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COLLIDING  $\gamma e$  AND  $\gamma \bar{e}$  BEAMS BASED ON  
THE SINGLE-PASS  $e^+e^-$  ACCELERATORS  
I I. POLARIZATION EFFECTS.  
MONOCHROMATIZATION IMPROVEMENT



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НОВОСИБИРСК

## A b s t r a c t

Polarization effects are considered in the colliding  $\gamma e$  and  $\gamma\gamma$  beams, which are proposed to be obtained on the base of the linear  $e^+e^-$  colliders (by backward Compton scattering of laser light on electron beams).

It is shown that using electrons and laser photons with such helicities  $\lambda$  and  $P_c$  that  $\lambda P_c < 0$  essentially improves the monochromatization. Characteristic energy of laser flash  $A_0$ , which is necessary to obtain the conversion coefficient  $k \sim 1$  at a definite monochromatization degree, is considerably less (sometimes for one order of magnitude) in the case  $2\lambda P_c = -1$  contrary to the case  $\lambda P_c = 0$ . Simultaneously the luminosities  $L_{\gamma e}$  and  $L_{\gamma\gamma}$  essentially increase.

Formulae are obtained which allow one to extract the polarization information about the  $\gamma e \rightarrow X$  and  $\gamma\gamma \rightarrow X$  reactions. Peculiarities connected with the specific scheme of the  $\gamma$  beam preparation are discussed.

Problems of the calibration of the  $\gamma e$  and  $\gamma\gamma$  collisions for the polarized beams are discussed.

## 1. Introduction

It was shown in refs. [1,2] that using the designed linear  $e^+e^-$  colliders VLEPP [3] and SLIC [4] one can obtain colliding  $\gamma e$  and  $\gamma\gamma$  beams with approximately the same energies ( $\geq 100$  GeV) and luminosities ( $\sim 10^{30} - 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ ) as in the  $e^+e^-$  collisions. The necessary intense  $\gamma$  beams are suggested to be obtained by Compton scattering of laser light which is focused on electron beams of these accelerators. A similar scheme was also proposed later in ref. [5] (this paper contains, however, some inaccuracies). In ref. [6] a free electron laser of the same beam is suggested to realize the scheme which has been proposed in ref. [1].

The present paper is the continuation of ref. [2] and deals with the polarization effects. It is naturally divided into two parts.

The result of sect. 3 seems to be the most important. When laser light and electrons with opposite helicities are used, the produced photon spectrum becomes harder. As a result, high enough monochromatization degree of the  $\gamma e$  and  $\gamma\gamma$  collisions can be obtained at relatively small energy of the laser flash (in comparison with the case of the nonpolarized beams).

Varying the polarizations of the initial beams one can obtain the polarized  $\gamma$  beam and therefore study all polarization effects in the  $\gamma e \rightarrow X$  and  $\gamma\gamma \rightarrow X$  reactions. These effects are considered in sects. 4-7 where we focus the attention on two main problems: 1) under what conditions the information on the polarization dependence of cross sections can be extracted in the most reliable way; 2) when can one neglect complications connected with polarization (such a complication takes place even for the nonpolarized initial beams).

The problems of calibration for polarized beams are discussed in appendix A.

## 2. General scheme and basic notations

Let us remind the main features of the scheme proposed in ref. [1]. The high energy photon is obtained by backward scattering of a laser photon with the energy  $\omega_0 \sim 1$  eV on an electron with the energy  $E \sim 100$  GeV. Almost all high energy photons travel at very small angles to the direction of an incident electron  $\theta \lesssim 10^{-5} - 10^{-4}$  rad.. We assume that the collision of electrons and laser photons is the head-on one (at small beam collision angle  $\alpha_0 \ll 1$  - see fig. 1 - the  $\alpha_0$  - dependence is negligible).

The energy of a scattered photon  $\omega$  decreases when its emission angle  $\theta$  increases

$$\omega = \frac{\omega_m}{1 + (\theta/\theta_0)^2}; \quad \omega_m = \frac{x}{x+1} E; \quad \theta_0 = \frac{m_e}{E} \sqrt{x+1}; \quad x = \frac{4E\omega_0}{m_e^2}. \quad (1)$$

Here  $\omega_m$  is the maximum energy of the scattered photon (at  $\theta = 0$ ).

In the interaction region a photon deviates from the initial electron trajectory by the distance  $\sim b\theta_0$  where  $b$  is the distance between the conversion point C and interaction point O (fig. 1). As a result, the  $\gamma$  beam area in the interaction region increases for the value  $\Delta S_{eff} \sim \pi(b\theta_0)^2$  in comparison with the area  $S_{eff} \sim \pi a_e^2$  which the electron beam would have had here:

$$\frac{\Delta S_{eff}}{S_{eff}} \sim \rho^2 \equiv \left( \frac{b\theta_0}{a_e} \right)^2. \quad (2)$$

While  $\rho < 1$  ( $b \lesssim 10$  cm) the photons with different energies are mixed in the interaction region. With the increase of  $\rho$  in the interaction region the hard photons (which have the smallest emission angles  $\theta$ ) get smaller transversal size than the softer ones, and the relative contribution of hard photons into luminosity increases. The collisions become more monochromatic, and the spectral luminosity distribution  $dL/dW$  improves ( $W$  is the invariant mass of the  $\gamma e$  or  $\gamma\gamma$  system,  $W_{\gamma e}^2 = 4\omega E$ ,  $W_{\gamma\gamma}^2 = 4\omega\tilde{\omega}$ , tilde denotes the quantities related to the second  $\gamma$  beam). The monochromatization degree  $\eta$  is determined (see ref. [2]) as a ratio of

the range of invariant mass  $\Delta W$  that contains half of the luminosity to the maximum value  $W_m$

$$\eta = \frac{\Delta W}{W}; \quad \int_{W_m - \Delta W}^{W_m} \frac{dL}{dW} dW = \frac{1}{2} L, \quad (W_m^{\gamma e} = \sqrt{4\omega_m E}, \quad W_m^{\gamma\gamma} = 2\omega_m). \quad (3)$$

The quantity  $\eta$  strongly depends on the value  $\rho$ .

For the neodymium glass or garnet laser ( $\omega_0 = 1.17$  eV) the energy interval considered  $E = 50 - 500$  GeV corresponds to  $x = 0.9 - 9$ . As a rule, the examples are given for the middle  $x = 2.69$  which corresponds to the abovementioned laser and  $E = 150$  GeV or  $E = 50$  GeV and  $\omega_0 = 3.51$  eV (the same laser with frequency tripling).

## 3. Improvement of the monochromatization of the $\gamma e$ and $\gamma\gamma$ collisions using laser and electron beams with opposite helicities polarizations

1. The energy spectrum of scattered photons is  $(d\sigma_c/d\omega)/\sigma_c$  where  $\sigma_c$  is the Compton cross section. For the nonpolarized beams this spectrum is rather broad - see curve a in Fig. 2. For the polarized beams the spectrum only varies if both electrons mean helicity  $\lambda$  and laser photons one  $P_c$  are nonzero (see i.e. [7])

$$\frac{1}{\sigma_c} \frac{d\sigma_c}{dy} \equiv f(x, y) = \frac{2\pi\alpha^2}{\sigma_c x m_e^2} \left[ \frac{1}{1-y} + 1-y - 4z(1-z) - 2\lambda P_c z x (2z-1)(2-y) \right], \quad (4)$$

$$y = \frac{\omega}{E} \leq y_m = \frac{\omega_m}{E} = \frac{x}{x+1}, \quad z = \frac{y}{x(1-y)} \leq 1.$$

The total cross section is

$$\sigma_c = \sigma_c^{np} + 2\lambda P_c \sigma_1,$$

$$\sigma_c^{np} = \frac{2\pi\alpha^2}{x m_e^2} \left[ \left(1 - \frac{y}{x} - \frac{8}{x^2}\right) \ln(x+1) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(x+1)^2} \right],$$

$$\sigma_1 = \frac{2\pi\alpha^2}{x m_e^2} \left[ \left(1 + \frac{2}{x}\right) \ln(x+1) - \frac{5}{2} + \frac{1}{x+1} - \frac{1}{2(x+1)^2} \right]. \quad (5)$$

In the region under consideration  $x = 0.9-9$  the ratio  $|\sigma_1/\sigma_c^{np}| < 0.17$  and  $\sigma_1 = 0$  at  $x \approx 2.5$ . Therefore, the total cross section  $\sigma_c$  deviates from that for the nonpolarized beams  $\sigma_c^{np}$  very slightly.

Contrary to the total Compton cross section the energy spectrum does essentially depend on the value  $\lambda P_c$ . At  $\lambda P_c < 0$  the number of the hardest photons increases while the number of the soft photons decreases (see the curve *b* in fig. 2). In other words, when  $(-\lambda P_c)$  increases the effective "pumping" of soft photons into hard ones arises. In the limit  $2\lambda P_c = -1$  the number of the hard photons nearly doubles. This fact results in the essential improvement of the monochromatization of the  $\gamma e$  and  $\gamma\gamma$  collisions.

On the contrary, at  $\lambda P_c > 0$  the number of the hard photons decreases (see the curve *c* in fig. 2) and the possibility of monochromatization becomes worse.

2. Let us consider the dependence of the spectral luminosity  $dL/dW$  and the monochromatization degree  $\eta$  (3) on the value  $\lambda P_c$  at different  $\rho^2$  (2).

At  $\rho^2 \ll 1$  we have for the  $\gamma e$  collisions  $dL_{\gamma e}/dW \propto W f(x, W^2/4E^2)$ , i.e. fig. 2 also shows the improvement of the  $dL_{\gamma e}/dW$  distribution with the increase of  $(-\lambda P_c)$ . The corresponding curves for the  $\gamma\gamma$  collision at  $\rho^2 \ll 1$  are shown in fig. 3. It is seen that the spectral luminosity  $dL_{\gamma\gamma}/dW$  in the hard part of spectrum increases considerably at  $2\lambda P_c = 2\tilde{\lambda} \tilde{P}_c = -1$ .

This effect becomes stronger when  $\rho^2$  increases. It is due to the fact that the growth of the distance  $\beta$  between conversion and interaction points is accompanied by the removal of soft photons from the interaction region. Therefore, the

relative role of the hard photons is increased. It is seen in fig. 4 where the spectral luminosity  $dL_{\gamma\gamma}/dW$  at different  $\rho^2$  is shown. At  $\rho^2$  fixed and  $\lambda P_c < 0$  the monochromatization degree  $\eta$  is better and the total luminosity  $L$  is larger than in the case of nonpolarized beams. These improvements are seen in figs. 5 and 6 where  $\eta$  and  $L$  are shown at  $2\lambda P_c = -1$  and  $\lambda P_c = 0$  both for the  $\gamma e$  and  $\gamma\gamma$  collisions. All these quantities are calculated using spectrum (4) (see eqs. (25) and (33) below) and assuming, as in ref. [2], that the electron beams are axial symmetric ones with Gaussian distribution of density (cf. (24)).

3. The most crucial quantity which determines the reality of laser conversion of electrons into photons is, apparently, the laser flash energy  $A$  (for the definite repetition rate  $\nu$ ), because the conversion coefficient  $k$  (i.e. the average number of high energy photons per one electron) is determined by the relation

$$k = A/A_0 \quad (\text{at } A < A_0). \quad (6)$$

The quantity  $A_0$  is calculated in ref. [2] for different assumptions about beam parameters.

In figs. 7 and 8 shown are the dependences of the characteristic energy  $A_0$  and the luminosity  $L$  on the monochromatization degree  $\eta$  for the  $\gamma e$  and  $\gamma\gamma$  collisions at  $2\lambda P_c = -1$  and 0. For the definiteness the calculations have been done here for Nd laser ( $\omega_0 = 1.17$  eV) and  $E = 150$  GeV ( $X = 2.69$ ) using the VLEPP parameters (see table 2 in ref. [2]). It is seen that the polarized beams give a large advantage.

So, in the  $\gamma e$  collisions for the monochromatization degree  $\eta_{\gamma e} = 10\%$  the quantities  $A_0 = 15$  J,  $L_{\gamma e} = 0.8$  kLee at  $2\lambda P_c = -1$  and  $A_0 = 40$  J,  $L_{\gamma e} = 0.5$  kLee at  $\lambda P_c = 0$ .

\*) The energy  $A_0$  is calculated according to eqs. (16) and (22) from ref. [2] assuming that the laser focal spot radius  $a_L$  is equal to the electron beam radius in the conversion region  $r_e$  at  $r_e > 20 \mu\text{m}$  and  $a_L = 20 \mu\text{m}$  at  $r_e < 20 \mu\text{m}$ . Note that the permissible duration of the laser flash  $\tau$  increases when  $\rho^2$  increases:  $\tau \propto \rho^2$ .

In other words, using polarized beams with  $2\lambda P_c = -1$  one can get good enough monochromatization and almost total permissible luminosity, for the laser flash with the energy close to the minimum one in fig. 7. Therefore, at the same laser flash energy  $A \lesssim 15 \text{ J}$  the luminosity at  $2\lambda P_c = -1$  is four times as large as that at  $\lambda P_c = 0$ .

For the  $\gamma\gamma$  collisions at  $2\lambda P_c = -1$  the asymptotic monochromatization degree  $\chi_{\gamma\gamma}^{\text{as}}(\rho \rightarrow \infty)$  improves for nearly two times. For  $2\lambda P_c = -1$  the monochromatization degree  $\chi_{\gamma\gamma} = 10\%$  can be obtained at not very large energy  $A_0 = 160 \text{ J}$  (while  $L_{\gamma\gamma} = 0.18 k^2 L_{ee}$ ). Meanwhile, for  $\lambda P_c = 0$  the value  $\chi_{\gamma\gamma} = 10\%$  is close to the asymptotic one, i.e. the corresponding value of  $A_0$  is very large and  $L_{\gamma\gamma}$  is very small. If one chooses  $\chi_{\gamma\gamma} = 15\%$ , the quantities  $A_0 = 50 \text{ J}$ ,  $L_{\gamma\gamma} = 0.31 k^2 L_{ee}$  at  $2\lambda P_c = -1$  and  $A_0 = 250 \text{ J}$ ,  $L_{\gamma\gamma} = 0.08 k^2 L_{ee}$  at  $\lambda P_c = 0$ . Therefore, at the same laser flash energy  $A \lesssim 50 \text{ J}$  the luminosity at  $2\lambda P_c = -1$  is 100 times as large as that at  $\lambda P_c = 0$ .

For the SLC at  $E = 50 \text{ GeV}$  we have  $x = 2.69$  for the Nd laser with frequency tripling. Therefore, the dependences  $L_{\gamma e}(\chi_{\gamma e})$  and  $L_{\gamma\gamma}(\chi_{\gamma\gamma})$  are the same as in figs. 7 and 8. However, because of the other parameters of the system the characteristic energies of the laser flash  $A_0$  are approximately 5 times as large as those in figs. 7 and 8. Using  $E = 50 \text{ GeV}$  and the Nd laser without frequency tripling one can obtain  $\chi_{\gamma\gamma} = 20\%$  at  $A_0 = 120 \text{ J}$  and  $400 \text{ J}$  (while  $L_{\gamma\gamma}/k^2 L_{ee} = 0.26$  and  $0.1$ ) for  $2\lambda P_c = -1$  and  $0$  correspondingly.

In the case under consideration the collisions are described by other than luminosity quantities connected with the beam polarizations. They are considered in sects. 5-7.

#### 4. $\gamma$ beam in the interaction region

1. Let us choose the reference system where  $Z$  axis is directed along the initial electron momentum. All the azimuthal angles are defined relative to one fixed  $X$  axis. We shall call it laboratory system.

Let  $\theta$  and  $\varphi$  be the polar and azimuthal angles of the emission of the final photon with momentum  $\vec{k}$  in this system. The polarization state of this photon is determined by Stokes parameters  $\xi_i$ ,  $i = 1, 2, 3$  relative to the laboratory system axes, among them  $\xi_2$  is the mean helicity and  $\rho = \sqrt{\xi_1^2 + \xi_3^2}$  is the degree of the linear polarization. For the convenience we also introduce  $\xi_0 \equiv 1$ .

The polarization state of the initial electron is determined by the polarization vector  $\vec{\zeta}$ , its longitudinal components

$$\zeta_{11} = \zeta_{22} = 2\lambda \quad (7)$$

where  $\lambda$  is the mean electron helicity, its transverse components is  $\vec{\zeta}_1$ . Below the quantity

$$\alpha = 2\tau |\vec{k}_1| / m_e = 2\tau \sqrt{(x+1)y(y_m-y)} \quad (8)$$

will appear, where  $\vec{k}_1$  is the transverse component of vector  $\vec{k}$ .

The polarization state of the laser photon is determined by quantities  $P_c, P_l$  and  $\gamma$  where  $P_c$  is the circular polarization degree or mean photon helicity,  $P_l$  is the linear polarization degree and  $\gamma$  is the azimuthal angle of the direction of the maximum linear polarization.

2. Using the results of refs. [8-10] we obtain (see appendix B) the differential Compton cross section and the Stokes parameters  $\xi_i$  for the high energy photons in the laboratory system

$$\frac{d\sigma_c}{dyd\varphi} = \frac{y_m \alpha^2}{2y^2} \frac{d\sigma_c}{d\Omega} = \frac{\alpha^2}{xm_e^2} \Phi_0, \quad (9)$$

$$\xi_i = \Phi_i / \Phi_0, \quad (10)$$

$$\Phi_j = \sum_{n=0}^j (C_{jn} \cos n\varphi + S_{jn} \sin n\varphi); \quad j=0,1,2,3. \quad (11)$$

The items of  $\Phi_j$  which do not depend on  $\varphi$  are equal

$$C_{00} = \frac{1}{1-y} + 1-y - 4z(1-z) - 2\lambda P_c \tau x (2\tau-1)(2-y); \quad (12)$$

$$C_{10} = 2\tau^2 P_l \sin 2\gamma; \quad C_{20} = 2\tau^2 P_l \cos 2\gamma;$$

$$C_{20} = 2\lambda \tau x [1 + (1-y)(2\tau-1)^2] - P_c (2\tau-1) \left( \frac{1}{1-y} + 1-y \right). \quad (12)$$

To write down of the rest items, we introduce instead of the real quantities  $C_{jn}$  and  $S_{jn}$  the complex ones:

$$A_{jn} = C_{jn} + i S_{jn}; \quad j=0,1,2,3; \quad n=1,2,3,4. \quad (13)$$

After that we have

$$A_{j1} = |\vec{\tau}_1| \times \left[ -P_c e^{i\beta}; \frac{iP_c}{1-y} e^{-i\beta}; (2\tau-1)e^{i\beta} + P_t \tau e^{i(2\gamma-\beta)}; \frac{P_c}{1-y} e^{-i\beta} \right],$$

$$A_{j2} = -4\tau(1-\tau) [P_t e^{2i\gamma}; i; 2\lambda P_t y e^{2i\gamma}; 1], \quad (14a)$$

$$A_{j3} = -|\vec{\tau}_1| P_t \times (1-\tau) e^{i(2\gamma+\beta)} [0; 0; 1; 0],$$

$$A_{j4} = 2P_t (1-\tau)^2 e^{2i\gamma} [0; i; 0; 1].$$

Note the useful relation

$$A_{1n} = i A_{3n}. \quad (14b)$$

One can prove (see appendix B) that

$$|C_{i0}| \leq C_{00}, \quad |A_{jn}| \leq \sqrt{2} C_{00}. \quad (14c)$$

When  $y \rightarrow y_m$ , the coefficients  $C_{jn}$  and  $S_{jn}$  vanish

$$C_{jn}, S_{jn} \propto (y_m - y)^{n/2}, \quad y \rightarrow y_m. \quad (15)$$

The energy spectrum of the photons (4) follows from eq. (9) after integration over  $\varphi$ :

$$f(x, y) = \frac{2\pi \alpha^2}{\lambda m_e^2 \sigma_c} C_{00}. \quad (9a)$$

For the nonpolarized electron and laser beams we have

$$\xi_1 = -l \sin 2\varphi, \quad \xi_2 = 0, \quad \xi_3 = -l \cos 2\varphi, \quad l = 4\tau(1-\tau) / [(1-y)^{-1} + 1-y - 4\tau(1-\tau)], \quad (10a)$$

i.e. the high energy photons are linear polarized in the di-

rection transverse to the scattering plane, and degree of this polarization  $\propto (y_m - y)$  at  $y \rightarrow y_m$ .

3. To obtain the luminosity of the  $\gamma e$  and  $\gamma\gamma$  collisions one should know the photon distribution over the distance  $\vec{r} = \vec{r}_1$  from the beam axis in the interaction region. Therefore, it is necessary to consider the photon motion from the conversion region to the interaction region (at the distance  $b$ ). The photon with the energy  $\omega = y\epsilon$  was emitted at the angle  $\theta = \theta_0 \sqrt{(\omega_m/\omega) - 1}$  to the initial direction of the electron momentum. Let  $\varphi$  be the azimuthal angle of the emission of this photon. Then the number of photons  $dN_\gamma(\vec{r}, \omega, \varphi)$  which have travelled to the point  $\vec{r}$  in the interaction region can be expressed by the number of electrons  $dN_e(\vec{r})$  which would have had travelled to this point and by cross section (9)

$$dN_\gamma(\vec{r}, \omega, \varphi) = dN_e(\vec{r} - b\vec{\theta}) \frac{k}{\sigma_c} \frac{d\sigma_c}{d\omega d\varphi} d\omega d\varphi, \quad (16)$$

$$\vec{\theta} = \theta_0 \sqrt{(\omega_m/\omega) - 1} (\cos \varphi, \sin \varphi)$$

(here  $k$  is the conversion coefficient).

4. The mean polarization of the  $\gamma$  beam is obtained from eq. (10) after averaging it over the azimuthal angle  $\varphi$  of the weight, proportional to the cross section (9) (of. [9,11])

$$\langle \xi_i \rangle = \frac{\langle \Phi_i \rangle}{\langle \Phi_0 \rangle} = \frac{C_{i0}}{C_{00}}. \quad (17)$$

Note that these functions do not depend on the transverse electron polarization.

As a result, all Stokes parameters are distinct from zero and, therefore, in principle the complete polarization measurements are possible. The functions  $\langle \xi_i \rangle$  are proportional to the polarization parameters of the electrons and (or) laser photons, i.e. varying  $P_c, P_t, \gamma$  and  $\lambda$  one can, in principle, extract the dependence of the observed quantities on  $\langle \xi_i \rangle$ .

The energy dependences of Stokes parameters  $\langle \xi_i \rangle$  for some interesting cases are shown in fig. 9.

If the laser light has the linear polarization, then

high energy photons are polarized in the same direction. It is seen from fig. 9b that their linear polarization degree decreases with the decrease of their energy. (If the electrons have longitudinal polarization, the photons obtain the additional circular polarization).

If the laser light has the circular polarization  $P_c$  or (and) the electrons have the longitudinal polarization  $2\lambda$ , then the  $\gamma$  beam has the circular polarization  $\langle \xi_2 \rangle$  as well. Moreover,  $\langle \xi_2(y=y_m) \rangle = -P_c$  for  $\lambda = 0$  or  $P_c = \pm 1$ . If the electrons are nonpolarized, then  $\langle \xi_2 \rangle$  changes its sign at  $y = x/(x+2)$ , i.e. near  $y_m = x/(x+1)$  (the value  $y = x/(x+2)$  corresponds to scattering angle  $\pi/2$  in the rest system of the initial electron). It is seen from fig. 9 that at  $\lambda P_c < 0$  the region expands in which  $\langle \xi_2 \rangle$  has the same sign as  $\langle \xi_2(y=y_m) \rangle$ .

## 5. Polarization effects in the $\gamma e$ collisions

### 5.1. The $\gamma e$ "cross sections"

The cross section for the collisions of the polarized photons with electrons when the arbitrary system of particles  $X$  is produced can be written in the form

$$d\sigma_{\gamma e \rightarrow X} = \sum_{i=0}^3 \xi_i d\sigma_i, \quad (18)$$

where  $\xi_{1,2}$  are Stokes parameters of the photon and  $\xi_0 \equiv 1$  is introduced for convenience. "The cross sections"  $d\sigma_i$  can be expressed by the amplitudes  $M_a$  for the process  $\gamma e \rightarrow X$  with the photon helicity  $a = \pm 1$ . Indeed,

$$d\sigma_{\gamma e \rightarrow X} = \sum_{a,b=\pm 1} \rho_{ab} M_a M_b^* d\Gamma, \quad d\Gamma = \frac{(2\pi)^4}{4k\rho} \delta(k+p-\sum p_f) \prod_f \frac{d^3 p_f}{2E_f(2\pi)}, \quad (19a)$$

where  $d\Gamma$  is the phase space volume

and  $\rho_{ab}$  is the photon density-matrix in the helicity basis

$$\rho_{++} = \frac{1}{2}(1+\xi_2), \quad \rho_{--} = \frac{1}{2}(1-\xi_2), \quad \rho_{+-} = \rho_{-+} = \frac{1}{2}(-\xi_3 + i\xi_1). \quad (19b)$$

Having substitutes eq. (19b) into (19a), we obtain

$$d\sigma_0 = \frac{1}{2} (|M_+|^2 + |M_-|^2) d\Gamma, \quad d\sigma_2 = \frac{1}{2} (|M_+|^2 - |M_-|^2) d\Gamma, \quad (19c)$$

$$d\sigma_1 = -\text{Im}(M_+ M_-^*) d\Gamma, \quad d\sigma_3 = -\text{Re}(M_+ M_-^*) d\Gamma.$$

Note that  $d\sigma_0$  is the  $\gamma e \rightarrow X$  cross section for nonpolarized photon. For nonpolarized electrons "the cross sections"  $d\sigma_1$ ,  $d\sigma_2$ ,  $d\sigma_3$  vanish after averaging over the final spin states and azimuthal angles. If electrons are longitudinally polarized, the cross section  $d\sigma_2$  does not vanish, generally speaking, after such averaging.

### 5.2. Luminosity and the number of events

The spectral luminosity of the  $\gamma e$  collisions is determined as usual (see sect. 7.2. in ref. [2]) by the relation (the integration over  $\vec{r}$  and  $\varphi$  are assumed)

$$dL_{\gamma e}(\omega) = \nu \int dN_\gamma(\vec{r}, \omega, \varphi) dN_e(\vec{r}) / d^2 r \quad (20)$$

where  $\nu$  is the repetition rate.

Stokes parameters averaged "over collisions"  $\langle \xi_i(\rho) \rangle$  are determined by the relation (cf. eq. (10))

$$\langle \xi_i(\rho) \rangle dL_{\gamma e} = \nu \int dN_\gamma(\vec{r}, \omega, \varphi) \xi_i(\omega, \varphi) dN_e(\vec{r}) / d^2 r \quad (21)$$

As a result, in accordance with eq. (18) the counting rate (the number of events per time unit for the reaction  $\gamma e \rightarrow X$ ) is equal to

$$d\dot{N}_{\gamma e \rightarrow X} = dL_{\gamma e}(\omega) \sum_{i=0}^3 \langle \xi_i(\rho) \rangle \cdot d\sigma_i, \quad (\langle \xi_0 \rangle \equiv 1). \quad (22)$$

In the general case Stokes parameters averaged over collisions  $\langle \xi_i(\rho) \rangle$  (21) do not coincide with those averaged over the  $\gamma$  beam  $\langle \xi_i \rangle$  (17) (e.g. due to axial asymmetry of beams). However, in some important cases this difference vanishes. In particular,

$$\langle \xi_i(\rho) \rangle = \langle \xi_i \rangle = C_{i0}/C_{00} \text{ at } \rho^2 \ll 1 \text{ or } \rho^2 \gg 1 \quad (23)$$

or for axial symmetric beams.

For the case of axial symmetric beams this result follows from eqs. (20), (21) and (16). The limiting cases  $\rho^2 \ll 1$  and  $\rho^2 \gg 1$  will consider in the next section in detail.

It is useful to note that luminosity (20) and quantities (21) do not depend on the electron transverse polarization if both the electron beams have the same shape, and  $dN_e(-\vec{r}) = dN_e(\vec{r})$ .

### 5.3. Axial symmetric electron beams with the Gaussian distribution

In this case the number of electrons in the interaction region would be

$$dN_e(\vec{r}) = \frac{N_e}{\pi a_e^2} e^{-r^2/a_e^2} d^2r. \quad (24)$$

Substituting expressions (24) and (16) into eq. (20) one obtains

$$dL_{\gamma e}(\omega) = k L_{ee} f(x, \frac{\omega}{E}) \exp\left[-\frac{\rho^2}{2} \left(\frac{\omega_m}{\omega} - 1\right)\right] \frac{d\omega}{E}, \quad (25)$$

where  $k$  is the conversion coefficient and  $L_{ee} = \gamma N_e^2 / 2\pi a_e^2$  is the luminosity of the basic  $e^+e^-$  collision without collision effects. Note that the only difference with ref. [2] is the new form (4) for  $f(x, y)$ . Just this formula was used in sect. 2.

## 6. Polarization effects in the $\gamma\gamma$ collision

### 6.1. The $\gamma\gamma \rightarrow X$ "cross sections"

As in the  $\gamma e$  collisions the cross section for the  $\gamma\gamma \rightarrow X$  reaction with the polarized photons can be written in the form

$$d\sigma_{\gamma\gamma \rightarrow X} = \sum_{i,j=0}^3 \xi_i \tilde{\xi}_j d\sigma_{ij} \quad (26)$$

where  $\xi_i$  and  $\tilde{\xi}_j$  are Stokes parameters for the first and second photons and  $\xi_0 = \tilde{\xi}_0 = 1$ . "The cross sections"  $d\sigma_{ij}$  one can express by the helicity amplitudes of the  $\gamma\gamma \rightarrow X$  reaction - see appendix C.

After averaging over the final spin states and azimuthal angles only 3 among 16 quantities  $d\sigma_{ij}$  do not vanish. It is the cross section for nonpolarized photons  $\sigma = \sigma_{00}$  and quantities  $\tau^e = \sigma_{22}$  and  $\tau = \sigma_{33} - \sigma_{11}$  (here the notations from review [12] are used). They are expressed in terms of the  $\gamma\gamma \rightarrow X$  cross sections for the scattering of photons with parallel  $\sigma_{||}$  or orthogonal  $\sigma_{\perp}$  linear polarizations or in the states whose total helicity is zero  $\sigma_0$  or two  $\sigma_2$  (in c.m.s. of the photons):

$$\sigma = \sigma_{00} = \frac{1}{2}(\sigma_{||} + \sigma_{\perp}) = \frac{1}{2}(\sigma_0 + \sigma_2); \quad \tau^e = \sigma_{22} = \frac{1}{2}(\sigma_0 - \sigma_2); \quad \tau = \sigma_{33} - \sigma_{11} = \sigma_{||} - \sigma_{\perp} \quad (27)$$

For all processes which we know at high energies considered, the quantity  $\tau$  is small in comparison with  $\sigma$ :

$$|\tau| \ll \sigma, \quad |\tau^e| \lesssim \sigma. \quad (27a)$$

### 6.2. Luminosity and the counting rate

The spectral luminosity of the  $\gamma\gamma$  collisions is determined by the relation (cf. eq. (20))

$$dL_{\gamma\gamma}(\omega, \tilde{\omega}) = \gamma \int dN_{\gamma}(\vec{r}, \omega, \varphi) d\tilde{N}_{\gamma}(\vec{r}, \tilde{\omega}, \tilde{\varphi}) / d^2r \quad (28)$$

(The tilde is used for the parameters of the second beam, for it in formulae similar to eqs. (16), (9-14) one should replace the angles  $\varphi$ ,  $\gamma$  and  $\beta$  by  $(-\tilde{\varphi})$ ,  $(-\tilde{\gamma})$  and  $(-\tilde{\beta})$  because the second beam moves opposite to the direction of the  $\vec{z}$  axis of laboratory system).

The products of Stokes parameters averaged "over collision-



ons"  $\langle \tilde{\xi}_i \tilde{\xi}_j \rangle$  are determined by the relations\*

$$\langle \tilde{\xi}_i \tilde{\xi}_j \rangle dL_{\gamma\gamma} = \nu \int dN_e(\vec{r}, \omega, \varphi) \xi_i(\omega, \varphi) \tilde{\xi}_j(\tilde{\omega}, \tilde{\varphi}) d\tilde{N}_e(\vec{r}, \tilde{\omega}, \tilde{\varphi}) / d^2r \quad (29)$$

As a result, in accordance with eq. (26) the counting rate for the  $\gamma\gamma \rightarrow X$  reaction is equal to

$$d\dot{N}_{\gamma\gamma \rightarrow X} = dL_{\gamma\gamma} \sum_{ij=0}^3 \langle \tilde{\xi}_i \tilde{\xi}_j \rangle d\sigma_{ij} \quad (31)$$

Taking into account eq. (27), it is convenient to use the linear combination

$$\Lambda = \frac{1}{2} \langle \tilde{\xi}_3 \tilde{\xi}_3 - \tilde{\xi}_1 \tilde{\xi}_1 \rangle \quad (29a)$$

In this case eq. (31) has the form

$$d\dot{N}_{\gamma\gamma \rightarrow X} = dL_{\gamma\gamma} (d\sigma + \langle \tilde{\xi}_2 \tilde{\xi}_2 \rangle d\tau^e + \Lambda d\tau + \dots) \quad (31a)$$

### 6.3. Limit cases

In the general case the mean product of Stokes parameters is not equal to the product of Stokes parameters of the single beams. However, in the limit cases of small and large values of the quantity  $\rho^2$  (2) such a factorization takes place:

$\langle \tilde{\xi}_i \tilde{\xi}_j \rangle \approx \langle \tilde{\xi}_i \rangle \cdot \langle \tilde{\xi}_j \rangle$ . It is a very useful relation because the mean polarization of the  $\gamma$  beam  $\langle \tilde{\xi}_i \rangle$  is determined by simple eqs. (17), (12) and it does not depend on the beam shapes. Therefore, under such circumstances one needs neither new measurements nor calculations using the beam characteristics to determine  $\langle \tilde{\xi}_i \tilde{\xi}_j \rangle$ .

Let us show this factorization and estimate its accuracy without making any assumptions about the electron density.

While the conversion point is near the interaction point, the broadening of the  $\gamma$  beam  $\sim b\theta_0$  is small as compared to the smallest transverse size  $a_m$  which the electron beam would have had. In this case the photons which reach the definite point in the interaction region can have arbitrary azimuthal angles, i.e. the angles  $\varphi$  and  $\tilde{\varphi}$  are not correlated

(in other words, one can neglect in (16), (29) the difference between  $\vec{r}-b\vec{\theta}$  and  $\vec{r}$  and between  $\vec{r}-b\vec{\tilde{\theta}}$  and  $\vec{r}$ ). Therefore, the integrations over  $\varphi$  and  $\tilde{\varphi}$  in eqs. (28), (29) become independent and lead to the averaging of  $\xi_i$  (and  $\tilde{\xi}_i$ ) over  $\varphi$  (and  $\tilde{\varphi}$ ) with the weight proportional to  $d\sigma_c/d\omega d\varphi$  (and  $d\tilde{\sigma}_c/d\tilde{\omega} d\tilde{\varphi}$ ), i.e. to the abovementioned factorization.

To estimate the accuracy of this factorization, one expands in eqs. (28), (29) the quantity  $dN_e(\vec{r}-b\vec{\theta})$  in eqs. (28), (29) in powers of  $b\vec{\theta}$ . After integration over  $\vec{r}$  the quantity  $b\theta/a_m \sim \rho_m = b\theta_0/a_m$  becomes the parameter of this expansion and it determines the accuracy of factorization. Moreover, the first term of this expansion is proportional to the expression  $\int b\vec{\theta} \cdot \vec{P}_i(\varphi) d\varphi$ , where  $\vec{\theta} \propto (\cos\varphi, \sin\varphi)$ . If the electrons have no transverse polarization, the quantity  $\vec{P}_i(\varphi)$  (11) have the terms with  $\cos n\varphi$  and  $\sin n\varphi$  at  $n = 0, 2, 4$  only. Therefore, the first terms of this expansion vanish after integration over  $\varphi$  and the corrections to the factorization have the second order of magnitude. As a result, at  $dN_e(-\vec{r}) = dN_e(\vec{r})$  we have

$$\langle \tilde{\xi}_i \tilde{\xi}_j \rangle = \langle \tilde{\xi}_i \rangle \langle \tilde{\xi}_j \rangle + \tilde{\xi}_i \tilde{\xi}_j O(\rho_m^2) + O(\rho_m^4); \quad \rho_m = b\theta_0/a_m \ll 1. \quad (32a)$$

If  $dN_e(-\vec{r}) \neq dN_e(\vec{r})$ , the items  $\sim \tilde{\xi}_i \rho_m, \sim \tilde{\xi}_i \rho_m$  should be added in the righthand part of this expression.

Let us consider now the case when the distance  $b$  from the conversion point to the interaction one is large enough and the intrinsic size of the  $\gamma$  beam  $\sim b\theta_0$  is larger than the largest transverse size of the electron beam  $a_m$ . In this case the main part of the  $\gamma\gamma$  collisions is constituted by the collisions of the most energetic photons (whose energies  $\omega$  and  $\tilde{\omega}$  are close to  $\omega_m$ ). But for this photons the coefficients  $C_{in}$  and  $S_{in}$  with  $n \geq 1$  are small (see eq. (15)). Therefore, only the independent on  $\varphi$  term  $C_{i0}$  (and  $S_{i0}$ ) in  $\vec{P}_i$  (11) gives the main contribution, and factorization takes place.

To estimate the accuracy of factorization in this case, note that here  $\rho_m \sim \rho \sim \frac{1}{2}$ , where  $\frac{1}{2}$  is the monochromatization degree, i.e.  $C_{in}, S_{in} \sim \rho^{n/2}$  in accordance with eq. (15).

Every  $\gamma$  beam seems to be emitted from one point so they have axial symmetry with the accuracy  $\sim \rho_M^{-2} = (a_M / b\theta_0)^2$ . Therefore, with the same accuracy after azimuthal averaging the combinations  $C_{in} \tilde{C}_{jn}$  and  $S_{in} \tilde{S}_{jn}$  give the contribution to  $\langle \tilde{\xi}_i \tilde{\xi}_j \rangle$  only. Moreover, coefficients  $C_{i1}$  and  $S_{i1}$  (14) with the lowest power of  $\eta$  are proportional to the degree of the electron transverse polarization  $\xi_1$ . As a result,

$$\langle \tilde{\xi}_i \tilde{\xi}_j \rangle = \langle \tilde{\xi}_i \rangle \langle \tilde{\xi}_j \rangle + \xi_1 \tilde{\xi}_1 O(\eta) + O(\eta^2), \quad b\theta_0 / a_M \gg 1, \quad (32b)$$

(the largest corrections seem to be  $\sim \xi_1 \eta^{1/2} \rho_M^{-1}$ , but they are unessential because in the case considered  $\eta \gg \rho_M^{-1}$ ).

#### 6.4. Gaussian beams

Assuming the electrons beams to have axial symmetry and the Gaussian distribution of density (24), one can calculate the dependence on  $\rho^2$  (2) in detail. Let us substitute eqs. (16) and (24) into (28), (29) and integrate over  $\tilde{r}$ ,  $\varphi$  and then over  $\psi = \varphi - \tilde{\varphi}$ . After integration over  $\tilde{r}$  the expression  $\int \Phi_i(\varphi) \tilde{\Phi}_j(\varphi - \psi) \exp(-\nu \cos \psi) d\varphi d\psi$  appears. When one integrates it over  $\varphi$  the only combinations  $C_{i0} \tilde{C}_{j0}$  and  $C_{in} \tilde{C}_{jn} - S_{in} \tilde{S}_{jn} = \text{Re}(A_{in} \tilde{A}_{jn})$  give nonzero contributions. Their main properties follow from eqs. (13-15)

$$|\text{Re}(A_{in} \tilde{A}_{jn}) / 2 C_{00} \tilde{C}_{00}| \leq 1; \quad \text{Re}(A_{i1} \tilde{A}_{j1}) \propto \xi_1 \tilde{\xi}_1; \quad (33a)$$

$$\text{Re}(A_{in} \tilde{A}_{jn}) / 2 C_{00} \tilde{C}_{00} \propto (y_m - y)^{n/2} (y_m - \tilde{y})^{n/2} \quad \text{at} \quad y, \tilde{y} \rightarrow y_m.$$

The mean value of  $\cos n\psi$  is expressed via modified Bessel functions:

$$\langle \cos n\psi \rangle = \frac{I_n(\nu)}{I_0(\nu)}; \quad I_n(\nu) = \int_0^{2\pi} \frac{d\psi}{2\pi} e^{-\nu \cos \psi} \cos n\psi; \quad \nu = \rho^2 \sqrt{\left(\frac{\omega_m}{\omega} - 1\right) \left(\frac{\omega_m}{\tilde{\omega}} - 1\right)}. \quad (33b)$$

As a result, one obtains

$$\frac{dL_{\gamma\gamma}}{d\omega d\tilde{\omega}} = k^2 L_{ee} \frac{d\sigma_c}{\sigma_c d\omega} \frac{d\tilde{\sigma}_c}{\tilde{\sigma}_c d\tilde{\omega}} I_0(\nu) \exp\left[-\frac{\rho^2}{2} \left(\frac{\omega_m}{\omega} + \frac{\omega_m}{\tilde{\omega}} - 2\right)\right] M;$$

$$M = 1 + A_1 \langle \cos \psi \rangle + A_2 \langle \cos 2\psi \rangle; \quad A_1 = (\tilde{\xi}_1 \xi_1) P_c \tilde{P}_c \frac{\partial e \partial \tilde{e}}{2 C_{00} \tilde{C}_{00}}; \quad (33c)$$

$$A_2 = \frac{1}{2} P_c \tilde{P}_c A(y) \tilde{A}(\tilde{y}) \cos 2(\gamma - \tilde{\gamma}); \quad A(y) = 4\eta(1-\eta) / C_{00},$$

where  $\partial e$  is defined by eq. (8), and quantities  $A_n = \text{Re}(A_{0n} \tilde{A}_{0n}) / 2 C_{00} \tilde{C}_{00}$  are limited:  $|A_n| \leq 1$  and  $M < 1$ . The difference between this expression and that for the nonpolarized beams (see eq. (40) in ref. [2]) is in the new spectrum  $f(x, y)$  (4) and the additional factor  $M$ . If the electron transverse polarization and linear polarization of laser photons are absent, then  $M = 1$ ; just this case was considered in ref. [2] and in sect. 3.

The mean value  $\langle \tilde{\xi}_i \tilde{\xi}_j \rangle$  is determined by the relations which can be obtained in the same manner as eq. (33):

$$\langle \tilde{\xi}_i \tilde{\xi}_j \rangle = \frac{1}{M} \left[ \langle \tilde{\xi}_i \rangle \langle \tilde{\xi}_j \rangle + \sum_{n=1}^4 \frac{\text{Re}(A_{in} \tilde{A}_{jn})}{2 C_{00} \tilde{C}_{00}} \langle \cos n\psi \rangle \right]. \quad (34)$$

These relations take place for the arbitrary axial symmetric beams, but  $\langle \cos n\psi \rangle$  has form (33b) in the Gaussian case (24) only.

Factorization relations (32) are broken in the case under consideration by items of the type  $[\text{Re}(A_{in} \tilde{A}_{jn}) / 2 C_{00} \tilde{C}_{00}] \langle \cos n\psi \rangle$  (see eqs. (33a, b)). The quantities  $\langle \cos n\psi \rangle$  depend on  $\nu$  (33b) only, and the characteristic value of  $\nu \lesssim \rho^2$ . At  $\nu < 1$  and  $n \geq 1$  one has  $\langle \cos n\psi \rangle < 0.11$ . Therefore, if the electron transverse polarization is absent (and there are no items with  $n = 1$ ), factorization (32a) at  $\rho < 1$  is fulfilled with the accuracy  $\lesssim 10\%$ . With the same accuracy we have  $M \approx 1$ . For  $\rho^2 \gg 1$  in the main region we have  $y_m - y \sim \eta$ ,  $y_m - \tilde{y} \sim \eta$  and, therefore, the quantities  $\text{Re}(A_{in} \tilde{A}_{jn}) / 2 C_{00} \tilde{C}_{00} \sim \eta^n$  (see eq. (33a)) are small; that results in eq. (32b) and  $M \approx 1$ .

#### 6.5. Two unexpected effects (for details see appendix D)

Let us consider the case when the laser light is nonpolarized but both electron beams have transverse polarizations. According to eqs. (33), (34) we have in this case  $\langle \tilde{\xi}_2 \rangle = \langle \tilde{\xi}_2 \rangle = 0$  but  $\langle \tilde{\xi}_1 \tilde{\xi}_1 \rangle \neq 0$ , i.e. the electron transverse polarizations

"transform" into the circular polarizations of photons.

There is another interesting effect in the  $\gamma\gamma$  collisions in the case when both laser beams have the same polarizations in every flash (e.g., it may be one laser beam divided into two beams) but for many flashes the average polarization of laser light is zero. Nevertheless, in this case the  $\gamma\gamma$  luminosity changes considerably in comparison with the luminosity for the truly nonpolarized laser beams.

### 7.1. $\gamma\gamma$ -collisions. Nonpolarized electron and laser beams

The discussion below supplements that in ref. [2]. We consider here once more unexpected effect which arises in nonpolarized axial symmetric beams. Even in this case a nontrivial polarization effect takes place i.e. between quantities  $\langle \xi_i \tilde{\xi}_j \rangle$  there are the nonzero one

$$\langle \xi_3 \tilde{\xi}_3 \rangle = -\langle \xi_1 \tilde{\xi}_1 \rangle = \Lambda = \frac{1}{2} \langle \tilde{\ell} \cos 2\psi \rangle, \quad \Lambda > 0. \quad (35)$$

Here  $\tilde{\ell}$  is given by eq. (10a). As a result, eq. (31) converts to the form (cf. (27))

$$d\dot{N}_{\gamma\gamma \rightarrow X} = dL_{\gamma\gamma} (d\sigma + \Lambda d\tau) = dL_{\gamma\gamma} \left[ \left(\frac{1}{2} + \Lambda\right) d\sigma_{\parallel} + \left(\frac{1}{2} - \Lambda\right) d\sigma_{\perp} \right]. \quad (36)$$

Therefore, the initial states with parallel linear photon polarization more contribute to the observable events than those with orthogonal polarizations. This effect is due to the fact that the final photons in the Compton scattering are polarized orthogonal to the scattering plane (10a). Of course, this polarization vanishes after averaging over the  $\gamma$  beam  $\langle \xi_1 \rangle = \langle \xi_3 \rangle = 0$ . However, at  $\beta^2 \gg 1$  the photons colliding in any point should be emitted at the close angles  $\theta \approx \tilde{\theta}$ ,  $\varphi \approx \tilde{\varphi}$ , i.e.  $\psi = \varphi - \tilde{\varphi} \approx 0$ , and hence the quantity  $\xi_3 \tilde{\xi}_3 - \xi_1 \tilde{\xi}_1 = \tilde{\ell} \cos 2\psi$  does not vanish after averaging. Therefore, at  $\beta^2 \neq 0$  the counting rate contains on the polarization dependence though each  $\gamma$  beam is nonpolarized at the average.

As a result to extract the nonpolarized cross section  $d\sigma = d\sigma_{00}$  from the data one should know the quantity  $\Lambda d\tau$ , i.e. the

problem is more complicated than usually.

However, the effect under consideration is not great. Indeed, the quantity  $\Lambda \sim \beta^2 \sim \beta^4$  at  $\beta \ll 1$  due to the factor  $\langle \cos 2\psi \rangle$  and  $\Lambda \sim (y_m - y)(y_m - \tilde{y})$  at  $y, \tilde{y} \rightarrow y_m$  due to factors  $\tilde{\ell}$ . Moreover, the quantity  $\Lambda$  is not great at other values of  $\beta$  as well. Thus, at  $x = 2.69$  one has  $\beta < 0.6$  for all values of  $y$ , i.e.  $\Lambda < 0.18$ . Taking into account eq. (27a) as well one can expect the contribution  $\Lambda d\tau$  to be negligible.

### 7.2. $\gamma\gamma$ -collisions. The case $\lambda P_c \neq 0, \tilde{\lambda} \tilde{P}_c \neq 0$ (Axial symmetric beams)

This subsection supplements naturally the discussion of sect. 3.

The case when helicities of both electrons and laser photons are nonzero is important because at  $\lambda P_c < 0$  and  $\tilde{\lambda} \tilde{P}_c < 0$  the monochromatization of the  $\gamma\gamma$  collisions improves considerably (sect. 3). We assume here for the sake of simplicity that the transverse electron polarization and the linear polarization of laser photons are absent. In this case the circular polarization degrees  $\xi_2$  and  $\tilde{\xi}_2$  for the high energy photons are independent of azimuthal angles  $\varphi, \tilde{\varphi}$ , and  $\langle \xi_i \tilde{\xi}_j \rangle = \xi_i \tilde{\xi}_j$  for  $i = j = 0, 2$ . Here,

$$\xi_2 = \langle \xi_2 \rangle = C_{20} / C_{00} \quad (37)$$

where  $C_{j0}$  is defined in eq. (12). As a result, eq. (31) converts to the form (cf. eqs. (36), (33c))

$$d\dot{N}_{\gamma\gamma \rightarrow X} = dL_{\gamma\gamma} (d\sigma + \Lambda d\tau + \xi_2 \tilde{\xi}_2 d\tau^2 + \xi_2 d\sigma_{20} + \tilde{\xi}_2 d\sigma_{02}), \quad (38)$$

$$\Lambda = \frac{1}{2} A(y) \tilde{A}(\tilde{y}) \cdot \langle \cos 2\psi \rangle$$

and luminosity  $dL_{\gamma\gamma}$  is given in eq. (33c) with  $M = 1$ .

The contribution of  $d\sigma_{20}$  and  $d\sigma_{02}$  vanishes after averaging over azimuthal angles and polarization of the final particles. It is absent at all for the pair production or for the inclusive processes  $\gamma\gamma \rightarrow a + \dots$  (see appendix C).

In the interesting case  $2\lambda P_c \approx -1$  at  $\omega \approx \omega_m$  we have  $|\xi_2| \approx 1, \Lambda \approx 0$ . Moreover, one can change the sign of  $\xi_2$  having conserved the product  $\lambda P_c$  i.e. without variation

of spectrum) by simultaneously changing the signs of  $\lambda$  and  $P_c$ . This allows one to measure the interesting quantity  $\tau^a = \frac{1}{2}(\sigma_0 - \sigma_2)$  (27) besides  $\sigma = \frac{1}{2}(\sigma_0 + \sigma_2)$ . In other words, the direct measurement of  $\sigma_0$  and  $\sigma_2$  is possible. Fig. 10 shows the dependence of the corresponding luminosities  $dL_0/dz$  and  $dL_2/dz$  on  $z = W_{\gamma\gamma}/2E$  where

$$dL_0 = \frac{1}{2}(1 \pm \xi_2 \tilde{\xi}_2) dL_{\gamma\gamma}; \quad dN_{\gamma\gamma \rightarrow X} = dL_0 d\sigma_0 + dL_2 d\sigma_2 + dL_{\gamma\gamma}(\Lambda d\tau + \dots) \quad (39)$$

It is seen that  $dL_0/dz \gg dL_2/dz$  at  $z \approx z_m$ , i.e. here  $d\sigma_0$  will be measured only. For  $\lambda = 0$  in the limiting cases  $x \ll 1$  or  $x \gg 1$  and  $\rho \ll 1$  or  $\rho \gg 1$  these luminosities were obtained in ref. [6].

### 8. Discussion

1. Spectrum of the high energy photons becomes considerably harder for the electrons and laser photons with the opposite helicities (see fig. 2). This leads to two advantages in the problem of obtaining certain monochromatization degree in contrast with the case of the nonpolarized initial beams

- i) the characteristic laser flash energy  $A_0$  is less and
- iii) the luminosities  $L_{\gamma e}$  and  $L_{\gamma\gamma}$  are larger.

For example, at fixed laser flash energy  $A$  and for the monochromatization degree  $\lambda_{\gamma\gamma} < 15\%$  the luminosity  $L_{\gamma\gamma}$  increases by two orders of magnitude or more in comparison with the case of nonpolarized initial beams.

2. The average values of Stokes parameter of the  $\gamma$  beam  $\langle \xi_i \rangle$  can be done of order of unity by variation of laser flash polarization (see Fig. 9). Therefore, in principle the complete polarization measurements are possible in the  $\gamma e \rightarrow X$  and  $\gamma\gamma \rightarrow X$  reactions.

3. In the experiments with the usual polarized beams it only suffices to know beam densities and averaged polarizations. This does not hold for  $\gamma\gamma$  collisions under consideration due to special method of creation of the  $\gamma$ -beams. Average products of the Stokes parameters  $\langle \xi_i \tilde{\xi}_i \rangle$  for two beams which determine measurable cross sections (31) do not

decay into the products of average Stokes parameters for every beam. Therefore, they demand the special calibration. We show this difficulty to vanish in two important limits (32) when  $\langle \xi_i \tilde{\xi}_j \rangle \approx \langle \xi_i \rangle \langle \tilde{\xi}_j \rangle$  and  $\langle \xi_i \rangle$  can be calculated if one knows photon energies and electron and laser beam polarizations only (see eqs. (17), (12)).

4. Even if the electrons and laser photons are nonpolarized the high energy photons are polarized orthogonal to the scattering plane. While the conversion point is near the interaction one the photons with different scattering planes collide in any point of the interaction region. That is why the polarization effects vanish after azimuthal averaging.

When the conversion points are far enough from the interaction one the colliding photons were scattered approximately at the same plane, i.e. they have approximately parallel linear polarizations. That is why the additional contribution to the cross section which is proportional to the product of photon polarizations does not vanish after averaging

$\langle \xi_3 \tilde{\xi}_3 - \xi_1 \tilde{\xi}_1 \rangle \neq 0$ . Therefore, the measurable cross section differs from that for nonpolarized photons even in the case when electron and laser beams are nonpolarized. The corresponding additional contribution is given by the term  $\Lambda d\tau$  (cf. eq. (36)) where  $\Lambda$  depends essentially on the beam shape. Therefore, it is necessary to have, generally speaking, the additional method of extracting quantities  $\sigma$  and  $\Lambda\tau$  from the sum  $\sigma + \Lambda\tau$  measured. Fortunately, the value  $\Lambda\tau$  is usually small and can be neglected for first measurements (cf. the discussion in subsect. 7.1).

5. The main numerical results were obtained above for the Nd laser  $\omega_0 = 1.17$  eV and  $E = 150$  GeV, i.e.  $x = 2.69$ . Let us discuss briefly the  $E$  and  $\omega_0$  dependence restricting ourselves by the value  $x < 4.8^*$ . For the sake of definiteness

\* If  $x > 4.8$ , then  $e^+e^-$  pairs are produced at the collision of laser photon with high energy one in the conversion region. This process decreases the number of high energy photons and changes their spectrum.

let us consider the  $\gamma\gamma$  collisions at  $2\lambda p_c = 2\tilde{\lambda}\tilde{p}_c = -1$  and  $\tilde{L}_{\gamma\gamma} = 10\%$  for VLEPP parameters.

In this case the characteristic laser flash energy  $A_0$  (6) increases if  $\omega_0$  is fixed and  $E$  grows. Simultaneously the luminosity  $L_{\gamma\gamma}$  increases (at fixed value of the conversion coefficient  $k$ ). Thus, for  $\omega_0 = 1.17$  eV when  $E$  varies from 50 GeV to 300 GeV, the quantity  $A_0$  varies from 70 J to 250 J, and  $L_{\gamma\gamma}/k^2 L_{ee}$  varies from 0.05 to 0.31 (cf. table 8 in ref. [2]).

At the interval considered  $0.9 < x < 4.8$  the increase of  $\omega_0$  at fixed value of  $E$  is accomplished by small decrease of  $A_0$  and the considerable increase of  $L_{\gamma\gamma}$ . Thus, at  $E = 50$  GeV when  $x$  varies from 0.9 to 5 (i.e.  $\omega_0$  varies from 1.17 eV to 7 eV) the value of  $A_0$  decreases from 70 J to 40 J, and  $L_{\gamma\gamma}/k^2 L_{ee}$  increases from 0.05 to 0.3. Besides, the maximum photon energy  $\omega_m$  increases from 24 GeV to 42 GeV.

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## APPENDIX A. PROBLEMS OF CALIBRATION

The calibration of the luminosity for nonpolarized initial beams and the main calibration processes for  $\gamma e^-$  - and  $\gamma\gamma$  - collisions were discussed in ref. [2]. Here we discuss briefly the new effects arising due to initial beam polarizations.

### A.1. $\gamma\gamma$ - collisions.

1. The  $\gamma\gamma \rightarrow \mu^+\mu^-e^+e^-$  process was discussed carefully in refs. [13]. Its total cross section is relatively large ( $5.7 \cdot 10^{-33}$  cm<sup>2</sup>) and energy independent. The mean muon production angles are not too small,  $\theta_\mu \sim m_\mu/\omega$ . The total  $\mu$ -pair energy coincides with the energy of "its" photon. The  $\gamma\gamma \rightarrow \mu^+\mu^-e^+e^-$  cross section is independent of circular photon polarization. The distribution of muons both in energy and in the production angle  $\theta_\mu$  deviate for less than 2.5% when the linear photon polarization varies. If the photons are linearly polarized then the asymmetry of  $\sim 10\%$  arises in the muon azimuthal angle distribution.

The  $\gamma\gamma \rightarrow e^+e^-e^+e^-$  process has larger cross section ( $6.5 \cdot 10^{-30}$  cm<sup>2</sup>) and its polarization dependence is also not large [13]. In the investigation of the processes with small cross sections  $\sim d^2/E^2$  one can use the processes  $\gamma\gamma \rightarrow l^+l^-$  ( $l = e, \mu, \tau$ ) with large production angles for comparison. Their cross sections are strongly dependent on circular photon polarization and have large azimuthal asymmetry connected with linear photon polarization.

In these processes the greatest cross sections really are (see [12, 14])

$$\frac{d\sigma_2}{dt} = \frac{\pi d^2}{W^4} \left( \frac{W^2}{p_\perp^2} - 2 \right); \quad \frac{d(\sigma_{33} + \sigma_{44})}{dt} = - \frac{2\pi d^2}{W^4}; \quad p_\perp \gg m_l. \quad (A.1)$$

(Here  $W$  is invariant  $\gamma\gamma$  mass,  $t$  is the squared transfer,  $4$  - momentum,  $p_\perp$  - transverse component of the lepton momentum,  $x$  axis is chosen in leptons production plane). Other

cross sections are small:

$$\frac{d\sigma_0}{d\sigma_2} \sim \frac{m_e^2}{P_L^2}; \quad \frac{d(\sigma_{33}-\sigma_{11})}{d(\sigma_{33}+\sigma_{11})} = \frac{m_e^4}{P_L^4}; \quad \frac{d\sigma_{03}}{d(\sigma_{33}+\sigma_{11})} = \frac{d\sigma_{30}}{d(\sigma_{33}+\sigma_{11})} = \frac{m_e^2}{P_L^2} \quad (\text{A.2})$$

2. The luminosity calibration. The event number in the

$\gamma\gamma \rightarrow X$  reaction is (see (31))

$$dN_{\gamma\gamma \rightarrow X} = dL_{\gamma\gamma} \sum_{ij=0}^3 \langle \tilde{\xi}_i \tilde{\xi}_j \rangle d\sigma_{ij}$$

Since the total cross section and the energy distribution for the main calibration process  $\gamma\gamma \rightarrow \mu^+\mu^-e^+e^-$  (as for  $\gamma\gamma \rightarrow e^-e^+e^-e^+$ ) are practically independent of the photon polarizations, then the measurement of this cross section will allow one to measure directly the luminosity  $dL_{\gamma\gamma}/d\omega d\tilde{\omega}$  besides beam polarizations.

3. The products of the Stokes parameters

One must know and have an opportunity to change the mean values of the products of the Stokes parameters  $\langle \tilde{\xi}_i \tilde{\xi}_j \rangle$  for separate measurement of the cross sections  $d\sigma_{ij}$ . In the most attractive case of circularly polarized laser photons and longitudinally polarized electrons the event number is given by eqs. (35), (36). They show us the variation of the signs both of helicities of the electron and laser photon results in the change of  $\gamma$ -beam polarization but  $\gamma$ -spectrum remains unchanged.

The value of the circular polarization  $\tilde{\xi}_2$  (and  $\tilde{\xi}_2$ ) does not depend on the beam geometry and is given by eq. (35). This allows one to provide the complete study of the cross section dependence on the circular photon polarization.

In general, the calibration of the polarizations  $\langle \tilde{\xi}_i \tilde{\xi}_j \rangle$  is necessary, as it is seen from eqs (29), (34). Meanwhile, the most physically interesting cases are that of  $\rho^2 \ll 1$  (maximum conversion coefficient) and  $\rho^2 \gg 1$  (good monochromatization). In both these cases according to eq. (32) the dependence on the density distributions becomes unessential and one must only know the polarizations of laser and electron

beams to determine  $\langle \tilde{\xi}_i \tilde{\xi}_j \rangle \approx \langle \tilde{\xi}_i \rangle \langle \tilde{\xi}_j \rangle$ . In this case one must decrease the electron transverse polarization degree to decrease the inaccuracy of relations (32).

A.2.  $\gamma e$  - collisions

Total cross sections and energy distributions for main calibrating processes  $\gamma e \rightarrow \ell^+ \ell^- e$  ( $\ell = \mu, e$ ) do not practically depend on photon polarization as well. Therefore, one can determine the luminosity  $dL_{\gamma e}/d\omega$  for all photon polarizations ( $1^\pm$  azimuthal angle distribution depends on linear photon polarization).

When the processes with small cross sections are studied, one can use for comparison the Compton effect which depends strongly on  $\lambda$  and  $P_C$  (9).

In general here the polarization  $\langle \tilde{\xi}_i(\rho) \rangle$  (21) calibration is necessary (due to the reasons similar to those for  $\gamma\gamma$  - collisions). This difficult task is simplified for  $\gamma e$  - collisions not only at the cases of  $\rho^2 \ll 1$  and  $\rho^2 \gg 1$ , but at the case of axial symmetry of the beams when the polarization does not depend on beams geometry.

A.3. Electron polarization control

One can control the electron polarization value simultaneously with the luminosity measurement (provided the handling of a laser polarization is simple).

Longitudinal electron polarization. The hard part of the high energy photon spectrum is rather sensitive to the  $\lambda P_C$  value (see fig. 2). Therefore, the observation of the deviation of this spectrum when the sign of  $P_C$  is changed yields the information about the  $\lambda P_C$  value.

Transverse electron polarization  $\xi_\perp$  seems not to be interesting in the considered problems. It is only essential to verify the fact that its contributions (depending on the beam geometry) are small enough not to break practically the simple factorization relations (32). We shall consider the beams to

be axially symmetric for definiteness. Then all the effects of electron transverse polarization are not large and are proportional to  $\zeta_1 \tilde{\zeta}_1 \langle \cos \psi \rangle$ , where  $\langle \cos \psi \rangle$  depends on density distribution in a beam. In particular there is a contribution to luminosity (33) proportional to  $\zeta_1 \tilde{\zeta}_1 P_c \tilde{P}_c \langle \cos \psi \rangle$ . Therefore observing the luminosity deviation at the variation of the sign of  $P_c$  and  $\tilde{P}_c$  (when  $\lambda$  and  $\tilde{\lambda}$  are known) one can find the value  $\zeta_1 \tilde{\zeta}_1 \langle \cos \psi \rangle$  with sufficient accuracy.

## APPENDIX B. THE COMPTON EFFECT FOR POLARIZED PARTICLES

This appendix contains the necessary formulae for the Compton effect  $e(\rho) + \gamma_0(k_0) \rightarrow \gamma(k) + e(\rho')$  when the polarizations of both photons and the initial electron are considered. Let us introduce the invariants

$$\chi = \frac{2\rho k_0}{m_e^2}, \quad y = 1 - \frac{\rho k}{\rho k_0}, \quad \tau = \frac{y}{x(1-y)}, \quad s_1 = \frac{a k_0}{m_e}, \quad s_2 = \frac{a k}{m_e},$$

$$\varepsilon = \frac{\varepsilon_{\mu\nu\gamma\sigma} a^\mu k_0^\nu \rho^\gamma k^\sigma}{x m_e^3} \quad (\text{B.1})$$

where  $a$  is electron spin 4-vector. Let  $J_i$  and  $X'_i$  be the Stokes parameters for laser and high energy photons in the reaction axes - x axis is orthogonal to the reaction plane,  $x \parallel \vec{k}_0 \times \vec{k}$  and it is the same for both photons, y axes belong to the reaction plane (see [7] e.g.). The Compton differential cross section has the form in our case

$$d\sigma_c = \frac{\alpha^2}{2x m_e^2} \left[ F_0 + \sum_{i=1}^3 F_i X'_i \right] \delta(\rho + k_0 - k - \rho') \frac{d^3k}{\omega} \frac{d^3\rho'}{E'}$$

$$F_0 = \frac{4}{1-y} + 1-y - 4\tau(1-\tau) + 4\tau(1-\tau)X_3 + 2\tau[(1-2\tau)s_1 + s_2]X_2,$$

$$F_1 = 2(1-2\tau)X_1 - \frac{4\tau\varepsilon}{1-y}X_2,$$

(B.2)

$$F_2 = (1-2\tau)\left(\frac{1}{1-y} + 1-y\right)X_2 + 2\tau[s_1 + (1-2\tau)s_2] + 2\tau[(1-y)s_1 - (1-2\tau)s_2]X_3 +$$

$$+ 4\tau\varepsilon X_1,$$

$$F_3 = 4\tau(1-\tau) + 2[1-2\tau(1-\tau)]X_3 - 2\tau[(1-2\tau)s_1 - \frac{s_2}{1-y}]X_2.$$

The cross section (B.2) satisfies the symmetry relations which arise from  $\xi \leftrightarrow \eta$  symmetry (see [7]). Namely, the quantity  $F_0 + \sum_i F_i X_i'$  does not vary under the replacements  $X_1 \leftrightarrow X_1', X_2 \leftrightarrow -X_2', X_3 \leftrightarrow X_3'$  and  $k_0 \leftrightarrow -k_0$  (or  $S_1 \leftrightarrow -S_1, t-y \rightarrow 1/(1-y), \varepsilon \leftrightarrow \varepsilon/(1-y)$ ). Eq. (B.2) was obtained by us from eq. (87.10) in textbook [7] using the Reduce-2 system for algebraic computations (10 minutes on ES 1033). This cross section was computed earlier in the electron rest system [8], in the lab. system [9] and in the covariant form [10]. The result (B.2) in the corresponding system coincides with that of ref. [8] (The formulae of refs. [9, 10] contain misprints). In laboratory system the invariants have the form ( $\theta \ll 1$ ):

$$X = \frac{4E\omega_0}{m_e^2}, \quad y = \frac{\omega}{E}, \quad \alpha = \frac{y}{x(1-y)}, \quad S_1 = \lambda X,$$

$$S_2 = \lambda X(1-y)(1-2\alpha) - |\vec{S}_1| \frac{\alpha}{2\alpha} \cos(\varphi - \beta),$$

$$\alpha = 2\alpha \frac{|\vec{k}_1|}{m_e} = 2\alpha \sqrt{(x+1)y(y_m-y)}, \quad (B.3)$$

$$\varepsilon = -\frac{\vec{\xi}[\vec{k}_0, \vec{k}]}{2\omega_0 m_e} = -|\vec{S}_1| \frac{\alpha}{4\alpha} \sin(\varphi - \beta),$$

$$\delta(p+k_0-k-p') \frac{d^3k}{\omega} \frac{d^3p'}{E'} = dy d\varphi.$$

The azimuth of linear laser photon polarization at the laboratory frame is  $\gamma$ , and it is  $\varphi - \gamma - (\pi/2)$  in the reaction frame. Therefore,

$$X_1 = P_\perp \sin 2(\gamma - \varphi), \quad X_2 = P_\parallel, \quad X_3 = -P_\perp \cos 2(\gamma - \varphi). \quad (B.4)$$

At small emission angles  $\theta \ll 1$  considered the final photon moves almost along  $z$  direction and  $[\vec{k}_0, \vec{k}]$  azimuth is  $\varphi - (\pi/2)$  at the laboratory frame. Therefore, the Stokes parameters observed for final photon  $\xi_i'$  at this frame are connected with  $X_i'$  by the relations

$$X_1' = -\xi_1' \cos 2\varphi + \xi_3' \sin 2\varphi, \quad X_2' = \xi_2', \quad X_3' = -\xi_3' \cos 2\varphi - \xi_1' \sin 2\varphi. \quad (B.5)$$

From (B.2-5) one can obtain

$$\frac{d\sigma_c(\xi_i')}{dy d\varphi} = \frac{\alpha^2}{2\lambda m_e^2} \sum_{i=0}^3 \Phi_i \xi_i', \quad \xi_0' \equiv 1 \quad (B.6)$$

where  $\Phi_i$  are defined in (11-14). One can see that Stokes parameters for final electron itself are  $\xi_i = \Phi_i / \Phi_0$ . After summing up over final electron spin states we obtain cross section (9) from (B.6). Finally, after the integration of (B.6) over  $\varphi$  we obtain:

$$\frac{d\sigma_c(\xi_i')}{dy} = \frac{\pi \alpha^2}{\lambda m_e^2} \sum_{i=0}^3 C_{i0} \xi_i' \quad (B.7)$$

where  $C_{i0}$  are defined in (12). This yields mean  $\gamma$ -beam Stokes parameters (17).

The values  $C_{i0}$  and  $A_{jn}$  (12-14) satisfy some relations which arise from the inequality  $\sum_{i=1}^3 \xi_i^2 \leq 1$ . Really, rewriting it in the form  $\sum_{i=1}^3 \Phi_i^2 \leq \Phi_0^2$  and integrating it over  $\varphi$  one can find:

$$\sum_{i=1}^3 C_{i0}^2 + \frac{1}{2} \sum_{i,n \geq 1} |A_{in}|^2 \leq C_{00}^2 + \frac{1}{2} \sum_{n=1}^4 |A_{0n}|^2. \quad (B.8)$$

Using this result and the relations  $|A_{01}| \leq |A_{11}| = |A_{31}|$ ,  $A_{03} = A_{04} = 0$ ,  $|A_{02}| \leq |A_{12}| = |A_{32}|$  one can obtain inequalities (14c).



APPENDIX C.  $\gamma\gamma$  - "CROSS SECTIONS"

One can write the cross section for the  $\gamma\gamma \rightarrow \lambda$  reaction with polarized photons in the form

$$d\sigma_{\gamma\gamma \rightarrow \lambda} = \sum_{i,j=0}^3 \xi_i \tilde{\xi}_j d\sigma_{ij} \quad (C.1)$$

Here  $\xi_i, \tilde{\xi}_j$  are Stokes parameters of the first and second photons resp. ( $\xi_0 = \tilde{\xi}_0 = 1$ ). On the other hand, this cross section can be expressed by density matrices  $\rho_{ab}$  (19b) and the amplitude  $M_{ab}$  for the process  $\gamma\gamma \rightarrow \lambda$  with the photons helicities  $a, b = \pm 1$ :

$$d\sigma_{\gamma\gamma \rightarrow \lambda} = \sum_{abcd=\pm 1} \rho_{ac} \tilde{\rho}_{bd} M_{ab} M_{cd}^* d\Gamma, \quad (C.2)$$

$$d\Gamma = \frac{(2\pi)^4}{4kk} \delta(k+\tilde{k}-\sum p_f) \prod_f \frac{d^3 p_f}{(2\pi)^3 2E_f}$$

Comparing (C.1) and (C.2), it is easy to express  $d\sigma_{ij}$  through  $M_{ab}$  (we use the notations  $\sigma, \tau, \tau^a$  from ref. [12] also as well)

$$\begin{aligned} d\sigma &\equiv d\sigma_{00} = \frac{1}{4} (|M_{++}|^2 + |M_{--}|^2 + |M_{+-}|^2 + |M_{-+}|^2) d\Gamma, \\ d\tau^a &\equiv d\sigma_{22} = \frac{1}{4} (|M_{++}|^2 + |M_{--}|^2 - |M_{+-}|^2 - |M_{-+}|^2) d\Gamma, \\ d\tau &\equiv d(\sigma_{33} - \sigma_{11}) = \text{Re}(M_{--}^* M_{++}) d\Gamma, \\ d[\sigma_{33} - \sigma_{11} + i(\sigma_{31} + \sigma_{13})] &= M_{--}^* M_{++} d\Gamma, \\ d[\sigma_{33} + \sigma_{11} + i(\sigma_{31} - \sigma_{13})] &= M_{+-}^* M_{-+} d\Gamma, \\ d(\sigma_{20} + \sigma_{02}) &= \frac{1}{2} (|M_{++}|^2 - |M_{--}|^2) d\Gamma, \\ d(\sigma_{20} - \sigma_{02}) &= \frac{1}{2} (|M_{+-}|^2 - |M_{-+}|^2) d\Gamma, \\ d(\sigma_{03} + i\sigma_{21}) &= -\frac{1}{2} (M_{+-}^* M_{++} + M_{-+}^* M_{--}) d\Gamma, \\ d(\sigma_{30} + i\sigma_{12}) &= -\frac{1}{2} (M_{+-}^* M_{--} + M_{-+}^* M_{++}) d\Gamma, \\ d(\sigma_{23} + i\sigma_{01}) &= \frac{1}{2} (M_{+-}^* M_{++} - M_{-+}^* M_{--}) d\Gamma, \\ d(\sigma_{32} + i\sigma_{10}) &= \frac{1}{2} (M_{+-}^* M_{--} - M_{-+}^* M_{++}) d\Gamma. \end{aligned} \quad (C.3)$$

All the quantities  $d\sigma_{ij}$  are real. The complex form is used for shortness only.

The "cross sections"  $d(\sigma_{33} \pm \sigma_{11})$  arise from the interference of the amplitudes whose total helicities differ for 4 and the "cross sections"  $d\sigma_{01,10,03,30,12,21,25,32}$  - due to states whose helicities differ for 2.

When the pair production or inclusive processes  $\gamma\gamma \rightarrow a + \dots$  studied in the case of summing up over spin states of final particles only 8 independent "cross sections" remain of 16. If the  $X$  axis is in the reaction plane then they are  $d\sigma, d\tau^a, d\tau$  and  $d\tau^*$

$$\begin{aligned} d(\sigma_{33} + \sigma_{11}) &= M_{+-}^* M_{-+} d\Gamma, \quad d(\sigma_{03} + i\sigma_{21}) = -M_{+-}^* M_{++} d\Gamma, \\ d(\sigma_{30} + i\sigma_{12}) &= -M_{+-}^* M_{--} d\Gamma. \end{aligned} \quad (C.4)$$

Finally, after averaging over azimuths of emission angles and spins of the final particles, only  $\sigma, \tau^a$  and  $\tau$  do not vanish.

\*) The terms  $d\sigma_{12}$  and  $d\sigma_{21}$  were missed in eq. (4.23) of reviewed [14] when the similar relation for equivalent photons were written.

APPENDIX D

This appendix is devoted to the discussion of some polarization effects in the scheme to obtain colliding  $\gamma\gamma$ -beams considered. Their description is obtained from eqs. (33), (34).

D1. Electrons are polarized transversally, laser radiation is unpolarized

In this case the  $\gamma$ -beam mean polarization is zero according to (17). However, at  $\beta \sim 1$  for  $\gamma\gamma$ -collisions  $\langle \tilde{\xi}_1 \tilde{\xi}_2 \rangle \neq 0$  and the number of events (31) is equal to

$$dN_{\gamma\gamma \rightarrow X} = dL_{\gamma\gamma} (d\sigma + \langle \tilde{\xi}_1 \tilde{\xi}_2 \rangle d\tau^2 + \Lambda d\tau) \quad (D.1)$$

where  $\Lambda$  is defined by eq. (36) and (cf. (33a))

$$\langle \tilde{\xi}_1 \tilde{\xi}_2 \rangle = \frac{1}{2} (\tilde{\xi}_1 \tilde{\xi}_1) \alpha \tilde{\alpha} (2\gamma-1)(2\gamma-1) \langle \cos 2\psi \rangle / C_{00} \tilde{C}_{00}. \quad (D.2)$$

So, transverse electron polarization is "transferred" to circular photon polarization. But this effect is very small at  $\chi < 5$  considered. One can see that  $|\alpha(2\gamma-1)/C_{00}| < 0.13$ , i.e.

$$|\langle \tilde{\xi}_1 \tilde{\xi}_2 \rangle| < 0.01$$

D2. Correlation between two lasers

Let us consider the scheme when the conversion is obtained by one laser pulse divided into two parts, or when one laser source switches on steps for of both lasers that for every flash the polarizations are the same  $P_c = \tilde{P}_c$ ,

$$P_t = \tilde{P}_t, \quad \gamma = -\tilde{\gamma}.$$

It is interesting to consider the case when every flash is polarized, but the polarization vanishes after averaging over all flashes, i.e.  $\langle P_c \rangle = \langle P_t \cos 2\gamma \rangle = \langle P_t \sin 2\gamma \rangle = 0$

$$, \text{ but } \langle P_c \tilde{P}_c \rangle = \langle P_c^2 \rangle,$$

$\langle P_t \tilde{P}_t \rangle = \langle P_t^2 \rangle$  does not vanish. We shall consider the angle  $\gamma$  to be random so that  $\langle P_t^2 \cos^2 2\gamma \rangle = \langle P_t^2 \sin^2 2\gamma \rangle = \frac{1}{2} \langle P_t^2 \rangle$ ,  $\langle P_c P_t \cos 2\gamma \rangle = \langle P_c P_t \sin 2\gamma \rangle = 0$ . We discuss

the case when transverse electron polarization is absent. As a result we have, according to eqs. (33), (34):

$$dN_{\gamma\gamma \rightarrow X} = dL_{\gamma\gamma}^{(cor)} [d\sigma + \langle \tilde{\xi}_1 \tilde{\xi}_2 \rangle^{(cor)} d\tau^2 + \Lambda^{(cor)} d\tau]. \quad (D.3)$$

The luminosity  $dL_{\gamma\gamma}^{(cor)}$  deviates essentially from that for unpolarized beams  $dL_{\gamma\gamma}^{np}$  (c.f. (40) ref. [2]):

$$\frac{dL_{\gamma\gamma}^{(cor)}}{dL_{\gamma\gamma}^{np}} = 1 + 4\lambda\tilde{\lambda} \langle P_c^2 \rangle B(y)B(\tilde{y}) + \frac{1}{2} \langle P_t^2 \rangle A(y)A(\tilde{y}) \langle \cos 2\psi \rangle, \quad (D.4)$$

$$B(y) = \alpha \times (2\alpha-1)(2-y) [(1-y)^{-1} + 1-y-4\alpha(1-\alpha)]^{-1}.$$

The item from  $\langle P_t^2 \rangle$  is not large here (c.f. the discussion in connection with eq. (36)). But at  $y \approx y_m$  the function  $B(y) \approx 1$ . Therefore, the item from  $\langle P_c^2 \rangle$  influences the luminosity essentially. Particularly, at  $\lambda\tilde{\lambda} > 0$  the luminosity increases and the ratio (D4) can be close to 2. The values  $\langle \tilde{\xi}_1 \tilde{\xi}_2 \rangle^{(cor)}$ ,  $\Lambda^{(cor)}$  which determine the polarized photon contribution also prove not to be small.

So, the correlations change strongly the result in comparison with the really unpolarized beams or uncorrelating ones).

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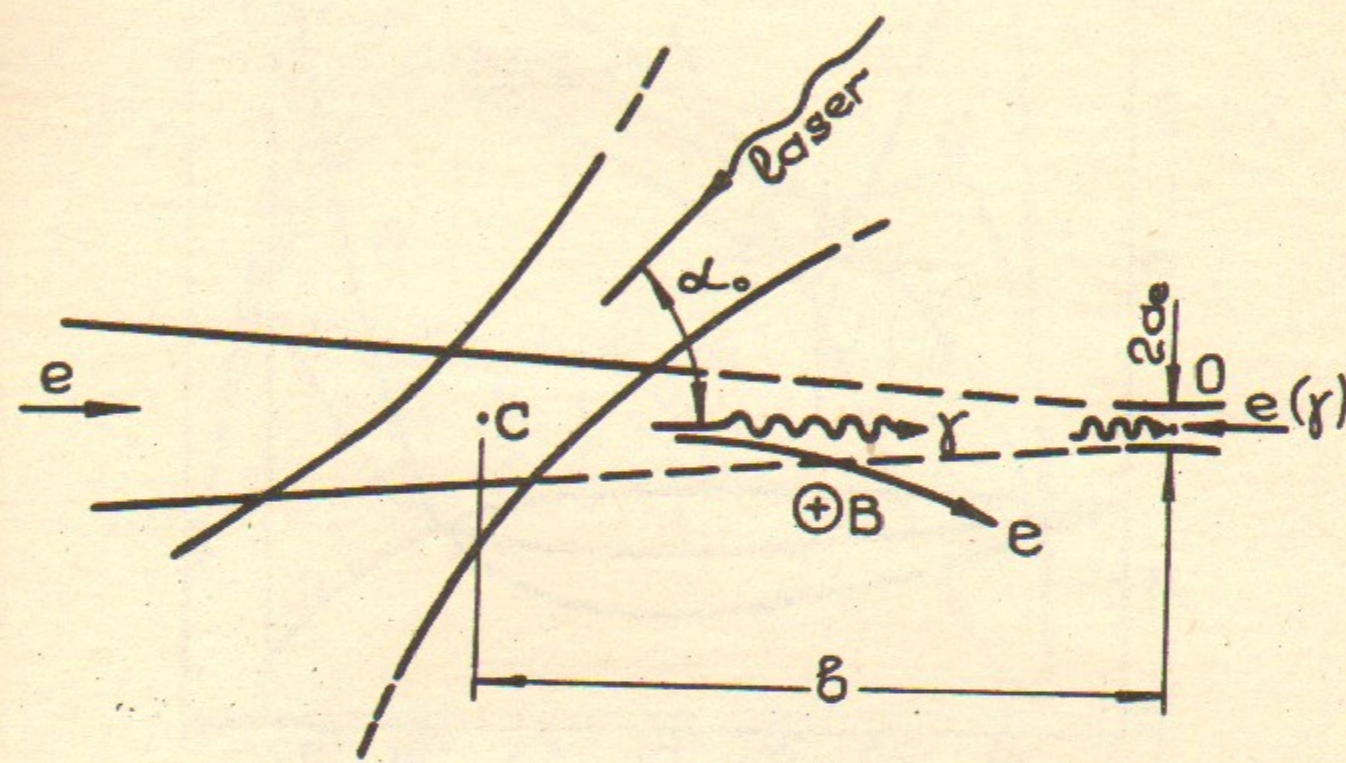


Fig. 1. The scheme of obtaining colliding  $\gamma e$  and  $\gamma\gamma$  beams.

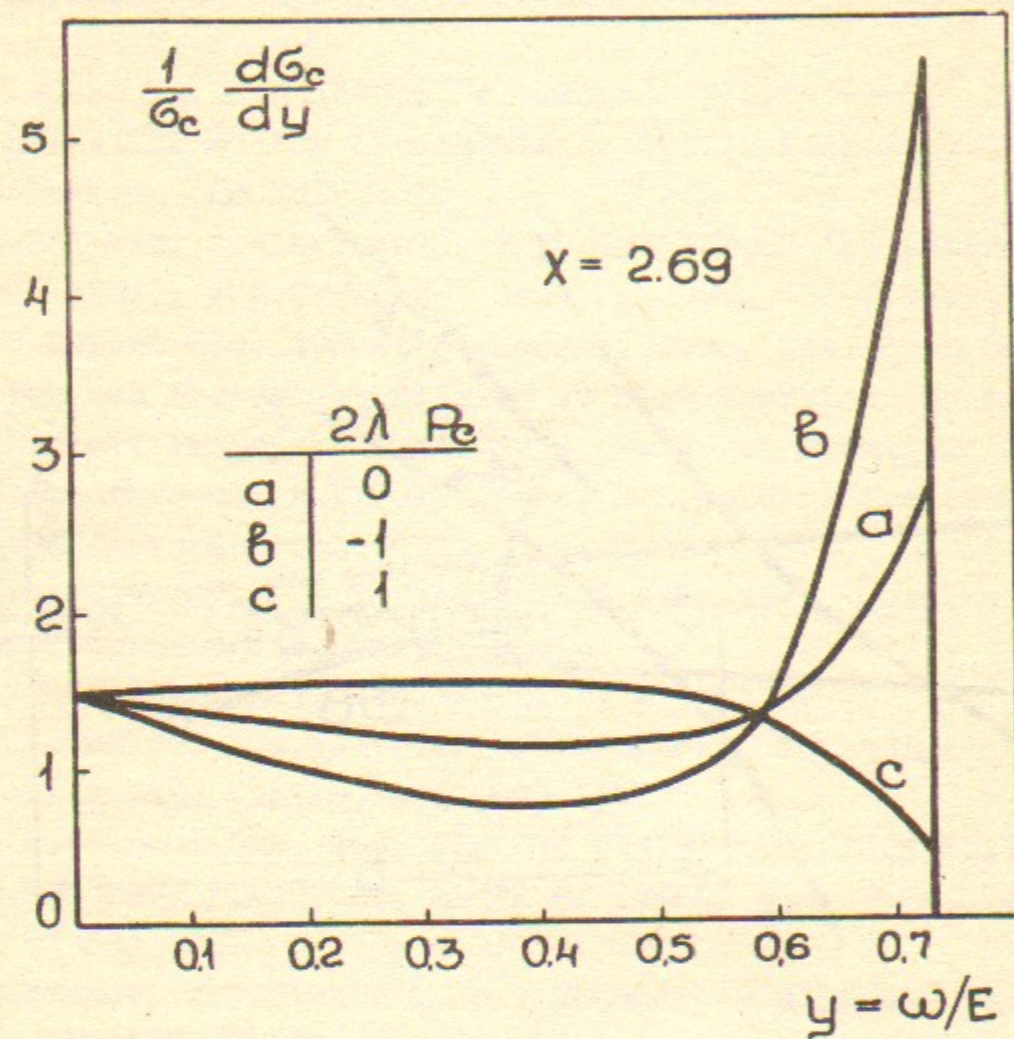


Fig. 2. Photon energy spectrum for different values of average electron  $\lambda$  and laser photon  $P_c$  helicities.

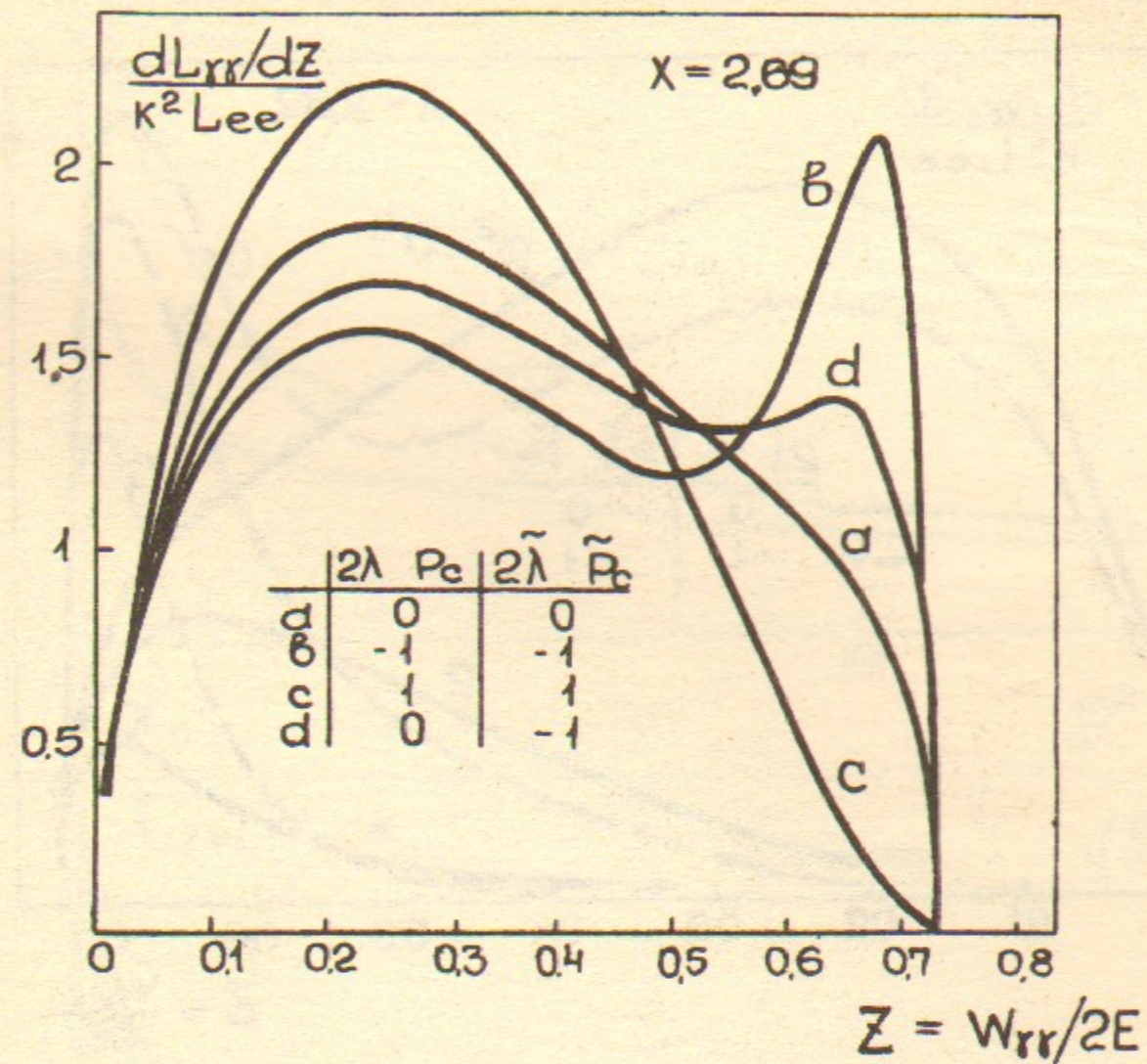


Fig. 3. Spectral luminosity of  $\gamma\gamma$  -collisions for different values  $\lambda$  and  $P_c$  at  $\beta^2 \ll 1$  (tilde denotes the quantities related to the second beam).

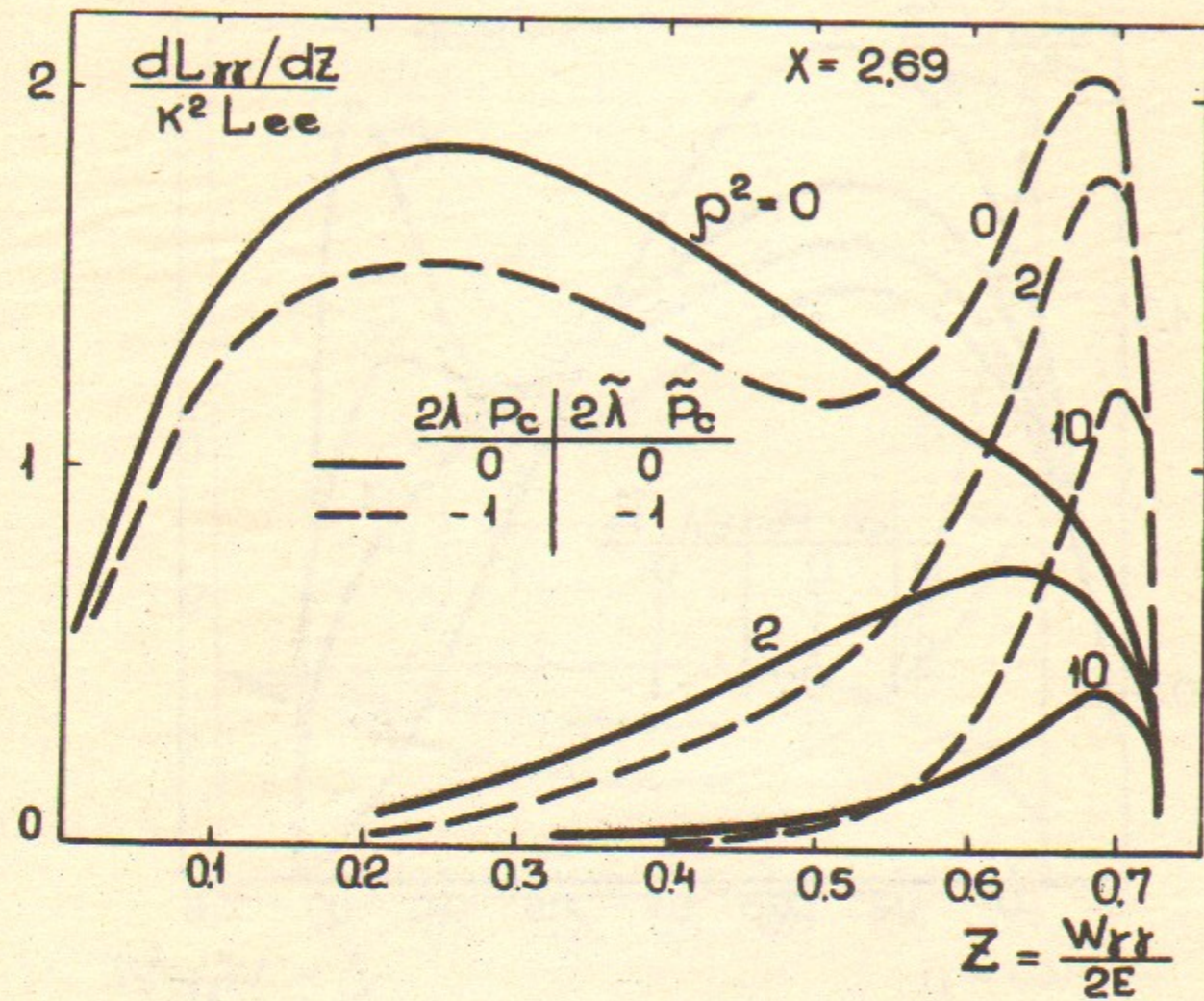


Fig. 4. Spectral luminosity of  $\gamma\gamma$ -collisions for different  $p^2$  and  $2\lambda P_c = 2\tilde{\lambda} \tilde{P}_c$ .

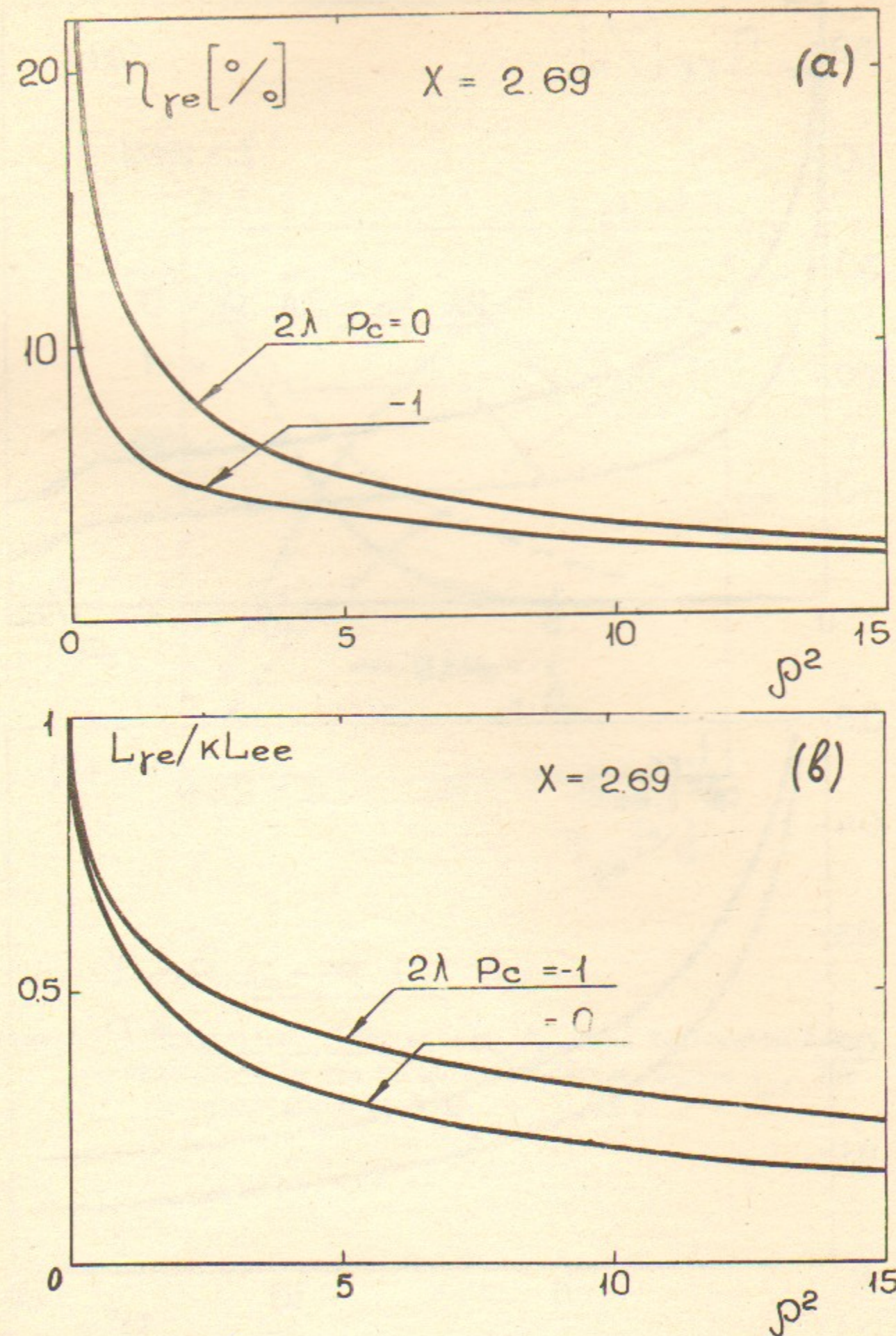


Fig. 5. The  $p^2$ -dependence of monochromatization degree (a) and total luminosity (b) for  $\gamma e$ -collisions at  $2\lambda P_c = -1$  and 0.

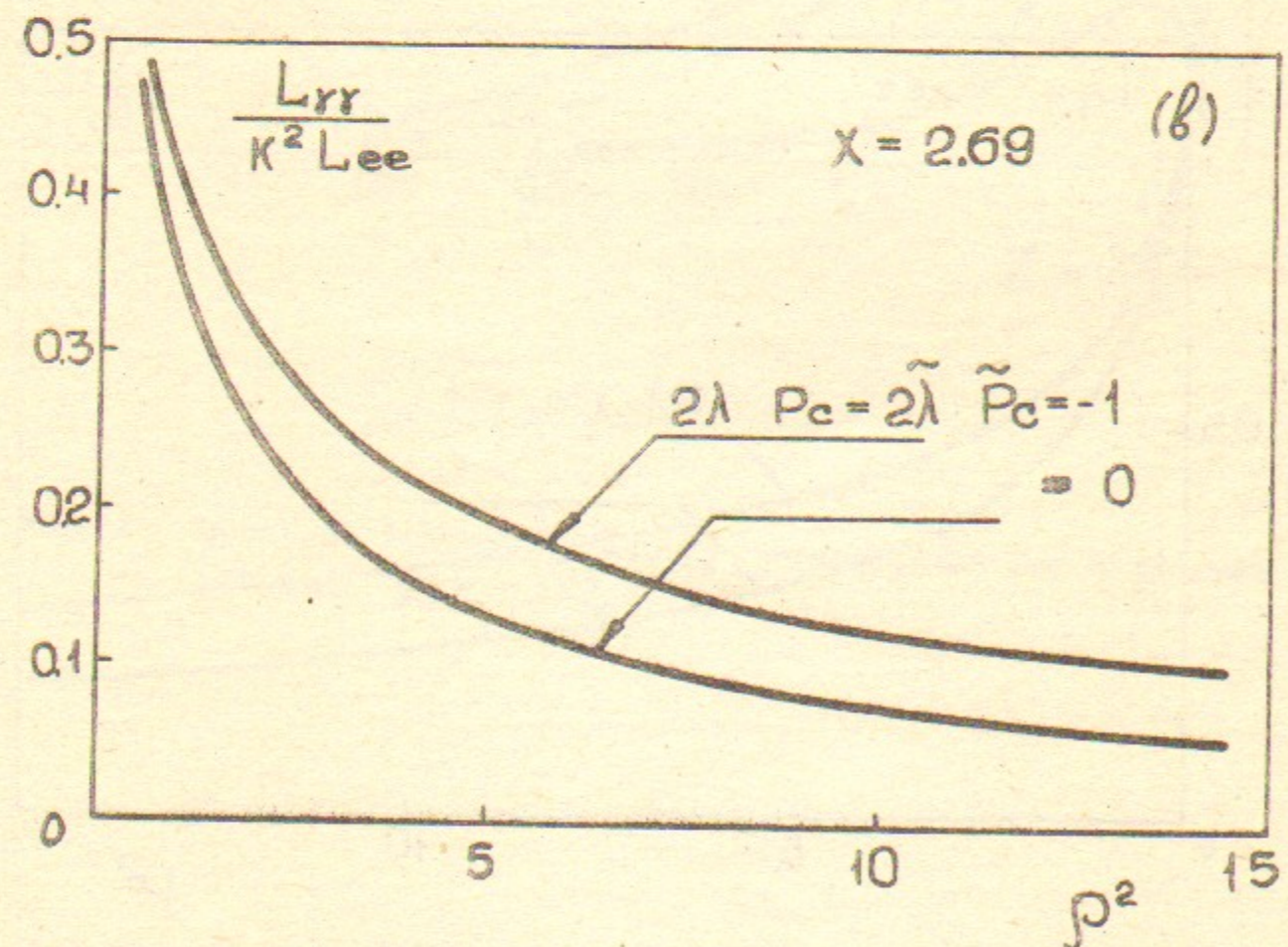
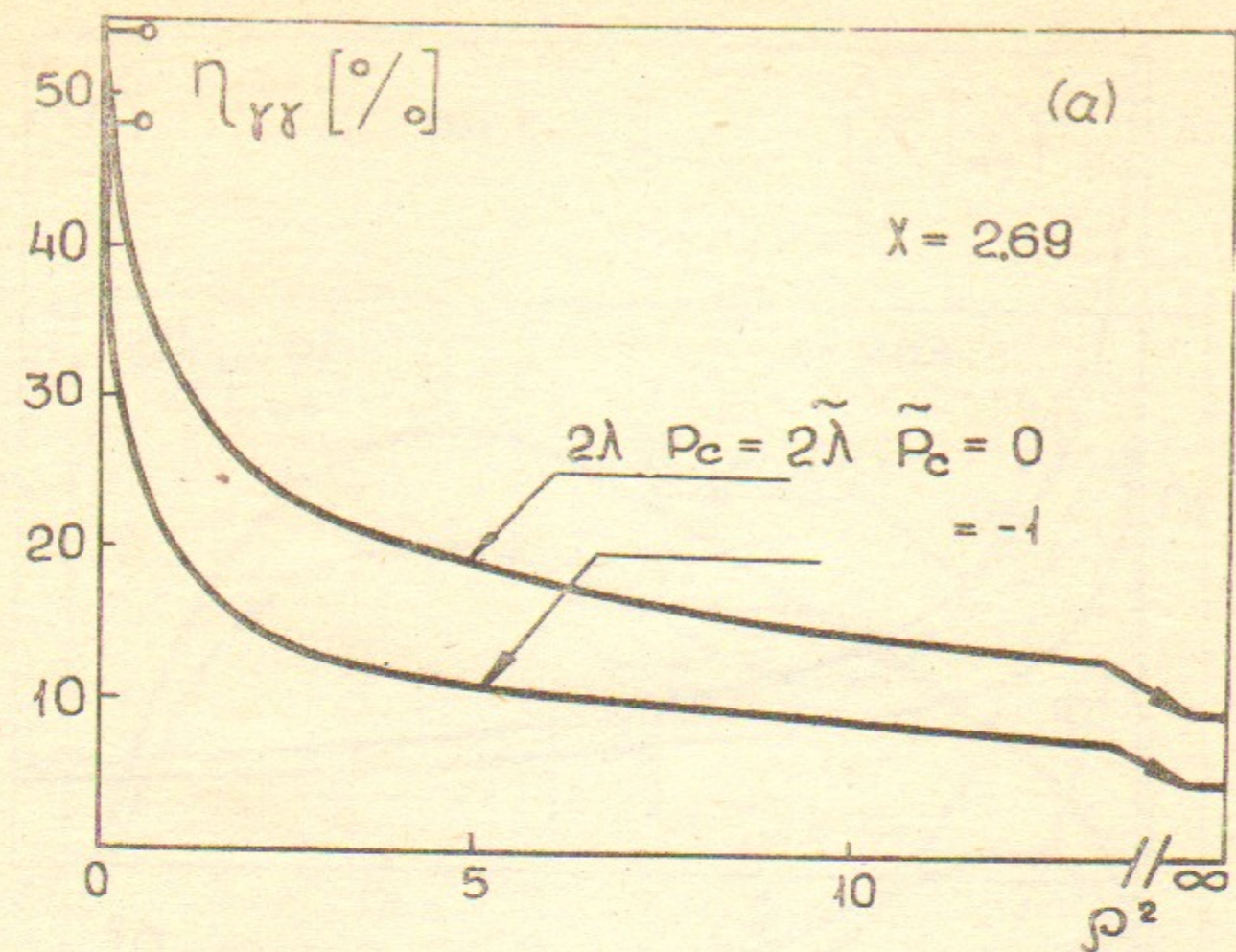


Fig. 6. The dependence of monochromatization degree (a) and total luminosity (b) for  $\gamma\gamma$ -collisions.

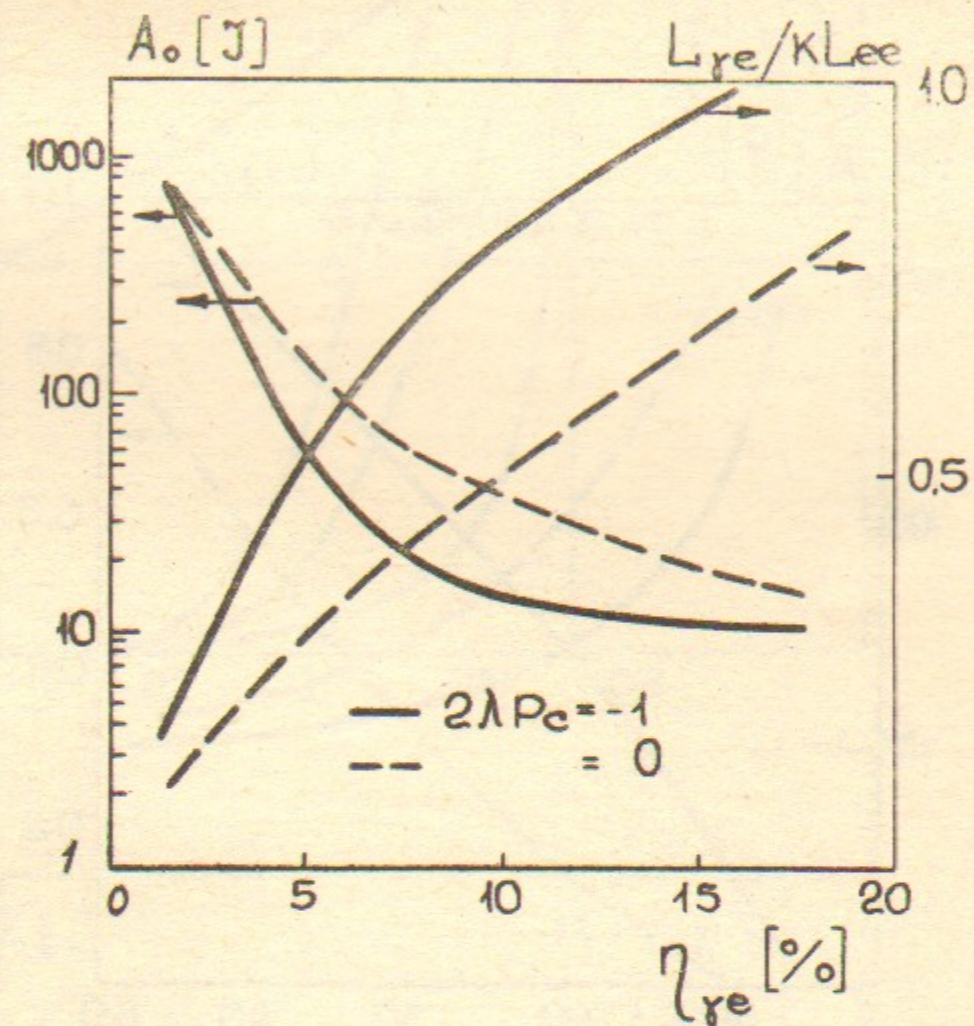


Fig. 7. Average laser flash energy  $A_0$  and luminosity  $L_{\gamma e}$  dependence on the monochromatization degree  $\eta_{\gamma e}$  for  $\gamma e$ -collisions at  $2\lambda P_c = -1$  and  $0$ .

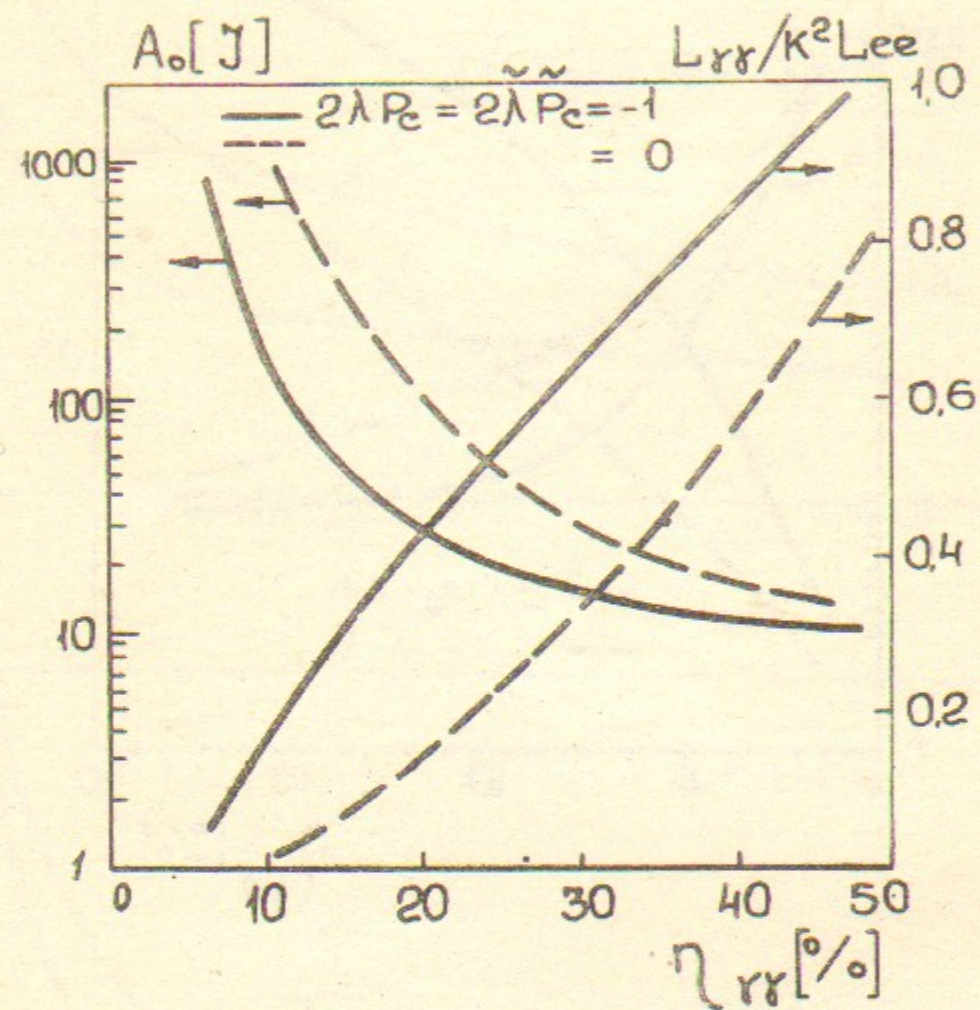


Fig. 8. Average laser flash energy  $A_0$  and luminosity  $L_{\gamma\gamma}$  dependence on the monochromatization degree  $\eta_{\gamma\gamma}$  for  $\gamma\gamma$ -collisions.

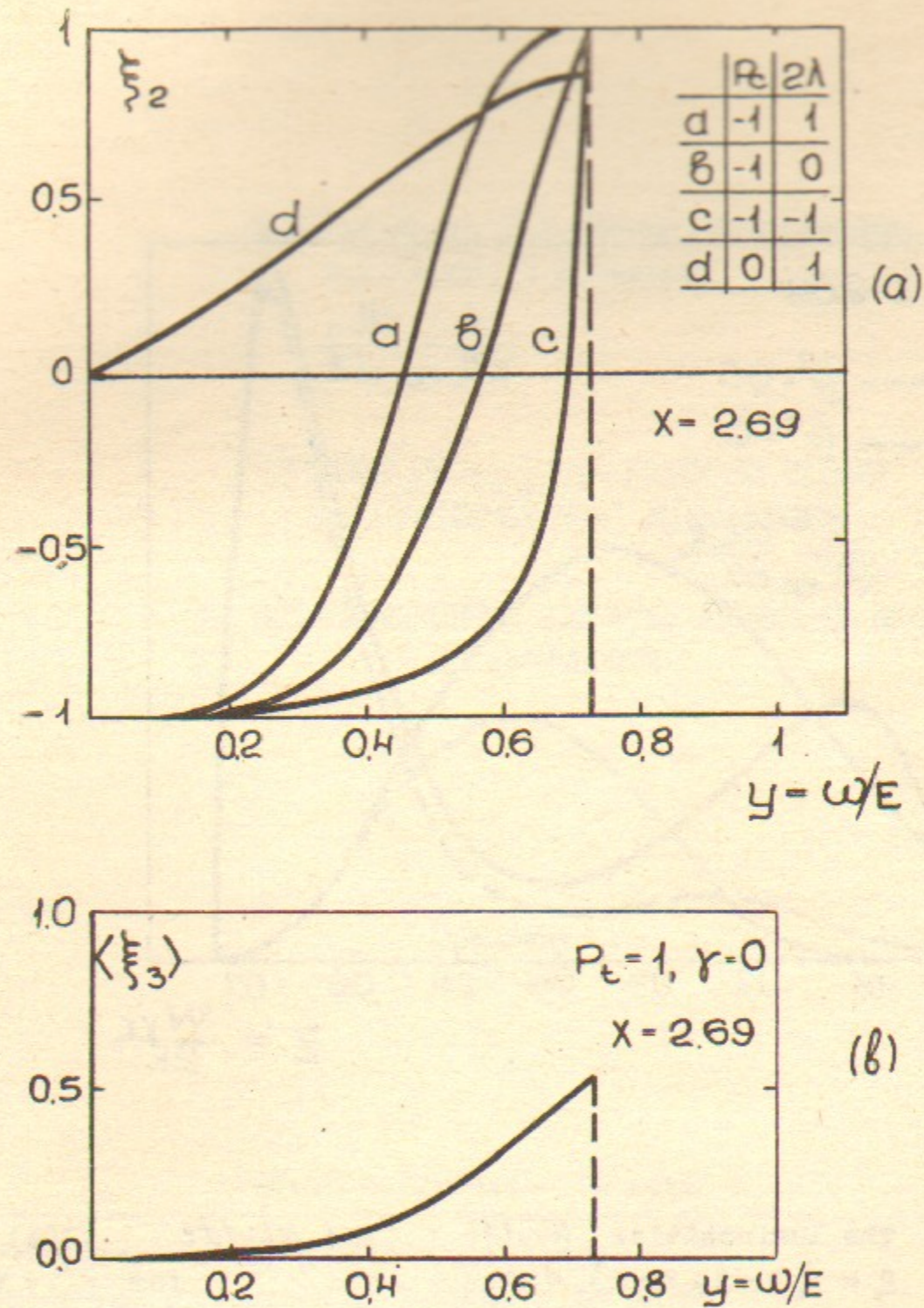


Fig. 9. a) The circular photon polarization degree vs  $\omega/E$  for different laser photon polarizations; b) The average linear photon polarization degree vs.  $\omega/E$  for linearly polarized laser photons.

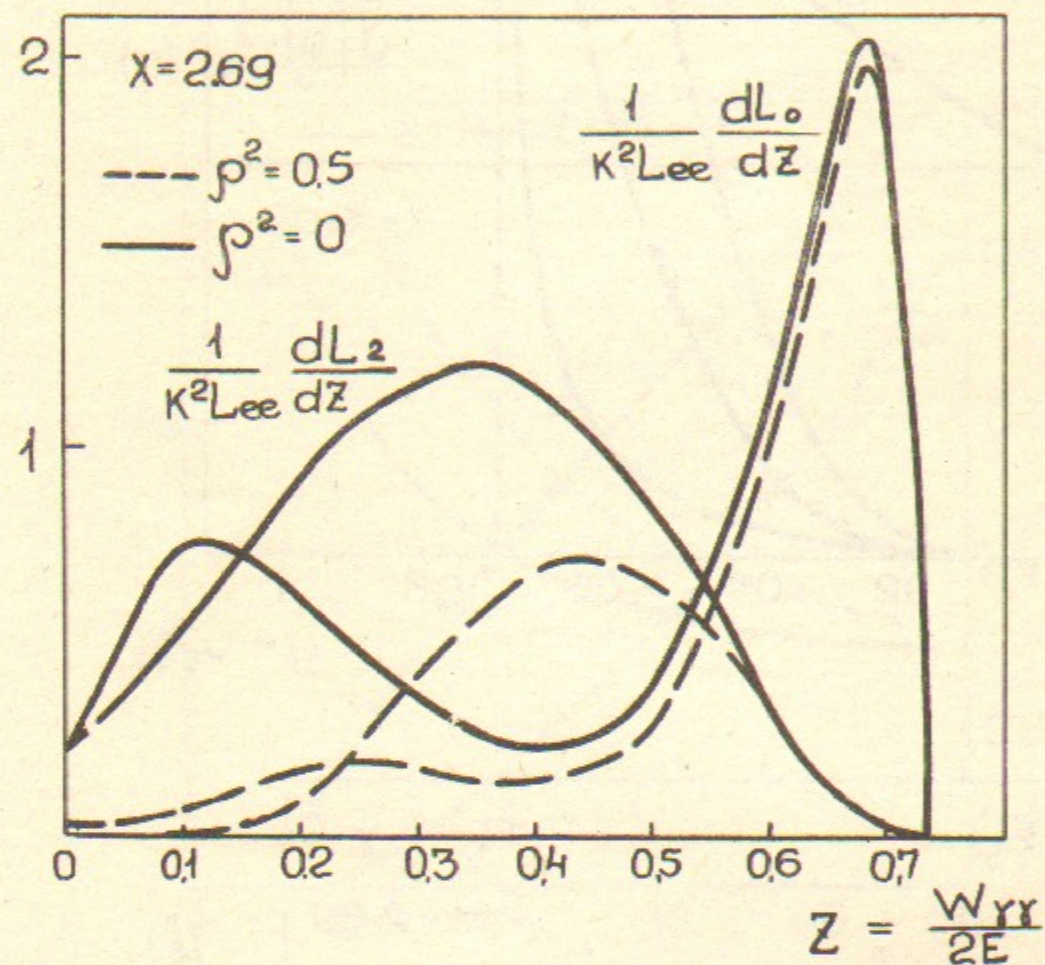


Fig. 10. The luminosities  $dL_0/dz$  and  $dL_2/dz$  (36b) at  $\rho_c = \tilde{\rho}_c = -2, \lambda = -2\tilde{\lambda} = 1$  and 0.5. for  $\rho^2 = 0$

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ВСТРЕЧНЫЕ  $\gamma e$  и  $\gamma\gamma$  - ПУЧКИ НА ОСНОВЕ  
ОДНОПРОЛЕТНЫХ  $e^+e^-$  УСКОРИТЕЛЕЙ

II. ПОЛЯРИЗАЦИОННЫЕ ЭФФЕКТЫ, УЛУЧШЕНИЕ МОНО-  
ХРОМАТИЧНОСТИ

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