alpha = 0.19$ , obtained in Ref. [10]. (Let us warn the reader, that these estimates correspond to the sound velocity of hadronic matter  $C^2 = 1/3$ ).



## References

1. O.V.Zhirov, "Hadron Nucleus Collisions. 1.Relativistic Cascade models". Preprint IYAF 81-30, Novosibirsk, 1981.
2. D.Sivers, S.J.Brodsky and R.Blankenbecler, Phys.Rep.23C, 1976.
3. V.M.Shekhter, in Proc. of 5 Intern. Sem. on High Energy Phys. Problem, p.246, Dubna, 1978;  
V.V.Anisovich, Yu.M.Shabelsky and V.M.Shekhter, Nucl.Phys.B133, 477, 1978.
4. N.N.Nikolaev, "Phys. of Elem. Part. and Atomic Nuclei", 12, 162, 1981.
5. V.B.Kopeliovich, Yadern.Fiz.26, 168, 1977.
6. M.I.Strikman and L.L.Frankfurt, Nucl.Phys.B148, 107, 1979.
7. V.K.Lukyanov and A.I.Titov, "Phys. of Elem. Part. and Atom. Nuclei", 10, 815, 1979.
8. S.Fredriksson, Preprint TH.2720-Cern, CERN, 1979.
9. G.Berlad, A.Dar and G.Eilam, Phys.Rev.D22, 1547, 1980.
10. L.D.Landau, Izv.Akad.Nauk SSSR, Ser.fiz., 17, 51, 1953;  
S.Z.Belenky and L.D.Landau, Uspekhi Fiz. Nauk, 56, 309, 1955.
11. E.V.Shuryak, Yadern.Fiz., 24, 630, 1976.
12. O.V.Zhirov and E.V.Shuryak, Yadern.Fiz.28, 485, 1978.
13. N.Masuda and R.M.Weiner, Phys.Rev.D15, 1515, 1978; *ibid*, p.1542.
14. N.Masuda, Phys.Rev.D20, 2314, 1979.
15. B.N.Kalinkin and Shmonin, "Phys. of Elem. Part. and Atomic Nuclei", 11, 630, 1980.
16. V.A.Matveev, R.M.Muradyan and A.N.Tavkhelidze, Nuovo Cim.Lett.7, 719, 1973.  
S.J.Brodsky and G.Farrar, Phys.Rev.Lett., 31, 1153, 1973.
17. E.L.Feinberg, ZhETF, 28, 241, 1955.
18. N.I.Bozhko et al, Preprint IHEP 79-78, Serpukhov, 1979.
19. J.Hamberstone and J.S.Wallarce, Nucl.Phys., A141, 362, 1970.
20. D.Cutts et al, Phys.Rev.Lett.43, 319, 1978.
21. D.A.Garbutt et al, Phys.Lett.B67, 355, 1977.
22. V.V.Abramov et al, Preprint IHEP 79-131, Serpukhov, 1979.
23. L.M.Barkov et al, Preprint IHEP 79-92, Serpukhov, 1979.
24. N.A.Nikiforov et al, Preprint ITEP-37, Moskov, 1980.
25. R.Weiner, Phys.Rev.Lett.32, 630, 1974; Phys.Rev.D13, 1363, 1976;  
R.Weiner and M.Westrom, Nucl.Phys.A286, 282, 1977.
26. U.Becker et al, Phys.Rev.Lett.37, 1731, 1976;
- D.Antreasyan et al, Phys.Rev.D19, 764, 1979.
27. V.Flaminio et al, Compilations CERN-HERA: 79-01, 79-02, 79-03, 1979.
28. D.I.Blokhintzev, ZhETF, 32, 350, 1957;  
J.Novakovsky and F.Cooper, Phys.Rev.D9, 771, 1974;  
M.I.Morawcsic and M.Teper, Phys.Rev.D16, 1593, 1977.
29. A.A.Tyapkin, "Phys. of Elem. Part. and Atom. Nuclei", 8, 544, 1977.
30. O.V.Zhirov, Preprint IYAF 79-114, Novosibirsk, 1979.
31. I.N.Sisakyan, E.L.Feinberg and D.S.Chernavsky, ZhETF, 52, 545, 1967.
32. E.V.Shuryak, Preprint IYAF 74-108, Novosibirsk, 1974;  
Yadern.Fiz.20, 549, 1974.
33. K.Guettler et al, Phys.Lett.B64, 111, 1976; Nucl.Phys.B116, 77, 1976.
34. E.G.Boos et al, Nucl.Phys.B143, 232, 1978.
35. E.V.Shuryak and O.V.Zhirov, Phys.Lett.B89, 253, 1980.
36. B.Andersson, G.Jarlskog and G.Damgaard, Nucl.Phys.B112, 413, 1976.
37. V.V.Anisovich, Yadern.Fiz.28, 761, 1978.
38. O.V.Zhirov, Yadern.Fiz.30, 1098, 1979.
39. E.V.Shuryak, Phys.Rep.61C, 71, 1980.
40. V.V.Anisovich, Phys.Lett.B57, 87, 1975.
41. J.E.Elias et al, Phys.Rev.D22, 13, 1980.
42. O.V.Zhirov, Preprint IYAF 81-31, Novosibirsk, 1981.

Figure captions

Fig. 1. The yields and  $p_t^2$ - slope parameter for cumulative production of deuteron, predicted by different models (see text). The solid curves 1 and 2 are the predictions of flucton model for the weight of flucton state  $W_{fl} = 0.05$  and  $0.1$ , respectively.

Fig. 2. The dependence of  $\tilde{\alpha} = d \ln(E \frac{dN}{d^3p}) / d \ln A$  on the rapidity  $y$ , predicted by coherent tube model. The solid and dotted curves correspond to  $n = 1$  and  $3$  respectively (see text), and absorption is completely neglected. The dashed ones take into account the intranuclear interactions provided that the particles are produced equiprobably along the tube length. The points correspond to data:  $(\circ, \Delta)$  - [22,21] for  $p_t = 0.5 - 1$  GeV/c and  $(\bullet)$  are the same data otherwise;  $(+)$  - [18] and  $(*)$  - [23].

Fig. 3. The CTM predictions versus the data [24]. The solid lines assume no intranuclear interactions, while the dashed ones take them into accounts.

Fig. 4. The same as in Fig. 3, but for  $K^-/K^+$  ratio.

Fig. 5. The  $A$ -dependence of particle ratios parametrized by  $R(\frac{h}{\pi}) \propto A^{\alpha_{h/\pi}}$  [21].

Fig. 6. The dependence of  $\tilde{\alpha}$  on  $y$ , predicted by Landau model. Curves 1 and 2 correspond to  $\beta = 1$  and  $2$ , respectively. The data notation are the same, as in Fig. 2.

Fig. 7. The influence of transverse expansion on the  $\tilde{\alpha}$  for different values of  $\eta_0$ . The data are taken from:  $\circ$  - [21] and  $\bullet$  - [22].

Fig. 8. The dependence of  $\tilde{\alpha}$  on  $y$ , predicted by additive quark model. Curves 1 and 2 correspond to the formation length of particles  $l_f = \infty$  and  $0$  respectively (see, for details, text and Ref. [1]). The data notation are the same as in Fig. 2.

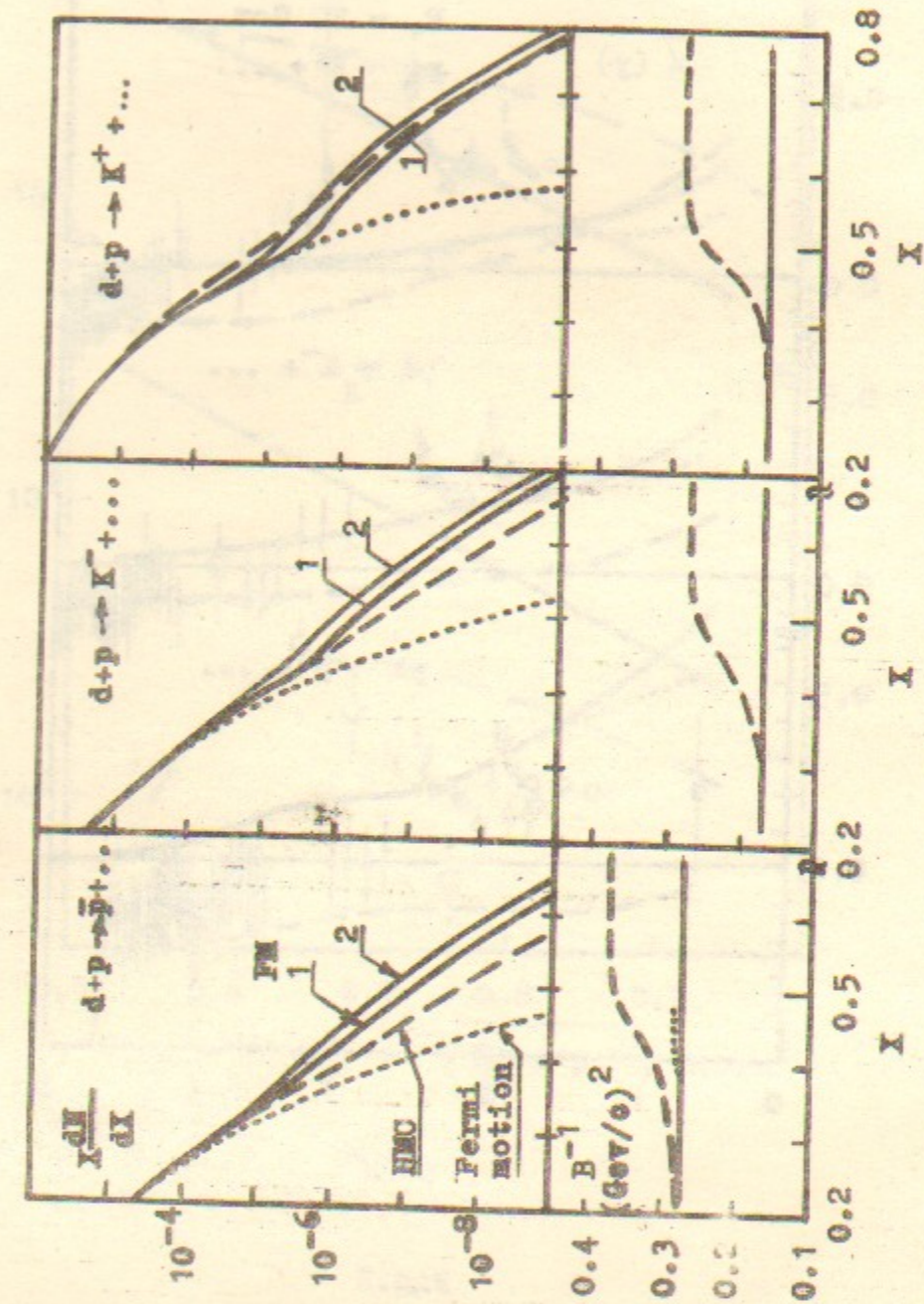


Fig. 1

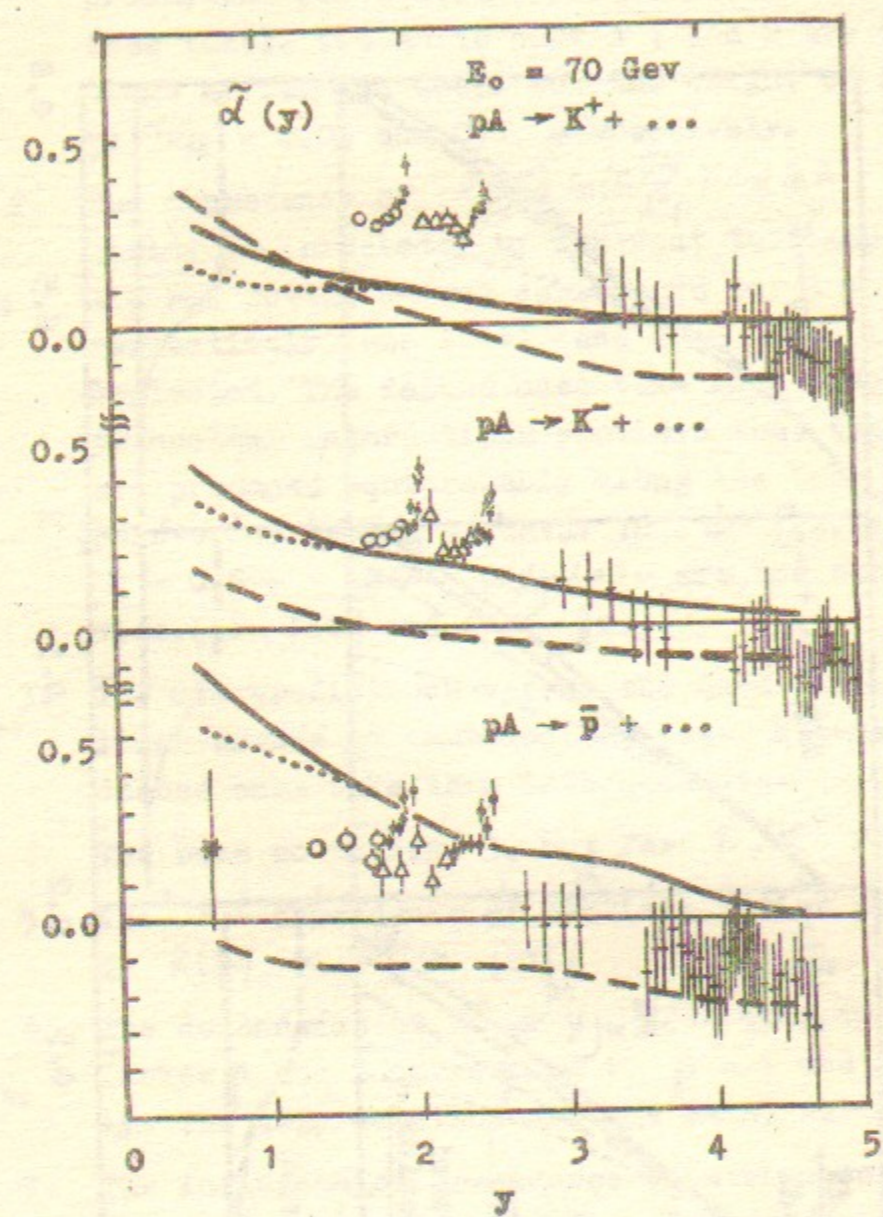


Fig.2

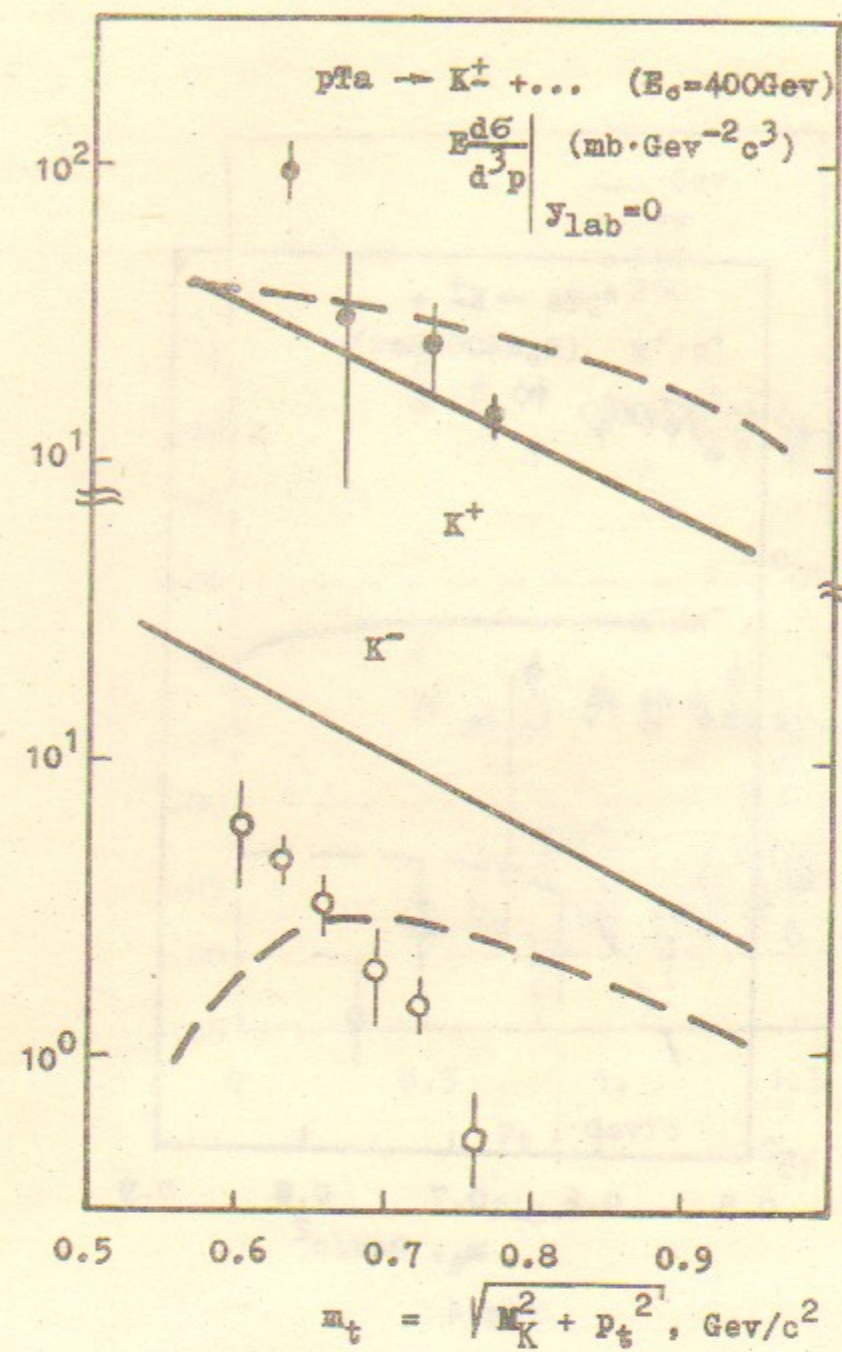


Fig.3

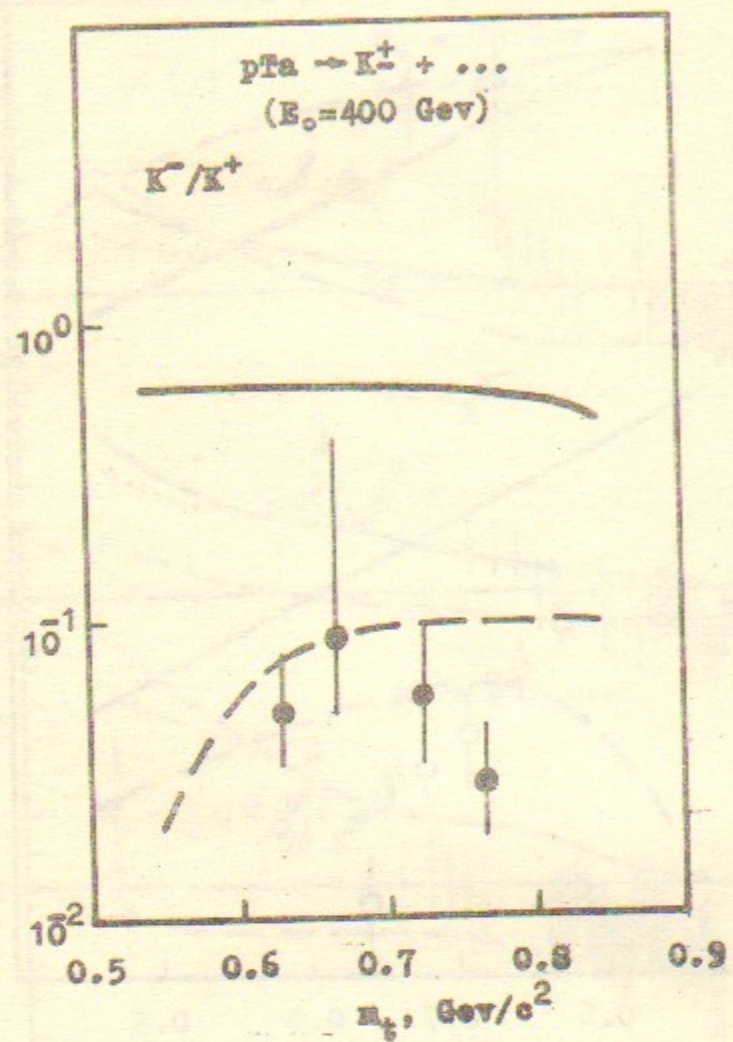


Fig.4

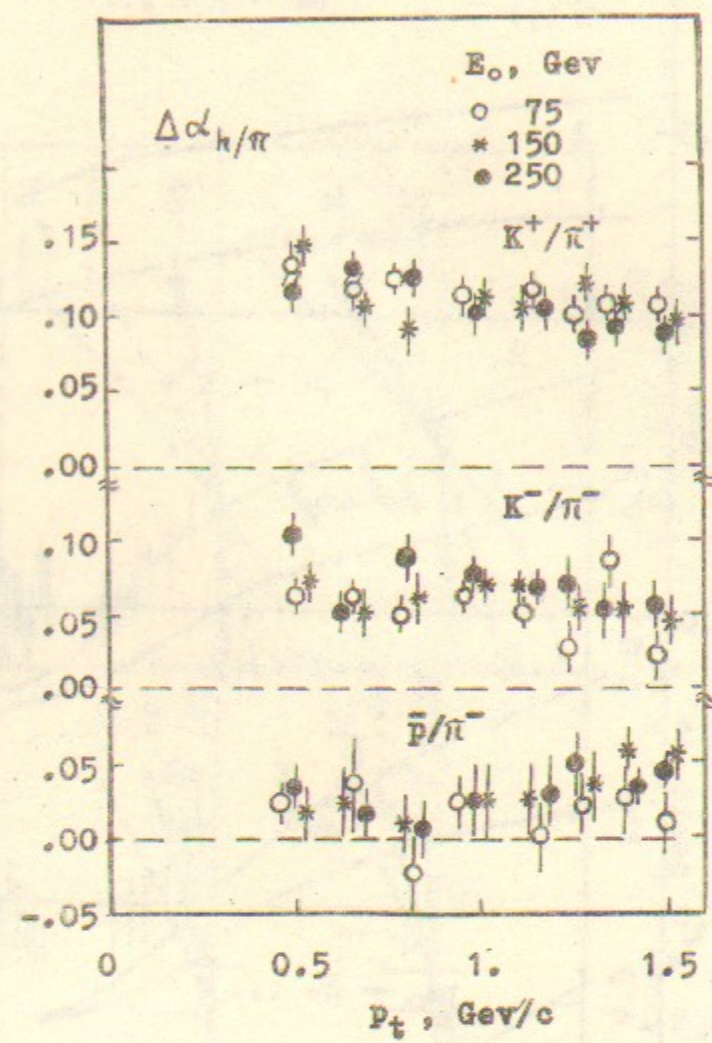


Fig.5

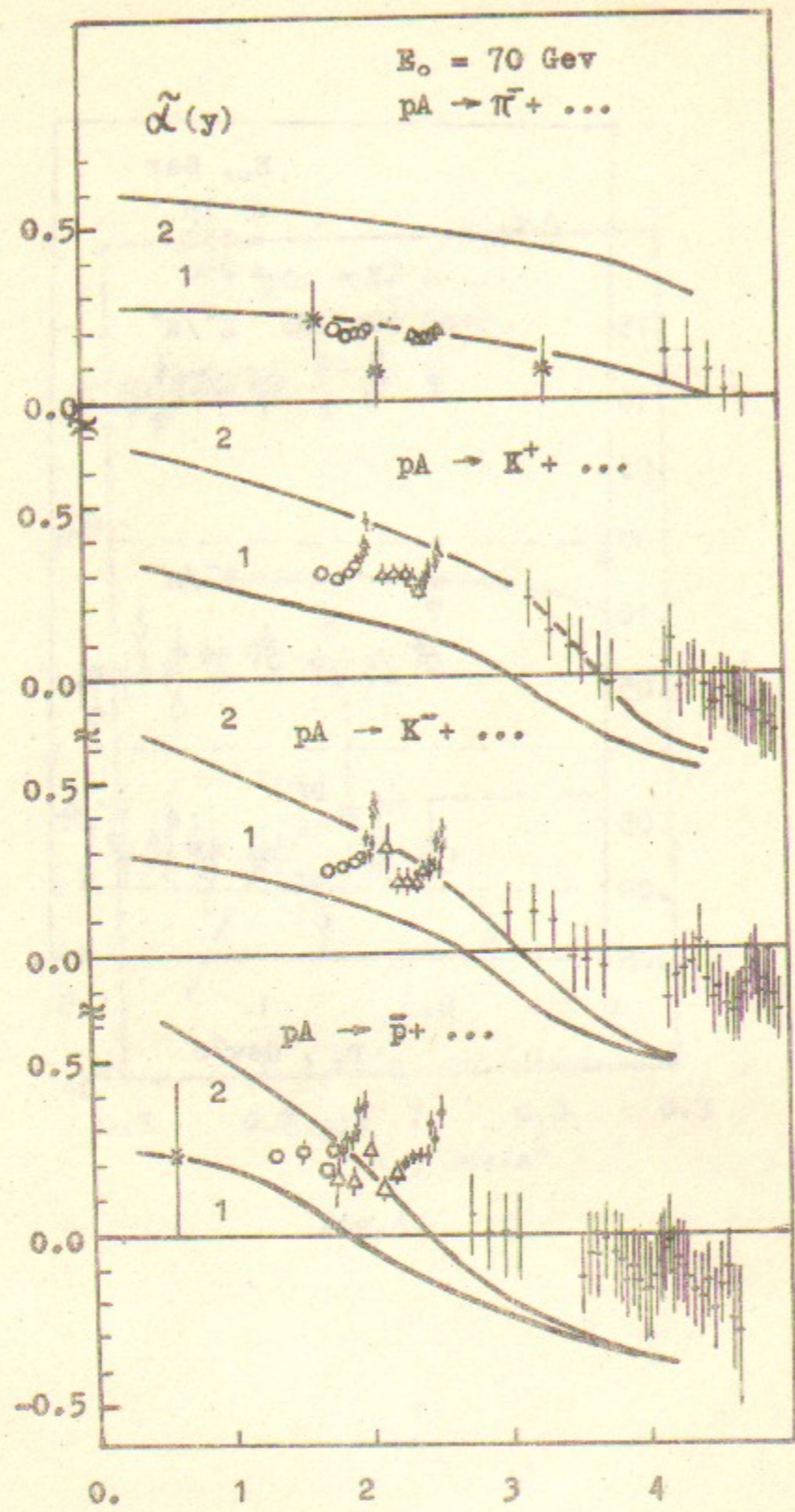
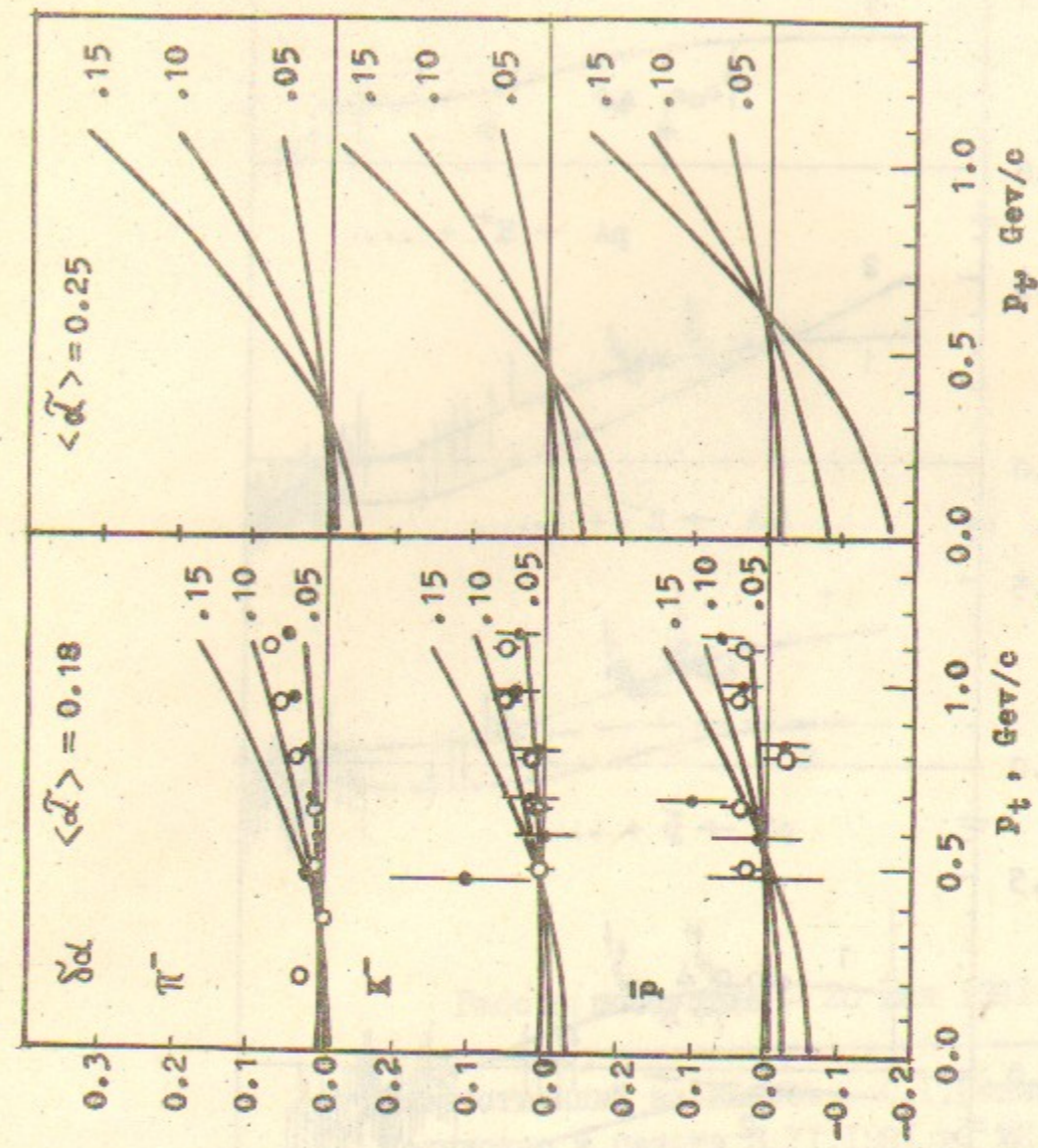


Fig.6



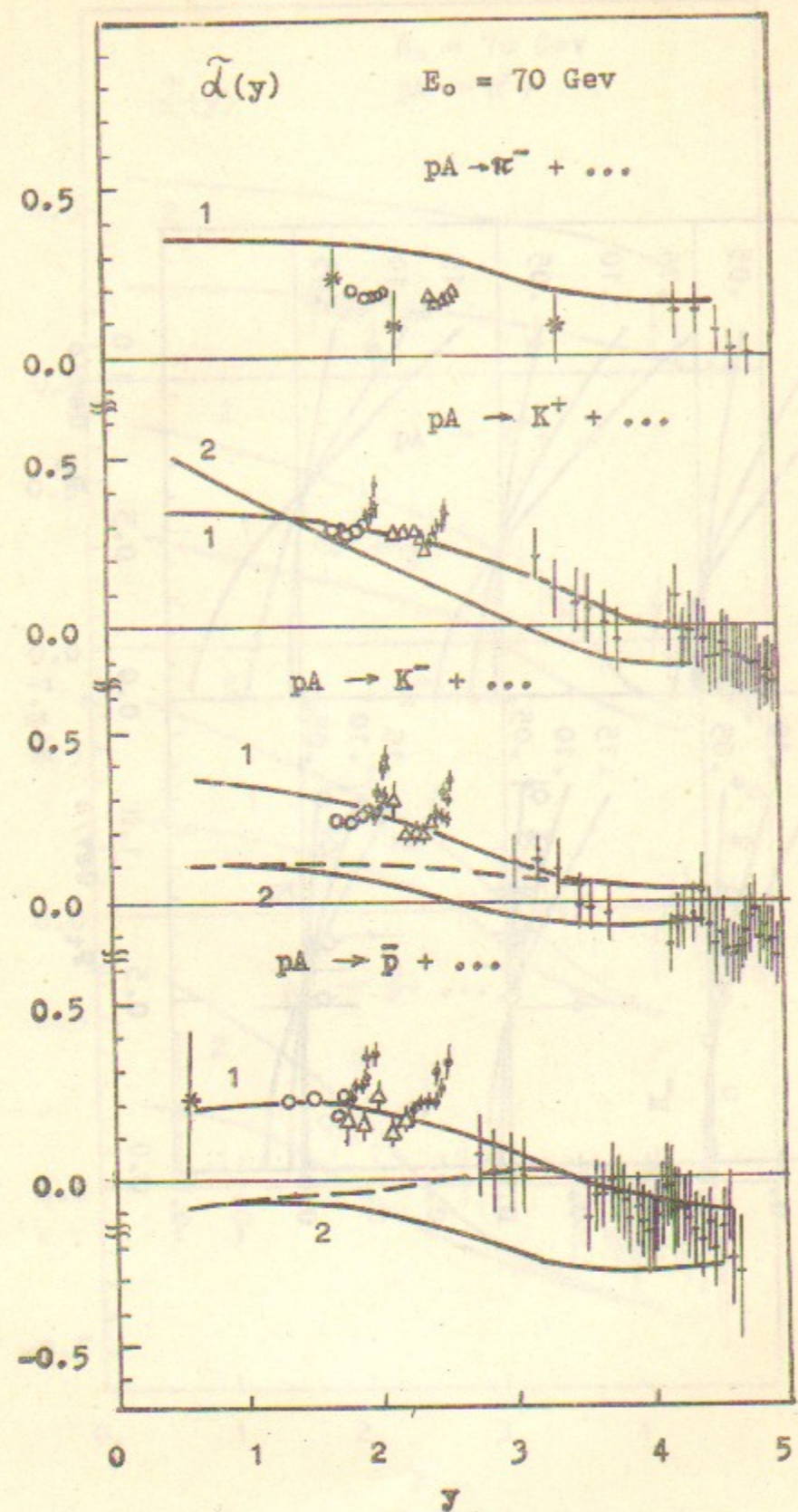


Fig.8

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