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V.L.Chernyak,
A.R.Zhitnitsky and I.R.Zhitnitsky

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DECAY WIDTHS

by

V.L.Chernyak, A.R.Zhitnitsky and I.R.Zhitnitsky

Institute of Nuclear Physics

630090, Novosibirsk, USSR

A B S T R A C T

The theoretical calculation of the $\psi \to \rho\pi$ and $\psi \to \pi\gamma$ decay widths is presented. The results obtained are in a reasonable agreement with the experimental data.

I. Introduction.

General properties of the heavy quarkonium exclusive decays have been considered in /1-3/. It has been pointed out in /3/ that the $\Psi \to \rho\pi$ decay amplitude has the behaviour $\sim M^{-3}$ (M is the heavy quark mass), while the highest possible behaviour of the two-particle decay amplitude is $\sim M^{-2}$ at $M \to \infty$ (for instance, for $\chi(3415) \to \pi\pi$). The $\Psi \to \rho\pi$ amplitude is suppressed because the ρ- meson has transverse polarization (i.e. its helicity $|\lambda| = 1$) while the π- meson helicity is, of course, equals zero. Therefore, the light quark sould turn overover its helicity in Fig.1 diagram, and this leads to additional suppression $\sim \mu/M \ll 1$ of the amplitude.* If the helicity is turned over due to the quark propagator in Fig.1, then μ is the quark mass ($m_u \sim 4\,MeV, m_d \sim 7\,MeV, m_s \sim 150\,MeV$) The helicity can be turned over at large distances as well, i.e. "inside" the meson wave function due to a spontaneous chiral symmetry breaking. For the u - and d - quarks this last mechanism is the main one, of course.

There are an another mechanism of the $\Psi \to \rho\pi$ decay, Fig.2 connected with the 3-particle ρ -meson wave function. The quark helicities are unchanged in this case and the quarks have opposite helicities, while $|\lambda_\rho| = 1$ is due to the gluon helicity. The additional suppression of the $\Psi \to \rho\pi$ amplitude

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* This suppression is analogous to that in the $\gamma - \rho\pi$ electromagnetic form factor /4-6/.

** The virtuality of the quark propagator in the Fig.1 is $\sim M^2$ and therefore its mass here is "the small distance mass", i.e. "the current mass".

is evident in this case due to the presence of three particles in the ρ - meson wave function.

We calculate in Sect.II the contributions to the $\Psi \to \rho\pi$ decay amplitude of the diagrams like those at Fig.1. We note that the ρ- meson mass m_ρ plays here the role of the mass μ (see above), and so the characteristic mass " μ " is, in some sense, large in this process. At first sight, it seems natural to expect $\mu \sim f_\pi \sim 150\, MeV$. The quantity f_π is not connected, however, with the mass scale of the chiral symmetry breaking. The more reasonable quantity is: $\mu \sim \sqrt{2}\, m_{const}$, where $m_{const} \sim 300-350\, MeV$ is "the constituent quark mass" (the factor $\sqrt{2}$ is due to: $\rho_{\lambda=0} \sim (\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$, $\rho_{\lambda=1} \sim (\uparrow\uparrow)$ and to turn over the helicity can each of two quarks). Because numerically $m_\rho \gtrsim 2 m_{const} \simeq 5 f_\pi$, the suppression factor

$$m_\rho/E_\rho \simeq m_\rho/M_c \simeq 0.5 \tag{1}$$

turns out to be not very small for the case of the C-quark ($M_c \simeq 1.5\, GeV$ is the C-quark mass).*

The naive estimate of the ratio $\chi_0(3415) \to \pi\pi / \Psi \to \rho\pi$ gives:

$$Br(\Psi \to \rho\pi) \Big/ Br(\chi_0 \to \pi\pi) \sim \frac{\alpha_s}{\pi}\left(\frac{\mu}{M_c}\right)^2 \sim 10^{-3} \text{ at } \mu \sim f_\pi . \tag{2}$$

Here the factor α_s/π is due to a difference in the number of gluons and unequal phase spaces, and the factor $(\mu/M_c)^2$ is due to a helicity flip in the $\Psi \to \rho\pi$ decay. The experiment gives for the ratio (2) the number /7/: $(1.2 \pm 0.1)\% / (0.9 \pm 0.2)\%$.
We show bellow that instead of (2) one has really:

$$Br(\Psi \to \rho\pi) \Big/ Br(\chi_0 \to \pi\pi) \sim \left(\frac{\alpha_s}{\pi} \ln^2 \frac{M_c^2}{\bar{m}^2}\right)\left(\frac{m_\rho}{M_c}\right)^2 \sim O(1) \tag{3}$$

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* The $\Upsilon \to \rho\pi$ decay will be considerably suppressed already as compared, for instance, with the $\Upsilon \to B(1300)\pi$ decay.

in agreement with the experimental data.

II. The contribution to the $\Psi \to \rho\pi$ decay due to a two-particle meson wave functions.

Let us define the invariant decay amplitude M_0 as follows:

$$M(\Psi \to \rho^+ \pi^-) = \varepsilon_{\mu\nu\lambda\sigma}\, \rho^\mu \Psi^\nu P_1^\lambda P_2^\sigma\, M_0 . \tag{4}$$

Here: ε -is the unit antisymmetric tensor, ρ_μ and Ψ_ν are the polarization vectors of the ρ and Ψ - mesons, P_1 and P_2 are the momenta of the π^- meson and ρ^+ meson.

We calculate in this Sect. the contribution to the $\Psi \to \rho\pi$ amplitude connected with the wave functions /1,3/ (Fig.1)

$$\langle \rho_\perp^+(P_2) | \bar{u}(z)\gamma_\mu d(-z) | 0 \rangle = f_\rho\, \overset{*}{\rho}{}_\perp^\mu\, m_\rho \int_{-1}^{1} d\xi\, \varphi_\rho(\xi) \exp(i z P_2),$$

$$\langle 0 | \bar{d}(z)\gamma_\mu\gamma_5 u(-z) | \pi^+(P_1) \rangle = i f_\pi P_1^\mu \int_{-1}^{1} d\xi\, \varphi_\pi(\xi) \exp(i z P_1), \tag{5}$$

$$\langle 1 \rangle_\rho \equiv \int_{-1}^{1} d\xi\, \varphi_\rho(\xi) = 1, \quad \langle 1 \rangle_\pi \equiv \int_{-1}^{1} d\xi\, \varphi_\pi(\xi) = 1.$$

The Ψ meson wave function has the form /3/:

$$\langle 0 | \bar{c}_\beta(z) c_\alpha(-z) | \Psi(P) \rangle \simeq \frac{1}{4}\varphi_V(0)\left[2 M_c \hat{\Psi} - \sigma_{\mu\nu}\Psi^\mu P^\nu\right]_{\alpha\beta} . \tag{6}$$

The direct calculation gives:*

$$M_0 = -\pi \alpha_s^3 \frac{2^7 \cdot 5}{81} \varphi_V(0) \frac{f_\pi f_\rho m_\rho}{M_\Psi^5} \left\langle \frac{1}{1-\xi^2} \right\rangle_\pi \left\langle \frac{1}{1-\xi^2} \right\rangle_\rho \ln(-M_\Psi^2/\bar{m}^2), \tag{7}$$

$$\ln(-M_\Psi^2/\bar{m}^2) = \ln(M_\Psi^2/\bar{m}^2) - i\pi .$$

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* Each of the one loop diagrams like those in Fig.1 include the double logarithmic terms $\sim \ln^2 M_\Psi^2$. These contributions cancel, as usually, in the sum of the diagrams (see /8,1/).

where M_ψ is the ψ- meson mass, $\bar{m}$ - is the characteristic infrared cut off *;

$$\left\langle \frac{1}{1-\xi^2} \right\rangle_{\pi,\rho} \equiv \int_{-1}^{1} d\xi\, \varphi_{\pi,\rho}(\xi) \frac{1}{1-\xi^2} .$$

The decay width $\psi \to e^+e^-$ expressed through the wave function $\varphi_V(0)$ (6) has the form:

$$\Gamma(\psi \to e^+e^-) = \frac{(4\pi\alpha)^2}{27\pi} \frac{|\varphi_V(0)|^2}{M_\psi} \tag{8}$$

Therefore,

$$\frac{\Gamma(\psi\to\rho\pi)}{\Gamma(\psi\to e^+e^-)} = \frac{25\cdot 64}{243} \frac{\bar{\alpha}_s^6}{\alpha^2} \left| \frac{f_\pi f_\rho m_\rho}{M_\psi^3} \left\langle \frac{1}{1-\xi^2}\right\rangle_\pi \left\langle \frac{1}{1-\xi^2}\right\rangle_\rho \ln\left(-\frac{M_\psi^2}{\bar{m}^2}\right) \right|^2 . \tag{9}$$

The characteristic virtuality of the light meson wave functions in the charmonium decays is $\bar{\mu} \simeq 500$ MeV /3/. The π and ρ- meson wave functions at this virtuality have been found in /9,10/ and have the form:

$$\varphi_\pi(\xi) \simeq \frac{15}{4}(1-\xi^2)\xi^2 \;; \quad \varphi_\rho(\xi) \simeq \frac{15}{4}(1-\xi^2)(0.3\xi^2+0.14). \tag{10}$$

We use the following values of the parameters in (9): $\bar{\alpha}_s \simeq 0.35$ (see bellow), $f_\pi \simeq 133$ MeV, $f_\rho \simeq 200$ MeV, $\bar{m} \simeq 300$ MeV.

With these values the ratio (9) is

$$\Gamma(\psi\to\rho\pi) \Big/ \Gamma(\psi\to e^+e^-) \simeq 7\cdot 10^{-2} . \tag{11}$$

The experimental value /7/ is:

$$\Gamma(\psi\to e^+e^-) \Big/ \Gamma_{tot}(\psi) \simeq 7\%$$

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* It is legitimate to keep the imaginary part in (7), as is clear from the dispersion relation for M_0 in M_ψ^2.

We have therefore from (11):

$$\Gamma(\psi\to\rho\pi) \Big/ \Gamma_{tot}(\psi) \simeq 0.5\% . \tag{12}$$

This number is two times smaller than the experimental value /7/: $(1.2 \pm 0.1)\%$.

The main uncertainty in (9) is connected with the quantity $\bar{\alpha}_s^6$. Our choice: $\bar{\alpha}_s \simeq 0.35$ is based on the following considerations. Considering the $\chi_0(3415) \to \pi\pi$ decay /3/ we pointed out that because the longitudinal momentum distribution of quarks in the π- meson wave function is wide enough (see (10)), the characteristic virtuality of one of two virtual gluons in the χ_0 -decay is: $\bar{q}_1^2 \simeq (500\text{ MeV})^2$, while another one has: $\bar{q}_2^2 \simeq (M_{\chi_0} - 0.5)^2\,\text{GeV}^2 \simeq 9\,\text{GeV}^2$. Analogous situation takes place in the $\psi\to\rho\pi$ decay. The gluon in Fig.1 do not entering the loop has: $\bar{q}_1^2 \simeq (500\text{MeV})^2$, and correspondingly, $(\alpha_s)_1 \simeq 0.43$ (we use $\Lambda \simeq 100$ MeV). The logarithmic contribution of the loop in Fig.1 comes from the wide region: $K_{min} < K < K_{max}$ (K is the gluon momentum), $K_{min} \simeq 500$ MeV, $K_{max} \simeq (M_\psi - 1.0) \simeq 2$ GeV. Therefore, the effective coupling: $\alpha_s^2(K_{max}/2) < \tilde{\alpha}_s^2 < \alpha_s(K_{min})\cdot\alpha_s(K_{max})$, $\alpha_s(K_{min}) \simeq 0.43$, $\alpha_s(K_{max}) \simeq 0.23$ and $(\tilde{\alpha}_s)^2 \simeq 0.09 - 0.1$.

Finally:

$$(\bar{\alpha}_s)^3 \simeq \alpha_s(500\text{MeV})\,\alpha_s(500\text{MeV})\,\alpha_s(2\text{GeV}) \simeq (0.35)^3 . \tag{13}$$

Let us dwell now on the role of the SU(3)-symmetry breaking effects and compare the decays $\psi \to (K\bar{K}^* + \bar{K}K^*)$ and $\psi\to\rho\pi$.

The ratio has the form:

$$\frac{\Gamma(\psi\to K\bar{K}^* + \bar{K}K^*)}{\Gamma(\psi\to\rho\pi)} = \left| \frac{M_0(K^{*+}K^-)}{M_0(\rho^+\pi^-)} \right|^2 \frac{4}{3}\, 0.85 , \tag{14}$$

where the factor 0.85 is the phase space correction and 4/3 is an isotopic factor. The ratio of the amplitudes is:

$$\frac{M_0(\overset{*}{K}{}^+K^-)}{M_0(\rho^+\pi^-)} = \left(\frac{\tilde{\alpha}_K}{\tilde{\alpha}_\pi}\right)^6 \left| \frac{m_{\overset{*}{K}} f_{\overset{*}{K}} f_K}{m_\rho f_\rho f_\pi} \; \frac{\left\langle \frac{1}{1-\xi^2} \right\rangle_{\overset{*}{K}} \left\langle \frac{1}{1-\xi^2} \right\rangle_K}{\left\langle \frac{1}{1-\xi^2} \right\rangle_\rho \left\langle \frac{1}{1-\xi^2} \right\rangle_\pi} \right|^2 . \tag{15}$$

The $\overset{*}{K}$ and K - meson wave functions have been found in /10/ and have the form (at virtuality $\mu^2 \simeq (500\,\mathrm{MeV})^2$):

$$\varphi_K(\xi) \simeq \frac{15}{4}(1-\xi^2)(0.6\xi^2+0.08); \quad \varphi_{\overset{*}{K}}(\xi) \simeq \frac{15}{4}(1-\xi^2)(0.1\xi^2+0.18). \tag{16}$$

One can see from (10),(16) that the $K, \overset{*}{K}$ - meson wave functions are more narrow than π, ρ - wave functions. Therefore, <u>the characteristic gluon virtualities $\overline{q_1^2}$ and $\overline{q_2^2}$ (see above) will be larger somewhat in the $K\overset{*}{K}$ decay as compared with that of $\pi\rho$,and correspondingly, $\tilde{\alpha}_K < \tilde{\alpha}_\pi$ in (15).</u> If one takes for the $K\overset{*}{K}$ decay: $\overline{q_1^2} \simeq \overline{q_2^2} \simeq (700\,\mathrm{MeV})^2$ then $\tilde{\alpha}_K \simeq 0.31$ (compare with (13)). The ratios of the parameters entering (15) are:

$$f_K/f_\pi \simeq 1.20\,, \quad f_{\overset{*}{K}}/f_\rho \simeq 1.05\,, \quad m_{\overset{*}{K}}/m_\rho \simeq 1.15 . \tag{17}$$

The ratio of the integrals is:

$$\frac{\left\langle \frac{1}{1-\xi^2} \right\rangle_{K,700\,\mathrm{MeV}} \cdot \left\langle \frac{1}{1-\xi^2} \right\rangle_{\overset{*}{K},700\,\mathrm{MeV}}}{\left\langle \frac{1}{1-\xi^2} \right\rangle_{\pi,500\,\mathrm{MeV}} \cdot \left\langle \frac{1}{1-\xi^2} \right\rangle_{\rho,500\,\mathrm{MeV}}} \simeq 0.7 . \tag{17'}$$

As a whole, we have:

$$\Gamma(\psi \to K\overset{*}{K} + \overset{*}{K}K) \Big/ \Gamma(\psi \to \rho\pi) \simeq 0.6 \tag{18}$$

Of course, the number (18) can not be taken too seriously. It is important, however, that it is the difference in the form of the π (ρ) and K ($\overset{*}{K}$) meson wave functions which

leads to $\tilde{\alpha}_K < \tilde{\alpha}_\pi$ and (17'). The naive estimate à la S.Brodsky and P.Lepage /11/ gives:

$$\frac{\Gamma(\psi \to K\overset{*}{K} + \overset{*}{K}K)}{\Gamma(\psi \to \rho\pi)} \simeq \left| \frac{m_{\overset{*}{K}} f_{\overset{*}{K}} f_K}{m_\rho f_\rho f_\pi} \right|^2 \frac{4}{3}\, 0.85 \simeq 2.4\,, \tag{19}$$

that contradicts to the experiment /7/.*

III. <u>3-particle wave function contributions.</u>

As was pointed out in the introduction, there are contributions like that at Fig.2 connected to the 3-particle component of the ρ -meson wave function,** in addition to the Fig.1 diagrams. It seems at first sight that the Fig.2 contributions are diminant as they are described by the Born diagram while the Fig.1 diagram contains the loop. Using the QCD sum rules /12/ we show bellow that the 3-particle ρ -meson wave function is very small numerically and, as a result, the Fig.2 contributions are smaller than those of Fig.1.

The 3-particle ρ -meson wave function is defined by the thre-local matrix element ***:

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* Let us point also that the ratio $\chi_0(3415) \to \rho\rho / \chi_0(3415) \to \pi\pi$ estimated by the method of S.Brodsky and P.Lepage is equals to: $\simeq (f_\rho/f_\pi)^4 \simeq 5$,i.e. $Br(\chi_0 \to \rho\rho) \simeq 5\%$ and this contradicts to the experimental data /7/: $Br(\chi_0 \to \rho\rho) < Br(\chi_0 \to \pi\pi) \simeq 1\%$.

** It is not difficult to see that the analogous contribution due to the 3-particle component of the π - meson wave function

$$\langle 0 | \bar{d}\, \sigma_{\mu\nu} \gamma_5 G^a_{\nu\lambda} \frac{\lambda^a}{2} u | \pi(p) \rangle = p_\mu p_\lambda f_{3\pi} \varphi_{3\pi}(z_i p)$$

is zero besause in this case the sum of the gluon spin projections in Fig.2 is: $\left| \sum_{i=1,2,3} S_z^i \right| = 3$.

*** (see the next page)

$$\langle 0|\bar d(z_1)\gamma_\lambda\gamma_5 g_s G^a_{\mu\nu}(z_3)\frac{\lambda^a}{2}u(z_2)|\rho^+_\perp(p)\rangle = p_\lambda \varepsilon_{\mu\nu\alpha\beta}\, p^\alpha \rho^\beta_\perp \cdot f_{3A}\cdot \varphi_{3A}(z_i p), \tag{20}$$

$$\varphi_{3A}(z_i p)=\int_0^1 dx_1\,dx_2\,dx_3\,\delta\left(1-\sum x_i\right)\varphi_{3A}(x_i)\exp\left\{i\sum x_i(z_i p)\right\},$$

$$\varphi_{3A}(z_1p=z_2p=z_3p=0)=1,$$

where $\rho^\mu_\perp$ -is the ρ- meson polarization vector with the helicity $|\lambda_\rho|=1$, $g_s^2/4\pi=\alpha_s$, f_{3A} is the constant which determines the scale of the wave function and is analogous to f_π or f_ρ (f_{3A} has the dimensionality (mass)^2). The dimensionless wave function $\varphi_{3A}(x_1,x_2,x_3)$ in (20) describes the distribution of quark and gluon longitudinal momenta in the ρ- meson, $p_i=x_i p$.

The diagrams like that in Fig.2 give the contribution to the decay amplitude M_0 (see (7)):

$$M_0^{(3)}=(4\pi\tilde\alpha_s)^2\frac{10}{27}\frac{f_\pi f_{3A}\varphi_V(0)}{M_\psi^5}\left\langle\frac{1}{x_1x_2}\,\frac{1}{1+\xi(2x_1-1)}\,\frac{1}{1-\xi(2x_2-1)}\right\rangle, \tag{21}$$

$$\langle O(x_i,\xi)\rangle \equiv \int_0^1 dx_1\,dx_2\,dx_3\,\delta\left(1-\sum x_i\right)\varphi_{3A}(x_i)\int_{-1}^1 d\xi\,\varphi_\pi(\xi)\,O(x_i,\xi).$$

One has from (21),(8):

$$\frac{\Gamma^{(3)}(\psi\to\rho\pi)}{\Gamma(\psi\to e^+e^-)}=\frac{50\pi^2}{27}\,\frac{\tilde\alpha_s^4}{\alpha^2}\left|\frac{f_\pi f_{3A}}{M_\psi^3}\langle\dots\rangle\right|^2. \tag{22}$$

The wave functions $\varphi_\pi(\xi)$ and $\varphi_V(0)$ are defined by (5),(6).

*** (from page 9). Besides, there is the 3-particle wave function of the ρ- meson which is defined by

$$\langle 0|\bar d\gamma_\lambda i g_s G^a_{\mu\nu}\frac{\lambda^a}{2}u|\rho_\perp(p)\rangle = p_\lambda(p_\mu\rho^\nu_\perp - p_\nu\rho^\mu_\perp)f_{3V}\,\varphi_{3V}(z_ip)$$

and corresponds to the p-wave state of two quarks. We expect, therefore, that φ_{3V} is no more important than φ_{3A} (see Appendix).

Using the QCD sum rules /12/ we have found (see Appendix) that: $f_{3A}\simeq 0.6\cdot10^{-2}\,\mathrm{GeV}^2$ and that the wave function $\varphi_{3A}(x_i)$ don't differs greatly from its asymptotic form: $\varphi^\infty_{3A}(x_i)=360x_1x_2x_3^2$ (x_3 is the longitudinal momentum fraction carried by the gluon, x_1 and x_2 -by the quarks). Using the wave functions: $\varphi_{3A}(x_i)\simeq\varphi^\infty_{3A}(x_i)$ and $\varphi_\pi(\xi)\simeq\frac{15}{4}(1-\xi^2)\xi^2$ one obtains for the integral in (21): $\langle\dots\rangle\simeq 45$.

Substituting all that into (22) one has:

$$\frac{\Gamma^{(3)}(\psi\to\rho\pi)}{\Gamma(\psi\to e^+e^-)}\simeq 0.5\,\tilde\alpha_s^4 \lesssim 1.3\cdot10^{-2} \quad \text{at} \quad \tilde\alpha_s\le0.4. \tag{23}$$

Therefore, this contribution is small as compared with (11).

IV. The $\psi\to\pi^0\gamma$ decay.

The contribution of the diagram in Fig.3 with the photon exchange into the $\psi\to\pi\gamma$ decay amplitude is:

$$M(\pi^0\gamma)=\varepsilon_{\mu\nu\lambda\sigma}\gamma^\mu\psi^\nu p_1^\lambda p_2^\sigma M_\gamma;\quad M_\gamma=\frac{4\sqrt2}{9}|e|^3\frac{f_\pi\varphi_V(0)}{M_\psi^3}\left\langle\frac{1}{1-\xi}\right\rangle_\pi, \tag{24}$$

where γ_μ -is the photon polarization vector, $e^2/4\pi=\alpha\simeq1/137$.

The virtuality of the quark propagator in Fig.3 is: $\bar{k}^2\simeq\frac12(1-\xi)M_\psi^2\simeq1.5\,\mathrm{GeV}^2$. At such a virtuality: $\langle1/1-\xi\rangle_\pi\simeq2.3$.

So, the Fig.3 contribution to the decay probability is:

$$\frac{\Gamma_\gamma(\psi\to\pi^0\gamma)}{\Gamma(\psi\to e^+e^-)}=\frac{4\pi\alpha}{9}\,\frac{f_\pi^2}{M_\psi^2}\left|\left\langle\frac{1}{1-\xi^2}\right\rangle_\pi\right|^2\simeq10^{-4}, \tag{25}$$

i.e. $\Gamma_\gamma(\psi\to\pi^0\gamma)/\Gamma_{tot}(\psi)\simeq7\cdot10^{-6}$

which is considerably smaller than the experimental value /7/:

$$\Gamma(\psi\to\pi^0\gamma)/\Gamma_{tot}(\psi)=(7.3\pm4.7)\cdot10^{-5}. \tag{26}$$

The two-loop diagrams shown in Fig.4 contribute as well into $\psi\to\pi^0\gamma$ decay. Each of the Fig.4 diagrams gives the contribution $\sim \ln^4 M_\psi$. However, the contributions $\sim \ln^4 M_\psi$ and $\sim \ln^3 M_\psi$ cancel, as usually, in the sum of the diagrams. What is more interesting, the contributions $\sim \ln^2 M_\psi$ cancel as well. We estimated numerically the Fig.4 diagram contributions into the invariant amplitude (see (24)):

$$M_{2\,loop} \lesssim \tilde{\alpha}_s^3 |e| \frac{f_\pi \varphi_\psi(0)}{M_\psi^3} \frac{1}{4\pi} \ln M_\psi^2/\bar{m}^2 \simeq \tilde{\alpha}_s^3 M_\gamma << M_\gamma . \qquad (27)$$

Therefore, the contributions into the $\psi\to\pi^0\gamma$ amplitude described by the Fig.4 diagrams and connected with the small distances are small. There are, however, the contributions shown in Fig.5 and corresponding to that region of variables in Fig.4, where the quark emitting the photon has small virtuality. The propagator of such a quark can not be taken as a perturbation theory propagator. If we restrict ourselves by the ρ- meson contribution, then we come, evidently, to the vector dominance model (VDM) and have:

$$M_{VDM}(\pi^0\gamma) = -\frac{|e|}{\sqrt{2}} \frac{f_\rho}{m_\rho} M_0 , \qquad (28)$$

where the decay amplitude $M_0(\psi\to\rho^0\pi^0)$ is defined in (4),(7). One can see from (28),(7) and (24) that <u>the signs of the VDM and Fig.3 contributions coincide</u>. In the numerical calculation bellow we use the experimental value of the amplitude M_0 corresponding to: $\Gamma(\psi\to\rho\pi)/\Gamma_{tot}(\psi) \simeq 0.4\%$ (compare with (12)). Then the total contribution of the Fig.3 and Fig.5 diagrams into the $\psi\to\pi^0\gamma$ decay width is equal to:

$$\frac{\Gamma(\psi\to\pi^0\gamma)}{\Gamma(\psi\to e^+e^-)} \simeq \left| \left(\frac{0.4\%}{7\%}\right)^{1/2} \frac{|e| f_\rho}{\sqrt{2}\, m_\rho} + 1\cdot 10^{-2} \right|^2 \simeq 5.3\cdot 10^{-4} . \qquad (29)$$

Finally, one has from (29):

$$\Gamma(\psi\to\pi^0\gamma)/\Gamma_{tot}(\psi) \simeq 3.7\cdot 10^{-5} .$$

This number does not contradict to the experimental value (26).

V. Conclusions.

The calculation of the $\psi\to\rho\pi$ decay width is a more complicated problem then the calculation of the meson form factors, $\chi_0(3415)\to\pi\pi$ decay width, etc. The reason is in the suppression of the $\psi\to\rho\pi$ decay amplitude in the $M_c\to\infty$ limit and that is why there are a number of decay mechanisms which compete with each other. There appears, in particular, the contribution due to 3-particle component of the ρ -meson wave function.

Using the method of QCD sum rules /12/ we find out the properties of various mesonic wave functions and using these wave functions we calculate the $\psi\to\rho\pi$ decay width. Our results for the $\psi\to\rho\pi$ decay width are in a reasonable agreement with the experimental data.*

We have shown also that the positive interference between the VDM (Fig.5) and photon exchange (Fig.3) contributions plays an essential role in the explanation of the $\psi\to\pi^0\gamma$ decay properties.

We are grateful to I.B.Khriplovich for the useful discussion.

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* We have for the $\psi'(3685)\to\rho\pi$ decay:

$$Br(\psi'\to\rho\pi) \simeq \left(\frac{M_\psi}{M_{\psi'}}\right)^6 Br(\psi\to\rho\pi) \frac{Br(\psi'\to e^+e^-)}{Br(\psi\to e^+e^-)} \simeq 6\cdot 10^{-2}\ \% .$$

APPENDIX

We describe bellow the application of the QCD sum rules/12/ to the determination of the 3-particle ρ -meson wave function properties.*

The matrix element in (20) is determined by the constant f_{3A} in the local limit $z_1 = z_2 = z_3 = 0$. In order to determine the value of this constant we consider the correlator:

$$T = i\int dx\, e^{iqx} \langle 0 | T J_\nu(x) J_{\nu'}(0) | 0 \rangle =$$

$$(\sigma q)^2 \left[(\sigma q)(\sigma_\nu q_{\nu'} + \sigma_{\nu'} q_\nu) - (\sigma q)^2 g_{\nu\nu'} \right] J(q^2) + \ldots \tag{A1}$$

$$J_\nu(0) = \left[\bar{u}(0) g_s G^a_{\mu\nu}(0) \frac{\lambda^a}{2} \gamma_\lambda \gamma_5 d(0) \right] \sigma_\mu \sigma_\lambda , \quad \sigma^2 = 0 ,$$

where σ_μ is an arbitrary lightlike vector. The QCD sum rule for the constant f_{3A} has the form (Figs.6,7,8):

$$f^2_{3A} \exp\left\{ -m^2_\rho / M^2 \right\} = \frac{\alpha_s}{720\pi^3} M^4 \left\{ 1 - \left(1 + \frac{s_0}{M^2} \right) e^{-s_0/M^2} \right\} + \tag{A2}$$

$$\frac{5\pi^2}{18} \frac{\ln M^2/\mu^2}{M^4} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle + \frac{140}{3}\pi^3 \frac{\langle 0 | \sqrt{\alpha_s}\, \bar{u} u | 0 \rangle^2}{M^6} + \ldots$$

The standard treatment gives:

$$f_{3A} \simeq 0.6 \cdot 10^{-2}\ \mathrm{GeV}^2 . \tag{A3}$$

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* The method of using the QCD sum rules for determination of wave function properties is described in /9/. In this Appendix we give therefore the results only.

The smallness of f_{3A} is due to the following points: i) the scale of the power corrections in (A2) (Figs.7,8) is not large; ii) the Born contribution,Fig.6,is small due to two loops.

We have considered also the analogous sum rules for the wave function moments (see /9/). The results are as follows. For the wave function $\varphi_{3A}(z,\xi) = \varphi_{3A}(z,-\xi)$ normalized by

$$\langle 1 \rangle \equiv \int_{-1}^{1} d\xi \int_0^1 dz\, \varphi_{3A}(z,\xi) = 1 , \quad \xi = x_1 - x_2 , \quad z = x_3 \tag{A4}$$

we have obtained:

$$\langle z \rangle_{1\,\mathrm{GeV}} = 0.84 \cdot \frac{3}{7} ; \quad \langle \xi^2 \rangle_{1\,\mathrm{GeV}} = 1.5 \cdot \frac{1}{14} . \tag{A5}$$

The wave function $\varphi_{3A}(z,\xi)$ depends (due to the loop logarithmic corrections in higher orders of the perturbation theory) on the normalization point μ and at $\mu \to \infty$ has the form:

$$\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{-\gamma_0} \varphi_3(z,\xi)_\mu \to 45 z^2 \left[(1-z)^2 - \xi^2 \right] \equiv \varphi^\infty_{3A}(z,\xi) , \tag{A6}$$

where γ_0 is the anomalous dimensionality of the current in (A1). One has for the asymptotic wave function φ^∞_{3A} : $\langle z \rangle = 3/7$; $\langle \xi^2 \rangle = 1/14$. One can see from (A5) that at $\mu \simeq 1\,\mathrm{GeV}$ the quark distribution in $\xi = x_1 - x_2$ is somewhat more wide,and the momentum fraction carried by the gluon is somewhat smaller,as compared with the φ^∞_{3A} wave function. The differences are not very large,however,and besides the integral in (21) is not very sensitive to the precise form of $\varphi_{3A}(z,\xi)$. Therefore,we use in the text: $\varphi_{3A} \simeq \varphi^\infty_{3A}$.

We have considered also the sum rules for the wave function $\varphi_{3V}(x_i)$ (see footnote at page 9) and convince ourselves that $f_{3V} < f_{3A}$.

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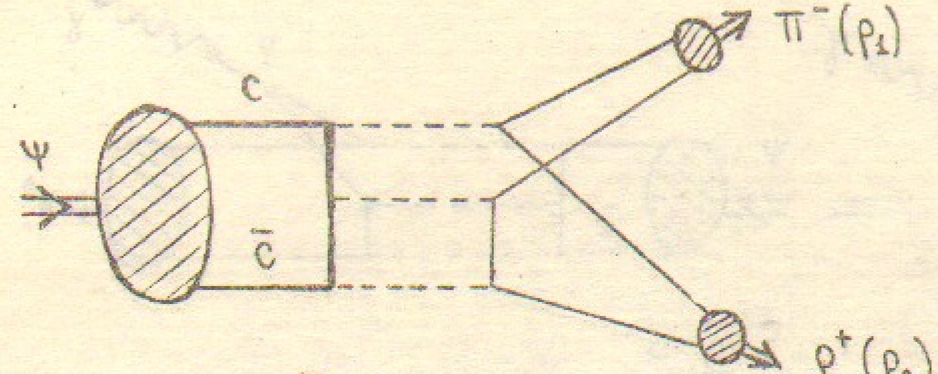


Fig. 1

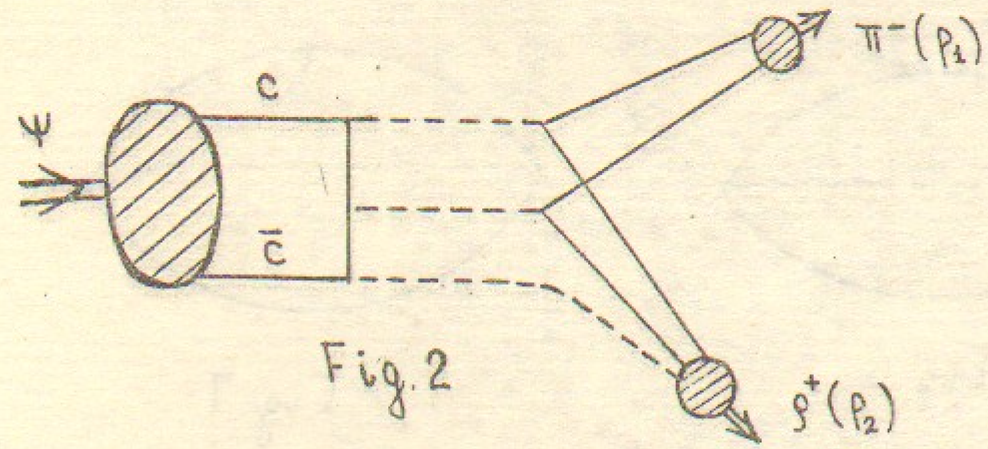


Fig.2

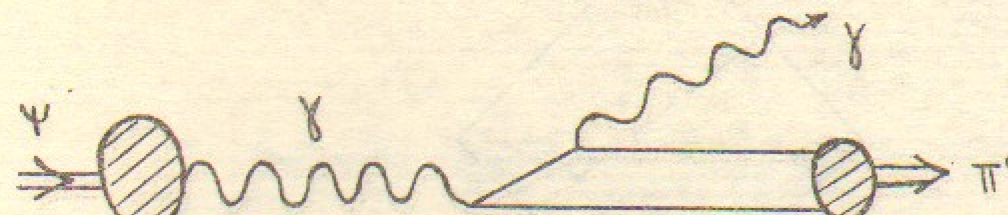
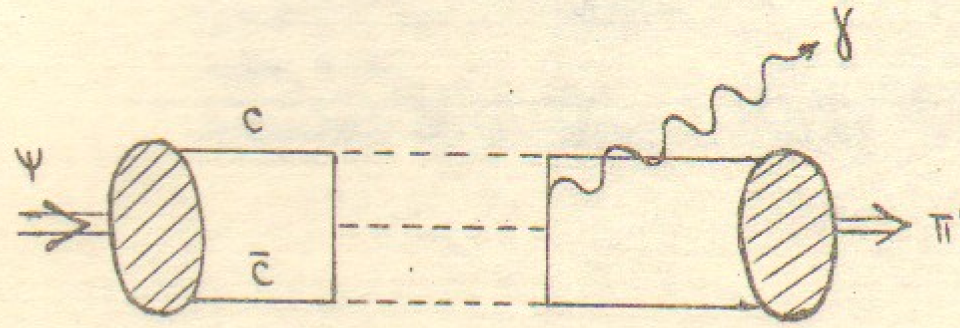


Fig. 3



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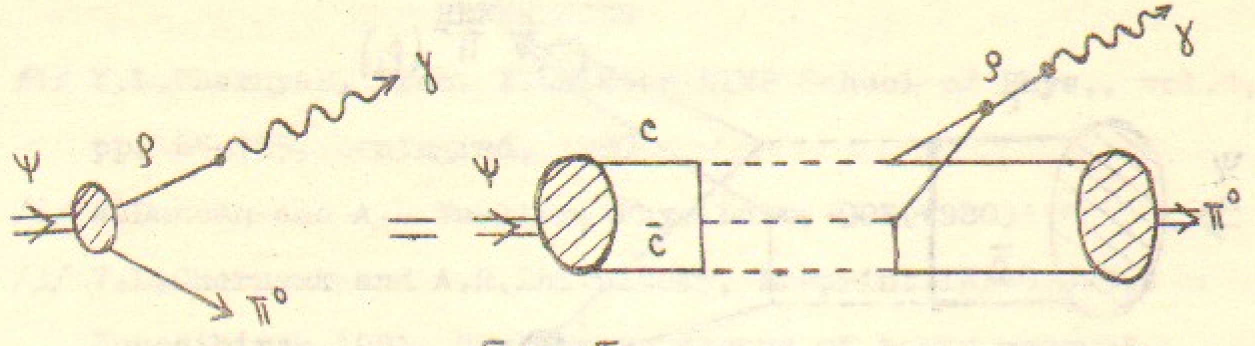

Fig. 5

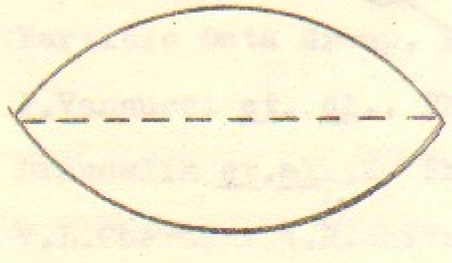

Fig. 6

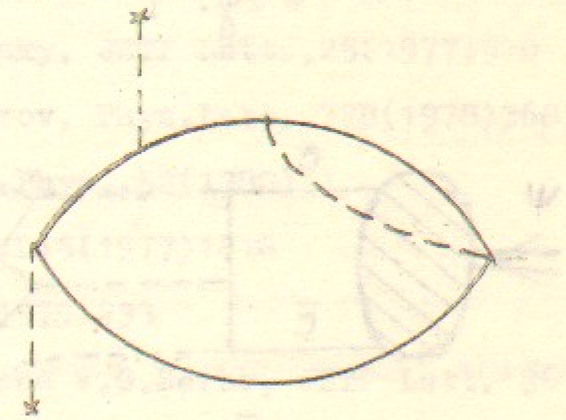

Fig. 7

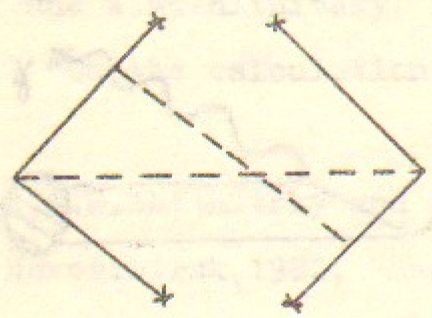

Fig. 8

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