

ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ  
СО АН СССР

V.N.Baier, I.N.Meshkov, A.N.Skrinsky

ON EXPERIMENTS ON COULOMB-NUCLEAR INTERFERENCE  
IN  $p\bar{p}$  SCATTERING AT SUPERHIGH-ENERGY COLLIDING  
BEAMS

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A b s t r a c t

The possibility of setting up the experiments on proton-antiproton scattering at small angles in the region of Coulomb-nuclear interference on superhigh-energy colliding  $p\bar{p}$ -beams is discussed. It is shown that the implication of high-energy electron cooling allows one to obtain the required resolution. The latter condition becomes very hard at superhigh energies.

1. One of the most important characteristics of hadron physics is the energy dependence of the total cross section  $\sigma^{tot}(s)$  of a hadron interaction. In view of this, the measurements of such a cross section were carried out, as soon as an opportunity arose, at all existing accelerators. Equally with direct investigation of  $\sigma^{tot}(s)$ , of great interest is a measurement of the real part of the elastic scattering amplitude  $Re F(s, t)$ . The reasons are the following: 1) This quantity may be related, by means of dispersion relations, to the total cross section  $\sigma^{tot}(s)$  and, with both these quantities independently measured, the dispersion relations and, hence, the physical principles underlying the derivation of these relations (first of all microcausality) can be tested. 2) If the dispersion relations are regarded as the valid ones, then with  $Re F$  measured, they can be used in finding the energy dependence of  $\sigma^{tot}(s)$  in the range of higher energies (the energy is more higher, the higher is the accuracy of measurements). 3) The measurement of  $Re F_{pp}$  is of great significance with a view to the comparison with  $Re F_{pp}$  and verification of the various consequences of the Pomanchuk theorem (behaviour of  $Re F_{pp} / Im F_{pp}$ , comparison of  $d\sigma_{pp}/dt$  with  $d\sigma_{pp}/dt$ , etc.).

A standard way of finding  $Re F$  is the measurement of the elastic scattering in the region in which the interference of strong and electromagnetic amplitudes is essential. Extensive studies of the Coulomb-nuclear interference have been carried out in the direct beam experiments both in  $pp$  /1/ and  $\pi^+p$ -collisions /2/, as well as in  $K^+p$ ,  $\pi^+p$ ,  $p\bar{p}$  collisions /2a/.

At high energies achievable on colliding beams, the interference region lies within very small scattering angles. The proton-proton scattering at the highest energies has been

studied with the CERN ISR in the experiments performed up to  $\sqrt{S} = 63 \text{ GeV}$  /3/. From the measured values of  $Re F$ , it has been concluded that  $\sigma_{\text{tot}}(S)$  will continue growing (as the squared logarithm of  $S$ ) up to  $\sqrt{S} = 500 \text{ GeV}$ . After the antiproton storage ring in CERN had been put into operation, the first experiments on studying the antiproton-proton scattering at the ISR with  $\sqrt{S} = 53.2 \text{ GeV}$  /4/ in the interference region have been carried out.

It should bear in mind that the investigation of  $p\bar{p}$  ( $pp$ ) scattering in the interference region becomes more complicated with increasing the energy of colliding beams. For this reason, undoubted interest is of non-traditional approaches to the experiments of such a kind. In the present work we would like to emphasize that the electron cooling makes it possible to carry out the antiproton-proton scattering experiments in the interference region at superhigh energies. In this case, the focusing system of a storage ring is assumed to be used as the analyzer of the detection system.

2. In order to define the parameters of the system, let us use the estimates of the cross section of the process which are described in detail in the Appendix. The curves  $d\sigma/dt$  and  $R = (\frac{d\sigma}{dt}(p) - \frac{d\sigma}{dt}(p=0)) / \frac{d\sigma}{dt}(p=0)$  at  $\rho = 0.14$  in Fig.1 allow one to conclude that the maximum interference takes place within the  $0.001+0.0015 \text{ (GeV/c)}^2$  range of the values of a parameter  $|t|$ , which is equal to

$$t = -2\rho^2(1 - \cos\theta_s) \approx -\rho^2\theta_s^2, \quad (1)$$

where  $\rho$  is the particle momentum in the c.m. frame. Note that in the experiment /3/ the maximum interference also took place at  $|t| = 0.001+0.002 \text{ (GeV/c)}^2$ . Therefore, one can estimate it

basing upon a very weak dependence of the position of a maximum of interference as a function  $t$  of energy  $S$ . This means that in the experiment under discussion several tens of points should be resolved within the  $0.001-0.015 \text{ (GeV/c)}^2$  interval which corresponds to that of scattering angle  $\theta_s$  in the c.m. frame:

$$0.034/\gamma < \theta_s < 0.13/\gamma, \quad (2)$$

where  $\gamma = \sqrt{S} / 2M$ .

To obtain the predictions, similarly to /3/, concerning the behaviour of  $\sigma_{\text{tot}}^{p\bar{p}}$  up to the values  $\sqrt{S}$ , which exceed more than by ten times the values of  $\sqrt{S}$  at which  $\rho$  has been measured, it is necessary to detect about  $10^6$  events at every energy, that gives the required statistical accuracy.

As has already been mentioned, the complexity of the experiment under discussion is, first of all, associated with the necessity for detection of the particles elastically scattered at a small angle. To do this, it is necessary to provide:

- fairly small emittances of  $p\bar{p}$  beams and such values of the beta-functions in the collision region which enable the required high angular resolution, by a factor of 5-10 better than  $\theta_{s\text{min}}$ , to be obtained;
- the required resolution of a detector over the scattering angle  $\theta_s$ ;
- the possibility of separating the elastic scattering events on the background of inelastic processes with small energy losses;
- the corresponding fixed nature of the geometry of the experiment (mainly the geometrical stability of the beam parameters).

The requirements listed above have to be correlated with

the conditions for achieving a luminosity which makes it possible to collect the indicated statistics (the total counting rate in the interference region is of the order of 1 Hz).

3. For the elastic scattering events to be selected, a detector is to be installed inside the vacuum chamber of a storage ring at the section which is removed, over the phase of vertical betatron oscillations, from the collision region by

$$\Delta_{zd} = \frac{\pi}{2} (2n + 1), \quad n \text{ is an integer,} \quad (3)$$

that provides the maximum vertical displacement of particle scattered at the angle  $\Theta_s$ . In this case, the limitations on the spatial resolution of a detector are not so hard. At the same time, in order to remove the inelastically scattered particles (which have lost even a small fraction of their energy) from the flux of particles entering the detector, it seems reasonable to place the diafrags cutting from the side of low energy, for example, after detectors, or/and the thin counters of anti-coincidences, thereby producing a maximum of the dispersion  $\eta_{cl}$  on the orbit section which is removed, over the phase of radial betatron oscillations, from the collision region by

$$\Delta_{x1}^{cl} = \pi(2n + 1). \quad (4)$$

As a result, those particles will be extracted from the beam which have lost an energy

$$\frac{\Delta E}{E} \geq \frac{\Delta X_{cl}}{\sqrt{\beta_{cl}}} \cdot \left( \frac{\eta_{cl}}{\sqrt{\beta_{cl}}} + \frac{\eta_o}{\sqrt{\beta_{x0}}} \right)^{-1}, \quad (5)$$

where  $\Delta X_{cl}$  is the radial aperture on the extraction region,

$\beta_{cl}$  is the value of the radial beta-function on the extraction region,  $\beta_{x0}$  and  $\eta_o$  are the values of the radial beta- and dispersion functions on the collision region. The minimum value

of  $\Delta E$  for extracted particles may be made less than the rest energy of a pion. The suggested method of extraction naturally assume the detection of useful scattering events in vertical plane. In this case, the particles scattered simultaneously at a small radial angle, which is comparable with the vertical scattering angle to be measured, should not reach the detector. For this purpose, similar cutting diafrags should be placed on the section with a large value of the radial beta-function, which is

$$\Delta_{x2}^{cl} = \frac{\pi}{2} (2n + 1) \quad (6)$$

apart the collision region over the phase of radial betatron oscillations.

4. It is clear that the detection and extraction systems have to be installed in pairs and symmetrically with respect to the collision region.

Detectors may be made like a telescope of thin coordinate-sensitive semiconducting counters. These counters are included into a logical circuit which is intended for selection of elastically scattered proton-antiproton pairs. The vertical arrangement of counters is preferable also for decreasing the background caused by the particles which radially escape from the beam due to intra-beam scattering. For the same purpose, the r-z coupling of particle motion in a storage ring need to be eliminated, too.

The experimental conditions impose particular limitations on the parameters of the beam and focusing system at the collision region. The first of these limitations is to provide the angular resolution in the region of a minimum scattering angle

$\Theta_{smin}$ .

It is obvious that, in this case,

the angular beam spread in the collision region should not exceed  $\theta_{smin}/k_1$ , and hence the requirement for the vertical beam emittance  $\epsilon_z$  and the value of the  $\beta$ -function  $\beta_{z0}$  at the collision point is of the form

$$\frac{\epsilon_z}{\beta_{z0}} \leq \frac{\theta_{smin}^2}{k_1^2} \approx \frac{1.1 \cdot 10^{-3}}{k_1^2 \gamma^2}, \quad k_1 \sim 5 \div 10 \quad (7)$$

The second requirement is to provide the possibility of extracting the particles scattered at the angle  $\theta_{smin}$ , out of the beam. To do this, it is necessary that the deviation of a scattered particle should exceed a half-height of the beam by a factor of  $k_2$ :

$$z_{sd} \geq k_2 \sqrt{\epsilon_z \beta_{zd}}, \quad k_2 \sim 5 \div 10$$

or

$$\frac{\epsilon_z}{\beta_{z0}} \leq \frac{\theta_{smin}^2}{k_2^2} \approx \frac{1.1 \cdot 10^{-3}}{k_2^2 \gamma^2} \quad (8)$$

The final form of the latter condition is naturally independent of a value of the beta-function in the detection region, and both limitations are close to each other.

The coordinate resolution  $\Delta z_d$  of a detector should be not worse than

$$\Delta z_d = \frac{\theta_{smin}}{k_1} \cdot \sqrt{\beta_{z0} \beta_{zd}} \approx \frac{3 \cdot 10^{-2}}{k_1 \gamma} \sqrt{\beta_{z0} \beta_{zd}} \quad (9)$$

5. The luminosity necessary in this experiment can be obtained from the value of the function  $d\sigma/dt$  represented in Fig.1. The integral of this function within the limits of variation of the parameter  $|t|$  is equal to

$$\Delta\sigma \approx 4 \text{ mb.}$$

If it is assumed that the transverse size of the detector in radial direction provides the detection of particles scattered

into a solid angle

$$\Delta\Omega = 0.2\pi \cdot \Delta\theta_s,$$

then the luminosity required for a total counting rate of 1 Hz equals

$$L_{min} \approx 2.5 \cdot 10^{27} \text{ cm}^{-2} \cdot \text{s}^{-1}.$$

6. The luminosity of colliding beams per one collision region is

$$L = 2\pi f_0 \frac{n_b}{n_0 r_p^2} \cdot \frac{\gamma^2 \epsilon_z}{\beta_{z0}} \cdot \left(1 + \sqrt{\frac{\beta_{z0}}{J\beta_{x0}}}\right)^2 J \Delta\nu_x^2 \quad (10)$$

where  $f_0$  is the revolution frequency of particles in a storage ring,  $n_b$  is the number of bunches in every beam,  $n_0$  is the number of collision regions,  $r_p$  is the classical proton radius,

$$J = \frac{\epsilon_x}{\epsilon_z} \quad (11)$$

$\Delta\nu_x, \Delta\nu_z$  are the shifts of frequencies of betatron oscillations caused by the colliding beam:

$$\Delta\nu_x = \frac{r_p N_b}{2\pi \gamma \epsilon_x} \cdot \frac{n_0}{1 + \sqrt{\frac{\beta_{z0}}{J\beta_{x0}}}}, \quad \frac{\Delta\nu_x}{\Delta\nu_z} = \sqrt{\frac{\beta_{x0}}{J\beta_{z0}}} \quad (12)$$

The limitations on the emittances (7) and (8) imply that the maximum luminosity achievable in the experiment is energy-independent and is determined by the geometry and the required resolution over the momentum transferred:

$$L_{max} = 2\pi f_0 \frac{n_b}{n_0 r_p^2} \cdot \left(\frac{\theta_{smin}}{k_{1,2}}\right)^2 \cdot \left(1 + \frac{\beta_{z0}}{\beta_{x0}}\right)^2 \Delta\nu_{max}^2 \quad (13)$$

where  $\Delta\nu_{max}$  is the value of the shift of betatron frequencies which is admissible in the regime of colliding beams with very small values of the emittances. The parameter  $J$  in this formula

is equal to 1.

7. The time of existence for so small emittances is determined by a lot of perturbing factors such as collision effects, noises of a rf system, instabilities of magnetic fields and so on, as well as by the effect of intra-beam scattering. With this effect taken into account, the time of luminosity existence is /5/

$$\tau_{185} \approx \frac{\delta^3 l_b \nu^{5/2}}{c r_p^2 L_c} \cdot \frac{\epsilon_x^2 \sqrt{\epsilon_z}}{N_b R_o^{1/2}} \quad (14)$$

Here  $l_b$  is the bunch length,  $R_o$  is the mean radius of a storage ring,  $L_c$  is the Coulomb algorithm.

The influence of perturbations may be radially suppressed by introducing the electron cooling at the experiment energy /6/. The characteristic time of cooling by a circulating electron beam /5,6/ is

$$\tau_c \approx 6 \cdot 10^{-3} \frac{\delta^5 l_b \epsilon_x^2 \sqrt{\epsilon_z \beta_{zc}}}{\eta c r_p^2 N_e \beta_{xc}} \quad (15)$$

where  $\eta$  is a part of the storage ring perimeter which is occupied by the cooling section,  $N_e$  is the number of electrons in the cooling bunch whose length is equal to that of the proton (antiproton) one,  $\beta_c$  are the values of beta-functions on the cooling section.

It is obvious that the parameters of the electron cooling system should provide a fairly small value of  $\tau_c$  so that, at least,

$$\tau_c < \tau_{185} \quad (16)$$

This condition for the experiment under consideration may be represented as an inequality containing only the geometrical

parameters of  $p\bar{p}$  and  $e^-$  storage rings, the quantity  $N_e$  and the required luminosity. Excluding the remaining parameters from eqs. (14) and (15) by means of eqs. (10-13), we find

$$\frac{\tau_{185}}{\tau_c} = 2.1 \cdot 10^3 \eta N_e \frac{k \nu^{5/2}}{\gamma J^{1/4} L_c} \cdot \frac{\beta_{xc}}{\sqrt{\beta_{zc}}} \cdot \sqrt{\frac{n_b f_o}{R_o \mathcal{L}}} \cdot \frac{1}{\beta_{z0}^{3/4} \cdot \beta_{x0}^{1/4}} \quad (17)$$

where  $k = \max\{k_1, k_2\}$ . With the other given parameters, it follows the limitation on the quantities  $\beta_{x0}, \beta_{z0}$ .

8. It is the sequence in which the parameters of the experiment (see Table 1) have been obtained. It is assumed that

$$k_1 = k_2 = 5, L_c = 10, n_b = n_o = 1, \mathcal{L} = 2.5 \cdot 10^{27} \text{ cm}^2 \cdot \text{s}^{-1}, \\ l_b = 1 \text{ m}, \tau_{185}/\tau_c \geq 1.5.$$

Note that the choice of a value of the beta-functions on the cooling section  $\beta_{xc}, \beta_{zc}$  and on the detection section  $\beta_{zd}$  has strong influence on the remaining parameters. Only increasing  $\beta_{xc}$  and  $\beta_{zd}$  and decreasing  $\beta_{zc}$ , one can reach the satisfactory conditions: small cooling time at a rather large displacement of the particles being detected from the beam axis in the detector region:

$$z_{sd} = \theta_s \sqrt{\beta_{z0} \beta_{zd}} \quad (18)$$

that make realistic the requirements for the coordinate resolution (9).

Table 1. Parameters of the experiment

Storage ring	SPS CERN	Doubler Fermilab	UNK IHEP
Parameters of a storage ring			
Particle energy, GeV	270	1000	3000
Mean orbit radius, km	1.1	1	3.3
Betatron oscillation frequency	27.6	30	30
Revolution frequency, Hz	43	45	15
Conditions of the experiment			
Values of beta-functions in the collision region, m	$\beta_{x0}$ 12.5 $\beta_{z0}$ 2.5	12.5 2.5	12.5 2.5
Beam emittances $\epsilon_x = \epsilon_z$ , cm-rad	$1.4 \cdot 10^{-7}$	$1 \cdot 10^{-8}$	$1.1 \cdot 10^{-9}$
Shifts of betatron oscillation frequencies	$\Delta\nu_x$ $2.4 \cdot 10^{-3}$ $\Delta\nu_z$ $1 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$ $1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$ $1.8 \cdot 10^{-3}$
Number of particles in a bunch	$5.2 \cdot 10^9$	$1.4 \cdot 10^9$	$0.8 \cdot 10^9$
Time of intra-beam scattering, c	4500	1600	170
Parameters of the electron cooling system			
Values of beta-functions on the cooling section, m	$\beta_{xc}$ 1100 $\beta_{zc}$ 10	1000 10	3300 10
Number of particles in the electron bunch	$1 \cdot 10^{11}$	$3 \cdot 10^{11}$	$1 \cdot 10^{12}$
Cooling time, c	2500	900	80
Parameters of the detection system			
Values of vertical beta-function, m	1100	1000	3300
Vertical beam size, mm	1.2	0.3	0.2
Displacement of a particle scattered at angle $\theta_{s, min}$ , mm	6.5	1.6	1.0

Appendix

In the small momentum transfer region it is accepted to describe the elastic proton-antiproton scattering cross section by the interference formula:

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2 G_p^2 G_{\bar{p}}^2}{t^2} + \frac{(1+\rho^2)e^{-\beta|t|} (\sigma_{\bar{p}p}^{tot})^2}{16\pi} + \frac{\alpha}{|t|} G_p G_{\bar{p}} \sigma_{\bar{p}p}^{tot} e^{-\frac{\beta|t|}{2}} (\rho \cos \delta + \sin \delta), \quad (A.1)$$

where  $t = -2p^2(1 - \cos \theta_s) \approx -p^2 \theta_s^2$ ,  $\rho = |\bar{\rho}|$  is the particle momentum in the c-frame,  $\theta_s$  is the scattering angle,  $\sigma_{\bar{p}p}^{tot}(s)$  is the total  $p\bar{p}$ -interaction cross section,  $\beta$  is the parameter of a slope of the diffraction cone; it is assumed that

$$d\sigma_{\bar{p}p}^{st}/d|t| = (d\sigma_{\bar{p}p}^{st}/dt)_{t=0} e^{-\beta|t|}; \quad \rho = \text{Re } F_{\bar{p}p}(s, 0) / \text{Im } F_{\bar{p}p}(s, 0); \quad G_p(t) = G_{\bar{p}}(t) = \left(1 + \frac{|t|}{0.71 \text{ GeV}^2}\right)^{-2}$$

are the electromagnetic formfactors of the proton and antiproton;  $\delta = -\alpha (\ln \frac{0.08}{|t|} - C)$ ,  $C = 0.577$  is the phase of the Coulomb amplitude /7,8/;  $\alpha = 1/137$ . Formula (1) holds under the condition that 1)  $\rho$  is t-independent (within the interference region); 2) the spin effects are not essential. The cross section (A.1) includes 3 parameters which can be experimentally found:  $\rho$ ,  $\beta$ ,  $\sigma_{\bar{p}p}^{tot}$ ; if  $\sigma_{\bar{p}p}^{tot}$  is measured independently, only two parameters:  $\rho$  and  $\beta$  should be found.

For the cross section to be estimated at the energies of proton accelerators of the new generation (SPS in CERN, Doubler in Fermilab, UNK in IHEP), it is necessary to extrapolate the parameters  $\sigma_{\bar{p}p}^{tot}$ ,  $\beta$ ,  $\rho$  in eq.(A.1). One of the way is to use the parametrizations available (see Refs. /1,9/):

$$\sigma_{\bar{p}p}^{tot} = (38.4 + 0.49 \ln^2 \frac{s}{122}) \text{ mb} \quad (A.2)$$

where  $s = 4E_c^2 = 2mE_L$  is taken in  $(\text{GeV})^2$ ,

$$\beta_{pp} = [6.9 + 0.77 \ln s] (\text{GeV})^{-2}. \quad (A.3)$$

An alternative method is to use the Lipkin parametrization /10/ for  $\sigma_{\bar{p}p}^{tot}$  and  $\rho$ , basing upon the quark model. This parametrization well describes the data available on hadron interaction at high energy. Note that either variants are well consistent with each other; for example, at  $\sqrt{s} = 600 \text{ GeV}$  we have  $\sigma_{\bar{p}p}^{tot} = 65 \text{ mb}$  and  $\rho = 0.14$ , according to the extrapolation formulas, and the Lipkin parametrization gives  $\sigma_{\bar{p}p}^{tot} = 63 \text{ mb}$  and  $\rho = 0.18$ , respect-



ively and, hence, the distinction between the variants have no influence on the results of the present paper.

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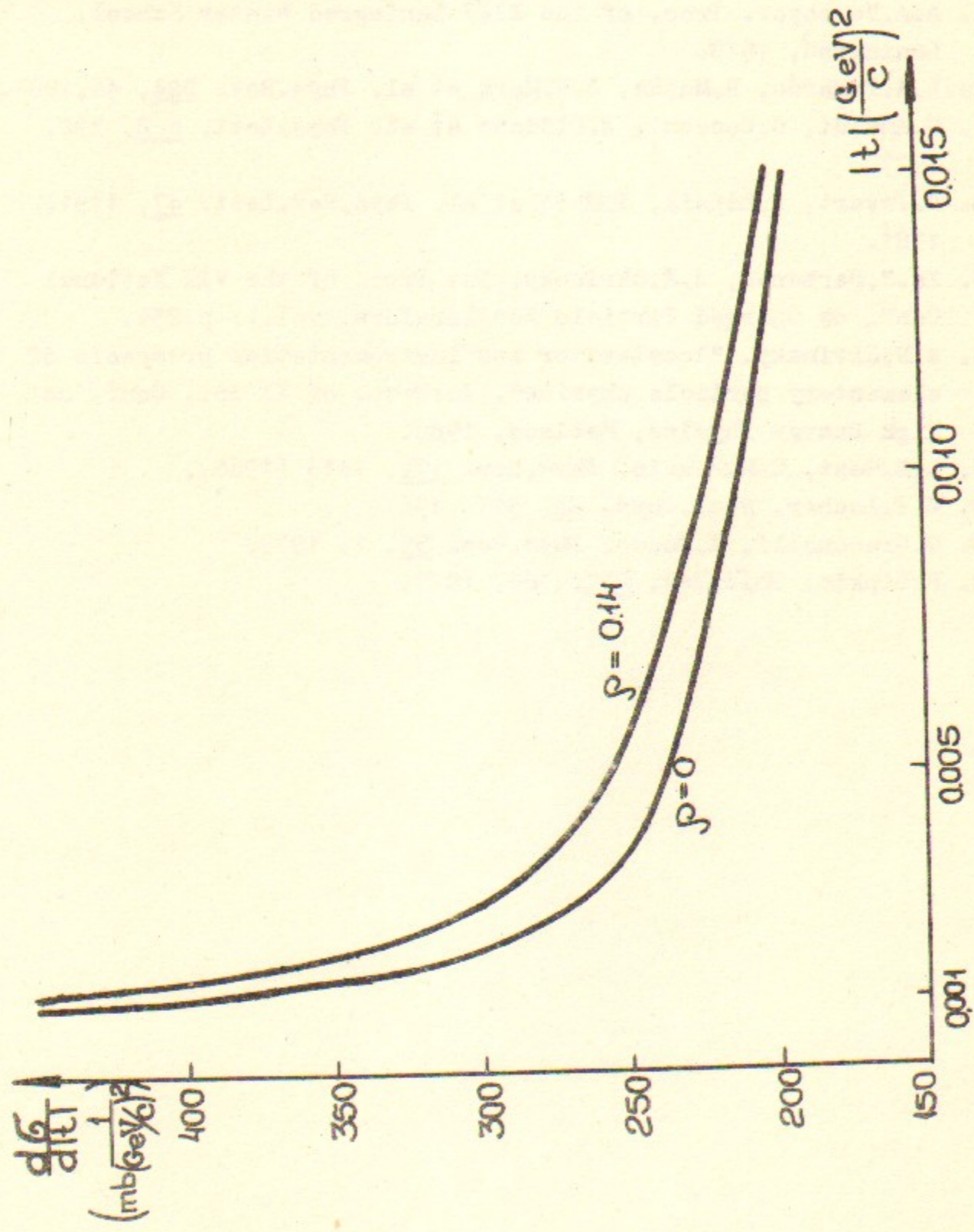


Fig. 1(a)

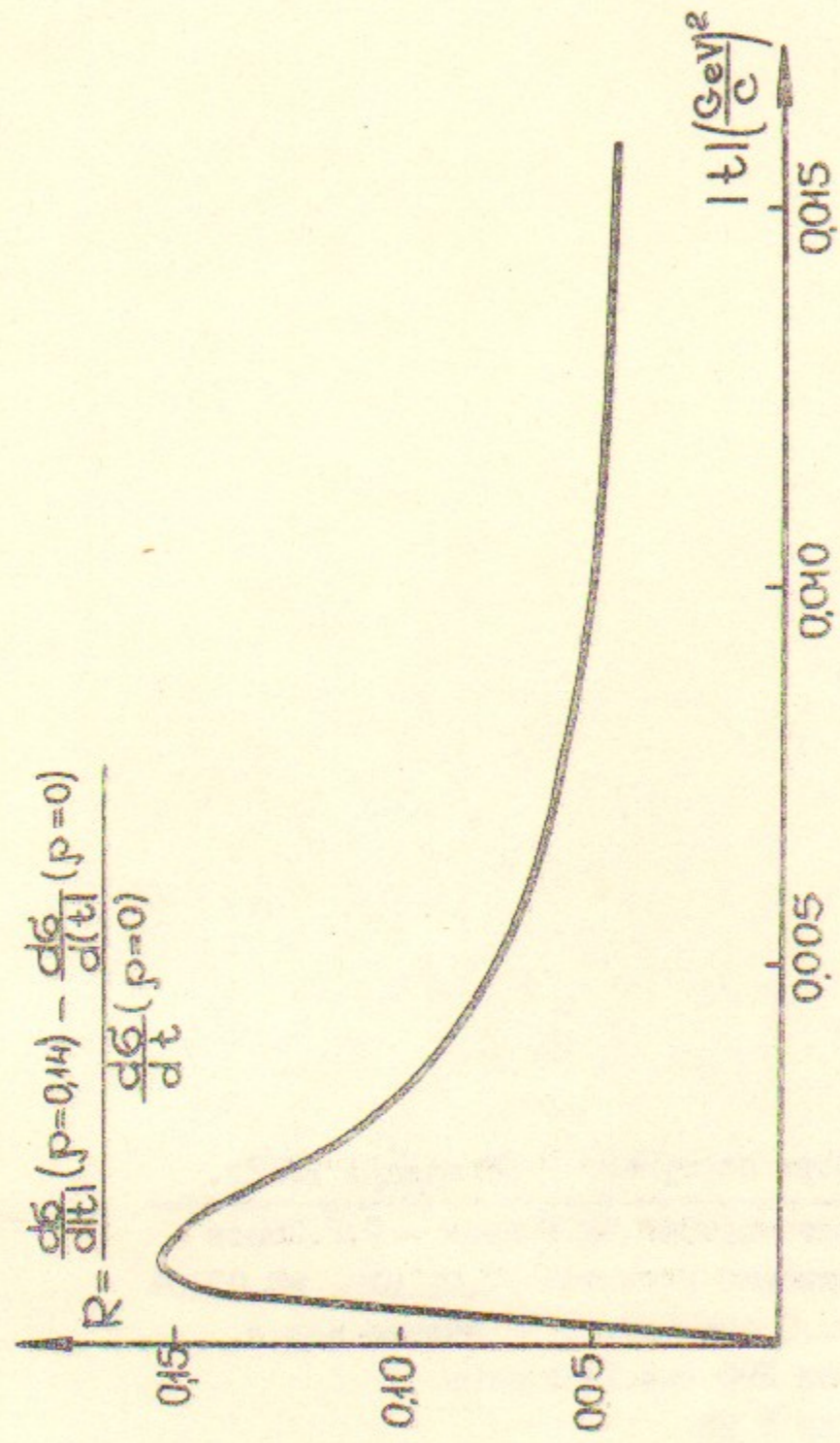


Fig. 1(b)

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