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# MESON WAVE FUNCTIONS AND SU(3) SYMMETRY BREAKING

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MESON WAVE FUNCTIONS AND SU(3)-
SYMMETRY BREAKING

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ABSTRACT

The wave functions of the pseudoscalar and vector mesons ( $K$, $\rho$, $K^*$, $\varphi$ ) are found using the QCD sum rules and used for the calculation of the heavy quarkonium two-particle decay widths.

## I. Introduction.

To describe quantitatively the heavy particle exclusive decays /1-3/ one needs to know the corresponding wave functions of the light mesons ( $\pi, K, \rho \dots$ ). The two-particle wave functions (w.f.) $\varphi_i(\xi)$ describe the distribution of the meson longitudinal momentum between the constituents, $P = P_1 + P_2$, $P_1 = xP$, $P_2 = (1-x)P$, $\xi = 2x - 1$, $P_z \to \infty$.

In the previous paper /4/ we have calculated few first moments $\langle \xi^n \rangle \equiv \int_{-1}^{1} d\xi\, \xi^n \varphi_\pi(\xi)$ of the $\pi$ -meson w.f. using the method of the QCD sum rules developed in /5/. The model $\pi$-meson w.f. has been proposed in /4/ which fulfils sumiltaneously QCD sum rules and leads to the predictions for the strong ( $\chi_{0,2} \to \pi\pi$ ) /3/, electromagnetic ( $\psi, \psi' \to \pi\pi$ ) /3/ and weak ( $D \to K\pi$ ) /6/ decays of heavy mesons which agree with the experimental data.

It is the goal of the present paper to extend the above method /4/ for finding the w.f. of strange (K-meson) and vector particles ( $\rho, K^* \dots$ ). In Sect.II we calculate the K-meson w.f. taking into account the SU(3)-symmetry breaking effects. In Sect. III the properties of the vector meson w.f. are described. In Sect.IV a number of applications are presented and comparison with the experimental data on the charmonium level decays is given.

## II. K-meson wave function.

The w.f. we are interested in is defined as follows:

$$\langle 0 | \bar{s}(z)\gamma_\mu\gamma_5 u(-z) | K^+(q) \rangle_{\mu^2} = i q_\mu f_K \varphi_K(zq, \mu^2) \quad ; \quad z^2 = 0 , \tag{1}$$

$$\varphi_K(zq, \mu^2) = \int_{-1}^{1} d\xi\, \varphi_K(\xi, \mu^2) \exp\{ i\xi(zq) \}, \quad \varphi_K(\xi) \equiv \varphi_K^+(\xi) + \varphi_K^-(\xi),$$

$$\varphi_K^{\pm}(\xi) = \pm\varphi_K^{\pm}(-\xi), \tag{1}$$
$$\int_{-1}^{1} d\xi\, \varphi_K(\xi) = 1.$$

Here $\mu$ -is the normalization point, the w.f. $\varphi_K^{+}(\xi)$ and $\varphi_K^{-}(\xi)$ are the SU(3)-symmetric and antisymmetric parts of the K-meson w.f. $\varphi_K(\xi)$. In the SU(3)-symmetry limit $\varphi_K^{-}(\xi)\to 0$ and $\varphi_K^{+}(\xi)\to\varphi_\pi(\xi)$, $f_K\to f_\pi$. In this Sect. we consider $\varphi_K^{+}(\xi)$ w.f. ( $\varphi_K^{-}(\xi)$ will be considered elsewhere).

Our approach is as follows. Using the QCD sum rules we calculate the ratios $\langle\xi^n\rangle_K/\langle\xi^n\rangle_\pi$, $n=2,4$ of the K- and $\pi$ -meson w.f. moments

$$\langle\xi^n\rangle_{\pi,K} \equiv \int_{-1}^{1} d\xi\, \xi^n \varphi_{\pi,K}(\xi). \tag{2}$$

We choose then the form of the K-meson w.f. analogous to that of $\pi$ -meson /4/: $\varphi_K(\xi)=\frac{15}{4}(1-\xi^2)(A\xi^2+B)$ and find then the coefficients A and B using the values of the moments $\langle\xi^n\rangle_K/\langle\xi^n\rangle_\pi$ and the $\pi$ -meson w.f. described in /4/: $\varphi_\pi(\xi,\mu\simeq 500\,\mathrm{MeV})\simeq\frac{15}{4}(1-\xi^2)\xi^2$.

The application of the QCD sum rules for the calculation of the w.f. moments is described in details in /4/. We therefore describe bellow mainly the results.

Let us consider the correlator:

$$T_n = i\int dx\, e^{iqx}\langle 0|T\{\bar{s}(x)\hat{z}\gamma_5(iz_\mu\overleftrightarrow{D}_\mu)^n u(x),\ \bar{u}(0)\hat{z}\gamma_5 s(0)\}|0\rangle = \tag{3}$$
$$(zq)^{n+2}\, I_n(q^2),\quad \overleftrightarrow{D}=\vec{D}-\overleftarrow{D},\quad \vec{D}_\mu=\vec{\partial}_\mu - igB_\mu^a\frac{\lambda^a}{2}.$$

The asymptotic behaviour of $I_n(q^2)$ at $Q^2=-q^2\to\infty$ is determined in a standard way /5/ and has the form (after "borelezation")



$$\frac{1}{\pi M^2}\int_0^\infty ds\, e^{-s/M^2}\, \mathrm{Im}\, I_n(s) = \frac{1}{4\pi^2}\frac{3}{(n+1)(n+3)} + \frac{1+3n}{1+n}\frac{\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle}{12M^4} +$$
$$\frac{16}{9}\pi\frac{\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle\langle 0|\sqrt{\alpha_s}\,\bar{s}s|0\rangle}{M^6} + \frac{16}{81}\pi(1+2n)\frac{\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2+\langle 0|\sqrt{\alpha_s}\,\bar{s}s|0\rangle^2}{M^6} - \tag{4}$$
$$\frac{3}{4\pi^2}\frac{1}{1+n}\frac{m_s^2}{M^2}\left(\frac{\alpha_s(M)}{\alpha_s(\mu)}\right)^{8/9} + \frac{m_s\langle 0|\bar{s}s|0\rangle}{M^4} + \frac{1}{3}\frac{m_s^3\langle 0|\bar{s}s|0\rangle}{M^6} -$$
$$\frac{n}{6}\frac{m_s}{M^6}\left\{\langle 0|ig\bar{s}\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a s|0\rangle - 4m_s^2\langle 0|\bar{s}s|0\rangle\right\}.$$

The spectral density $\mathrm{Im}\, I_n(s)$ is taken in the form:

$$\frac{1}{\pi}\mathrm{Im}\, I_n(s) = f_K^2\langle\xi^n\rangle_K\,\delta(s-m_K^2) + \frac{3}{4\pi^2(n+1)(n+3)}\theta(s-s_{nK}). \tag{5}$$

The parameters $s_{nK}$ in (5) determine the duality intervals /5/ in the corresponding channels.

In order to single out the SU(3)-symmetry breaking effects of interest we subtract from (4) the corresponding sum rules for the $\pi$ -mesom w.f. /4/. (The $\pi$ -meson w.f. is considered known). So the corresponding sum rules have the form*:

$$\frac{f_K^2}{f_\pi^2}e^{-m_K^2/M^2} = 1 + \frac{M^2}{4\pi^2 f_\pi^2}\left\{e^{-s_{0\pi}/M^2} - e^{-s_{0K}/M^2}\right\} + \frac{m_s\langle\bar{s}s\rangle}{f_\pi^2 M^2} - \frac{3}{4\pi^2}\frac{m_s^2}{f_\pi^2}\left(\frac{\alpha_s(M^2)}{\alpha_s(\mu^2)}\right)^{8/9}. \tag{6}$$

$$\frac{f_K^2}{f_\pi^2}\frac{\langle\xi^2\rangle_K}{\langle\xi^2\rangle_\pi}e^{-m_K^2/M^2} = 1 + \frac{M^2}{20\pi^2 f_\pi^2\langle\xi^2\rangle_\pi}\left\{e^{-\frac{s_{2\pi}}{M^2}} - e^{-\frac{s_{2K}}{M^2}}\right\} + \frac{m_s\langle\bar{s}s\rangle}{f_\pi^2\langle\xi^2\rangle_\pi M^2} - \tag{7}$$
$$\frac{1}{3}\frac{m_s}{M^4}\frac{\langle ig\bar{s}\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a s\rangle}{f_\pi^2\langle\xi^2\rangle_\pi} - \frac{1}{4\pi^2}\frac{m_s^2}{f_\pi^2\langle\xi^2\rangle_\pi}\left(\frac{\alpha_s(M)}{\alpha_s(\mu)}\right)^{8/9}.$$

$$\frac{f_K^2\langle\xi^4\rangle_K}{f_\pi^2\langle\xi^4\rangle_\pi}e^{-m_K^2/M^2} = 1 + \frac{3M^2}{140\pi^2 f_\pi^2\langle\xi^4\rangle_\pi}\left\{e^{-\frac{s_{4\pi}}{M^2}} - e^{-\frac{s_{4K}}{M^2}}\right\} + \frac{m_s\langle\bar{s}s\rangle}{f_\pi^2\langle\xi^4\rangle_\pi M^2} - \tag{8}$$
$$\frac{2}{3}\frac{m_s}{M^4}\frac{\langle ig\bar{s}\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a s\rangle}{f_\pi^2\langle\xi^4\rangle_\pi} - \frac{3}{20\pi^2}\frac{m_s^2}{f_\pi^2\langle\xi^4\rangle_\pi}\left(\frac{\alpha_s(M)}{\alpha_s(\mu)}\right)^{8/9}.$$

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* see the next page

We use the following values of the matrix elements entering (6)-(8): $m_s\langle\bar{s}s\rangle \simeq -1.7\cdot10^{-3}\,GeV^4$; $\langle ig\bar{s}\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a s\rangle \simeq -(0.4\,GeV)^5$ /7,8/.

The standard treatment of (6)-(8) (see /4/) gives:

$$f_K^2/f_\pi^2 \simeq 1.4\,, \qquad s_{0K} \simeq 1.1\,GeV^2\,. \tag{9a}$$

$$\left(\frac{f_K^2\langle\xi^2\rangle_K}{f_\pi^2\langle\xi^2\rangle_\pi}\right)_{\overline{M_2^2}\simeq1.5\,GeV^2} \simeq 1.1\,, \quad \left(\frac{\langle\xi^2\rangle_K}{\langle\xi^2\rangle_\pi}\right)_{1.5\,GeV^2} \simeq 0.8\,, \quad s_{2K}\simeq 2\,GeV^2. \tag{9b}$$

$$\left(\frac{f_K^2\langle\xi^4\rangle_K}{f_\pi^2\langle\xi^4\rangle_\pi}\right)_{\overline{M_4^2}\simeq2.2\,GeV^2} \simeq 1\,, \quad \left(\frac{\langle\xi^4\rangle_K}{\langle\xi^4\rangle_\pi}\right)_{2.2\,GeV^2} \simeq 0.7\,, \quad s_{4K}\simeq 2.7\,GeV^2. \tag{9c}$$

It is worth noting that the value $\Delta_s = m_s\langle\bar{s}s\rangle \simeq -1.7\cdot10^{-3}\,GeV^4$ which was used above is $\sim 25\%$ smaller then the number $\Delta'_s = (m_s\simeq150\,MeV)\cdot(\langle\bar{s}s\rangle\simeq-(0.25\,GeV)^3)$ /5/. The smaller value $\Delta_s$ is preffered because it gives (see (9a)): $f_K/f_\pi \simeq 1.2$ (experimentally $f_K/f_\pi = 1.2 \div 1.25$ ) while $\Delta'_s$ leads to $f_K/f_\pi = 1.09$. The arguments in favor $\Delta_s$ are given also in /9/.

The numbers (9a)-(9c) represent the main result of this Section. Now we use these numbers for finding the coefficients A and B in the K-meson w.f. written in the form (see the beginning of this Sect.):

$$\varphi_K(\xi,\mu^2) = \frac{15}{4}(1-\xi^2)\left[A(\mu^2)\cdot\xi^2 + B(\xi^2)\right]; \quad \varphi_\pi(\xi,\mu^2\simeq0.25\,GeV^2) \simeq \frac{15}{4}(1-\xi^2)\xi^2, \tag{10}$$

$$A(\mu^2)+5B(\mu^2)=1, \quad A(\mu_2^2) = A(\mu_1^2)\left[\alpha_s(\mu_2^2)\big/\alpha_s(\mu_1^2)\right]^{50/9b}, \quad b = 11-\tfrac{2}{3}n_f \simeq 9.$$

Using (9b) we obtain:

$$\varphi_K(\xi,\mu^2=1.5\,GeV^2) = \frac{15}{4}(1-\xi^2)(0.45\xi^2+0.11). \tag{11}$$

$$\varphi_K(\xi,\mu^2=0.25\,GeV^2) = \frac{15}{4}(1-\xi^2)(0.6\xi^2+0.08). \tag{12}$$

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* We suppose that $|\langle\bar{s}s\rangle - \langle\bar{u}u\rangle| \lesssim \frac{1}{5}|\langle\bar{u}u\rangle|$ and neglect it.



We have also the possibility to check up the selfconsistency of the whole approach since the coefficients $A(\mu^2)$ and $B(\mu^2)$ can be found using (9c) as well. This gives* (compare with (11)):

$$\varphi_K(\xi,\mu^2=1.5\,GeV^2) = \frac{15}{4}(1-\xi^2)(0.4\xi^2+0.12). \tag{13}$$

In summary, we have found that while $f_K > f_\pi$ the K-meson w.f. is more narrow than the $\pi$ -meson w.f. As it will be shown latter, the general tendency is as follows: the larger is the corresponding constant $f_i$ , the more narrow is the distribution of quarks in $\xi$ in this state (i.e. if $f_i > f_j$ then $\langle\xi^2\rangle_i < \langle\xi^2\rangle_j$ ).

III. Vector meson wave functions.

In this Sect. the w.f. we dealt with are defined by the matrix element:

$$\langle 0|\bar{\psi}_2(z)\gamma_\mu\psi_1(-z)|V_\lambda(q)\rangle_{\mu^2} = \varepsilon^\lambda_\mu(q)\, m_V f_V \varphi_V(zq,\mu^2) + \dots \tag{14}$$

where $\varepsilon^\lambda_\mu$ -is the polarization vector, $m_V$ -is the vector particle mass, the constant $f_V$ is analogous to $f_\pi$ and $f_K$. The QCD sum rules for the moments $\langle\xi^n\rangle$ of the $\rho$ -meson w.f. have the form:

$$\frac{1}{\pi M^2}\int_0^\infty ds\, e^{-s/M^2}\, Im\, I_n(s) = \frac{3}{4\pi^2(n+1)(n+3)} + \frac{1+3n}{1+n}\frac{1}{M^4}\langle\frac{\alpha_s}{\pi}G^2\rangle + \tag{15}$$

$$\frac{16}{9}\pi\langle\sqrt{\alpha_s}\,\bar{u}u\rangle^2\frac{1}{M^6}\left[\frac{2}{9}(2n+1)-1\right]$$

The sum rule with $n=0$ has been considered in /5/. The value of $f_\rho \simeq 200\,MeV$ determined from this sum rule (with

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* The dependence of the w.f. moments $\langle\xi^n\rangle_{\mu^2}$ on $\mu^2$ is determined by the renormalization group, see /4/.

$\langle\sqrt{\alpha_s}\,\bar{u}u\rangle^2 \simeq 1.8\cdot 10^{-4}\,GeV^6$, $\langle\frac{\alpha_s}{\pi}G^2\rangle \simeq 1.2\cdot 10^{-2}\,GeV^2)$ /5/ is in excelent agreement with the experimental data. Using the standard fitting procedure one obtains from (16) with $n=4$:

$$\left[\langle\xi^4\rangle_\rho\right]_{M^2=1.5\,GeV^2} = 0.15. \tag{17}$$

We don't use here the value $\langle\xi^2\rangle$ because at $n=2$ there are strong cancellations between the various contributions into the main power correction $\langle\sqrt{\alpha_s}\,\bar{u}u\rangle^2$ ( the sum of coefficients is $\sim 10$ times smaller than each term). In such a situation we expect that the higher power corrections ( $\langle\bar\Psi\Psi\cdot\bar\Psi\sigma_{\mu\nu}G_{\mu\nu}\Psi\rangle$ etc.) will be important. Indeed, it is impossible to obtain more or less acceptable fit from (16) with $n=2$ as it stands.

Considering the K-meson w.f. in the previous Sect. we have shown that the coefficients $A(\mu^2)$ and $B(\mu^2)$ in the model w.f. of the form (10) can be found by using not only the value $\langle\xi^2\rangle$ but the value $\langle\xi^4\rangle$ as well (see (11),(13)). Therefore, using (17) we obtain:

$$\varphi_\rho(\xi)_{\mu^2=1.5\,GeV^2} \simeq \frac{15}{4}(1-\xi^2)(0.25\xi^2+0.15);\ \varphi_\rho(\xi)_{\mu^2=0.25\,GeV^2} \simeq \frac{15}{4}(1-\xi^2)(0.3\xi^2+0.14). \tag{18}$$

The sum rule for the $\overset{*}{K}$ -meson has the form (4) but now the term $\frac{16}{9}\pi\langle\sqrt{\alpha_s}\,\bar{u}u\rangle\langle\sqrt{\alpha_s}\,\bar{s}s\rangle/M^6$ enters with an opposite sign. Therefore, we have: $f^2_{\overset{*}{K}}/f^2_\rho = 1.1$;

$$\left(\frac{f^2_{\overset{*}{K}}}{f^2_\rho}\,\frac{\langle\xi^4\rangle_{\overset{*}{K}}}{\langle\xi^4\rangle_\rho}\right)_{M^2=1.5\,GeV^2} = 0.9;\quad \left.\frac{\langle\xi^4\rangle_{\overset{*}{K}}}{\langle\xi^4\rangle_\rho}\right|_{M^2=1.5\,GeV^2} = 0.8. \tag{19}$$

The only difference between the sum rules for the $\varphi$ and $\overset{*}{K}$ mesons is the additional factor 2 in the SU(3)-breaking terms in the first case. We have * : $f^2_\varphi/f^2_\rho = 1.3$,

$$\left(\frac{f^2_\varphi}{f^2_\rho}\,\frac{\langle\xi^4\rangle_\varphi}{\langle\xi^4\rangle_\rho}\right)_{M^2=1.5\,GeV^2} = 0.8;\quad \left.\frac{\langle\xi^4\rangle_\varphi}{\langle\xi^4\rangle_\rho}\right|_{M^2=1.5\,GeV^2} = 0.6. \tag{20}$$



Using the $\rho$ -meson w.f. (18) and (19),(20) we obtain:

$$\varphi_{\overset{*}{K}}(\xi,\mu^2=1.5\,GeV^2) = \frac{15}{4}(1-\xi^2)(0.1\xi^2+0.18), \tag{21}$$

$$\varphi_\varphi(\xi,\mu^2=1.5\,GeV^2) = \frac{15}{4}(1-\xi^2)(-0.05\xi^2+0.21). \tag{22}$$

As one can see from (22) the $\varphi$ -meson w.f. coincides with the asymptotic one: $\varphi^\infty(\xi) = 3/4\,(1-\xi^2)$.
Comparison of $\pi, K, \rho, \overset{*}{K}$ and $\varphi$ -meson w.f. shows that $\langle\xi^2\rangle_\varphi < \langle\xi^2\rangle_{\overset{*}{K}} < \langle\xi^2\rangle_\rho < \langle\xi^2\rangle_K < \langle\xi^2\rangle_\pi$ while $f_\varphi > f_{\overset{*}{K}} > f_\rho > f_K > f_\pi$.

As for the expected accuracy of the results obtained above, we discuss separately two questions: 1) the accuracy of the moment values $\langle\xi^2\rangle_K/\langle\xi^2\rangle_\pi \ldots$ ; 2) the accuracy of the model wave functions.

1) The point is that the QCD sum rules for the constants $f_\pi, f_K, m_\rho \ldots$ /5/ give the predictions with the accuracy $\lesssim 10\%$. We expect therefore the accuracy no worse than 15-20% for the quantities $\langle\xi^2\rangle_i$ and $\langle\xi^4\rangle_i$. The accuracy of the ratios like $\langle\xi^2\rangle_K/\langle\xi^2\rangle_\pi$, $\langle\xi^4\rangle_K/\langle\xi^4\rangle_\pi$ is evidently somewhat better. We expect it to be about 5-10%.

2) The precise form of the model w.f. chosen above have, of course, lower accuracy (especially in the region $|\xi|\to 1$ ). The arguments in favour of such a form are its simplicity and agreement of the predictions and the experimental data (see /3,6,10/ and bellow).

IV. <u>Applications and conclusions.</u>

The w.f. obtained above allow one to predict quantitatively various quarkonium decay widths. In particular, $\chi_0 \to M_i M_i$

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* (from the previous page)

The values $f_{\overset{*}{K}}$ and $f_\varphi$ has been obtained previously in /5/.

( $\chi_0$ is ${}^3P_0$ -charmonium state, $M_i = \pi, K, \rho \ldots$) decay width has the form /2,3/:

$$Br(\chi_0 \to M_i M_i) = \left| \pi \tilde{\alpha}_s \frac{16\sqrt{2}}{27} \frac{f_i^2}{M_0^2} I^i_{\chi_0} \right|^2 ,$$

$$I^i_{\chi_0} = \int_{-1}^{1} \frac{d\xi_1}{1-\xi_1^2} \varphi_i(\xi_1) \int_{-1}^{1} \frac{d\xi_2}{1-\xi_2^2} \varphi_i(\xi_2) \frac{1}{1-\xi_1\xi_2} \left[ 1 + \frac{1}{4} \frac{(\xi_1-\xi_2)^2}{1-\xi_1\xi_2} \right] , \tag{23}$$

where $M_0$ is $\chi_0$- mass, $\varphi_i(\xi)$ are the corresponding meson w.f. Using (10),(12) we have:

$$\left[ \frac{Br(\chi_0 \to K^+K^-)}{Br(\chi_0 \to \pi^+\pi^-)} \right]^{1/2} = \frac{f_K^2}{f_\pi^2} \frac{I^K_{\chi_0}}{I^\pi_{\chi_0}} \simeq 0.85 \simeq \left[ \frac{Br(\chi_2 \to K^+K^-)}{Br(\chi_2 \to \pi^+\pi^-)} \right]^{1/2} \tag{24}$$

( $\chi_2$ is ${}^3P_2$- charmonium state). This number does not contradicts to the experimental data /11/ *. Let's note once more that the SU(3)-symmetry breaking influences not only the constants, $f_K \neq f_\pi$ ,but the w.f. $\varphi_K(\xi) \neq \varphi_\pi(\xi)$ as well. The K-meson w.f. $\varphi_K(\xi)$ is more norrow than $\varphi_\pi(\xi)$ and just this property compensates nearly the effect due to $f_K > f_\pi$.

Analogously,using the $\pi$ - and $\rho$ -meson w.f. we have*:

$$\left[ \frac{Br(\chi_0 \to \rho^+\rho^-)}{Br(\chi_0 \to \pi^+\pi^-)} \right]^{1/2} = \frac{f_\rho^2}{f_\pi^2} \frac{I^\rho_{\chi_0}}{I^\pi_{\chi_0}} \simeq 0.70 . \tag{25}$$

We expect therefore: $Br(\chi_0 \to \rho\rho) \simeq 0.5\%$ (this branching ratio was not measured up to now). Let us point out also that the $\chi_2$ -state with the helicity $|\lambda| = 2$ can decay into pair of the $\rho_\perp$ -mesons with $|\lambda| = 1$ while $\chi_0$ -state and $\chi_2$ -state

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* Let us note that the naive estimates like those in /12/ give:

$$\frac{Br(\chi_0 \to K^+K^-)}{Br(\chi_0 \to \pi^+\pi^-)} \simeq \left( \frac{f_K}{f_\pi} \right)^4 \simeq 2 ; \quad \frac{Br(\chi_0 \to \rho^+\rho^-)}{Br(\chi_0 \to \pi^+\pi^-)} \simeq \left( \frac{f_\rho}{f_\pi} \right)^4 \simeq 5$$

and this contradicts to the experiment /11/.



with $\lambda = 0$ decay only into $\rho_L$ -mesons with the helicity $\lambda = 0$. The decay $\chi_2^{|\lambda|=2} \to \rho_\perp \rho_\perp$ is connected with the tensor w.f. of the $\rho_\perp$ -meson

$$\langle 0 | \bar{d}(z) \sigma_{\mu\nu} u(-z) | \rho_\perp(q) \rangle = (\varepsilon^\perp_\mu q_\nu - \varepsilon^\perp_\nu q_\mu) f_T \varphi_T(zq) .$$

The properties of this w.f. are not considered yet. We expect that $Br(\chi_2 \to \rho\rho)/Br(\chi_2 \to \pi\pi) > Br(\chi_0 \to \rho\rho)/Br(\chi_0 \to \pi\pi)$.

As for the $\pi, K, \rho \ldots$ -meson electromagnetic form factors, their asymptotic behaviour is determined by the integrals of the form: $I_i = \int_{-1}^{1} d\xi \, d\xi \, \varphi_i(\xi)/1-\xi$ . So,we have (in the charmonium region $q^2 \simeq 10\,GeV^2$):

$$\frac{F_K(q^2)}{F_\pi(q^2)} = \frac{f_K^2}{f_\pi^2} \frac{I_K^2}{I_\pi^2} \simeq 1 ; \quad \frac{F_\rho(q^2)}{F_\pi(q^2)} = \frac{f_\rho^2}{f_\pi^2} \frac{I_\rho^2}{I_\pi^2} \simeq 1 . \tag{26}$$

In summary,we have shown that using the method of QCD sum rules /5/ it is possible to find out the main characteristic properties of the meson wave functions. The predictions obtained with help of these wave functions are in agreement with the experimental data. We have find some regularity in the wave functions properties: the larger is the corresponding dimensional constant $f_i$ ( $f_\rho, f_K, f_\pi \ldots$),the more narrow is the dimensionless wave function $\varphi_i(\xi)$ ,i.e. if $f_i > f_j$ then $\langle \xi^2 \rangle_i < \langle \xi^2 \rangle_j$ . The various exclusive amplitudes have the form of the product of $f_i$ and $I(\varphi_i)$ where $I(\varphi_i)$ are the corresponding integrals of the wave functions. Therefore, these two effects work in an opposite directions (if $f_i > f_j$ then $I(\varphi_i) < I(\varphi_j)$ ) and have a tendency to compensate each other. And which of these two effects will be more significant depends on the process under consideration.

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