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ON THE CALCULATION OF THE π-MESON WAVE FUNCTION

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A B S T R A C T

First few moments, $\langle \xi^{2n} \rangle = \int_{-1}^{1} d\xi\, \varphi_\pi(\xi)\, \xi^{2n}$, $n = 0, 1, 2, 3$ of the π -meson wave function $\varphi_\pi(\xi)$ (which determine the distribution of quark longitudinal momentum inside the π -meson, $P_1 = x P_\pi$, $P_2 = (1-x) P_\pi$, $\xi = x - (1-x) = 2x - 1$, $P_\pi \to \infty$) are calculated,using the sum rules like those of /6/.

Based on these results,we discuss qualitative behaviour of the wave function $\varphi_\pi(\xi)$ and propose the model wave function which gives good description of the π -meson decay modes of charmed particles,as shown in our next papers /7,8/.

I. Introduction.

During last few years the significant progress has been reached in understanding of asymptotic behaviour of exclusive processes in QCD framework. This area includes the asymptotic behaviour of: a) meson and baryon form factors; b) large angle elastic scattering; c) exclusive electroproduction; d) threshold behaviour of the deep-inelastic structure functions; e) heavy meson decays, etc. /1-5/.

The main object with the help of which all the above processes are described is the meson (baryon) wave function. Defined in an appropriate way (see bellow) the meson wave function $\varphi(x, \mu^2)$ has the meaning of the amplitude for the meson to decay (in the $P_\pi \to \infty$ reference frame) to the quark-antiquark pair with the fractions of the longitudinal momentum "X" and "1-X" and with virtualness up to „μ^2". At large enough μ^2 (say, $\mu^2 > \mu_0^2 \simeq (500\,MeV)^2$) the function $\varphi(x, \mu^2)$ can be found with the help of the renormalization group, if the function $\varphi(x, \mu_0^2)$ is known. The function $\varphi(x, \mu_0^2)$ depends mainly on the large distance interaction and can not be found within the perturbation theory.

The main goal of the present paper is to obtain the information about the properties of the wave function $\varphi(x, \mu_0^2)$ for which we use the sum rule method developed in papers/6/. Using the sum rules for the corresponding correlators, we find the values of the first few moments of the π -meson wave function

$$\langle \xi^{2n} \rangle = \int_{-1}^{1} d\xi\, \xi^{2n}\, \varphi_\pi(\xi, \mu_0^2), \quad n = 0,1,2,3, \qquad (1)$$

where $\xi P_\pi = x P_\pi - (1-x) P_\pi = (2x-1) P_\pi$ is the quark-antiquark relative longitudinal momentum. The corresponding sum rules are obtained in Sect.II and the values of the wave function moments are found there as well. Using these results, in Sect.III we discuss the characteristic qualitative properties of the π-meson wave function and propose the model wave function which fulfil the sum rules.

We use then this model wave function to describe the strong decays of the heavy mesons and the π-meson form factor ($\chi_0(3415) \to \pi^+\pi^-$, $\chi_2(3555) \to \pi^+\pi^-$, $\psi(3100) \to \pi^+\pi^-$, etc.) /7/, and weak decays of charmed mesons ($D^0 \to K^-\pi^+$, $D^0 \to \bar{K}^0\pi^0$, etc.)/8/. We show that the selfconsistent description of all the above processes is possible. This means that the model wave function proposed in this paper simultaneously fulfils sum rules and gives good description of the heavy mesons strong, electromagnetic and weak decays.

II. <u>Calculation of the π-meson wave function moments.</u>

The π-meson wave function connected with lowest twist operators is defined as follows:

$$\langle 0 | \bar{d}(z) \gamma_\nu \gamma_5 \exp\left\{ i g \int_{-z}^{z} d\sigma_\mu B_\mu(\sigma) \right\} u(-z) | \pi^+(q) \rangle_{\mu^2} \equiv \tag{2}$$

$$\equiv \sum \frac{(-1)^n}{n!} \langle 0 | \bar{d}(0) \gamma_\nu \gamma_5 (z_\mu \overleftrightarrow{D}_\mu)^n u(0) | \pi^+(q) \rangle_{\mu^2} = i q_\nu \tilde{\varphi}(zq, \mu^2) + \ldots,$$

$$z^2 = 0.$$

Here: $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu$, $\overrightarrow{D}_\mu = \overrightarrow{\partial}_\mu - i g B_\mu^a \frac{\lambda^a}{2}$ - is the covariant derivative, $d(0)$ and $u(0)$ are quark field operators,

"μ^2" is the normalization point of the operators in (2). The function $\tilde{\varphi}(\xi, \mu^2)$ defined by

$$\tilde{\varphi}(zq, \mu^2) = \int_{-1}^{1} d\xi \, e^{i\xi(zq)} \tilde{\varphi}(\xi, \mu^2) \tag{3}$$

has the meaning of decay amplitude into quark-antiquark pair with the fractions of longitudinal momentum X and (1-X), $\xi = 2x-1$ and with a virtuality up to "μ^2". The wave function $\tilde{\varphi}(\xi, \mu^2)$ enter the expressions for an asymptotic behaviour of all exclusive reactions in which the π-meson takes place. Therefore, one needs to know the wave function $\tilde{\varphi}(\xi, \mu^2)$ in order to do quantitative calculations.

The dependence of $\tilde{\varphi}(\xi, \mu^2)$ on μ^2 at $\mu^2 > \mu_0^2$ is determined by the renormalization group (μ_0^2 is the characteristic scale beginning with which the perturbation theory is applicable) /1-5/:

$$\tilde{\varphi}(\xi, \mu^2) = \frac{3}{4}(1-\xi^2) \sum_{k=0}^{\infty} f_k C_k^{3/2}(\xi) \left[\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)} \right]^{\gamma_k / b}, \quad b = 11 - \frac{2}{3} N_f, \tag{4}$$

$$\gamma_k = \frac{4}{3}\left[1 - \frac{2}{(k+1)(k+2)} + 4 \sum_{j=2}^{k+1} \frac{1}{j} \right], \quad f_k = \int_{-1}^{1} d\xi \, C_k^{3/2}(\xi) \tilde{\varphi}(\xi, \mu_0^2),$$

where $\{ C_k^{3/2}(\xi) \}$ is the system of the Gegenbauer polinomials normalized by

$$\int_{-1}^{1} d\xi \frac{3}{4}(1-\xi^2) C_k^{3/2}(\xi) C_n^{3/2}(\xi) = \delta_{kn}, \quad C_0^{3/2}(\xi) = 1, \tag{5}$$

γ_k -are the corresponding anomalous dimensions.

Therefore, for determination of $\tilde{\varphi}(\xi, \mu^2 > \mu_0^2)$ it is sufficient to know $\tilde{\varphi}(\xi, \mu_0^2)$. The wave function $\tilde{\varphi}(\xi, \mu_0^2)$ is determined mainly by large distance interactions and can not be directly calculated at present. The experimental information about $\tilde{\varphi}(\xi,\mu^2)$ available now is exhausted by the knowledge of the matrix element $\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 u(0)|\pi^+(q)\rangle = i f_\pi \cdot q_\mu$, $f_\pi = \int_{-1}^{1} d\xi\, \tilde{\varphi}(\xi,\mu^2) = 133\,MeV$ from $\pi\to\mu\nu$ decay. In the following we use the dimensionless wave function $\varphi(\xi,\mu^2)$

$$\tilde{\varphi}(\xi,\mu^2) = f_\pi\, \varphi(\xi,\mu^2), \qquad \langle \xi^0\rangle = \int_{-1}^{1} d\xi\, \varphi(\xi,\mu^2) = 1.$$

Our goal is to calculate approximately a number of wave function moments

$$\langle \xi^{2n}\rangle_{\mu_0^2} = \int_{-1}^{1} d\xi\, \xi^{2n} \varphi(\xi,\mu_0^2), \quad n=0,1,2,3. \tag{6}$$

(In the limit of the isotopic symmetry: $\varphi(\xi,\mu^2)=\varphi(-\xi,\mu^2)$). As one will see bellow, the knowledge of few first moments of the wave function gives stringent enough restrictions on its possible form and allows to understand the qualitative behaviour of $\varphi(\xi,\mu^2)$.

To calculate approximately the values of moments (6) we use the sum rule method developed in the papers /6/. This method allows to obtain the information about spectral dencities of two point functions. In number of cases the sum rules appear to be sensitive to separate resonance contribution and allow, therefore, to determine the value of the matrix element $\langle 0|O_n|Res\rangle$, where O_n -is the corresponding local operator, $|Res\rangle$ -is the one particle state.

As one can see from (2), the moments (6) are determined by the matrix elements of operators:

$$\langle 0|\bar{d}(0)\hat{z}\gamma_5 z_\nu (iz_\mu \overleftrightarrow{D}_\mu)^n u(0)|\pi^+(q)\rangle_{\mu^2} = i f_\pi (zq)^{n+1} \langle \xi^n\rangle_{\mu^2}, \; z^2=0 \tag{7}$$

In order to find these matrix elements we consider the correlators

$$T_{n_1n_2}(q^2;zq) = i\int dx\, e^{iqx}\langle 0|T\,O_{n_1}(x)O_{n_2}(0)|0\rangle = (zq)^{n_1+n_2+2}\, I_{n_1n_2}(q^2), \tag{8}$$

$$O_n = \frac{1}{\sqrt{2}}\bar{u}\hat{z}\gamma_5(iz_\mu \overleftrightarrow{D}_\mu)^n u - (u\to d)\; ; \; z^2=0 .$$

The spectral den$ity in (8) has the form:

$$\frac{1}{\pi}\,\mathrm{Im}\, I_{n_1n_2}(q^2) = f_\pi^2 \langle \xi^{n_1}\rangle_{q^2} \langle \xi^{n_2}\rangle_{q^2}\, \delta(q^2-m_\pi^2) + \ldots \tag{9}$$

The asymptotic behaviour of $I_{n_1n_2}(q^2)$ at large $(-|q^2|)$ has the form:

$$I_{n_1n_2}(q^2) \to \int_{-1}^{1} d\xi\cdot\xi^n \cdot \Big\{ -\frac{1}{4\pi^2}\ln(-q^2)\cdot\frac{3}{4}(1-\xi^2) + \frac{1}{12}\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle \cdot$$

$$\cdot\frac{1}{q^4}\cdot\frac{1}{2} - \frac{32}{9}\pi\frac{\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2}{q^6}\Big[\frac{1}{2}\delta(1-\xi)+\frac{1}{2}\delta(1+\xi)\Big] - \tag{10}$$

$$\frac{64}{81}\pi\frac{\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2}{q^6}\Big[\frac{1}{2}\delta(1-\xi)+\frac{1}{2}\delta(1+\xi)-\delta'(1-\xi)-\delta'(1+\xi)\Big]\Big\}+\ldots$$

$$I_{n_1n_2}(q^2) \to -\frac{1}{4\pi^2}\ln(-q^2)\frac{3}{(n+1)(n+3)} + \frac{1}{12}\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle\frac{1}{q^4}\frac{1}{n+1} -$$

$$-\frac{32}{9}\pi\frac{1}{q^6}\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2 - \frac{64}{81}\pi\frac{1}{q^6}\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2(1+2n)+\ldots,\; n=n_1+n_2.$$

Four terms in (10) correspond to the contributions of four types of diagram, Figs. 1-4. After rotation into the Euclidean space and "borelezation" /6/, sum rules take the form:

$$\frac{4\pi}{M^2}\int_0^\infty ds\, e^{-s/M^2}\, \mathrm{Im}\, I_{n,n_2}(s) \to \frac{3}{(n+1)(n+3)}\Big\{ 1 + \tag{11}$$

$$\frac{n+3}{9}\pi^2\frac{1}{M^4}\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle + \frac{64}{243}\pi^3\frac{1}{M^6}\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2(n+1)(n+3)(11+4n)\Big\}.$$

We use the following values for the matrix elements entering into (11) (see /6/):

$$\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2 \simeq 1.83\cdot 10^{-4}\,GeV^6, \quad \langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle \simeq 1.2\cdot 10^{-2}\,GeV^4. \tag{12}$$

Then the sum rules for the correlators I_{20}, I_{40}, and I_{60} take the form:

$$\frac{20\pi}{M^2}\int ds\, e^{-s/M^2}\,\mathrm{Im}\, I_{20}(s) \to 1+\left(\frac{0.26\,GeV^2}{M^2}\right)^2+\left(\frac{0.75\,GeV^2}{M^2}\right)^3,$$

$$\frac{140\pi}{3M^2}\int ds\, e^{-s/M^2}\,\mathrm{Im}\, I_{40}(s) \to 1+\left(\frac{0.306\,GeV^2}{M^2}\right)^2+\left(\frac{1.12\,GeV^2}{M^2}\right)^3, \tag{13}$$

$$\frac{84\pi}{M^2}\int ds\, e^{-s/M^2}\,\mathrm{Im}\, I_{60}(s) \to 1+\left(\frac{0.34\,GeV^2}{M^2}\right)^2+\left(\frac{1.48\,GeV^2}{M^2}\right)^3.$$

The sum rule for the I_{00} correlator has been obtained and treated in /6/. The value for f_π obtained in this way agrees well with the experimental one. It is important to note that the intermediate states in the correlators I_{n0} and I_{00} are the same. Therefore, we take the same model for the I_{n0} -spectral densities as that was used in /6/ for I_{00}. Exactly, we put

$$\frac{1}{\pi}\mathrm{Im}\, I_{n0}(s) \simeq f_\pi^2\langle\xi^n\rangle\,\delta(s) + \frac{3}{4\pi^2(n+1)(n+3)}\,\theta(s-s_{n0}) \tag{14}$$

where the parameters s_{n0} determine the duality intervals in corresponding correlators. From (13),(14) one has:

$$30\pi^2 f_\pi^2\langle\xi^2\rangle + M^2 e^{-s_{20}/M^2} \simeq M^2\left\{1+\left(\frac{0.26\,GeV^2}{M^2}\right)^2+\left(\frac{0.75\,GeV^2}{M^2}\right)^3\right\} \tag{15a}$$

$$\frac{140\pi^2}{3} f_\pi^2\langle\xi^4\rangle + M^2 e^{-s_{40}/M^2} \simeq M^2\left\{1+\left(\frac{0.306\,GeV^2}{M^2}\right)^2+\left(\frac{1.12\,GeV^2}{M^2}\right)^3\right\} \tag{15b}$$

$$84\pi^2 f_\pi^2\langle\xi^6\rangle + M^2 e^{-s_{60}/M^2} \simeq M^2\left\{1+\left(\frac{0.34\,GeV^2}{M^2}\right)^2+\left(\frac{1.48\,GeV^2}{M^2}\right)^3\right\}. \tag{15c}$$

The treatment of (15) has been made in the following way. In each of sum rules (15) the value of M^2 has been varied in such limits that power corrections at r.h.s. in (15) were between 5-10% and 30-35%.* Then the parameters $\langle\xi^{2n}\rangle$ and s_{n0} were choosen so that to obtain the best fit to the theoretical curve at r.h.s. in each of sum rules (15). As a result:

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* At smaller values „M^2" the neglected terms with higher powers „M^{-n}" become significant, at larger values „M^2" the sum rules (15) lose their sensitivity to the π -meson contribution.

$$\langle \xi^2 \rangle_{\bar{M}^2_{20} \simeq 1.5\,GeV^2} \simeq 0.36 \;, \quad s_{20} \simeq 1.66\;GeV^2$$

$$\langle \xi^4 \rangle_{\bar{M}^2_{40} \simeq 2.2\,GeV^2} \simeq 0.22 \;, \quad s_{40} \simeq 2.36\;GeV^2 \tag{16}$$

$$\langle \xi^6 \rangle_{\bar{M}^2_{60} \simeq 3\,GeV^2} \simeq 0.16 \;, \quad s_{60} \simeq 3.1\;GeV^2$$

It is worth noting that proceeding in the same way we obtain $f_\pi \simeq 129\;MeV$ (the experimental value is $f_\pi \simeq 133\;MeV$) from the I_{00} -sum rule and the value $f_\rho \simeq 190\;MeV$ (the experimental value is $f_\rho \simeq 200\;MeV$) from an analogous sum rule for the isovector-vector current. We expect, therefore, that the above found values $\langle \xi^2 \rangle$ and $\langle \xi^4 \rangle$ are correct within 15-20% uncertainty. (The accuracy of the predictions decreases as n increases).*

Because in sum rules we have not introduced explicitly the logarithmic corrections due to anomalous dimensions, the values of $\langle \xi^{2n} \rangle$ obtained above correspond to normalizations at suitable intermediate points: $\bar{M}^2_{20} \simeq 1.5\;GeV^2$, $\bar{M}^2_{40} \simeq 2.2\;GeV^2$, $\bar{M}^2_{60} \simeq 3\;GeV^2$,respectively.**

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* Note that the inclusion of the A_1 -meson in addition to π -meson do not change nearly the values of $\langle \xi^{2n} \rangle$.

** As the values $\bar{M}^2_{n0}$ are large enough, $\alpha_s(M^2)$ varies already slowly in this region and so the precise knowledge of $\bar{M}^2_{n0}$ is not needed. More punctual treatment of the anomalous dimension effects gives essentially the same results.

As will be shown in /7/, the characteristic virtuality of the pion wave function in charmonium decays is: $\mu_0^2 \simeq (500\;MeV)^2$. We, therefore, renormalize the above found values $\langle \xi^{2n} \rangle$ to this point. The renormalization is performed with the help of formulae (4):

$$\langle \xi^2 \rangle_{\mu_0^2} = \left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(\bar{M}^2_{20})}\right]^{\frac{50}{9b}} \langle \xi^2 - \tfrac{1}{5} \rangle_{\bar{M}^2_{20}} + \tfrac{1}{5}\langle 1 \rangle \;;$$

$$\langle \xi^4 \rangle_{\mu_0^2} = \left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(\bar{M}^2_{40})}\right]^{\frac{364}{45 \cdot b}} \langle \xi^4 - \tfrac{2}{3}\xi^2 + \tfrac{1}{21} \rangle_{\bar{M}^2_{40}} + \tag{17}$$

$$+ \frac{2}{3}\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(\bar{M}^2_{40})}\right]^{\frac{50}{9b}} \langle \xi^2 - \tfrac{1}{5} \rangle_{\bar{M}^2_{40}} + \frac{3}{35}\langle 1 \rangle \;;$$

$$\langle \xi^6 \rangle_{\mu_0^2} = \left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(\bar{M}^2_{60})}\right]^{\frac{26}{3 \cdot b}} \langle \xi^6 - \tfrac{15}{13}\xi^4 + \tfrac{45}{143}\xi^2 - \tfrac{5}{429} \rangle_{\bar{M}^2_{60}} +$$

$$+ \left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(\bar{M}^2_{60})}\right]^{\frac{364}{45 \cdot b}} \cdot \frac{15}{13} \langle \xi^4 - \tfrac{2}{3}\xi^2 + \tfrac{1}{21} \rangle_{\bar{M}^2_{60}} + \left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(\bar{M}^2_{60})}\right]^{\frac{50}{9 \cdot b}} \cdot \frac{845}{1859} \cdot$$

$$\cdot \langle \xi^2 - \tfrac{1}{5} \rangle_{\bar{M}^2_{60}} + 0.048 \langle 1 \rangle .$$

We use:

$$\alpha_s(\mu^2) = \frac{4\pi}{b} \frac{1}{\ln \mu^2/\Lambda^2} \;, \quad b = 9 \,, \quad \Lambda = 100\;MeV . \tag{18}$$

From (15),(17),(18) we have:

$$\langle \xi^2 \rangle_{\mu_0^2 = (500\,MeV)^2} \simeq 0.41 \;; \tag{19}$$

$$\langle \xi^4 \rangle_{\mu_0^2 = (500\,MeV)^2} \simeq 0.27 \;;$$

$$\langle \xi^6 \rangle_{\mu_0^2 = (500\,MeV)^2} = 0.20 \;.$$

The expressios (19) present the main result of this paper. In the next Chapter we use them to construct the model wave function of the pion.

We do not consider here the next ($n \geq 4$) moments of the wave function. The reasons are as follows.

1) As one can see from (11), the coefficients at operators of larger dimensions grow more quickly with n_1 (n_2). Therefore, at larger „n" one generally needs to take into account the operators with higher dimensions in the operator expansion.

2) Because the contribution of the diagram in Fig.1 decreases with "n" (see (11)), the relative importance of power terms in sum rules increases (see point "1"). This leads to increase of the characteristic values of M^2 at which the power corrections are, say, $\sim 20\,\%$ of the perturbation theory contribution. At larger values of M^2 the background contribution play more important role in l.h.s. of sum rules (11), while the role of the π-meson contribution decreases. Therefore, as "n" increases the sum rules become less sensitive to the pion contribution.

We do not use the sum rules for the correlators $I_{22}, I_{24}, I_{44}\ldots$ The matter is that in the correlator I_{nn} intermediate states with the spins $0 \leq S \leq n+1$ contribute as well (while in the correlator I_{n0} the intermediate states have $S=1$ only, except for the pion being the Goldstone particle). That is why in the sum rules for I_{nn} as compared to that of I_{n0} :a) the background (i.e. all other contributions except for the pion) can play much more important role, and its form can vary noticeably with "n"; b) the role

of the pion contribution and sensitivity of sum rules to it is smaller. If we ignore the presence of additional contributions with spins $0 \leq S \leq n+1$ to the correlator I_{nn} (absent in I_{n0}), and choose the same form for the spectral densities $\mathrm{Im}\, I_{nn}$ and $\mathrm{Im}\, I_{n0}$ (see (14)), then naturally we obtain on this way larger values for $\langle \xi^n \rangle$. The reason is that we force the pion by itself to fill a duality interval. Indeed, the value $\langle \xi^2 \rangle_{1.5\,GeV^2}$ obtained in this way from the sum rule I_{22} is: $\langle \xi^2 \rangle_{1.5\,GeV^2} \simeq 0.47$, and this value does not contradicts to (16) yet. However, the value of $\langle \xi^4 \rangle$ obtained on this way from I_{44} already exceeds the value (16) considerably.

III. Discussion.

In this Section using the above found $\langle \xi^n \rangle$ values we describe qualitatively the behaviour of the pion wave function and propose the model for it.

Strickly speaking, the knowledge of few first moments is clearly unsufficient for reconstruction of the wave function. If however, we use additional physical considerations, it is not difficult to reconstruct the qualitative behaviour of the wave function.

As a starting point we choose the exactly known wave function $\varphi_\infty(\xi) = \varphi(\xi, \mu^2 \to \infty) = \frac{3}{4}(1-\xi^2)$, Fig.5a. This wave function corresponds clearly to the contribution of diagram shown in Fig.1, because at $\mu^2 \to \infty$ due to the asymptotic freedom one can neglect power and logarithmic corrections. The wave function $\varphi_\infty(\xi)$ give too small values for the moments $\langle \xi^n \rangle$ as compared to (19):

$$\langle \xi^2 \rangle_\infty = 0.20; \quad \langle \xi^4 \rangle_\infty = 0.086; \quad \langle \xi^6 \rangle_\infty = 0.048 \tag{20}$$

This is natural, as the power corrections (Figs.2-4) add to the loop contribution (Fig.1) in (15) and therefore increase the values of $\langle \xi^n \rangle$.

Now, in what way one should change the wave function $\varphi_\infty(\xi)$ in order to increase the moment values $\langle \xi^n \rangle$, $n=2,4,6$ and to keep the normalization $\langle \xi^0 \rangle = 1$ intact? Of course, there are many possibilities. However, we put some limitations based on physical considerations. Namely, we expect that $\varphi(\xi,\mu) \to 0$ at $|\xi| \to 1$ (the point $|\xi|=1$ corresponds to the case when the whole pion momentum is carried by one quark, while another one is "wee"). Besides, it is reasonable to expect that the ground state (pion) wave function is positive, $\varphi(\xi, \mu^2 > \mu_0^2) \geq 0$*. Then, among functions with such properties the simplest way to increase the values of the moments $\langle \xi^n \rangle$ as compared to (20), is to take the function $\varphi(\xi)$ such that $\varphi(\xi) < \varphi_\infty(\xi)$ at small $|\xi|$ and $\varphi(\xi) > \varphi_\infty(\xi)$ at, say, $|\xi| > 0.5$ In such a way we come to the qualitative behaviour of the wave function $\varphi_1(\xi)$ depicted at Fig.5b.

Most characteristic (and unexpected) property of the wave function $\varphi(\xi)$, Fig.5b, is the existence of maximum at $|\xi| \neq 0$ and minimum at $\xi \simeq 0$. (Let us remind in connection with this that for the nonrelativistic quarks the wave function is: $\varphi(\xi) \simeq \delta(\xi)$, i.e. each quark carries one half of the whole momentum). This means that as a rule greater part of the pion momentum is carried by one quark.

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* This property is due to analogy to the nonrelativistic quantum mechanics, though the node theorem is absent, of course, in QCD.

Based on these considerations, we propose the following model wave function for the pion:

$$\varphi(\xi, \mu_0^2 \simeq (500\,MeV)^2) = \frac{15}{4}\xi^2(1-\xi^2). \tag{21}$$

The corresponding values of the moments of this function are equal to

$$\langle \xi^0 \rangle = 1, \ \langle \xi^2 \rangle = 0.43, \ \langle \xi^4 \rangle = 0.24, \ \langle \xi^6 \rangle = 0.15 \tag{22}$$

and agree well with the values (19) found above. The function (21) has the maximum at $|\xi| \simeq 0.7$, and at this point ~ 85% of the pion momentum is carried by one quark, and the rest ~ 15% by another one.

Of course, the considerations described above can not be considered as a derivation of the wave function (21). We expect however, that the model wave function (21) reproduces correctly all characteristic features of the true pion wave function. As will be shown in subsequent papers /7,8/, the use of the wave function (21) for description of pionic decay modes of heavy mesons gives the results which agree with experiment.

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FIGURE CAPTIONS

Fig.5a Asymptotic form of the π -meson wave function

$$\varphi_\infty(\xi) = \varphi(\xi, \mu^2 \to \infty)$$

Fig.5b The qualitative behaviour of the π -meson wave function $\varphi(\xi, \mu_0^2)$, $\mu_0^2 \simeq (500\,\mathrm{MeV})^2$.

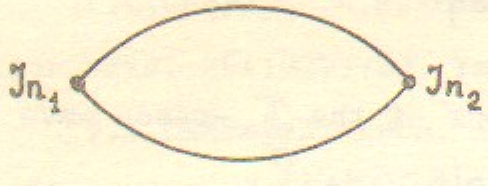

Fig.1

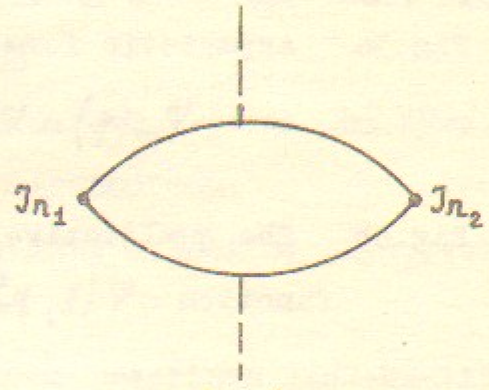

Fig.2

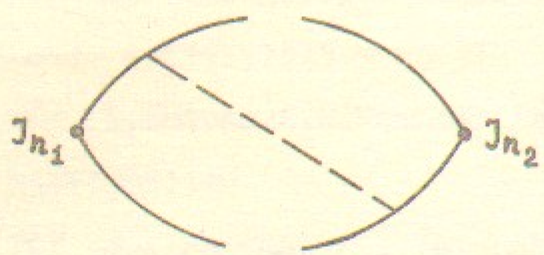

Fig.3

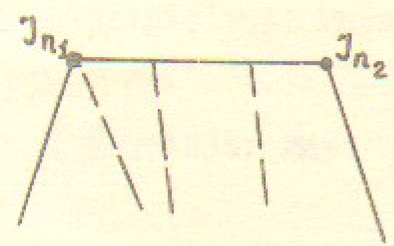

Fig.4

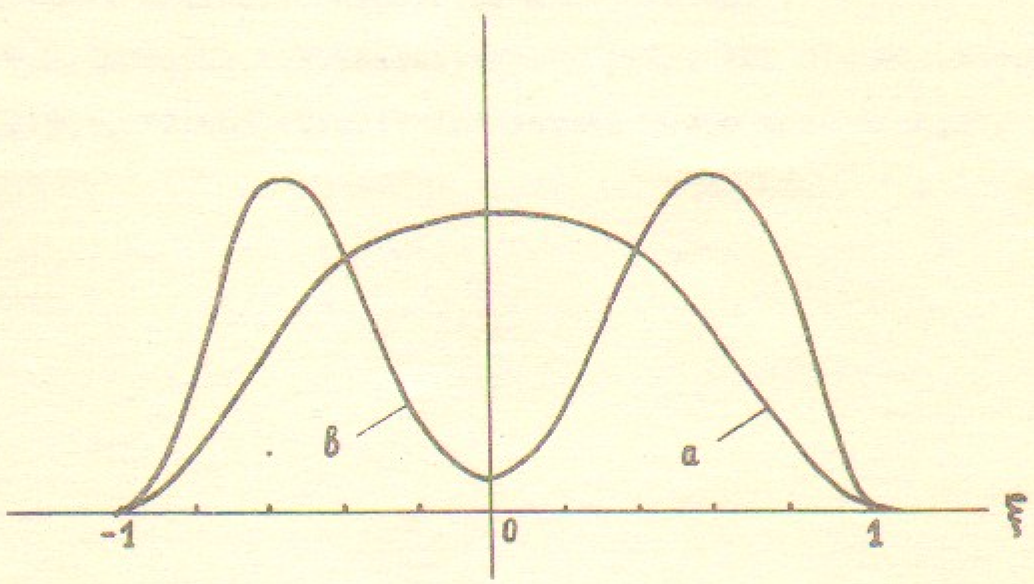

Fig.5

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