ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
СО АН СССР

V.N.Baier and Yu.F.Pinelis

SMALL-SIZE VACUUM FLUCTUATIONS AND QCD SUM RULES

PREPRINT 82-115

НОВОСИБИРСК

SMALL-SIZE VACUUM FLUCTUATIONS AND QCD SUM RULES

V.N.Baier and Yu.F.Pinelis

Institute of Nuclear Physics

630090, Novosibirsk 90, USSR

Abstract

A nonperturbative mechanism of an asymptotic freedom breaking has been analysed for the ρ, J/ψ, Υ-resonances which is connected with short-wave vacuum fluctuations (VF) of $\rho \sim \rho_c \ll 1/\Lambda$ ($\Lambda \approx 0.1$ GeV) in size. A method for investigation of nonperturbative effects in the physical region has been suggested. The description of the $e^-e^+ \to (\text{hadrons})_c$, $e^-e^+ \to (\text{hadrons})_b$, $e^-e^+ \to (\text{hadrons})_{I=1}$ processes for the physical values of q^2 is not only in agreement with a small size of VF due to which a general structure of the total cross section is formed, but also gives a large value $\langle \frac{\alpha_s}{\pi} G^2 \rangle \sim 0.1\ \text{GeV}^4$ of gluon vacuum condensate. An approach to scaling is accompanied by large-scale oscillations of the total cross sections in agreement with experiment. It is shown that the sum rules in Euclidean region for the ψ and Υ-families work quite well for the considered mechanism of the asymptotic freedom breaking. In the sum rules the Coulomb effects play a very important role for the Υ and ψ-families and they should be adequately considered. The given mechanism of asymptotic freedom breaking is shown to lead to a dynamical freezing of the coupling constant α_s, starting from rather small distances. It is shown within approach, that interaction between heavy quark and antiquark may be described by a potential, which is local, static, energy- and quark mass-independent, what is common for the phenomenological potential models.

1. Introduction

Although the main problem of QCD - explanation of the mechanism of confinement - is not yet solved, by now some important steps have taken in understanding of the nonperturbative QCD. A real progress in this field has started with the discovery the solutions of classical Euclidean field equations - - the instantons /1/. In the papers /2/, /3/ it has been realized that instantons should play an important role in the formation of the structure of nonperturbative QCD vacuum, spontaneous breaking of chiral symmetry and generation of dynamical mass of quarks, renormalization of the coupling constant as well as in the solution of the U(1) - problem and in the generation of pseudogoldstone bosons.

At present, one can obtain quantitative results only in the dilute instanton gas approximation (DGA) /3-6/ not taking into account the interaction of instantons between each other. It is important that not all the processes in QCD take place at "extremely" large distances, $\tau \sim 1/\Lambda$ $(\Lambda_{\overline{MS}} \approx 0.1\,GeV)$, at which this approximation is sure not to work. There is /3,5-10/ an intermediate region of distances $\tau \sim \rho_c$ $(\rho_c \ll 1/\Lambda)$ with which connected are, in particular, chiral symmetry breaking /3,5,6,9/, large scale /7/ in gluon channels with quantum numbers $0^{\pm}$, formation of large intervals of variability because of small-size vacuum fluctuations (VF) /10/.

In this paper we present some results that are the extension of the paper /10/. The concrete consideration of the process $e^{+}e^{-} \to$ charm in the physical region and the comparison with experiment enable one to conclude that the VF, which form the general structure of spectral density, are small size VF (we have connected them with instantons) and also that the vacuum gluon condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ is large in magnitude ($\langle \frac{\alpha_s}{\pi} G^2 \rangle \sim 0.1\ GeV^4$).

In Refs. /11/ the phenomenological method has been suggested for obtaining the mass and widths of lowest states in different channels beyond the framework of perturbation theory - the method of sum rules. The basic assumption of the method /11/ consist in that the dominance region of the lowest state and the region of asymptotically free (with an accuracy up to small power corrections which fix, however, the scale in the problem) behaviour of the Euclidean amplitude are partly overlapped. It is clear that such a possibility depends, certainly, on a relative magnitude of matrix elements of the corresponding local operators, which in turn is determined by a characteristic size ρ_c of the VF forming the amplitude under discussion. Omittion of the higher power corrections implies that $1/\rho_c \ll E_z$ (E_z is the distance from the left border of the dominance region of the lowest state to the threshold), i.e. the VF must be of long-wave nature in this case.

The conclusions of Ref. /10/ imply the other physical pattern in which, for example, $1/\rho_c \sim E_z \sim 1$ GeV for heavy quarks. In this case, one should not consider only the contribution from the operator of lower dimension and it is necessary to sum up the infinite subsequence of power corrections associated with gluon small-size VF, $\rho_c \sim 1/E_z \ll 1/\Lambda$. The leading infinite subsequence has been summed in Ref. /10/. In this paper the case of just heavy quarks, which interact, mainly, with the gluonic degree of freedom of the vacuum, has been considered. Therefore, the effects of the condensate of light quarks are negligible. There are some arguments that the quark degree of freedom is connected with the spatial scale d, larger than ρ_c, where the d-characteristic distance between instantons /5,6,9/(if DGA is true, then $d \gg \rho_c$). Therefore, in case of the processes with light quarks one can expect that $1/\rho_c > E_z > 1/d$. This explains the smaller duality intervals as compared to those for heavy quarks.

In this case at $\rho_c \sim (1\ \text{GeV})^{-1}$ the Euclidean sum rules, in which the operator expansion is not used, are well satisfied although the magnitude of $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ is essentially larger than in Ref. /11/. Deviations from asymptotic freedom change smoothly with virtuality in the Euclidean region of dominance of lowest state. The procedure of extraction of information on the vacuum structure, basing upon these smooth deviations from asymptotic freedom, is ambigous in many respects. Moreover, the contribution

of small-size VF (we have connected them with instantons) fastly decreases with increasing the virtuality, that can obscure the larger value of $\langle \frac{\alpha_s}{\pi} G^2 \rangle$. In the physical region, where the amplitude varies fastly, information on the vacuum structure is more direct. If the VF are long-wave, $\rho_c \sim 1/\Lambda$ then the consideration in physical region gives rise to the spectral density concentrated very closely to the threshold, on an interval of the order of Λ. If the characteristic VF are small size, $\rho_c \sim (1\text{GeV})^{-1}$, large intervals of variability, much larger than Λ, then appear in a natural manner for $e^+e^- \to (\text{hadrons})_{c,b}$, $e^+e^- \to (\text{hadrons})_{I=1}$.

If the second possibility ($\rho_c \ll 1/\Lambda$) is realized in the nature, the coefficients in the approximation of deviations of polarization operator from the asymptotic freedom in the Euclidean region by the inverse powers of virtuality already make no sense of vacuum average values of the operators of lowest dimensions but represents some effective quantities.

Below, apart from an analysis of $e^+e^- \to (\text{hadrons})_f$ (f is the flavor of heavy quark and antiquark) in the physical region, the Euclidean sum rules for this process (of the type of those in Refs /11,12/, and the e^+e^- - annihilation into hadrons from light quarks with isospin I = 1 **directly in the physical region are considered.**

In this case however these sum rules don't use the operator expansion with account of contributions of the lowest dimension operators only as in sum rules of Refs /11,12/. We have considered non-perturbative effects in the Euclidean region too, especially in order to do difference of our mechanism of the breaking of the asymptotic freedom for vector resonances from longwave mechanism of the Refs /11,12/ more transparent. Moreover, it is remarkable fact that the success of the potential model can be explained just within the framework of physical pattern with small-size VF. The reasonability of the very concept of a potential in QCD is a direct consequence of the smallness of characteristic size of gluon VF.

But the potential has no sense in the pattern with long-wave VF /12/, and the success of the potential model not only in the explanation but also in the prediction of a large number of experimental facts seems to be absolutely incredible for this case.

Note that the consideration of all these problems gives

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = 0.09 \pm 0.03 \text{ GeV}^4 \tag{1}$$

$$\rho_c^{-1} = 0.7 \pm 0.2 \text{ GeV}$$

2. Heavy quarks in the physical region

In studying the non-perturbative effects in physical region we cannot restrict ourselves to the finite number of terms in the series of power corrections. It is necessary to sum up a certain infinite subsequence of power corrections /10/. In the non-relativistic approximation not only the operator $\vec{\mathcal{E}}^2(0)$ but the operators $\vec{\mathcal{E}}(0)\mathcal{D}_0^{2n}\vec{\mathcal{E}}(0)$ $(n \geq 1)$, where $\vec{\mathcal{E}}$ is the vacuum chromoelectric field corresponding to the vector-potential B_μ $(\mathcal{D}_0 = i\partial_0 + gB_0)$, should be taken into account.

The heavy quark and antiquark $Q, \bar{Q}$, produced in e^+e^- annihilation, moving away one from another and increasing therefore their total color dipole moment, perturbate the non-perturbative QCD vacuum, thereby creating a gluon-like excitation – the non-perturbative gluon G (Fig. 1). Then, the quarks Q, $\bar{Q}$, being previously in the singlet state, come to the octet state. If $Q, \bar{Q}$ are non-relativistic, they have time to interact with the non-perturbative gluon G, having the non-zero total color charge in the octet state.

With increasing of energy E, some fraction of E is taken off by the gluon G, that increases the time of interaction and, hence, the $Q\bar{Q}$ production cross section. As will be seen below, this amplification is very significant; at $E \sim 1/\rho_c$ (ρ_c is the characteristic size of VF) the deviations of the $Q\bar{Q}$ production cross section from the perturbative one with taking into account operators $\vec{\mathcal{E}}(0)\mathcal{D}_0^{2n}\vec{\mathcal{E}}(0)$ $(n = 0, 1, 2, \ldots)$

is considerably larger than for the case when only the operator $\vec{\mathcal{E}}^2(0)$ is taken into account.

It is worth noting that the main non-perturbative effect for heavy $Q\bar{Q}$ at comparatively short distances is connected with the mixing with the state $Q\bar{Q}G$, i.e. with the admixture of the other Fock component.

The sum of infinite subsequence of the most significant power corrections proves to be representable in the form of a multiplicative combination of the quark matrix element and the correlator of gluon vacuum fields. Further, the difference of the total cross section $\sigma_f^{e^+e^-}(E)$ for the process $e^+e^- \to (\text{hadrons})_f$ (f is the flavor of the produced heavy quark and antiquark) from the perturbative cross section $\sigma_{f,0}^{e^+e^-}(E)$ is connected with $\operatorname{Im} \Delta G(\vec{0},\vec{0};E)$:

$$\Delta\sigma_f^{e^+e^-}(E) = -\frac{Q_f^2 e^4}{8m^4} \operatorname{Im} \Delta G(\vec{0},\vec{0};E) \tag{2}$$

($\Delta G(\vec{r}\,', \vec{r}; E$) from Ref. /10/). Neglecting the Coulomb interaction, one has for the quark matrix element $M_{p_0}(\vec{0},\vec{0};E)$ /10/

$$M_{p_0}(\vec{0},\vec{0};E) = \frac{m^3}{\pi}\Big[i\Big(\frac{\kappa^3}{\lambda^8} - \frac{3}{2}\frac{\kappa}{\lambda^6} + \frac{3}{8\kappa\lambda^4} + \frac{1}{16\kappa^3\lambda^2}\Big) - \frac{1}{\lambda^8}\Big((\lambda^2-\kappa^2)^{3/2}\vartheta(p_0-E) + i(\kappa^2-\lambda^2)^{3/2}\vartheta(E-p_0)\Big)\Big] \tag{3}$$

where $\kappa^2 = mE$, $\lambda^2 = mp_0$. The cut, which starts in $E = 0$, contributes only to the imaginary part of the matrix element. At the point $E = p_0$ the second cut occurs, which is also contributes, above the threshold, only to the imaginary part of $M_{p_0}(\vec{0},\vec{0};E)$. Then,

$$\Delta G(\vec{0},\vec{0};E) = \int_0^\infty d\tau\, K_E(\tau)\, M_\tau(\vec{0},\vec{0};E) \tag{4}$$

where

$$M_\tau(0,0;E) = \frac{-im^2}{\pi\kappa^3}\int_{-\infty}^{+\infty}\frac{dz}{2\pi z}e^{iEz\tau}\Big[\frac{(1-z)^{3/2}-1}{z^3} + \frac{3}{2z}\Big(\frac{1}{z}-\frac{1}{4}\Big) - \frac{1}{16}\Big] \tag{5}$$

Integration in (5) is carried out below the real axis. The only singularity of the integrand is the cut along the real axis from $z=1$ to $z=\infty$. The contour of integration may be deformed to the upper half-plane, reducing (5) to an integral over the cut:

$$M_\tau(\vec{0},\vec{0};E)=-\frac{2m^2}{\pi K^3}\int_1^\infty\frac{dz}{2\pi}e^{iE\tau z}\frac{(z-1)^{3/2}}{z^4} \tag{6}$$

As a result, one has

$$\Delta G(\vec{0},\vec{0};E)=-\frac{2m^2}{\pi^2K^3}\int_0^\infty d\tau\, K_{\mathcal{E}}(\tau)\int_0^\infty\frac{d\xi\,\xi^4}{(1+\xi^2)^4}e^{iE\tau(1+\xi^2)} \tag{7}$$

In formulas (4) and (7) $K_{\mathcal{E}}(\tau)$ is the gluon vacuum correlator

$$K_{\mathcal{E}}(\tau)=\frac{4}{3}\pi\alpha_s\left\langle\vec{\mathcal{E}}(0)Pe^{ig\int_0^\tau B_0(\xi)d\xi}\vec{\mathcal{E}}(\tau)Pe^{ig\int_\tau^0 B_0(\xi)d\xi}\right\rangle \tag{8}$$

It has been argued in Ref. /10/ that the VF which saturate the correlator (8) may be represented by instantons. In this paper the correlator $K_{\mathcal{E}}(\tau)$ has been calculated at Euclidean times in the one-instanton approximation. However, in order to describe the total cross section, a correlator in the Minkowski space is required. The problem of analytic continuation is not trivial in this case. Emphasize that a consistent procedure of instanton calculations in physical region can be performed, in principle, from the very beginning to the end, in terms of <u>real</u> fields (see Ref. /13/ and B.P., will be published). Naive analytic continuation from the Euclidean space to the Minkowski one ($\tau\to i\tau$) leads to a complex singular vector-potential (for $SU_I(2)$ subgroup)

$$B_\mu(\vec{x},t)=\frac{\eta^{(\pm)}_{\mu\nu a}x^\nu\tau_a}{\vec{x}^2+\rho^2-\tau^2+i\varepsilon} \tag{9}$$

where $x^\mu=(i\tau,\vec{x})$, τ_a is the Pauli matrix. Although this procedure offers the possibility of obtaining correct answers with the use of the way of getting round the singularities as in (9), it is only an auxiliary one. Here we are limited

to the use of this procedure in its literal form. As a result, we have, for $K_{\mathcal{E}}(\tau)$:

$$K_{\mathcal{E}}(\tau)=2\int_0^\infty\frac{d\rho}{\rho^5}D(\rho)\mathcal{K}_{\mathcal{E}}\left(\frac{\tau}{\rho}\right) \tag{10}$$

where

$$\mathcal{K}_{\mathcal{E}}\left(\frac{\tau}{\rho}\right)=\sum_{\delta=0,\pm1}\mathcal{K}^{\delta}_{\mathcal{E}}\left(\frac{\tau}{\rho}\right), \tag{11}$$

$$\mathcal{K}^{\delta}_{\mathcal{E}}\left(\frac{\tau}{\rho}\right)=\frac{10\pi^2 b^7}{3}\int_0^1 d\xi\frac{\xi^4(1-\xi^2)}{(b+\sqrt{1-\xi^2})^7}F\left(\frac{7}{2},2+\delta\xi;4;\frac{4b\sqrt{1-\xi^2}}{(b+\sqrt{1-\xi^2})^2}\right) \tag{12}$$

$b=2\rho/\tau$, F is a hypergeometric function. The integrand has a singularity of the form $(b-\sqrt{1-\xi^2})^{2\delta\xi-3}$ at $\xi=\xi_0=\sqrt{1-b^2}$, i.e. at $\tau=2\sqrt{z_0^2+\rho^2}$, z_0 being the distance to the instanton centre. Regularization is performed in such a way as shown in (9). At $\tau=2\rho$ the correlator $\mathcal{K}_{\mathcal{E}}(\frac{\tau}{\rho})$ is singular and its regularization should be also performed according to (9). When $\tau>2\rho$ it has an imaginary part.

The behaviour of the correlator in the physical region drastically differs from that of $\mathcal{K}_{\mathcal{E}}(\frac{\tau}{\rho})$ in the Euclidean region. In Fig. 2, where $\mathcal{K}_{\mathcal{E}}\equiv3\mathcal{K}^0_{\mathcal{E}}(\frac{\tau}{\rho})$ is presented, one can see the singularity at $\tau=2\rho$, while in the Euclidean region /10/ we have a monotonously damping $\mathcal{K}_{\mathcal{E}}(\frac{\tau}{\rho})$. Just such behaviour of $\mathcal{K}_{\mathcal{E}}(\frac{\tau}{\rho})$ (11), (12) results in a non-monotonous energy dependence of the total cross section; if the correlator damp monotonously, then the oscillations would be absent in the cross section at all. At large times τ (both physical and Euclidean) the correlator $\mathcal{K}_{\mathcal{E}}(\tau/\rho)$ decreases rapidly - as $(1/\tau^4)\ln\frac{\tau}{\rho}$, at small τ $\mathcal{K}_{\mathcal{E}}(\frac{\tau}{\rho})=\frac{4\pi^2}{3}(1\pm\frac{3}{5}\frac{\tau^2}{\rho^2})$ (for physical and Euclidean τ respectively). The characteristic scale of variation of $\mathcal{K}_{\mathcal{E}}(\tau/\rho)$ is $\tau/\rho\sim1$.

The difference between $R_f(E)=\frac{\sigma_f^{tot}(E)}{\sigma_{\mu\mu}(E)}$ and perturbative value of this ratio, due to the leading subsequence of operators, $\vec{\mathcal{E}}(0)\mathcal{D}_0^{2n}\vec{\mathcal{E}}(0)$ ($n=0,1,2,..$), is representable as follows:

$$\Delta R_f(E) = \Delta R_f^0(E)\, \Phi(E) \quad ,$$

$$\Delta R_f^0(E) = -\frac{3\pi^2 Q_f^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2^7 m^4 v^5}\left(1-\frac{16\alpha_s}{3\pi}\right)\frac{w}{1-e^{-w}} \quad , \tag{13}$$

$$\Phi(E) = \frac{2^8 E}{3\pi}\int_0^\infty \frac{d\xi\, \xi^4}{(1+\xi^2)^4}\int_0^\infty d\tau\, \mathrm{Im}\left[\frac{K_E(\tau)}{K_E(0)}\, e^{iE\tau(1+\xi^2)}\right] \tag{14, 15}$$

where $w=\frac{4\pi\alpha_s}{3v}$, $v\simeq\sqrt{E/m}$. The same factor, which takes into account the Coulomb interaction in final state, is included in ΔR_f^0 as well as in the perturbative term R_f^0 (see also Sec.3). In this Section we restrict ourselves to substitution $\mathcal{K}\to 3\mathcal{K}_E^0(\tau)$. The correction of order α_s /15/ is taken into account in (14). At fairly large E, formula (13) is transformed into the nonrelativistic version of the result for ΔR_f in Ref. /10/, which corresponds to the contribution from operator $\vec{E}^2$ ($\Phi_{E\to\infty}\to 1$). Remark that in this case $\Delta R_f(E)$ is negative, although, one would think, the VF are suppressed, when the quark and the antiquark go away one from another, that should lead to attraction and, hence, to $\Delta R_f(E)>0$. Explanation is that at high energies (when only the contribution from operator $\vec{E}^2$ "survives") the interaction, generated by VF, is, no doubt, non-potential and, hence, the above considerations are unapplicable. With a decrease of E the function $\Delta R_f(E)$ oscillates and, that is not so trivial, is positive in the near-threshold region.

The change of the sign of $\Delta R_f(E)$ is connected with switching on operators $\vec{E}(0)\mathcal{D}_0^{2n}\vec{E}(0)$, $n\geq 1$ in addition to $\vec{E}^2$. Just these operators ensure the potential character of the interaction ($\Delta R_f>0$ in agreement with intuitive consideration) in the near threshold region, where $\Delta R_f\sim\int_0 d\tau\, K_E(\tau)$ (see also Sec.5), that differs from long-wave mechanism /11/, where $\Delta R_f<0$ /10/.

In the function $\Phi(E)$ the energy E is met with the characteristic size ρ_c of VF forming the non-perturbative structure of the total cross section. It is obvious that the larger ρ_c is, the faster are oscillations and irregularities in cross section to which the corresponding terms will lead, but the global structure of cross section is due to the small-size VF. Since we are interested in the VF with small size ρ_c, averaging

needs to perform at energy intervals less than $1/\rho_c$ for direct comparison with experiment.

In the case of small-size VF, the spectrum of states $Q\bar{Q}G$ should lies much above the states $Q\bar{Q}$ because the energy of the non-perturbative gluon in the octet intermediate state is of the order of $1/\rho_c \gg \Lambda$. We identify these states, $Q\bar{Q}G(1^{--})$, with the so-called vibrational levels introduced, in Ref. /16/, in the model of a quark-confining string (QCS). It is of interest that after fitting of the states J/ψ and ψ' the predicted position of vibrational levels /16/ is, indeed, rather high: one level is at around 4 GeV and two close levels are at around 4.4 GeV, i.e. in the non-relativistic limit of the QCS model the spacing between vibrational levels and the levels with no string vibrations turns out to be of the order of 1 GeV. In Ref. /16/, in order to test the validity of the non-relativistic approximation in the QCS model, the relativistic splittings of low lying states have been studied. They have turned out to be small, of the order of 0.1 GeV. As we shall see in section 5,6 large splitting of states $Q\bar{Q}$ and $Q\bar{Q}G$ is a necessary condition for reasonability of the very concept of a potential, at least, for not too high levels.

Because the function $\Phi(E)$ in (13) is independent on the mass of a quark, deviations of the total cross sections from the perturbative ones, which have been averaged over fast oscillations, for $e^+e^-\to(\text{hadrons})_c$ and $e^+e^-\to(\text{hadrons})_b$ should differ, in the accepted approximation, only by normalization:

$$\frac{\Delta R_b(E)}{\Delta R_c(E)} = \frac{Q_b^2}{Q_c^2}\left(\frac{m_c}{m_b}\right)^{3/2}\simeq 0.04 \tag{16}$$

It is interesting that this conclusion doesn't use any specific form VF forming the gluon correlator $K_E(\tau)$.

It agrees with the recent experimental data on $e^+e^-\to(\text{hadrons})_b$ /17/; in experiment there are no oscillations, beginning already from 10.6 GeV: $\Delta R_b = 0.0\pm 0.08$ (R is constant within this errors). If the family $\Upsilon(b\bar{b})$

will be discovered, the total cross section of $e^+e^- \longrightarrow (\text{hadrons})_t$ will take a perturbative value soon after the threshold of open flavor t : $\Delta R_t(E) \sim \left(\frac{m_c}{m_t}\right)^{3/2} \Delta R_c(E) < 0{,}04$ at $m_t > 20\,\text{GeV}$ $(R_c(E) \lesssim 2)$.

The attempts have recently been made to observe vibrational levels for the Υ -family /18/, whose position is predicted in /19/: 10.45 GeV and 10.80GeV. With a growth of the mass of a quark the mixing of the $Q\bar{Q}$ and $Q\bar{Q}G$ states becomes weak rapidly (see (16)), so the lepton widths of these levels should be very small, that complicates their observation.

Note that for the process $e^+e^- \to (\text{hadrons})_c$ in the region where non-perturbative effects are significant, it is desirable to take into account relativistic corrections, since the mass of c-quark is not too large ($m_c \approx 1.5$ GeV).

If one neglects the contributions from operators $\vec{E}(0)\mathcal{D}_0^{2n}\vec{E}(0)$ with $n \geqslant 1$, then $\Delta R_f(E)$ is very small in the energy range of interest ($E \sim 1/\rho_c$). Taking into account operators $\vec{E}(0)\mathcal{D}_0^{2n}\vec{E}(0)$ with $n \geqslant 1$ introduces a large factor of amplification. At low energies the contribution from operators $\vec{E}(0)\mathcal{D}_0^{2n}\vec{E}(0)$ to $R_f(E)$ diverges as $E^{-3/2}$. In this region ($E \to 0$) formula (13) losses its validity because the operators of larger dimension, $\vec{E}\mathcal{D}_0^{n_1}\vec{E}\mathcal{D}_0^{n_2}\vec{E}\mathcal{D}_0^{n_3}\vec{E}$, etc., begin work; the one instanton approximation becomes unvalid as well, . After the bump in the near-threshold region, there is a deep at higher energies, that is a consequence of the global parton-hadron duality.

The other interesting result is a concentration of the spectral density in the near-threshold region (even for $\rho_c^{-1} \sim 1$ GeV). For light quarks a first zero of $\Delta R^{I=1}(s)$ is at $\sqrt{s} \sim 0{,}9$ GeV, while for heavy quarks the first zero of $\Delta R_f(E)$ is at $\sqrt{s} - 2m \equiv E \sim 0{,}3$ GeV. It is very interesting that this fact agrees with the mass value $m_c = 1.53$ GeV obtained in Sec.3 and gives $m_{J/\psi} - 2m_c \ll m_\rho$.

So, it follows from this Section and Sec.3 that there is a principal difference in mass generation of vector resonances for light and heavy quarks. For the ρ meson mass an interpretation is possible in terms of two constituent quark masses ($m_\rho \sim 2m_{dyn} \approx 0{,}7$ GeV), while for J/ψ - resonance it is not the case. One of the reasons for this fact consists in that the light quark and antiquark go away at large distances (comparing with ρ_c), where nearly independent generation of constituent masses occurs, whereas the heavy quark and antiquark are at the distances comparable with ρ_c . Moreover, the interaction of Q and $\bar{Q}$ with VF is stronger than for the case of light quarks because of their nonrelativistic motion near the threshold.

In calculating $K_E(\tau)$, we have used the expression for $\mathcal{D}(\rho)$ in a somewhat more general form than in /10/:

$$\mathcal{D}(\rho) = \mathcal{D}_0(\rho)\,\vartheta(\rho_c - \rho) + \mathcal{D}_\beta\left(\frac{\rho_c}{\rho}\right)^\beta \vartheta(\rho - \rho_c) \tag{17}$$

Parameter $\mathcal{D}_\beta$ can be fixed, for example, using the quantity $\langle \frac{\alpha_s}{\pi} G^2 \rangle$; in the one-instanton approximation

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = \frac{16\,\mathcal{D}_\beta}{(4+\beta)\,\rho_c^4} \tag{18}$$

where $\mathcal{D}_0(\rho)$ is not taken into account.

The dependence $R_c(E) = R_c^0(E) + \Delta R_c(E)$ is plotted in Fig. 3 for various values of ρ_c, $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, for $\beta = 0$. For direct comparison with experiment, it is necessary to carry out the smearing procedure using, for example, the sum rules of the type /20/. Note that the dependence of $R_c(E)$ on β is weak.

Emphasize only that if $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012\ \text{GeV}^4$ /11/, than the smeared value of $\Delta R_c(E)$, which can be estimated with use of Eqs.(13)-(15), is very small for the case of long-wave VF with $\rho_c = 1/(0.2\ \text{GeV})$, that contradicts to the experiment. In this case, the total spectral density proves to be concentrated in an extremely narrow near-threshold region $\Delta \simeq 0{,}05$ GeV, if one extrapolate the instanton contribution to such energies. However, the experimental data on $R_c(E)$, even when averaged over large energy interval $\Delta = 0.7$ GeV /21/, substantially differ from the perturbative value $R_c^0(E)$ up to energies $E \sim 1.5$ GeV. Although we have used the results of the calculations with the instantons, the estimate of the characteristic interval for variation of the heavy quarks production total cross sections $\Delta \sim 1/\rho_c$ seems to be model-independent. One can expect, however, that additional numerical factors can appear in different channels.

It is worth noting that an analysis of the polarization operator $\Pi(E)$ in the Euclidean region is rather unsensitive to a size of VF which cause asymptotic freedom breaking because of smooth behaviour of the $\Pi(E)$. Deviations from asymptotic freedom can be described by some effective quantities in this region. But in the physical region the determination of the characteristic size ρ_c of dominating VF (and their amplitude) is performed basing on the characteristic frequency of variation with energy of total cross section. We obtain that $\Delta \sim \rho_c^{-1} \sim$ 0.7 GeV.

Thus, just small-size VF can form the general structure of the total cross section for $e^+e^- \to (\text{hadrons})_f$, its large interval of variability.

3. Sum rules for heavy quarks

Let us construct appropriate sum rules in the under-threshold region. In the non-relativistic case the sum rules are convenient to formulate directly in terms of the Green's function in imaginary time.

In this case, the Coulomb effects should be taken into account adequately. The matter is that at $\kappa = \kappa_n = \frac{2m\alpha_s}{3n}$ $(\kappa^2 = -mE)$, n=1,2,... the poles of the Coulomb Green's function in the energy representation manifest for the Green's function in the imaginary time as exponential factors $e^{-E_n\tau}$ $\left(E_n = -\frac{\kappa_n^2}{m} = -\frac{\omega_c}{n^2}\right)$. For the Υ- and ψ-families, the characteristic Coulomb frequencies of motion Q and $\bar{Q}$ are not small: $\omega_c^{\Upsilon} \sim$ 0,5 GeV $\omega_c^{J/\psi} \sim$ 0,3 GeV. Consequently, taking into account the Coulomb factor is nesessary already on comparatively short distances $r \lesssim a_B$ ($a_B^{\Upsilon} \sim (1.5\ \text{GeV})^{-1}$, $a_B^{J/\psi} \sim (0.7\ \text{GeV})^{-1}$, a_B is the Bohr radius, here $\Lambda_{c\bar{c}}$ = 0,25 GeV, $\Lambda_{b\bar{b}}$ = 0.35 GeV, see eqs.(30),(31)).

We shall take into account also, according to the method of summation for the systems containing heavy quarks (Ref./10/), an infinite subsequence of leading operators $\vec{\mathcal{E}} \mathcal{D}_0^{2n} \vec{\mathcal{E}}$, n =0,1, 2,.... in the non-relativistic approximation. If the size ρ_c

of VF is small enough, description of the deviation of the Green's function from asymptotically free behavior will be erroneous without such a summation already at comparatively small times $\tau \sim \rho_c$. Summing over n, we obtain, from the Green's function $G(\vec{r}',\vec{r};E)$ in Ref./10/, the following sum rules (Coulomb effects are not taken into account here):

$$\Delta G_{exp} = -\left(1 - \frac{16\alpha_s}{3\pi}\right)\frac{2m^{1/2}}{\pi^{5/2}}\int_0^1 d\omega (1-\omega)^{3/2}\int_0^{\omega\tau} d\tau' K_{\mathcal{E}}(\tau')(\omega\tau - \tau')^{1/2} \qquad (19)$$

where the spectral representation

$$\Delta G_{exp} = \Delta G_{exp}(\vec{0},\vec{0};\tau) = G_{exp}(\vec{0},\vec{0};\tau) - G_0(\vec{0},\vec{0};\tau),$$

$$G_{exp}(\vec{0},\vec{0};\tau) = \frac{m^2}{6\pi^2 Q_f^2}\int_0^{\infty} dE\, e^{-E\tau} R_f(E),$$

$$G_0(\vec{0},\vec{0};\tau) = \frac{m^2}{6\pi^2 Q_f^2}\int_0^{\infty} dE\, e^{-E\tau} R_f^{0}(E) = 3\left(\frac{m}{4\pi\tau}\right)^{3/2}\left(1 - \frac{16\alpha_s}{3\pi}\right),$$

$$R_f^0(E) = \frac{9}{2} Q_f^2 \left(\frac{E}{m}\right)^{1/2} \vartheta(E)\left(1 - \frac{16\alpha_s}{3\pi}\right). \qquad (20)$$

is used for the Green's function.

The sum rules (19) are representable in the other form. Let us expand correlator $K_{\mathcal{E}}(\tau)$ in powers of τ ($\varkappa_0 = 1$):

$$K_{\mathcal{E}}(\tau) = K_{\mathcal{E}}(0)\sum_{n=0}^{\infty} \varkappa_n \tau^{2n}$$

Here the term containing τ^{2n} corresponds to operator $\vec{\mathcal{E}} \mathcal{D}_0^{2n} \vec{\mathcal{E}}$. Then eq.(19) takes the following form:

$$\Delta G_{exp} = -3\left(1 - \frac{16\alpha_s}{3\pi}\right)\left(\frac{m\tau^3}{16\pi^3}\right)^{1/2} K_{\mathcal{E}}(0)\sum_{n=0}^{\infty}\frac{\varkappa_n \tau^{2n}}{(1+2n)(2+2n)(3+2n)(4+2n)} \qquad (21)$$

Correlator $K_{\mathcal{E}}(\tau)$ is calculated, according to (8) and (10).

Let us now turn to the Coulomb effects for the sum rules (19). It is the simplest way to find out the Coulomb factor for $G_0(\vec{0},\vec{0};\tau)$ from the Coulomb Green's function in the Meixner form:

$$G(\vec{0},\vec{r};E) = \frac{m}{4\pi r}\Gamma(1-\nu) W_{\nu,1/2}(2\kappa r)$$

where $\nu = \frac{2m\alpha_s}{3\kappa}$. With the use of the expansion for the Whittaker function $W_{\nu,1/2}(2\kappa r)$ at small arguments we immediately obtain

$$G(\vec{0},\vec{0};E) = -\frac{m\kappa}{4\pi} - \frac{m}{2\pi}\frac{2}{3} m\alpha_s \ln\kappa + \frac{m}{2\pi}\sum_{n=1}^{\infty}\frac{\kappa_n^2}{\kappa - \kappa_n} \qquad (22)$$

the terms which are independent on energy (some of them are divergent at $r \to 0$) are omitted in the regularized Green's function, that corresponds to an usual renormalization for the polarization operator. The formula for $G(\vec{0},\vec{0};E)$ has earlier been derived in QED in Ref./22/(see also Ref./12/).

The Green's function $G(\vec{0},\vec{0};\tau)$ in imaginary time is derived from (22) by means of the inverse Laplace transform $\hat{\mathcal{L}}_\tau^{-1}$. It is worth emphasizing that the borelization operator $\hat{B}_\tau = \lim_{\varepsilon\to\infty, n\to\infty, \varepsilon/n = 1/\tau} \frac{\varepsilon^n}{(n-1)!}(-d/d\varepsilon)^n$ used in Refs./11,12/ is connected with the well-studied inverse Laplace transform $\hat{\mathcal{L}}_\tau^{-1}$:

$$\hat{B}_\tau = -\hat{\mathcal{L}}_\tau^{-1}\frac{d}{d\varepsilon} = \tau\hat{\mathcal{L}}_\tau^{-1}$$

The inverse Laplace transforms of three terms in (22) are tabular:

$$G(\vec{0},\vec{0};\tau) = \langle\vec{0}|e^{-\tau\left(\frac{p^2}{m}-\frac{4}{3}\frac{\alpha_s}{r}\right)}|\vec{0}\rangle\vartheta(\tau) = \left(\frac{m}{4\pi\tau}\right)^{3/2}C(\tau) \tag{23}$$

where $C(\tau)$ is a Coulomb multiplicative factor

$$C(\tau) = 1 + 2\pi^{1/2}\sigma + \frac{2}{3}\pi^2\sigma^2 + 4\pi^{1/2}\sigma^3\sum_{n=1}^{\infty}\frac{1}{n^3}e^{\sigma_n^2}(1+\operatorname{erf}\sigma_n) \tag{23a}$$

where $\sigma = \frac{2\alpha_s}{3}(m\tau)^{1/2}$, $\sigma_n = \frac{\sigma}{n}$, $\operatorname{erf} z = \frac{2}{\pi^{1/2}}\int_0^z dz' \exp(-z'^2)$. Parameter σ corresponds to an usual Coulomb parameter $\frac{4}{3}\frac{\alpha_s}{v}$ in the energy representation. Formula (23a) has been derived in Ref./12/ via direct use of the borelization operator.

Formula (23a) with bare α_s = const does not take into account, however, the leading logarithmic corrections $\alpha_s^m \ln^m(1/\Lambda r)$ m = 1,2,.... where r is the characteristic distance between Q and $\bar{Q}$. Their summation based on the renormalization group reduces to the appearance, in (23a), the running coupling constant $\alpha_s(r)$ instead of α_s = const. It is clear that at small times τ, Q and $\bar{Q}$ move almost freely and the characteristic $r \sim \left(\frac{\tau}{m}\right)^{1/2}$. In the case of large times τ, the lowest state (the n-th state is suppressed by the factor $e^{-(E_n-\omega_c)\tau}$) dominates in the Green's function (23) and, hence, $r \sim a_B$.

In the general case, the averages

$$\bar{r}_p(\tau) = \frac{\langle r^p(\tau)\rangle}{\langle r^{p-1}(\tau)\rangle}, \quad \langle r^p(\tau)\rangle = \hat{\mathcal{L}}_\tau^{-1}\langle r^p(E)\rangle,$$

$$\langle r^p(E)\rangle = \langle\vec{0}|\frac{1}{H_0^s - E}\, r^p \frac{1}{H_0^s - E}|\vec{0}\rangle. \tag{24}$$

can serve as appropriate quantities in our analysis. If $p \geq 0$ is an integer, then for $\langle r^p(E)\rangle$ we obtain

$$\langle r^p(E)\rangle = \frac{m^2\Gamma^2(p+2)}{4\pi(2k)^{p+1}}\sum_{n=1}^{\infty}\frac{n_{p+2}}{[(n-\nu)_{p+2}]^2} \tag{25}$$

where $(a)_p$ is Pochhammer symbol: $(a)_p = a(a+1)\ldots(a+p-1)$.

In a simple particular case of p = 0, we have, from eq.(25),

$$\bar{r}_0(\tau) = \left(\frac{\tau}{\pi m}\right)^{1/2}\frac{1+2\pi^{1/2}\sigma+\frac{2}{3}\pi^2\sigma^2+4\pi^{1/2}\sigma^3\sum_{n=1}^{\infty}\frac{1}{n^3}e^{\sigma_n^2}(1+\operatorname{erf}\sigma_n)}{1+\frac{2}{3}\pi^{3/2}\sigma+\frac{2}{45}\pi^{7/2}\sigma^3+2\sigma^2\sum_{n=1}^{\infty}\frac{1}{n^3}(3+\sigma_n^2)e^{\sigma_n^2}(1+\operatorname{erf}\sigma_n)} \tag{26}$$

At small τ: $\bar{r}_0(\tau) = \left(\frac{\tau}{\pi m}\right)^{1/2}$; at large τ: $\bar{r}_0(\tau) = a_B$. The function $\bar{r}_0(\tau)$ has the expected behaviour both at small and large τ. Equality (26) should be considered as a self-consistence equation for $\bar{r}_0(\tau)$ since α_s depends, in its right-hand side, on $\bar{r}_0(\tau)$, too. For a consistent determination of the argument of α_s in this problem, it is desirable to calculate and to sum up the corrections $\alpha_s(\alpha_s/v)^m$ (m = 1,2,...) to the Coulomb Green's function $G_0(\vec{0},\vec{0},\tau)$.

It is convenient to take the logarithm of the both sides of the sum rules because of the exponential Coulomb factors. Then in the spectral representation for $G_{exp}(\vec{0},\vec{0};\tau)$, one uses the following $R_f(E)$:

$$R_f(E) = 3\left[\sum_R \frac{3\pi}{2\alpha^2}\Gamma_R^{e^+e^-}\delta(E-E_R) + \frac{3}{2}\left(\frac{E}{m}\right)^{1/2}\left(1-\frac{16\alpha_s}{3\pi}\right)Q_f^2\vartheta(E-E_c)\right] \tag{27}$$

It is worth emphasizing that the sum rules for the Υ- and ψ-families depend weakly on the choice of the instanton density $D(\rho)$. Therefore, for our purposes it suffices to use the simplest model with peak of $D(\rho)$ at $\rho \sim \rho_c \ll \frac{1}{\Lambda}$ (see Ref./6/ and also Ref./9/). As a result, we have the sum rules in the form:

$$L(\tau) \equiv \ln\left[\frac{2\sqrt{\pi}(m_0+\Delta)^{1/2}}{Q_f^2\alpha^2\left(1-\frac{16\alpha_s}{3\pi}\right)}\sum_R \Gamma_R^{e^+e^-}e^{-(M_R-2m_0)\tau} + \frac{2}{\sqrt{\pi}}\int_{M_c-2m_0}^{\infty}d\varepsilon(\varepsilon-2\Delta)^{1/2}e^{-\varepsilon\tau}\right] =$$

$$= R(\tau) \equiv \ln C(\tau) - \frac{3}{2}\ln\tau - \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{8\pi(\pi\tau)^{3/2}}{9(m_0+\Delta)}O\left(\frac{\tau}{\rho_c}\right) - 2\tau\Delta, \tag{28}$$

where

$$O\left(\frac{\tau}{\rho_c}\right)=\int_0^{\tau/\rho_c} dz \left(\frac{\tau}{\rho_c}-z\right)^{1/2}\int_0^1 dw(1-w)^{3/2} w^{3/2}\psi(wz) \; ; K_{\mathcal{E}}(\tau)=K_{\mathcal{E}}(0)\psi\left(\frac{\tau}{\rho_c}\right) \quad (29)$$

In eq.(28), M_R are the masses of resonances; continium starts at M_c ($M_c = 2m + E_c$). For the convenience of the fittings, a fixed large quantity m_0 is separated in the mass of the quark $m = m_0 + \Delta$ and the parameter Δ varies ($m_0^b = 4.60$ GeV, $m_0^c = 1.45$ GeV). Then the function $L(\tau)$ depends weakly on the fitting parameters.

In the sum rules (28) we do not take into consideration the difference of the Coulomb renormalization of the nonperturbative term, which is proportional to $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, from the corresponding factor for $G_0(\vec{0},\vec{0};\tau)$. When Q, $\bar{Q}$ interact with VF, the quarks Q, $\bar{Q}$ exist for some time in the octet state, i.e. far from Coulomb singularities. Hence, the Coulomb factor for $G_0(\vec{0},\vec{0};\tau)$ grows more rapidly than for $\Delta G(\vec{0},\vec{0};\tau)$ with the increase of τ. In view of this, the results of the fittings for $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ here serve as the <u>lower</u> boundary for a true value of $\langle \frac{\alpha_s}{\pi} G^2 \rangle$. The sum rules with the difference between the Coulomb factors taken into account will be published elsewhere.

The factor $(1-\frac{16\alpha_s}{3\pi})$ can be fixed with respect to $\Lambda_{\overline{MS}} \approx 100$ MeV. Experimental data for the Υ - and ψ -families are taken from Madison Conference (1980) and Review of Particle Properties (1982) respectively. Let us note that in our analysis the best fit is obtained for the lepton widths $\Gamma_{\Upsilon}^{e^+e^-} = (0.97 \pm 0.06)$ KeV and $\Gamma_{J/\psi}^{e^+e^-} = (4.95 \pm 0.25)$ KeV.

We start with the sum rules for the Υ -family. If the Coulomb effects are not taken into account, the sum rules, as seen from curves $R_0^{\alpha_0}(\tau)$, $R_0^{\alpha_1}(\tau)$, $R_0^{\alpha_2}(\tau)$ and $R_0^{\alpha_3}(\tau)$ in Fig.4, cannot be consistent at any values of the parameters. If the factor $C(\tau)$ is taken into account, but the leading logarithmic corrections $\alpha_s^m \ln^m(1/\Lambda\tau)$ are neglected, i.e. at α_s = const in σ, a good enough fit, at 8 GeV$^{-1} > \tau >$ 4 GeV^{-1}, can be immediately found (curve $R_0^{\beta}(\tau)$) at $\alpha_s = 0.32$, $\Delta = 0.24$ Gev, $\langle \frac{\alpha_s}{\pi} G^2 \rangle =$ = 0 (!?). The sum rules in the form (28) have previously been analysed in Ref./12/ just under the assumption that α_s = const and without taking the operators $\vec{\mathcal{E}}\mathcal{D}_0^{2n}\vec{\mathcal{E}}$ into account (the Coulomb factor for operator $\vec{\mathcal{E}}^2$ has also been calculated in Ref./12/). But in that paper the range of small τ, which gives the information about m and Λ, has not been taken into account

and only one Υ resonance has been taken into consideration in $L(\tau)$. In this case, the fitting itself has little sence; the values of α_s, Δ coincide practically with those of α_s, Δ from naive Coulomb formulas as a result of such a fitting ($L(\tau)$ is determined by the values of M_{Υ} and $\Gamma_{\Upsilon}^{e^+e^-}$). This does not mean anyway that the spectrum of Υ -resonances is a Coulomb one. In addition, neglection of the corrections $\alpha_s^m \ln^m(1/\Lambda\tau)$ is by no means justified for this problem. Taking them into account change $R(\tau)$ very strongly.

In the following consideration we shall use $\overline{r_0(\tau)}$ from (26) as the argument of α_s (note that the ratios $\bar{r}_p(\tau)/\bar{r}_0(\tau) \simeq$ const for not too large p). The fitting curves $R(\tau)$ are dependent on the parameters: $\Delta, \Lambda, \langle \frac{\alpha_s}{\pi} G^2 \rangle$ and ρ_c. Only the parameters Δ and Λ are essential in the range of small τ. If we fix, for example, the quantity Δ, then Λ is already fitted with very good accuracy. Let us choose, from all curves $R_0(\tau)$ (for which $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0$), obtained in such a way, one $R_0^{min}(\tau)$, which corresponds to a minimally possible deviation of $R_0(\tau)$ on $L(\tau)$ at large τ (this curve provides a large interval of sewing if $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ and ρ_c are taken into account). The curve corresponds to the parameters (for average points):

$$\Lambda = (0.35 \pm 0.10) \text{ GeV} , \; m_b = (4.91 \pm 0.07) \text{ GeV} \quad (30)$$

At fixed Λ and Δ, in the range of large τ the fitting is performed already by means of the parameters $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, ρ_c. In the region of τ, where the deviation of $R_0^{min}(\tau)$ on $L(\tau)$ is starting, the dependence on ρ_c may be neglected since the operators $\vec{\mathcal{E}}\mathcal{D}_0^{2n}\vec{\mathcal{E}}$, n = 1,2,.... have not taken part in. At the larger τ the range of consistence between $R(\tau)$ and $L(\tau)$ can be enlarged by choosing the parameter ρ_c:

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0{,}06 \;{}^{+\,0.03}_{-\,0.02} \text{ GeV}^4 \; , \; \rho_c = (0.7 \pm 0.2\text{GeV})^{-1} \quad (1b)$$

(taking into account the Coulomb factors difference for the perturbative and nonperturbative terms in $R(\tau)$ results in increasing this value $\langle \frac{\alpha_s}{\pi} G^2 \rangle$). The corresponding curve $R_{\Omega}(\tau)$ is drawn in Fig.4. It is noteworthy that at small enough τ ($\tau \sim 1$ GeV^{-1}) the relativistic corrections enter into play, that leads to the difference of $R(\tau)$ and $L(\tau)$ in this region. From the side of large τ, our consideration is restricted by the growth of a relative magnitude of the nonperturbative term as compared to the perturbative one(this is indicated in Fig.4 by the cross).

The use of the one-loop expression for α_s in the fitting implies that the higher corrections over α_s are included in the redefinition of Λ .This is analogous to the introduction of $\Lambda_{e^+e^-}$ in the physical region.And we denote the introduced,in such a way parameter as $\Lambda_{Q\bar{Q}}$.The coupling constant α_s has been taken at the virtualities which are inherent for $G(\vec{0},\vec{0};\tau)$.One should, therefore, expect that $\Lambda_{Q\bar{Q}}$ is close to $\Lambda_{\widetilde{MOM}} \simeq 2.16\Lambda_{\overline{MS}}$.In this case, $\Lambda_{Q\bar{Q}}/2.16 \simeq 160$ MeV refers to the Euclidean region unlike $\Lambda_{\overline{MS}} \simeq 100$ MeV which is extracted from Υ-decay in the physical region.We would like to emphasize an important circumstance.The basic qualitative conclusion of this section: a large magnitude of $\langle\frac{\alpha_s}{\pi}G^2\rangle$ and a small size of ρ_c are both independent on the assumptions about,say, the instanton nature of VF and also on the form of density D(ρ).This conclusion is based on the substantial deviation of the curves $R_0^{min}(\tau)$ and $L(\tau)$ .If $\langle\frac{\alpha_s}{\pi}G^2\rangle$ = = 0.012 GeV^4 /11/ one cannot make consistent the theoretical and experimental curves for reasonable values of parameters.

As seen from comparison between the curves $R_{s2}(\tau)$ and $R_0^{\beta}(\tau)$ in Fig.4, taking into account the nonperturbative effects in the framework of the given mechanism of violation of asymptotic freedom has led to a dynamical freezing of the coupling constant α_s. In Ref./10/ we have obtained,for effective mass of the gluon,the estimate $\mu_g = \left(\frac{3\pi}{2}\langle\alpha_s G^2\rangle\right)^{1/4} \approx 0.65$ GeV (at $\langle\frac{\alpha_s}{\pi}G^2\rangle$ = 0.012 GeV^4),if $\langle\frac{\alpha_s}{\pi}G^2\rangle$ = 0.05 GeV^4,then $\mu_g \simeq 0.90$ GeV,that is in agreement with the empirical estimate in Ref./8/.One can expect that the coupling constant α_s will be frozen just at energies around $2\mu_g \approx 1.8$ GeV. From the other hand, the dynamical freezing of occurs in the range $\tau \sim$ (2 GeV)$^{-1}$ in our sum rules.In view of this, we assume that the given mechanism manifest itself there where the freezing of α_s is empirically observed.

An analysis which is similar to that for the Υ-family,can be carried out for the ψ-family as well.One should bear in mind, however, that the region of sewing in this case is narrower than for the Υ-family because the nonperturbative effects achieve a level,say, 25% faster than for the Υ-family; the relativistic effects also are more important. Here (in Fig.5) the curve $R_0^{min}(\tau)$ corresponds to the parameters

$$\Lambda = 0.25 \pm 0.06 \text{ GeV}, \quad m_c = 1.53 \pm 0.07 \text{ GeV} \qquad (31)$$

and,for the parameters characterizing VF,one obtains (from the curve $R_{s2}(\tau)$)

$$\langle\frac{\alpha_s}{\pi}G^2\rangle = 0.07 \pm^{0.04}_{0.03} \text{ GeV}^4, \quad \rho_c^{-1} = 0.7 \pm 0.2 \text{ GeV} \qquad (1c)$$

For $\langle\frac{\alpha_s}{\pi}G^2\rangle$ = 0.012 GeV^4 /11/it appears that there is no possibility to obtain an agreement between the theoretical curves and $L(\tau)$ (α_s= 0.23 in $L(\tau)$),see $R^{\gamma_1}(\tau)$, $R^{\gamma_2}(\tau)$.

The sum rules for the ψ-family have been considered earlier in Ref./11/ in the moment representation,where the Coulomb effects have been taken into account,however, in the lowest order only, although, $\frac{\alpha_s}{v} \sim \sqrt{n}\alpha_s\left(\frac{2m_c}{\sqrt{n}}\right) \sim 1$,for such n ,for which the fit was carried out.The Coulomb effects play a very important role for sum rules for the ψ-family also, that can be seen comparing the curves $R_0^{\alpha_1}$, $R_0^{\alpha_2}(\tau)$ and $L(\tau)$ in Fig.5. Let us note that the curve $R^{\beta}(\tau)$ which corresponds to the parameters similar to those used in Ref./11/ can be fitted to the"experimental" curve $L'(\tau)$ at α_s= 0.08.Recently,calculations of the contribution $O(G^3)$ to the ψ sum rules were carried out in an interesting paper/14/ where the Coulomb effects were taken into account,however, at the lowest order as well.

4. Light quarks in the physical region

It is of interest to check the received conclusions for the other channel where there are variety of experimental data, namely, in the isovector channel in the e^+e^--annihilation. The logic of consideration is nearly the same as for heavy quarks; if we are not interested in detailed information on the spectral density at small energy intervals, in this case, its structure comes from the VF of relatively small sizes, $\rho \sim \rho_c \ll 1/\Lambda$ of instanton nature. Using the analytic continuation into physical region, as in section 2, for the polarization operator in the one-instanton approximation /4,23,24/, one obtains

$$\Delta R^{I=1}(s) = -2\pi^2 \int_0^\infty \frac{d\rho}{\rho} D(\rho)\pi\left(J_2N_2 + 4J_1N_1 + 3J_0N_0\right) \qquad (32)$$

where the argument of Bessel functions J_ℓ, N_ℓ is equal to $\rho\sqrt{s}$. At asymptotically large s, two regimes: $\rho \gg 1/\sqrt{s}$ and $\rho \sim 1/\sqrt{s}$ are possible. Second regime can be realized using the instantons of any small size, whose density, therefore, is $D_0(\rho)$ /2/. The first regime can occur for the instantons whose characteristic size is ρ_c ($\rho_c \sim (1\,GeV)^{-1}$) and whose density is amplified due to the interaction with large-scale VF. In this case, ($s \to \infty$) ($D(\rho)$ from Ref. /10/),

$$\Delta R^{I=1}(s) = \frac{3}{2}\pi^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \rho_c^4 \frac{\sin(2\rho_c\sqrt{s})}{(\rho_c\sqrt{s})^4} \qquad (33)$$

i.e. the instantons lead to damping oscillations of the total cross sections with characteristic frequency $\sim 1/\rho_c$. The second regime has been considered in Ref. /24/ as a dominant one. But the values of the parameter used in this paper are $\Lambda_{PV} = 0.3 \div 0.7$ GeV, that has resulted in exceeding the answer by $2 \cdot 10^4 \div 4 \cdot 10^7$ times (as compared to the case $\Lambda_{PV} = 0.1\,GeV$). With the generally accepted now values of $\Lambda_{PV} \simeq \Lambda_{e^+e^-} \simeq \Lambda_{\overline{MS}} \simeq 0.1\,GeV$ the result /24/ (neglecting spontaneous breaking of chiral symmetry) becomes of the order of 1 at $\sqrt{s} \sim \sqrt{s_0} = 0.3\,GeV$, where, first, the DGA approximation is already unapplicable and, second, at such $\sqrt{s}$ the instantons of the large size $\rho \sim 1/\sqrt{s_0}$ work, whose density is not equal to $D_0(\rho)$, of course. At $\sqrt{s} = 0.7\,GeV$ the corresponding $\Delta R^{I=1}(s) \simeq -4 \cdot 10^{-5} < 0$ is negligibly small.

Spontaneous breaking of chiral symmetry in the instanton density associated with long-wave quark VF /11,7/ reduces to the substitution of $m_q\rho$, where m_q is the quark current mass, for

$$m_q(\rho_0)\rho \left(\frac{\ln(\rho_0\Lambda_{PV})}{\ln(\rho\Lambda_{PV})}\right)^{\delta} - \frac{2\pi^2}{3}\left\langle(\bar{q}q)_{\rho_0}\right\rangle \rho^3 \left(\frac{\ln(\rho_0\Lambda_{PV})}{\ln(\rho\Lambda_{PV})}\right)^{-\delta}$$

where $\delta = 4/(11 - 2N_f/3)$. In this case the second regime gives larger values of $\Delta R^{I=1}(s)$, which, however, are still very small. Let us stress that due to high powers of ρ in the instanton density the main contribution to the integral over ρ is given by the

instantons with $\rho \sim \frac{29}{4\sqrt{s}}$ ($\rho \sim \frac{17}{4\sqrt{s}}$) (taking (or not) into account the chiral symmetry breaking). For the second regime (comp. with /24/)

$$\Delta R^{I=1}(s) \simeq -1.33 \cdot 10^{14} \Lambda_{PV}^9 |\langle\bar{q}q\rangle|^3 \frac{1}{s^9} \ln^{\frac{25}{3}}\left(\frac{4\sqrt{s}}{29\Lambda_{PV}}\right) \simeq$$

$$\simeq -\left(\frac{1}{\sqrt{s}}\, 0.56\,GeV\right)^{18} \ln^{25/3}\left(\sqrt{s}/0.73\,GeV\right) < 0 \qquad (33a)$$

This means that if the instantons are reasonable even at $\rho \lesssim (0.3\ \mathrm{GeV})^{-1}$, then eq. (33a) is valid only at high energies $\sqrt{s} \gtrsim 2.2\,GeV$, when $|\Delta R^{I=1}(s)| \lesssim 3 \cdot 10^{-7}$.

We would like to note the interesting possibility of extracting information on the density of instantons $D(\rho)$ from comparison of the instantons contributions with difference of the experimental cross section from the perturbative one, by solving a some kind of inverse problem. The density $D_0(\rho)$ /2/ gives rise to a monotonously decreasing contribution /24/, small in magnitude; whereas in the experiment, oscillations with a period of the order of 0.7 GeV are observed. Already to obtain observed oscillations $\sigma_{tot}^{e^+e^-}$, it is necessary, at some ρ_c, to increase $D(\rho)$ as compared to $D_0(\rho)$; moreover, for the first regime to be accomplished, the increase should be fairly sharp. In our rough model for $D(\rho)$ in /10/, the increase has been represented as a jump of $D(\rho)$ at the point $\rho = \rho_c$. Because the period of oscillations in $\sigma_{tot}^{e^+e^-}$ is of the order $0.7\,GeV \gg \Lambda$, the required size of ρ_c should be small: $\rho_c \sim (1\,GeV)^{-1}$.

Recently, in Ref. /25/, attention, parallel with Ref. /10/, has been paid to the importance of studying the non-perturbative effects in physical region. In this paper, a model for $R^{I=1}(s)$ is considered which is based on introduction of singularities of the light quarks vector polarization operator in the complex x^2 - plane, and these singularities being more simple type than those of the one-instanton contribution. But the sense of these singularities is unknown unfortunately.

Fig. 6 presents the dependence $R^{I=1}(s) = R_0^{I=1} + \Delta R^{I=1}(s)$ at various values of β and $\langle \frac{1}{\pi}\alpha_s G^2 \rangle$ ($\rho_c = (0.5\,GeV)^{-1}$) as well as the experimental points (see, for example, Ref. /26/). The experimental cross section in this channel is more regular than in $e^+e^- \to$ (hadrons) and, hence, a direct comparison of $R^{I=1}(s)$ with $R^{I=1}_{exp}(s)$ is possible, in principle. The stricter procedure of comparison can be carried out by averaging both $R^{\mathrm{th}}(s)$ and $R^{I=1}_{exp}(s)$ in terms of, for example, the sum rules /20/. In contrast to heavy quarks, the light ones can escape at the larger distances in virtue of its relativistic nature and, therefore, are more sensitive to multiinstanton configurations, in particular, to the instanton-antiinstanton ones. These effects are of great importance in the near-threshold range, where formula (30) already does not hold. All this leads to decreasing the scale in this channel as compared to $1/\rho_c$, that corresponds effectively to larger ρ_c than for heavy quarks.

5. Potential in QCD because of small-size VF

One of the methods of describing the heavy quark systems is introducing the effective potential of interaction. In very successful potential models which describe not only the spectrum and widths of bound states but predict also a lot of experimental facts, the interaction potential has been introduced phenomenologically. Such a success of the phenomenological potential models should be explained by the consistent theory of strong interactions, i.e. by QCD. However, in the paper /12/ it was affirmed that if $\langle \frac{1}{\pi}\alpha_s G^2\rangle \neq 0$, the quark-antiquark interaction cannot be described in terms of the potential in its usual meaning. Furthermore, the effective potential has the following drawbacks: it is not local, energy- and mass-dependent. In that paper the effective potential is proportional to the mass of a quark, while the observed spectra of ψ and Υ families agree with the approximate independence of interaction on flavor (e.g., $m_{\psi'} - m_{J/\psi} \simeq m_{\Upsilon'} - m_{\Upsilon}$). In this case it is most unlikely that the potential model work so well with so weak support from the QCD. It is worth emphasize the important circumstance: these assertions /12/ have been made basing upon the assumption of Ref. /11/ on the long-wave nature of VF.

As we shall see, the conclusion on the very existence of local potential is opposite to the paper /12/ in the physical pattern with the determining role of small-size VF. Roughly speaking, the factor $\rho_c m \alpha_s^2$ serves as a parameter of locality. Moreover, the effective potential proves to be approximately independent of the flavor of the quark and of its energy, that is in well agreement with potential models. This and following sections have been previously presented in /27/.

As has already been indicated in /10/ and in sections 2 and 3 of the present paper, consideration of small-size VF requires to take into account not only operator $\vec{\mathcal{E}}^2$ but operators $\vec{\mathcal{E}}\mathcal{D}_0^{2n}\vec{\mathcal{E}}$ $(n \geq 1)$ as well. The corresponding correction to the Green function of a quark and antiquark /10/

$$\Delta G(\vec{r}\,', \vec{r}; E) = N_c \left\langle \vec{r}\,' \right| \frac{1}{E - H_0^s} \hat{V} \frac{1}{E - H_0^s} \left| \vec{r} \right\rangle \tag{34}$$

may be interpreted as the manifestation of the interaction potential:

$$\hat{V} = -\frac{1}{N_c}\int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} K_{\mathcal{E}}(p_0)\, \vec{r}\, \frac{1}{H_0^a - E + p_0}\, \vec{r} \tag{35}$$

($K_{\mathcal{E}}(p_0)$ is the Fourier transform of correlator $K_{\mathcal{E}}(\tau)$ (8)).

If the long-wave VF formed correlator $K_{\mathcal{E}}(p_0)$, the energy of non-perturbative gluon in the intermediate octet state can be neglected, and operator $\hat{V}$ then reduces to the contribution of operator $\vec{\mathcal{E}}^2$, considered in Refs. /12,28/. But, for real quarkonia - the ψ and Υ -families - only the VF with size $\rho \gg (\frac{4}{9} m \alpha_s^2)^{-1}$: $\rho \gg (0.3\,GeV)^{-1}$, $\rho \gg (0.5\,GeV)^{-1}$ respectively, may be regarded as such long-wave VF. It is difficult to agree with the fact that the VF with so large size play a dominant role in the dynamics of these systems. But, for heavy quarks $m \gg (\frac{4}{9}\rho_c \alpha_s^2 (\frac{3}{2m\alpha_s}))^{-1}$, $m \gg 20\,GeV$ at $\rho_c = (0.7\,GeV)^{-1}$, only the contribution of operator $\vec{\mathcal{E}}^2$ survives and the conclusions of the paper /12/ become valid. Note that the non-perturbative effects are very small for such systems with $m \gg 20\,GeV$; f.e. $\Delta E_{1s} / |E_{1s}| \ll 10^{-2}$, here $\Lambda = 0.3$ GeV.

Let us consider the matrix element of the operator $\hat{V}$ from $\vec{r}$ to $\vec{r}\,'$:

$$V(\vec{r}',\vec{r}) = -\frac{\vec{r}'\vec{r}}{N_c}\int_{-\infty}^{\infty}\frac{dp_0}{2\pi}K_{\mathcal{E}}(p_0)\left\langle\vec{r}'\left|\frac{1}{H_0^a - E + p_0}\right|\vec{r}\right\rangle \tag{36}$$

The case of a local potential means that

$$V(\vec{r}',\vec{r}) = V(\vec{r})\,\delta(\vec{r}'-\vec{r}) \tag{37}$$

where $V(\vec{r})$ is an usual potential. Note that and in the physical pattern with small-size VF playing a dominant role, a relative importance of three terms in the denominator of octet propagator depends on which distances are of interest for us and which levels are studied. Nevertheless, the Coulomb repulsion in the octet state can be, in practice always, neglected (the ratio of the repulsive potential $V_a(r) = \frac{\alpha_s}{2N_c r}$ to the attractive one in the singlet channel is equal to $(N_c^2-1)^{-1}$, so for J/ψ : $V_a(a_B) \approx 0{,}08\,GeV$). After this, $V(\vec{r}',\vec{r})$ is expressed via a free quark-antiquark Green function $G_0(\vec{r}',\vec{r};E) = \frac{m}{4\pi|\vec{r}'-\vec{r}|}e^{-(m(p_0-E))^{1/2}|\vec{r}'-\vec{r}|}$. If one is interested in not so small distances, such that the kinetic energy $\vec{p}^2/m \sim 1/m(\vec{r}'-\vec{r})^2$ is small as compared to the energy of non-perturbative gluon p_0, then $G_0(\vec{r}',\vec{r};E)$ is proportional to δ - function:

$$G_0(\vec{r}',\vec{r};E) \simeq \frac{1}{p_0 - E}\;\delta(\vec{r}'-\vec{r}) \tag{38}$$

Hence, the non-locality of the potential extends up to distances

$$|\vec{r}'-\vec{r}| \sim |m(p_0-E)|^{-1/2} \ll a_B \tag{39}$$

for not too high levels ($a_B = \left(\frac{2}{3}m\alpha_s\left(\frac{1}{a_B}\right)\right)^{-1}$) is the Bohr radius). In this case, the potential is energy-independent and reduces to an oscillator one:

$$V(r) = \frac{r^2}{N_c}\int_0^{\infty} d\tau K_{\mathcal{E}}(\tau) \tag{40}$$

where we transfer to Euclidean times, $K_{\mathcal{E}}(\tau) > 0$.

The integral of correlator $K_{\mathcal{E}}(\tau)$ which determines the force of the potential, can be readily estimated: $\lambda \equiv \int_0^{\infty} d\tau K_{\mathcal{E}}(\tau) \sim \sim \varsigma_c K_{\mathcal{E}}(0) = \frac{\pi^2}{6}\langle\frac{\alpha_s}{\pi}G^2\rangle\varsigma_c$ in comparison with its value $\lambda^0 = 1{,}3\frac{\pi^2}{6}\varsigma_c\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{4+\beta}{3+\beta}$ derived from formulas (10), (17).

The potential (40) appears from the first term in the expansion of the potential operator (35) in powers of $\frac{1}{p_0^n}\left(\frac{p^2}{m} - E\right)^n$. The corresponding terms of a series for the potential operator (35), at $n \geqslant 1$, are not local and cannot be interpreted literally as the manifestation of the potential of interaction. But, when one considers the interaction with VF using the sum rules in Euclidean region, such contributions give rise to a certain effective potential $V_{eff}(r)$ in the sum rules. Let us expand the effective potential $V_{eff}(r)$ in a series of r :

$$V_{eff}(r) = \sum_k \sigma_k r^k \tag{41}$$

and then find the first correction to the Born term for the time Green function at Euclidean τ due to interaction (41), by expressing this correction via σ_k :

$$G_{eff}(\vec{0},\vec{0};\tau) = \left(\frac{m}{4\pi\tau}\right)^{3/2}\left[1 - \tau\sum_k \sigma_k\Gamma\left(\frac{k}{2}+1\right)\left(\frac{\tau}{m}\right)^{k/2}\right] \tag{42}$$

Formula (42) has been derived in Ref. /29/ for k = 1,2 and in /30/ for any k. Now, let us use the connection (obtained in formula (21), sect. 3) of the Green function $G(\vec{0},\vec{0};\tau)$ with the correlator of gluon vacuum fields $K_{\mathcal{E}}(\tau)$. Equating the coefficients at the same powers of τ , one can see that $k = 4n+4$. Therefore (with neglection of Coulomb repulsion in octet state in the limit of large N_c), in the expansion of the effective, from the point of view of the sum rules, potential $V_{eff}(r)$, associated with the contribution from the operators of gluon vacuum field $\vec{\mathcal{E}}\mathcal{D}_0^{2n}\vec{\mathcal{E}}$, there are only the degrees r^4, r^8, r^{12} etc.:

$$V_{eff}(r) = \frac{2|K_\mathcal{E}(0)|}{m} \sum_{n=0}^{\infty} \frac{\varkappa_n (m r^2)^{2n+2}}{(1+2n)(2+2n)(3+2n)(4+2n)(2n+2)!} \quad (43)$$

It should not be forgotten that the result (43) corresponds to the first correction to Born term, and, hence, one can use it only if the corresponding terms in (42) are small as compared to the Born term. In view of this, the attempts to solve /30/ the Schrodinger equation for $Q\bar{Q}$ with a potential ∞r^4, corresponding to the first term in (43) have no sense. In (43) the expansion is performed with respect to the dimensionless parameter $\rho_c^{-1} m r^2$, i.e. with respect to the parameter inverse to the non-locality one. There is no locality on the distance r where $\rho_c^{-1} m r^2 \ll 1$. However, if the VF are of fairly small size, $\rho_c \sim (1\,GeV)^{-1}$, the region of non-locality, $r < (\rho_c/m)^{1/2}$, is narrow ($r < (1\,GeV)^{-1}$ for ψ -family and $r < (3\,GeV)^{-1}$ for Υ-family). Although there is an explicit dependence of the potential on the mass of a quark in (43), this formula is converted to the oscillator potential (40) outside the region of non-locality (when the parameter $\rho_c^{-1} m r^2$ is large). The oscillator potential (40) has sense irrespective to the sum rules for potential.

6. Energy corrections for heavy-quark systems

Now let's consider the corrections to the energies of levels $Q\bar{Q}$ which are due to the interaction (35) generated by gluon operators $\vec{\mathcal{E}} \mathcal{D}_0^{2n} \vec{\mathcal{E}}$. Neglecting relativistic corrections and the Coulomb repulsion in octet state we have, for the Coulomb level with the main quantum number n, $\left(E_n = -\frac{4m}{9n^2}\alpha_s^2\right)$

$$\Delta E_{nl} = -\frac{1}{N_c} \int_{-\infty}^{\infty} \frac{dp_0}{2\pi} K_\mathcal{E}(p_0) \langle n | \vec{r} \frac{1}{H_0^a + p_0 - E_n} \vec{r} | n \rangle \quad (44)$$

Calculating the quark matrix element, for example, for the lowest level 1S, one obtains

$$\Delta E_{1S} = \frac{m a_B^4}{N_c} \int_{-\infty}^{\infty} \frac{dp_0}{2\pi} K_\mathcal{E}(p_0) \frac{\delta^2}{(1+\delta)^6} \left[3 + 18\delta + 42\delta^2 + 42\delta^3 + 7\delta^4\right] \quad (45)$$

where $\delta = (1 + m p_0 a_B^2)^{-1/2}$, a_B is the Bohr radius. Since the characteristic $|p_0| \sim 1/\rho_c$, the kinetic energy $\vec{p}^2/m$, at $\rho_c \sim (1\,GeV)^{-1}$, can be neglected for the ψ - and Υ -families. Then, from (44) one gets approximately

$$\Delta E_{nl} = \frac{n^2}{2N_c}\left[5n^2 + 1 - 3l(l+1)\right] a_B^2 \int_0^{\infty} d\tau\, K_\mathcal{E}(\tau) \quad (46)$$

Correction ΔE_{nl} from (46) corresponds to potential (40). It is seen from (45) that for the lowest level $\Upsilon(9.46)$, at $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.1\ GeV^4$, $\rho_c^{-1} = 0.8$ GeV, $\beta = 5$ one has $\frac{\Delta E_\Upsilon}{|E_\Upsilon|} = 0.20$ (and from eq.(46) one has $\Delta E_\Upsilon / |E_\Upsilon| = 0.39$) for $\Lambda = 0.3$GeV.

If one takes into account only the operator $\vec{\mathcal{E}}^2$, this ratio will increase, for the same value of $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.1\ GeV^4$ by a factor 1.6, i.e. the summation of the infinite subsequence of power corrections would give rise to their partial cancellation. This is readily understood from the qualitative point of view: taking into account the energy $|p_0| \sim 1/\rho_c$ of the non-perturbative gluon in intermediate state QQG has led to the substitution of the energy denominator of the order $\omega_Q \sim \frac{4}{9} m \alpha_s^2$ for $1/\rho_c > \frac{4}{9} m \alpha_s^2$. We observe here that the influence of small-size VF is possible to imitate by the contribution of long-wave VF at noticeable smaller value of $\langle \frac{\alpha_s}{\pi} G^2 \rangle$. Corrections to the energies and to the widths of Coulomb levels which are due to only one operator $\vec{\mathcal{E}}^2$ have been calculated in Refs./12/, /28/.

The fact that, for the Υ-resonance, non-perturbative corrections prove to be small means that the Υ-resonance is almost Coulomb system. Moreover, we can obtain, from $\Delta E_\Upsilon / |E_\Upsilon| = 0.20$, that

$$m_b = \tfrac{1}{2}\left(M_\Upsilon + |E_\Upsilon| - \Delta E_\Upsilon\right) = 4.92\ \text{GeV},$$

what is in excellent agreement with Sec.3. For the J/ψ resonance we have, from Sec.3, $\Delta E_{J/\psi} = M_{J/\psi} + |E_{J/\psi}| - 2m_c = 0.35$ GeV $\sim |E_{J/\psi}|$ and eq.(45) gives an estimate $\Delta E_{J/\psi} = 0.38$ GeV. This means that the non-perturbative effects are very important for the J/ψ resonance. Validity of the potential (40) is limited, from the side of small distances τ, by the region of non-locality; from the side of large τ the limitation is associated, in particular, with the breaking of multipole expansion.

It should be noted that a priori the very fact of existence of the local potential of interaction between Q and $\bar{Q}$ in QCD is not trivial and rather surprising since it is required, for its existence, that not the initial operator of interaction $V(\vec{\tau},\tau) = \frac{g}{2}\vec{\tau}\cdot\vec{\mathcal{E}}_a(\vec{\tau},\tau)\,t^a$ with a free color indices be iterated in higher corrections to Born approximation, but a certain effective gauge-invariant colorless potential constructed by means of $V(\vec{\tau},\tau)$. The fact that the non-locality occurs in the second order perturbation theory over $V(\vec{\tau},\tau)$ is known from the consideration of the Stark-effect in QED. Non-triviality consists in a principal necessity of small-size VF for reasonability of the potential. The role of these fluctuations is not only to make the potential of interaction local but also to make it iterable in higher orders.

The important consequence and confirmation of the physical patterns with small-size VF, when we compared its consequences with the potential model, is the static character of the potential generated by them. In this section we often use the smallness of kinetic energy $\vec{p}^{\,2}/m$ as compared to $|p_0| \sim 1/\rho_c$. This just implies that the quark and antiquark are static in the intermediate octet state in which the potential is generated. This assertion is in good agreement with the potential model. However, one cannot neglect the motion of $Q,\bar{Q}$ in the colour singlet state. Let us stress the difference with the known consideration of the static potential from the Wilson loop (see i.e./3/), where quarks are considered to be static in the colour singlet state also.

So, as we have seen in this section, the assumption on a dominant role of small-size VF in the dynamics of $Q\bar{Q}$

-states enables one to explain, in terms of QCD, all the basic features of the phenomenological potential in the potential model. Whereas the assumption on the long-wave nature of VF /11/ gives rise to the impossibility of introducing the potential of interaction Q and $\bar{Q}$ in the framework of QCD.

7. Conclusions

We have assumed in the present paper, unlike /11/, that the non-perturbative mechanism of asymptotic freedom breaking is connected, mainly, with small-size VF rather than with long-wave VF. In the framework of this physical pattern we formulate the method of considering non-perturbative effects in physical region both for heavy (the method i based, in this case, on the summation of the infinite subsequence of power corrections) and light quarks; in addition, we succeed in demonstrating the consistence of the potential model with QCD, in particular, in explaning an approximate flavor-independence of the interaction between Q and $\bar{Q}$.

The logic of construction of the sum rules in the Euclidean region becomes the other to a great extent; in the region of dominance of lowest level the operator expansion is unapplicable since $E_2 \sim 1/\rho_c$. But, because the deviations from asymptotic freedom are moderate in this region and are a smooth function of virtuality, essential part of this deviations may be imitated by several terms of operator expansion. However, the appropriate coefficients cannot be identified, in a given physical pattern, with the vacuum averages of local operators of lower dimension: $\frac{\alpha_s}{\pi}G^2$, $\bar{\psi}\sigma G\psi$, G^3, etc. The average, with respect to the non-perturbative vacuum, of the squared gluon field prove to be here much larger than in /11/: $\langle\frac{\alpha_s}{\pi}G^2\rangle/\langle\frac{\alpha_s}{\pi}G^2\rangle_{SVZ} \simeq 8$. The analysis for $e^+e^- \to (\text{hadrons})_{c,b}$ and $e^+e^- \to (\text{hadrons})_{I=1}$ may be generalized to the other channels. In the present case, the operator expansion is substituted by the expansion over the number of instantons and antiinstantons are taken into account (which represent small-size VF). The physical reason for the possibility of such a consideration is the suppression of the density of instantons by light quarks /3/, /5/. It is just effect that makes reasonable the one-instanton approximation widely used in the present paper.

In our opinion, the possibility of forming a wide broad spectrum of resonances with characteristic masses $M \sim 1\,GeV \gg \Lambda$ in vacuum fields with large wavelength $\lambda \sim (0{,}2\,GeV)^{-1}$ seems to be rather strange from the qualitative point of view. Moreover, the analysis is insensitive to a specific shape of VF, in particular and homogeneous fields are admissible. The more natural possibility would be the reflection, in the masses and widths of resonances, of a specific shape of VF, which form them.

Note that if one proceed from the connection between the vacuum average $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ and the magnitude of condensate of vacuum quark-antiquark pairs, which is generated by a gluon field /11/,

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = -12\, m \langle \bar{\Psi}\Psi \rangle_m \qquad (47)$$

and one uses the quantity $\langle \bar{\Psi}\Psi \rangle_m$ known from PCAC, one obtains

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0{,}07\, GeV^4 \qquad (48)$$

In the estimate (48) we have extrapolated the equality (47), which holds at large enough masses m, to the mass of the constituent quark, m = 0.35 GeV.

We would like to note also that the magnitude $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, derived in today's popular calculations on the lattice, is larger than in /11/ (in pure gluodynamics, $SU(3)$) as well (Ref. /31/):

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.10 \pm 0.05\ GeV^4 \qquad (49)$$

It appears that the mechanism considered in this paper works quite well. In the QCD framework it is possible to explain many phenomenological and empirical facts and to avoid some difficulties connected with the long-wave mechanism /11/.

We are thankful to V.S.Fadin for useful discussions of the Coulomb problem. We are grateful to A.G.Grozin for giving some computational programms and advices in programming.

References

1. A.A.Belavin, A.M.Polyakov, A.S.Schwartz, Yu.S.Tyupkin. Phys. Lett. 59B (1975) 85.
2. G.'t Hooft. Phys. Rev. Lett. 37 (1976) 8; Phys. Rev. D14 (1976) 3432, D18 (1978) 219E.
3. C.G.Callan, R.Dashen, D.J.Gross. Phys. Rev. D17 (1978) 2717; D18 (1978) 4684; D19 (1979) 1826.
4. N.Andrei, D.J.Gross. Phys. Rev. D18 (1978) 468.
5. R.D.Carlitz. Phys. Rev. D17 (1978) 3225.
6. R.D.Carlitz, D.B.Creamer. Phys. Lett. 84B (1979) 215; Annals of Physics (N.Y.) 118 (1979) 429.
7. V.A.Novikov et al. Nucl. Phys. B165 (1980) 67; B191 (1981) 301.
8. G.Parisi, R.Petronzio. Phys. Lett. 94B (1980) 51.
9. E.V.Shuryak. Phys. Lett. 107B (1981) 103, Novosibirsk preprints INP-81-118, INP-81-134 (1981).
10. V.N.Baier, Yu.F.Pinelis. Novosibirsk preprint INP-81-141 (1981); Phys. Lett. 116B (1982) 179 .
11. M.A.Shifman, A.I.Vainshtein, V.I.Zakharov. Nucl. Phys. B147 (1979) 385, 448, 519; B163 (1980) 46.
12. M.B.Voloshin, Nucl. Phys. B154 (1979) 365, Yadernaya Fisika 36 (1982) 247; Moscow preprints ITEP-21 (1980), ITEP-30 (1981).
13. K.M.Bitar, S.-Jin Chang. Phys. Rev. D17 (1978) 486, D18 (1978) 435.
14. S.N.Nikolaev, A.V.Radyushkin. Dubna preprints E2-81-766 (1981); E2-82-521 (1982).
15. J. Schwinger, Sources, particles, fields. Addison-Wesley, 1973.
16. R.C.Giles, S.-H.H.Tye, Phys. Rev. Lett., 37 (1976) 1175.
17. D.Andrews et al. (CLEO Collaboration) Report CLNS 81/513/ /CLEO 81/06.

18. E.Rice et al. (CUSB Collaboration). Phys. Rev. Lett. 48 (1982) 906.

19. W.Buchmüller, S.-H.H.Tye. Phys. Rev. Lett. 44 (1980) 850.

20. E.Poggio, H.Quinn, S.Weinberg. Phys. Rev. D13 (1976) 1958.

21. R.M.Barnett, M.Dine, L.Mc Lerran. Phys. Rev. D22 (1980) 594.

22. M.A.Brown. ZhETF, 54 (1968) 1220;
R.Barbieri, P.Christillin, E.Remiddi. Phys. Rev. A8 (1973) 2266.

23. R.D.Carlitz, Ch. Lee. Phys. Rev. D17 (1978) 3238.

24. L.Baulieu et al. Phys. Lett. 77B (1978) 290.

25. M.S.Dubovikov, A.V.Smilga. Moscow preprint ITEP-42 (1982).

26. S.I.Eidelman, L.M.Kurdadze, A.I.Vainshtein. Phys Lett. 82B (1979) 278.

27. V.N.Baier, Yu.F.Pinelis, Report at the Conference of the Nuclear Department of Academy of Sciences of USSR, February, 1982, "Phenomenological potential and QCD sum rules".

28. H.Leutwyler, Phys. Lett. 98B (1981) 447.

29. A.I.Vainshtein et al. Yadernaya Fisika, 32 (1980) 1622.

30. J.S.Bell, R.A.Bertlmann. Nucl. Phys. B177 (1981) 218, B187 (1981) 285.

31. P.Hasenfratz. Talk at EPS International Conference on High-Energy Physics, Lisbon, 9-15 July 1981.

Figure captions

Fig 1: The leading subsequence of nonperturbative contributions for heavy quark Q and antiquark $\bar{Q}$ in e^+e^- annihilation.

Fig 2: The correlator $\mathcal{K}_E(\frac{\tau}{\rho})$: curve a) for Euclidean times; for Minkowski times we present $\mathcal{K}_E(\frac{\tau}{\rho}) = \underline{3}\mathcal{K}^o_E(\frac{\tau}{\rho})$,curve b) for Re $\mathcal{K}_E(\tau/\rho)$ and curve b') for Im $\mathcal{K}_E(\tau/\rho)$.

Fig 3: The ratio $R_c(E)$ with nonperturbative effects taken into account in one-instanton approximation.
Curve 1 is for $R^o_c(E)$ ($\langle\frac{1}{\pi}\alpha_s G^2\rangle = 0$) with due regard for the Coulomb interaction in final state, a smooth interpolation is made with $R^o_c(\sqrt{s})$ in the relativistic region;
curve 2 is for $R^o_c(E) + \Delta R^o_c(E)$, operators $\vec{E}\mathcal{D}_{o_4}^{2n}\vec{E}$ $(n \geqslant 1)$ are not taken into account; $\langle\frac{1}{\pi}\alpha_s G^2\rangle = 0.12$ GeV^4;
curve 3 for $R^o_c(E) + \Delta R_c(E)$, $\langle\frac{1}{\pi}\alpha_s G^2\rangle = 0.1$ GeV^4, $\rho_c^{-1} = 0.7$ GeV;
curve 4 is for $R^o_c(E) + \Delta R_c(E)$, $\langle\frac{1}{\pi}\alpha_s G^2\rangle = 0.15$ GeV^4, $\rho_c^{-1} = 0.9$ GeV;
the function $\Delta R_c(E)$ is taken from Eqs.(13)-(15), $m_c = 1.53$ GeV, $\alpha_s = 0.23$ in formfactor multiplier as in Sec.3; $\beta = 0$, $R_{light} \simeq 3\sum_{f=u,s,d} Q_f^2(1+\alpha_s/\pi) \simeq 2.1$. We put $\Lambda \simeq \Lambda_{c\bar{c}}/1.5 = 0.17$GeV ($\Lambda_{c\bar{c}} = 0.25$ GeV obtained in Sec.3) in $w = 4\pi\alpha_s(mv)/3v$.
Experimental data come from Ref./21/.

Fig 4: The sum rules for the Υ-family. For particular curves we use the following notations for $R(\tau)$: ($\langle\frac{1}{\pi}\alpha_s G^2\rangle$(GeV^4), ρ_c (GeV^{-1}), Δ (GeV), Λ (GeV)), the dash means independence.

$R_{sr}(\tau)$: (0.06, 1.43, 0.31, 0.35),
$R_o^{min}(\tau)$: (0, -, 0.31, 0.35),
$R_\gamma(\tau)$: (0.012, 5.0, 0.31, 0.35),
$R_o^{\beta}(\tau)$: (0, -, 0.24, -),
$R_o^{\alpha_o}(\tau)$: (0, -, - 0.20, -),
$R_o^{\alpha_i}(\tau)$: (0.012, -, Δ_i , -), $\Delta_1 = -0.2$, $\Delta_2 = -0.3$, $\Delta_3 = 0{,}0$
$L(\tau)$: $\alpha_s = 0.13$, $\Delta = 0.31$, $M_c = 11.0$ GeV.

Crosses correspond to the indicated ratios of the nonperturbative term to a perturbative one.

Fig 5: The sum rules for the Ψ-family, the notations are the same as in Fig.4.

$R_{sr}(\tau)$: (0.07, 1.43, 0.08, 0.25),
$R_o^{min}(\tau)$: (0, -, 0.08, 0.25),

$R^{\gamma_1}(\tau)$:(0.012, 5.0, 0.08, 0.25),
$R^{\gamma_2}(\tau)$:(0.020, 5.0, 0.019, 0.35),
$R_o^{\alpha_i}(\tau)$:(0, -, Δ_i , -), $\Delta_1 = -0.19$, $\Delta_2 = -0.25$,
$R^{\beta}(\tau)$:(0.020, 5.0, -0.195, -);
L (τ): $\alpha_s = 0.23$, $\Delta = 0.08$, $M_c = 4.3$ GeV,
L' (τ): $\alpha_s = 0.08$, $\Delta = -0.195$, $M_c = 4.3$ GeV.

Fig 6: The ratio $R^{I=1}(s)$ in one-instanton approximation ($\varrho_c^{-1} = 0.5$ GeV):
curve 1 for $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.1$ GeV^4, $\beta = 0$,
curve 2 for $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.20$ GeV^4, $\beta = 0$,
curve 3 for $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.20$ GeV^4, $\beta = 10$.
Experimental data come from /26/.
The dashed curve corresponds to the perturbative ratio
$R_o^{I=1} = 3/2(1 + \alpha_s(\sqrt{s})/\pi)$.

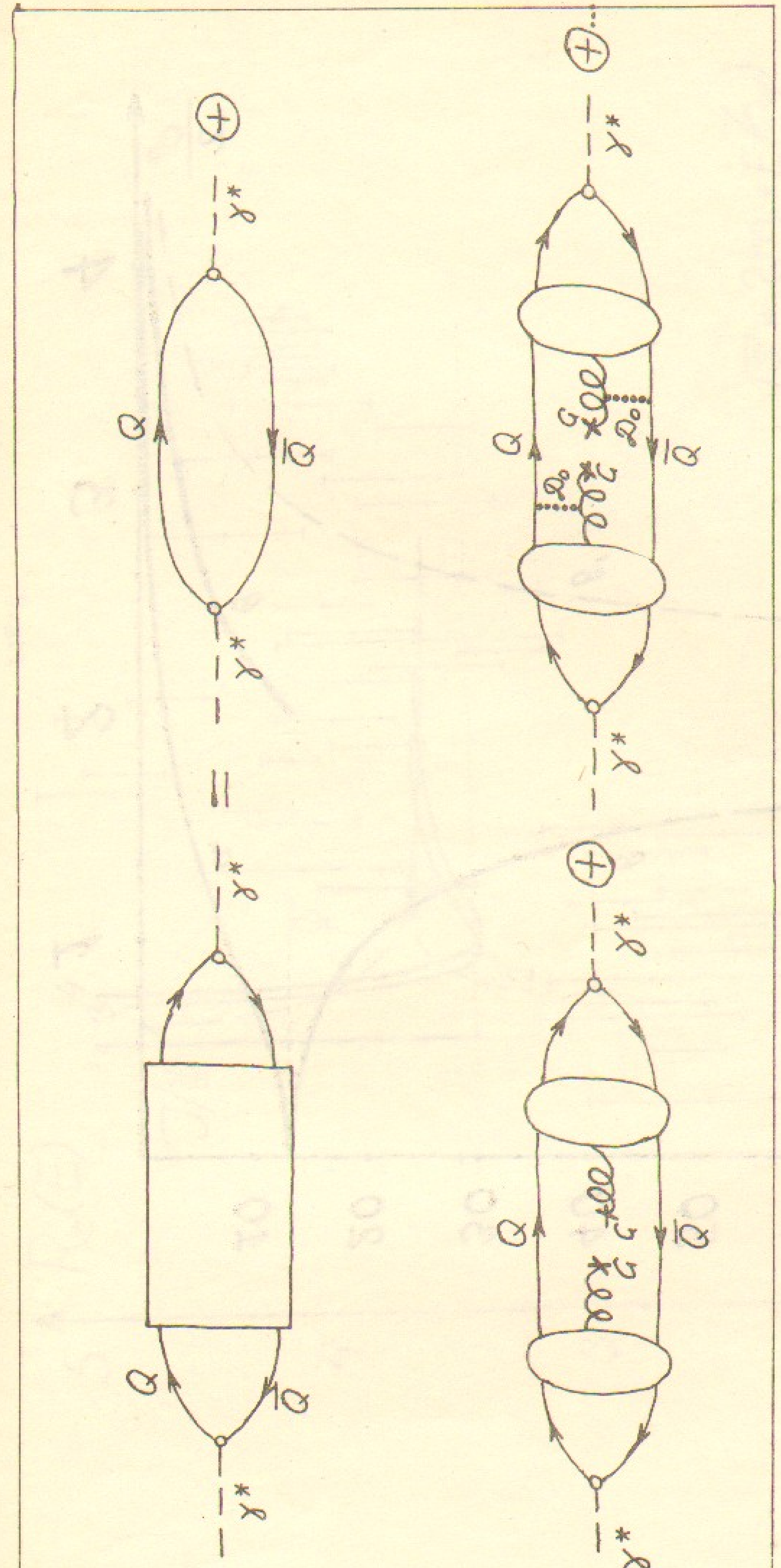
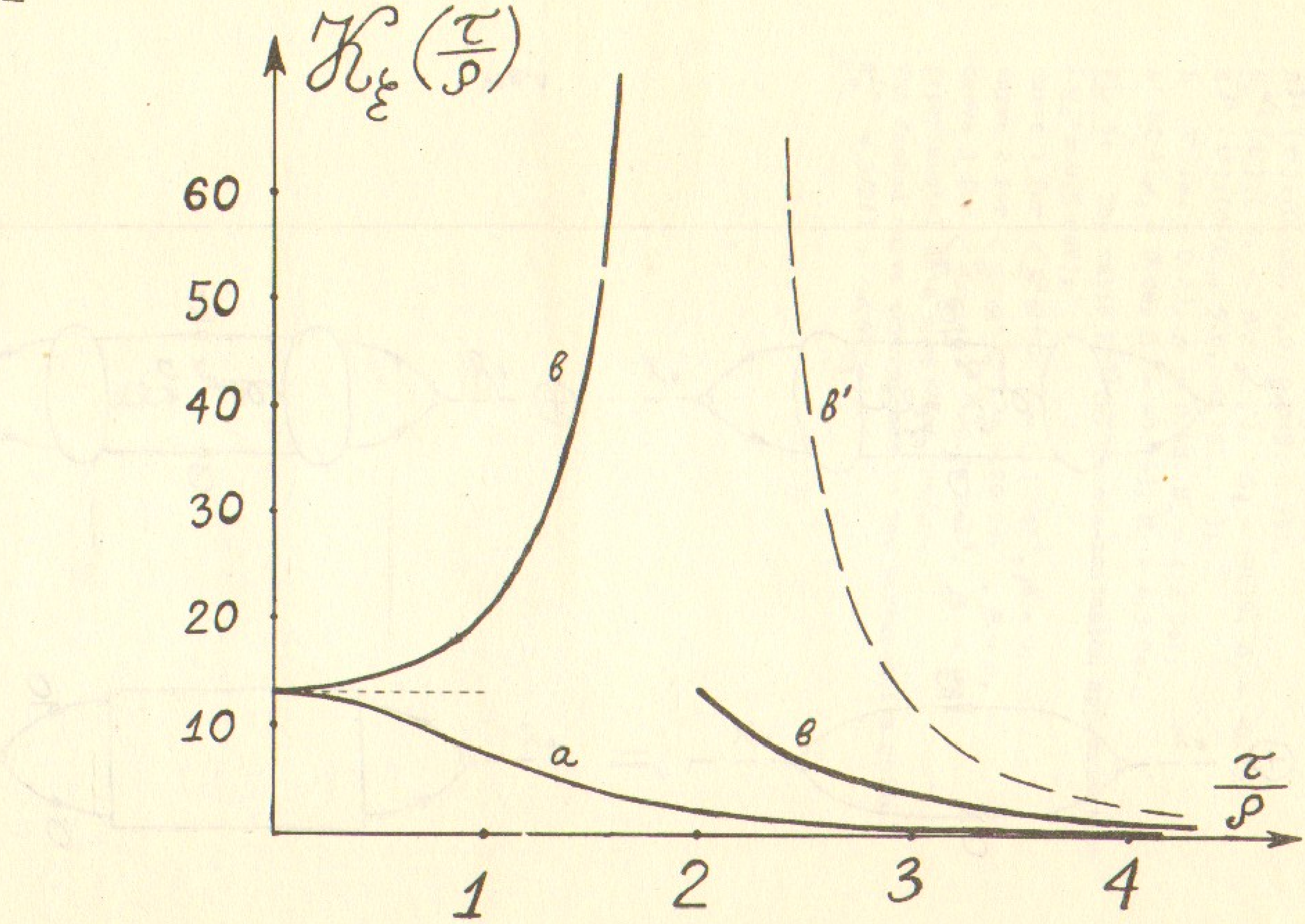
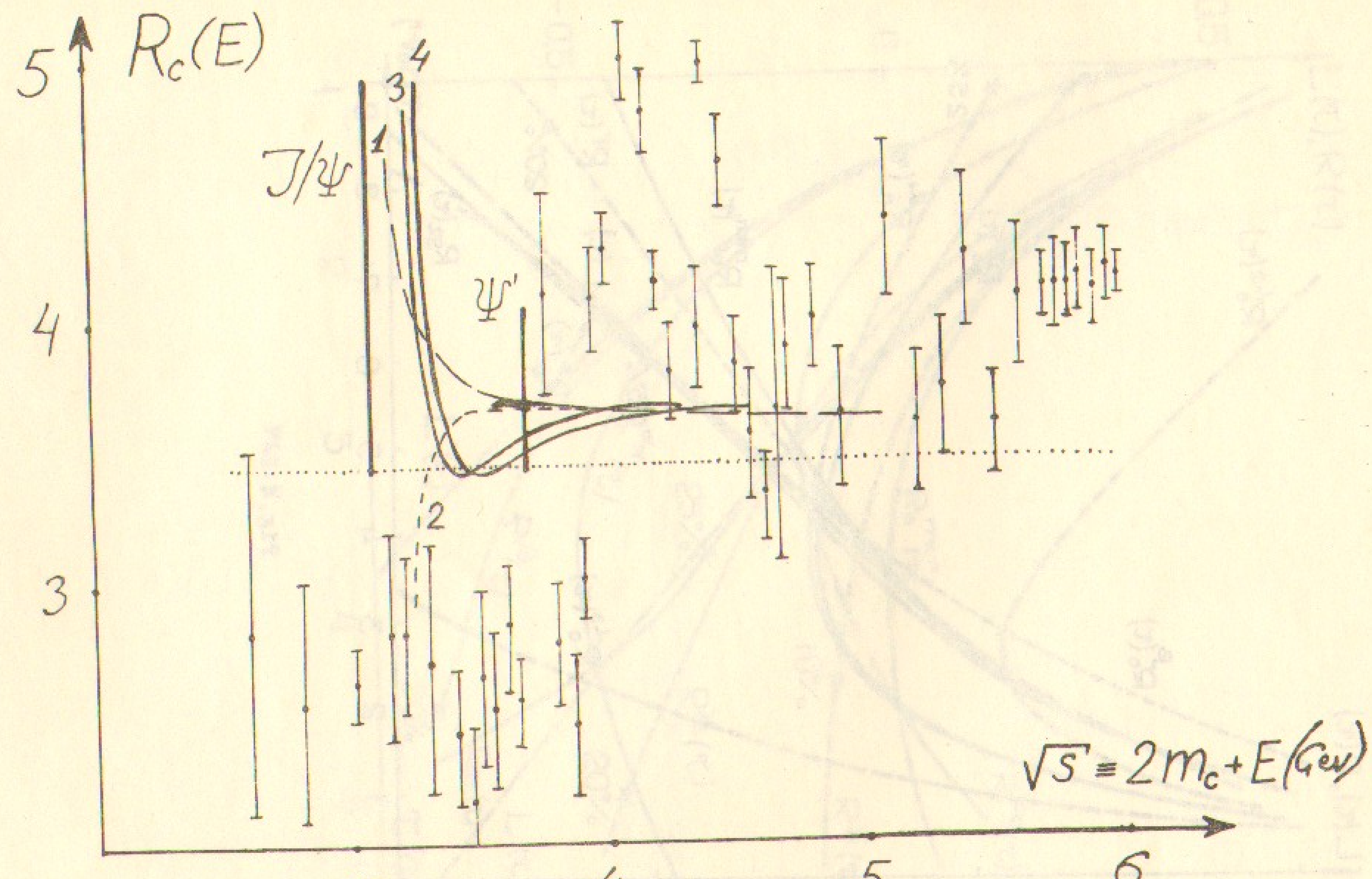


Fig. 1



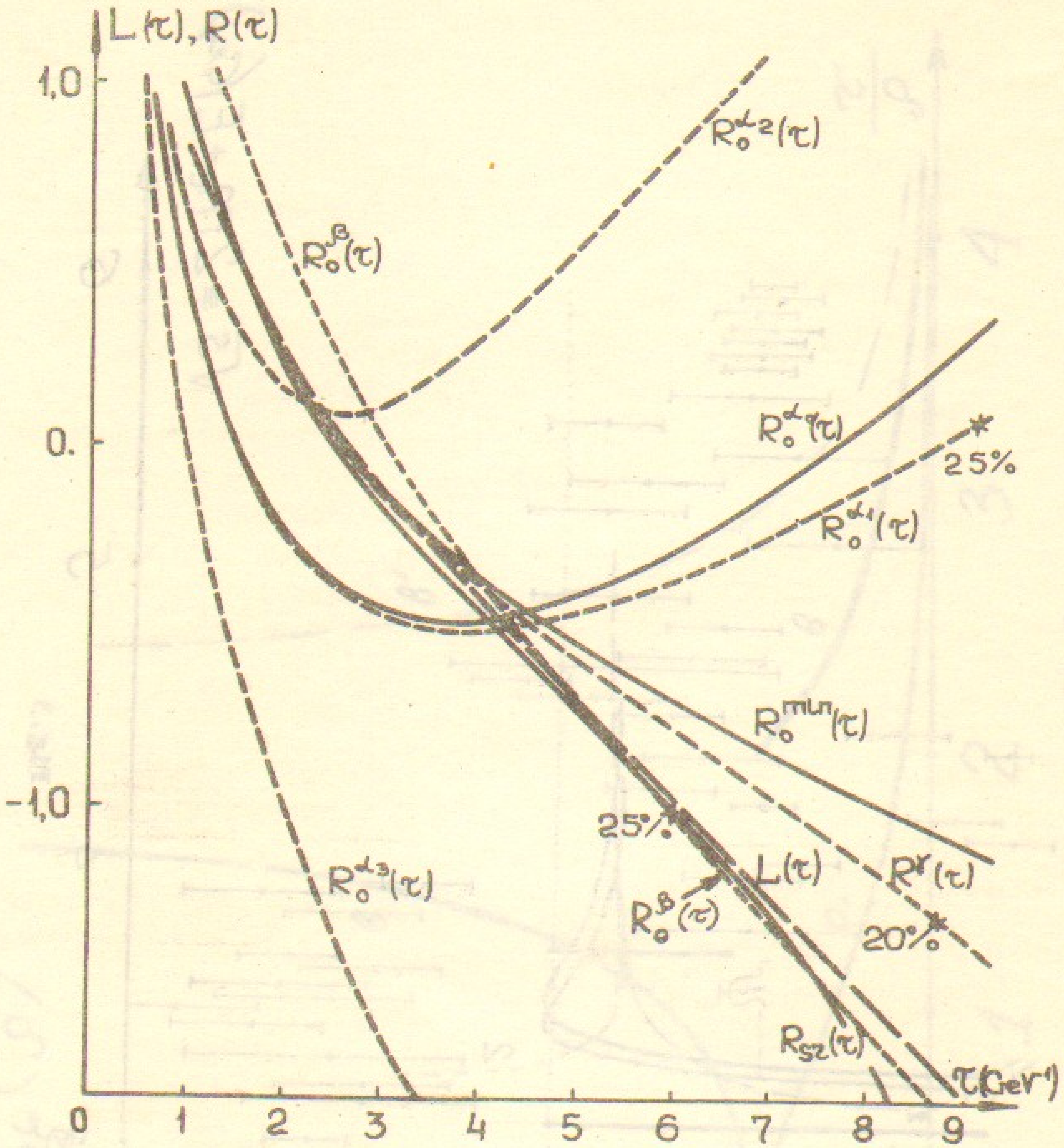
$\mathcal{K}_\xi\left(\frac{\tau}{\rho}\right)$
60
50
40
30
20
10
в
в'
a
в
$\frac{\tau}{\rho}$
1
2
3
4

Fig.2



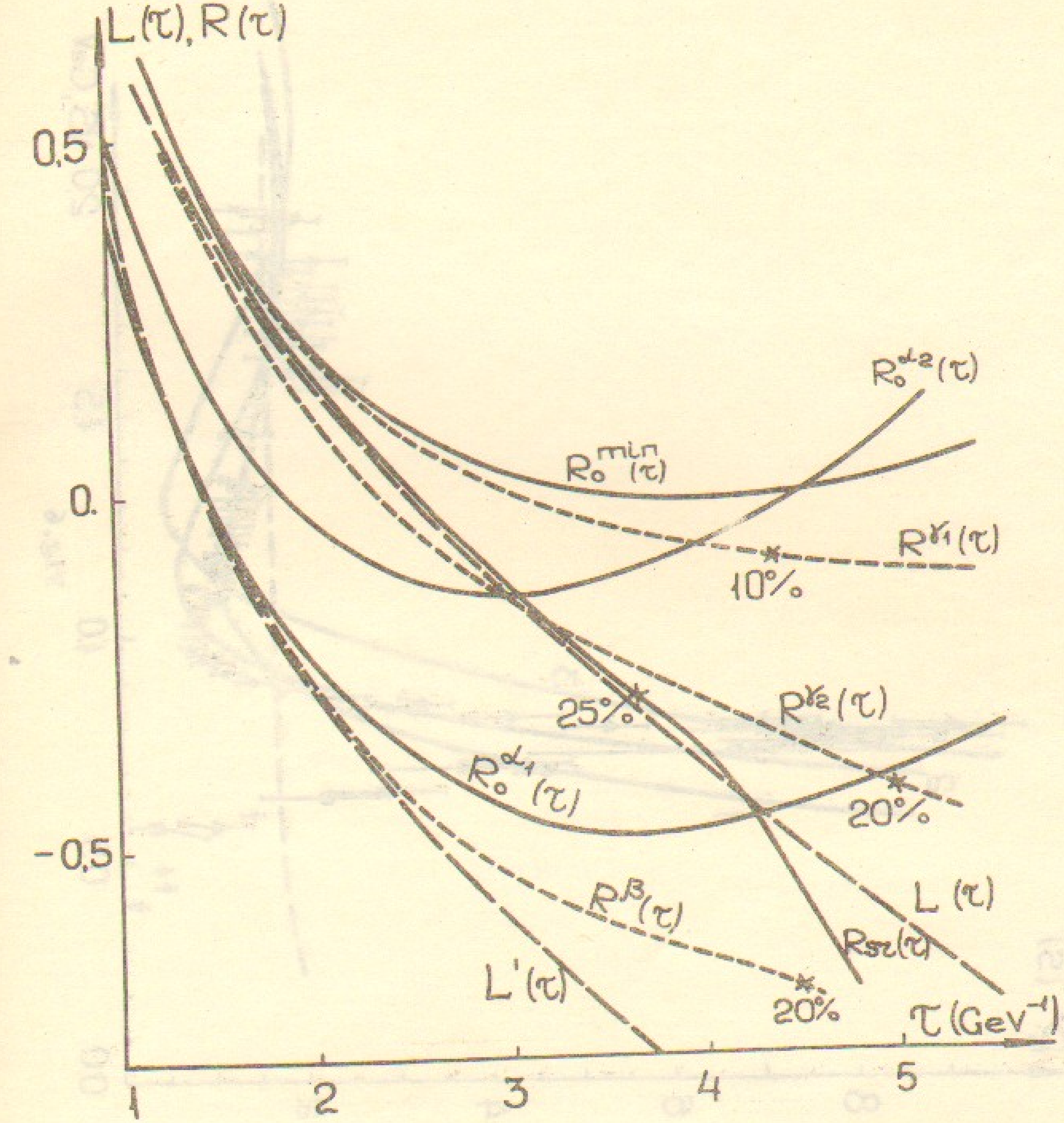
$R_c(E)$
5
4
3
J/Ψ
Ψ'
1
2
3
4
$\sqrt{s} = 2m_c + E$ (Gev)
3
4
5
6

Fig.3



L(τ), R(τ)
1,0
0.
-1,0
0
1
2
3
4
5
6
7
8
9
τ(GeV⁻¹)
$R_o^{\alpha_2}(\tau)$
$R_o^{\beta}(\tau)$
$R_o^{\alpha_q}(\tau)$
25%
$R_o^{\alpha_1}(\tau)$
$R_o^{min}(\tau)$
25%
$L(\tau)$
$R^{\gamma}(\tau)$
$R_o^{\beta}(\tau)$
20%
$R_o^{\alpha_3}(\tau)$
$R_{SZ}(\tau)$

Fig.4



L(τ), R(τ)
0,5
0.
-0,5
1
2
3
4
5
τ (GeV⁻¹)
$R_o^{\alpha_2}(\tau)$
$R_o^{min}(\tau)$
$R^{\gamma_1}(\tau)$
10%
25%
$R^{\gamma_2}(\tau)$
$R_o^{\alpha_1}(\tau)$
20%
$L(\tau)$
$R^{\beta}(\tau)$
$R_{SZ}(\tau)$
$L'(\tau)$
20%

Fig.5

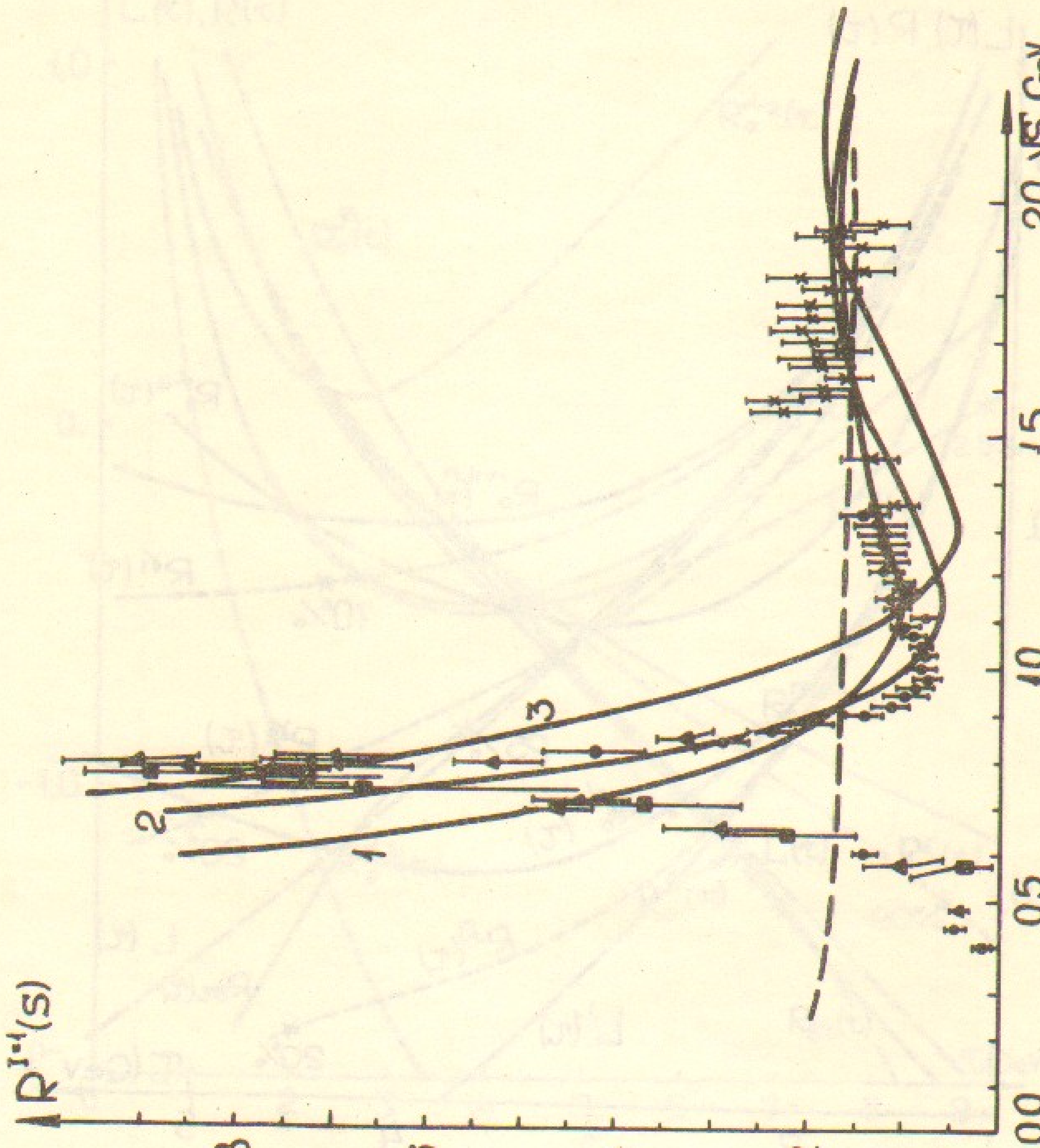


Fig.6

Preprint