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RADIATIVE POLARIZATION IN STORAGE RINGS

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RADIATIVE POLARIZATION IN STORAGE RINGS

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A b s t r a c t

In the work presented here some simple examples are given to illustrate the results of the theoretical studies performed earlier. The spin perturbations are written down in the linear approximation for calculation of the spin orbital coupling parameter.

C o n t e n t s

Introduction

The purpose of this work is to popularize the results given in /1-9/. Two throughly studied (calculated) projects for inserts into the storage ring VEPP-4 (Novosibirsk) are available now with qualitative recommendations of the magnetic field parameters, quadroupoles and other components introduced in the storage ring for getting the stable longitudinal polarization at the collision point /10,11/. The first project is the project of the so-called "siberian snake" at an energy 2 GeV. The second one (at 5 GeV) is the scheme of getting longitudinal polarization with recuperation of vertical polarization at the straight section output with the help of inserts of longitudinal fields and quadrupoles.

2. Spin motion equation

At large lifetimes of beams in the storage rings it becomes feasible the display of polarizing and depolarizing effects of rather weak "sources" of electro-magnetic field. An important case is a proper account of the influence of the particle radiation in the storage ring magnetic field on a spin. Both the direct changes in spin occur directly in the acts of irradiation and also changes because of the particle orbit deviations. The latter mechanism is linked to the dependence of the spin precession frequency on particle trajectory in inhomogeneous magnetic field.

Such a complex at first sight problem turns out to be completely solvable for particle motion in the storage rings with an arbitrary geometry of their magnetic fields. The theory enables describing any situations in the real storage rings. In particular, these can be inserts of various kind inserts into the straight sections of the storage rings ("siberian snakes", for instance, etc.). The description is based on the consept of the quntizing axis on an arbitrary particle trajectory in a storage ring.

The spin motion is described by the well known equation:

$$\dot{\vec{S}} = [\vec{\Omega}\vec{S}] , \qquad \vec{\Omega} = -\frac{q}{\gamma}\vec{H}_c + \frac{\gamma-1}{v^2}[\vec{v}\dot{\vec{v}}] , \tag{2.1}$$

where $\vec{H}_c = \gamma\vec{H}_\perp + \vec{H}_\parallel$ is a magnetic field in the particle rest frame, q is a giromagnetic ratio, $\gamma = (1-v^2)^{-1/2}$ is the relativistic factor (c = 1). The first term in $\vec{\Omega}$ is due directly to the particle magnetic moment $\vec{\mu} = q\vec{S}$, the second term proportional to the normal part of giromagnetic relation $q_0 = e/m$ ($[\vec{v}\dot{\vec{v}}] = q_0[\vec{v}(\vec{E}+[\vec{v}\vec{H}])]$) describes the Thomas precession and it is the consequence of the relativistic rotation kinematics. As is seen, the normal q_0 and anomalous $q_a = q - q_0$ parts of giromagnetic relation are not additively comprised in the equation. This fact is associated with the particle trajectory curved.

The formula for $\vec{\Omega}$ is convenient to be written in the form

$$\vec{\Omega} = \left(1+\gamma\frac{q_a}{q_0}\right)\frac{[\vec{v}\dot{\vec{v}}]}{v^2} - \frac{q}{\gamma}(\vec{H}\vec{v})\frac{\vec{v}}{v^2} + \frac{q}{\gamma^2 v^2}[\vec{v}\vec{E}] . \tag{2.2}$$

At $\gamma \gg 1$ the last term can be neglected. From this formula follows that during the motion in given fields at $\gamma \to \infty$ the anomalous part of angular precession frequency is not decreased.

It is convenient to replace in (2.1) to the generalized azimuth of particle motion θ as it is usually done in describing orbital motion in the storage ring. Then we have

$$\vec{S}' \equiv d\vec{S}/d\theta = [\vec{W}\vec{S}] , \qquad \vec{W} = \vec{\Omega}/\dot{\theta} . \tag{2.3}$$

3. Spin motion along the closed orbit

Let us start describing motion along the closed orbit. On the closed trajectory the spin angular precession frequency $\vec{W} = \vec{W}_s(\theta)$ is a periodic function of a generalized azimuth θ ($\theta/2\pi$ is a length along an equilibrium orbit in units of its circumference):

$$\vec{W}_s(\theta) = \vec{W}_s(\theta + 2\pi) .$$

It is evident that for solving the problem on polarization dynamics it is sufficient to build the solution of a spin motion

$$\vec{S}' = [\vec{W}_s\vec{S}] \tag{3.1}$$

on one period of a particle motion.

Let us consider a group of particles starting their motion with a certain azimuth θ with various spin initial conditions. Since the spin motion is a rotation (conserved a relative spin orientation) at any complex spin trajectory described by equation (3.1) the spin orientation transformation is reduced to the simple turn around a certain direction $\vec{n}_s$ ($\vec{n}_s^{\,2} = 1$) at the same angle φ . In two turns the spins apparently turned around $\vec{n}_s$ at an angle 2φ etc., i.e. the particle spins will rotate around $\vec{n}_s$. A rotation axis $\vec{n}_s$ and angle φ are determined by the field structure along the beam orbit and its energy. Let now polarization at the initial instant be directed along $\vec{n}_s$. Then having executed (during the particle rotation period), generally speaking the quite complex motion $\vec{n}_s(\theta)$ the polarization will return to its initial direction and in this way it will repeat at any orbit point in an arbitrary number of turns. The said above means that at any field and orbit configuration there always exists the polarization periodical motion executed at special initial condition.

A direction given by a periodical unit vector $\vec{n}_s$ determines the direction of the particle quantization axis on the closed orbit.

Two other eigen solutions $\vec{\eta}$ and $\vec{\eta}^*$ of equation (3.1) are complex and have the property

$$\vec{\eta}(\theta + 2\pi) = \vec{\eta}(\theta)\exp(-2\pi i \nu) , \tag{3.2}$$

where $\nu = \varphi/2\pi$ is the spin motion generalized frequency. The spin motion with any arbitrary initial orientation is a linear combination of the eigen solutions:

$$\vec{S} = S_n\vec{n}_s + S_\perp\vec{\eta}/2 + S_\perp^*\vec{\eta}^*/2$$

where $S_n = \vec{S}\vec{n}_s$ and $S_\perp = \vec{S}\vec{\eta}^*$ are constant. One can introduce two orthogonal (with respect to $\vec{n}_s$) orths $\vec{e}_1$ and $\vec{e}_2$ also periodical on azimuth. Then

$$\vec{\eta} = (\vec{e}_1 + i\vec{e}_2)\exp(-i\nu\theta) . \qquad (3.3)$$

In the system of movable orths $\vec{n}_s, \vec{e}_1, \vec{e}_2$ the spin motion is as simple as in the case of a homogeneous field:

$$\vec{S}' = [(\vec{W}_s - \vec{W}_b)\vec{S}] = \nu[\vec{n}_s \vec{S}] , \qquad (3.4)$$

where $\vec{W}_b = (\vec{e}_1'\vec{e}_2)\vec{n}_s + (\vec{e}_2'\vec{n}_s)\vec{e}_1 - (\vec{e}_1'\vec{n}_s)\vec{e}_2$ is an angular frequency of the movable system. Since the orths are fixed for each point of an orbit (given by the azimuth θ) the polarization change in one turn of a partice will be the same both in this and laboratory frames. The equation (3.4) describes just the spin rotation on the closed orbit with a frequency ν . It is easy to get the relation of the precession generalized frequency ν and an angular rotation velocity of the spin $\vec{W}_s$:

$$\nu = \vec{W}_s \vec{n}_s - \vec{e}_1' \vec{e}_2 . \qquad (3.5)$$

Except for special cases the frequency ν is not multiple to the integer number and therefore the spin periodical motion $\vec{n}_s(\theta)$ is the only one. At $\nu = K$ (spin resonance point) the motion is periodical at any initial condition and $\vec{n}_s$ will be whole uncertain.

4. Simple examples

An ideal storage ring

In such a storage ring there is no coupling between the z - and radial oscillations, z - and phase oscillations. At the motion along the closed orbit the field at any point of an orbit is vertical

$$\vec{H}_s / \langle H_z \rangle = \mathcal{K}\vec{e}_z ,$$

where $\mathcal{K}(\theta) = H_z / \langle H_z \rangle$ is the orbit dimensionless curvature ($\langle \mathcal{K} \rangle = \int_0^{2\pi} \mathcal{K} d\theta / 2\pi = 1$). In a circular storage ring $\mathcal{K} = 1$). Therefore in an ideal storage ring the azimuth periodicity property is posessed by the spin solution directed along the field:

$$\vec{n}_s = \vec{e}_z .$$

The orthogonal solution describes the rotation around $\vec{e}_z$

$$\vec{\eta} = (\vec{e}_x + i\vec{e}_y)\exp(-i\nu\tilde{\mathcal{K}}) , \qquad (4.1)$$

where $\tilde{\mathcal{K}} = \int_0^\theta \mathcal{K} d\theta$, $\vec{e}_x$ and $\vec{e}_y$ are the orths in the orbite plane and directed respectively along the normal and trajectory (accelerating basis)

$$\vec{e}_y = \vec{z}_s'/R = \vec{v}_s/v_s , \quad \vec{e}_x' = \mathcal{K}\vec{e}_y , \quad \vec{e}_y' = -\mathcal{K}\vec{e}_x .$$

A precession frequency ν for an ideal storage ring is equal to

$$\nu = \nu_0 \equiv \gamma q_a / q_0 .$$

The periodical orths $\vec{e}_1$ and $\vec{e}_2$ as is seen from relations (3.3) and (4.1) are

$$\vec{e}_1 + i\vec{e}_2 = (\vec{e}_x + i\vec{e}_y)\exp[-i\nu_0(\tilde{\mathcal{K}} - \theta)] \qquad (4.2)$$

and in a circular storage ring coincide with $\vec{e}_x$ and $\vec{e}_y$. In the storage ring with straight sections the spin component laying in the orbit plane is rotating around $\vec{n}_s$ with respect to $\vec{e}_x$ with a variable frequency $\nu_0\mathcal{K}$ and with constant frequency $\nu_0\langle\mathcal{K}\rangle = \nu_0$ with respect to the orth $\vec{e}_1$.

Near the values $\nu = K$ the spin motion is unstable and small perturbing fields strongly vary the direction $\vec{n}_s$. The energy values multiple to 440 MeV are resonant.

An ideal storage ring with introduce the field into the straight section for getting the longitudinal polarization at the point of collisions.

A proof of polarization stability (not obligatory directed along the field) opened up wide possibilities for control the spin at the beam collision points. It is easy to invent many ways for getting, for instance, longitudinal polarization at the collision point. The simple examples can be presented by the variants given in Fig. 1 and 2. Into the straight section of the storage ring $0 < \theta < \theta_0$ three sections are introduced (I,II,III) with radial fields (Fig. 1) and four sections (I,II,III,IV)(Fig. 2)(the latter variant is given in Ref. /12/).

The longitudinal polarization is performed at points (*). The arrows show the direction of $\vec{n}_s$.

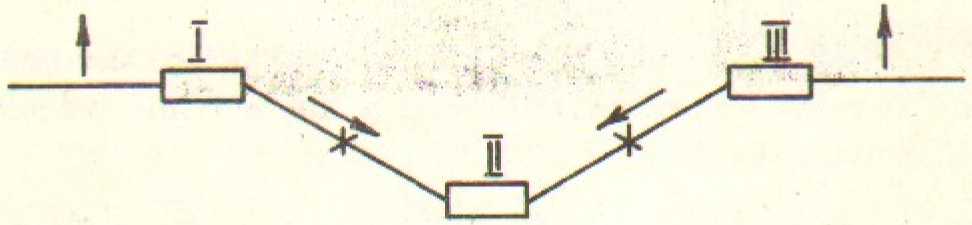

Fig. 1

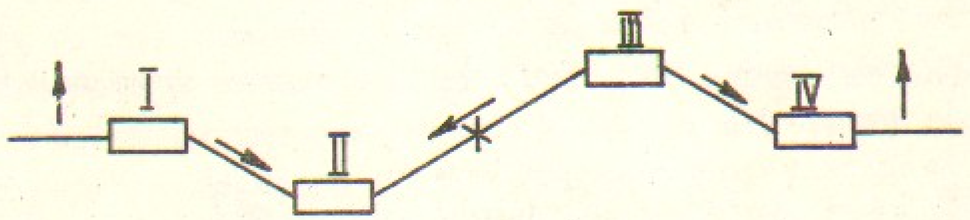

Fig. 2

The fields are introduced in such a way to restore the spin and orbit at the section output ($\nu_0 \gg 1$):

$$\int_0^{\theta_0} H_x d\theta = 0 \quad , \quad \int_0^{\theta_0} \theta H_x d\theta = 0 \quad . \qquad (4.3)$$

For the spin turn at an angle $\pi/2$ with respect to velocity by a constantly directed transverse field it is required at the length ℓ :

$$H_\perp(\kappa G) \simeq 23/\ell(m) . \qquad (4.4)$$

The periodical solution is still directed along z -axis in the main section of the storage ring $\theta_0 < \theta < 2\pi$. In the section with the radial magnetic fields $0 < \theta < \theta_0$ we have

$$\vec{n}_s(0 \div \theta_0) = \cos\varphi_\theta \, \vec{e}_z + \sin\varphi_\theta \, \vec{e}_y \, , \qquad (4.5)$$

where $\varphi_\theta = (\nu_0/\langle H_z \rangle)\int_0^\theta H_x d\theta$. The orthogonal solution is apparently the following:

$$\vec{\eta}(0 \div \theta_0) = \vec{e}_x + i\,(\cos\varphi_\theta \, \vec{e}_y - \sin\varphi_\theta \, \vec{e}_z)$$
$$\vec{\eta}(\theta_0 \div 2\pi) = (\vec{e}_x + i\,\vec{e}_y)\exp(-i\nu_0 \mathcal{K}) . \qquad (4.6)$$

We see that in one turn the solution $\vec{\eta}$ is turned at an angle $2\pi\nu_0$: $\vec{\eta}_{2\pi} = \vec{\eta}_0 \exp(-i2\pi\nu_0)$. That means the generalized frequency is also equal to $\nu_0 = \gamma q_a/q_0$. The periodical orths $\vec{e}_1$ and $\vec{e}_2$ are equal to

$$(\vec{e}_1 + i\,\vec{e}_2)_{0\div\theta_0} = \vec{\eta}(0 \div \theta_0) , \qquad (4.7)$$
$$(\vec{e}_1 + i\,\vec{e}_2)_{\theta_0\div 2\pi} = (\vec{e}_x + i\vec{e}_y)\exp[-i\nu_0(\mathcal{K} - \theta)] .$$

The common property for the variants with the radial field is that at the point of longitudinal polarization an incident velocity angle with respect to the horizontal plane depends on an energy and it is equal to $\pi/2\nu_0$. This may cause some inconveniences. A transition to the complex rotations in a magnetic field varied in its direction at the straight section removes this relation and gives additional freedom for the polarization and orbit control.

Let us give an example with the total restoration of the velocity and orbit position at the longitudinal polarization point. Let us introduce into the straight section the transverse helical magnetic field at a length ℓ_0 with one period:

$$H_x + i\,H_z = i\,|H_\perp^0|\exp(i\varkappa y + i\alpha) ,$$

where $|H_\perp^0| = const$, $\varkappa\ell_0 = 2\pi$, $\alpha = const$ is an angle between z -axis and initial field orientation. If the values of α and $|H_\perp^0|\ell_0$ are selected approximately $\alpha \simeq 0.26$, $|H_\perp^0|\ell_0 \simeq 2.07\pi/q_a \approx 95\ \kappa G \cdot m$ for electrons, then after passing this section the spin alters its vertical direction to the longitudinal one with restoration of its velocity. The orbit shift occured in the direction close to the vertical (by the value $\Delta z \approx \ell_0/\nu_0$) is easily compensated by introducing before the collision point the horizontal magnetic field with the zeroth value which does not distort the spin and velocity directions. The reverse turn of the spin to the vertical direction after collision point is similarly made by the opposite image magnetic fields.

At the present time there are many proposed variants of producing the longitudinal polarization with the spin and orbital motion restoration at the section output (see, example, Ref. /13 , 14 etc/).

An example with one "siberian snake"

Let us introduce into the storage ring section the fields bending after passing this section $0<\theta<\theta_0$ the spin at an angle π around the direction layed in the orbit plane. If one uses the longitudinal magnetic field, its required value $H_{\parallel}$ at length ℓ is equal to

$$H_{\parallel}(kG)\cdot \ell(m) \simeq 100\, E\,(GeV)\,. \qquad (4.8)$$

At high energies ($\nu_0 >> 1$) it is reasonable to use the transversal fields. There already are many variants of the spin turn at an angle π (see Ref. /15, 18/) which allow the simultaneous compensation for the orbit distortions (siberian snakes). Two most simple examples are given in Fig. 3 and 4.

In these variants the velocity direction is restored. The space shift occured is compensated for at the next section by a constantly directed (vertical) field with the zeroth mean value.

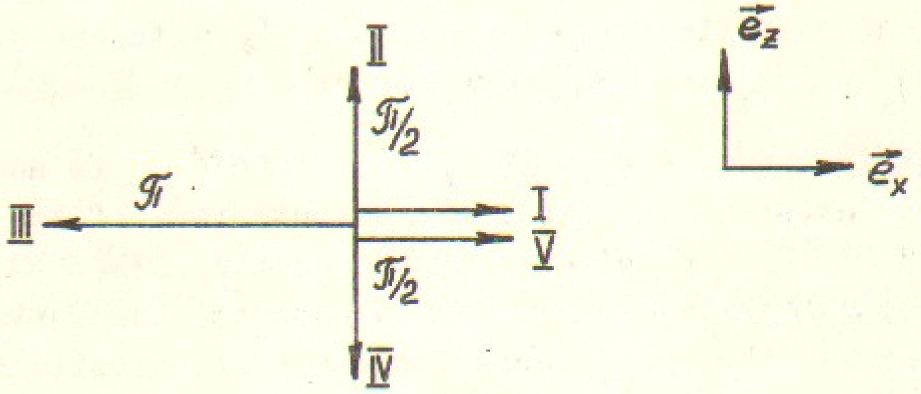

Fig. 3

The scheme of the spin turn around the velocity at an angle 180° by the transversal fields in the section with the particle velocity restoration. The picture plane is transversal to the velocity. Magnetic fields are introduced into five sections in a row. In the sections I,II,IV,V the spin is turned at 90° around the field direction, in the section III with radial field - at 180°.

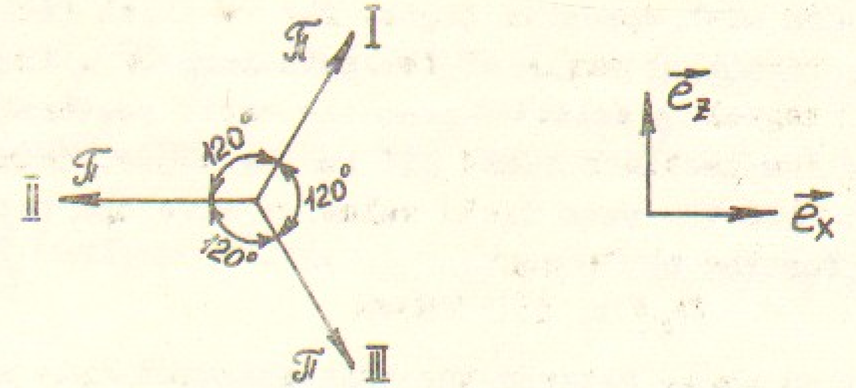

Fig. 4

The scheme of the spin turn around the radial direction at 180°. The field in section II is radial.

Let us also give an example with more economical field integral (Fig. 5). Into the section II the one helical period field is introduced

$$H_x + i H_z = H_\perp \exp(i \varkappa \theta)\,,$$

where $\varkappa$ and $H_\perp$ are constant, $\varkappa \ell_{II} = 2\pi$.

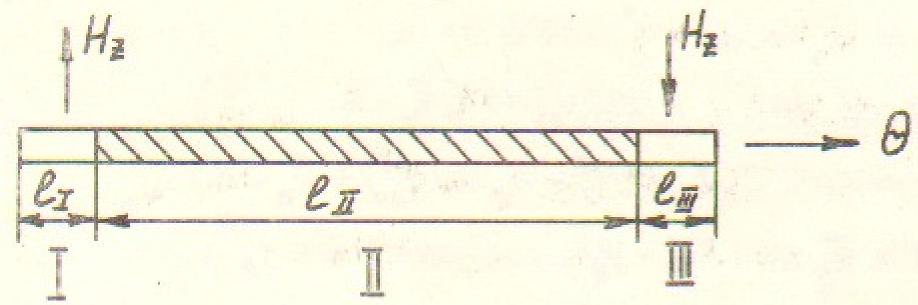

Fig. 5

Diagram of the snake with the helical field.

For the spin turn at an angle π after passing the section II it is required the field integral

$$H_\perp \ell \simeq 100\ kG\cdot m\ .$$

The magnetic field on the section II input can be oriented in radial direction. Then after passing the helical field the orbit will be also shifted along $\vec{e}_x$. This shift can be compensated for if in the sections I and III the vertical field are introduced with opposite signs. The vertical fields do not alter the resulting angle of the spin turn π. The vertical field integral is selected from the orbit restoration condition. Placing the sections I and III in the close vicinity of the section with the same field value we have the total field integral for the whole section

$$H_\perp \ell \simeq 130 \ KG \cdot m$$

α is an angle between the spin rotation axis and the velocity

$$\alpha \simeq -0{,}14 \ .$$

Let us give the formula for maxima of the orbit deviations in this scheme

$$z_{max} \simeq \frac{\ell_{\mathrm{II}}}{\pi \nu_0} \ , \qquad x_{max} \simeq \frac{3}{4} \frac{\ell_{\mathrm{II}}}{\pi \nu_0} \ .$$

Let us find out the polarization direction $\vec{n}_s$, the precession frequency ν and an orthogonal solution $\vec{\eta}$. Let into the section $0 < \theta < \theta_0$ the fields are introduced in such a way as shown in Fig. 3. Then, since the spin motion in each section is the rotation around the corresponding axis, we have*)

$$\vec{n}_s(0) = \vec{e}_x \sin \pi\nu_0 + \vec{e}_y \cos \pi\nu_0 \ ,$$
$$\vec{n}_s(\mathrm{I}) = \vec{e}_x \sin \pi\nu_0 + \cos \pi\nu_0 (\cos \varphi_\theta^{\mathrm{I}} \vec{e}_y + \sin \varphi_\theta^{\mathrm{I}} \vec{e}_z) ,$$
$$\vec{n}_s(\mathrm{II}) = \sin \pi\nu_0 (\vec{e}_x \cos \varphi_\theta^{\mathrm{II}} + \vec{e}_y \sin \varphi_\theta^{\mathrm{II}}) + \vec{e}_z \cos \pi\nu_0 \ ,$$
$$\vec{n}_s(\mathrm{III}) = \vec{e}_y \sin(\pi\nu_0 + \varphi_\theta^{\mathrm{III}}) + \vec{e}_z \cos(\pi\nu_0 + \varphi_\theta^{\mathrm{III}}) \ ,$$
$$\vec{n}_s(\mathrm{IV}) = -\sin \pi\nu_0 (\vec{e}_x \sin \varphi_\theta^{\mathrm{IV}} + \vec{e}_y \cos \varphi_\theta^{\mathrm{IV}}) - \cos \pi\nu_0 \vec{e}_z \ ,$$
$$\vec{n}_s(\mathrm{V}) = \cos \pi\nu_0 (\vec{e}_y \sin \varphi_\theta^{\mathrm{V}} - \vec{e}_z \cos \varphi_\theta^{\mathrm{V}}) - \sin \pi\nu_0 \vec{e}_x \ , \qquad (4.9)$$
$$\vec{n}_s(\theta_0 \div 2\pi) = \vec{e}_x \sin \nu_0 (\tilde{\varkappa} - \pi) + \vec{e}_y \cos \nu_0 (\tilde{\varkappa} - \pi) \ ,$$
$$\varphi_\theta^{\mathrm{I}} = \frac{\nu_0}{\langle H_z \rangle} \int_0^\theta |H^{\mathrm{I}}| d\theta \ , \quad \varphi_\theta^{\mathrm{II}} = \frac{\nu_0}{\langle H_z \rangle} \int_0^\theta |H^{\mathrm{II}}| d\theta \ \ldots$$

For the orthogonal solution we have

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*) In such cases the spin motion is more convenient to describe in a two-dimensional Paul representation. But we do not dwell upon this technique here, see Ref. /19, 3, 15, 20/.

$$\vec{\eta}(0) = \vec{e}_x \cos \pi\nu_0 - \vec{e}_y \sin \pi\nu_0 - i \vec{e}_z$$
$$\vec{\eta}(\mathrm{I}) = \vec{e}_x \cos \pi\nu_0 + \vec{e}_y (i \sin \varphi_\theta^{\mathrm{I}} - \sin \pi\nu_0 \cos \varphi_\theta^{\mathrm{I}}) - \vec{e}_z (i \cos \varphi_\theta^{\mathrm{I}} + \sin \pi\nu_0 \sin \varphi_\theta^{\mathrm{I}}) ,$$
$$\vec{\eta}(\mathrm{II}) = \vec{e}_x (\cos \pi\nu_0 \cos \varphi_\theta^{\mathrm{II}} - i \sin \varphi_\theta^{\mathrm{II}}) + \vec{e}_y (i \cos \varphi_\theta^{\mathrm{II}} + \cos \pi\nu_0 \sin \varphi_\theta^{\mathrm{II}}) - \sin \pi\nu_0 \ \vec{e}_z \ ,$$
$$\vec{\eta}(\mathrm{III}) = -i \vec{e}_x + \vec{e}_y (\cos \pi\nu_0 \cos \varphi_\theta^{\mathrm{III}} - \sin \pi\nu_0 \sin \varphi_\theta^{\mathrm{III}}) - \vec{e}_z (\sin \pi\nu_0 \cos \varphi_\theta^{\mathrm{III}} + \cos \pi\nu_0 \sin \varphi_\theta^{\mathrm{III}})$$
$$\vec{\eta}(\mathrm{IV}) = -\vec{e}_x (i \cos \varphi_\theta^{\mathrm{IV}} + \cos \pi\nu_0 \sin \varphi_\theta^{\mathrm{IV}}) + \vec{e}_y (i \sin \varphi_\theta^{\mathrm{IV}} - \cos \pi\nu_0 \cos \varphi_\theta^{\mathrm{IV}}) + \sin \pi\nu_0 \ \vec{e}_z \ ,$$
$$\vec{\eta}(\mathrm{V}) = -\cos \pi\nu_0 \vec{e}_x + \vec{e}_y (i \cos \varphi_\theta^{\mathrm{V}} - \sin \pi\nu_0 \sin \varphi_\theta^{\mathrm{V}}) + \vec{e}_z (i \sin \varphi_\theta^{\mathrm{V}} + \sin \pi\nu_0 \cos \varphi_\theta^{\mathrm{V}}) ,$$
$$\vec{\eta}(\theta_0 \div 2\pi) = -\vec{e}_x \cos \nu_0 (\tilde{\varkappa} - \pi) + \vec{e}_y \sin \nu_0 (\tilde{\varkappa} - \pi) + i \vec{e}_z \ .$$

The periodical orths $\vec{e}_1$ and $\vec{e}_2$ are as always equal to $\vec{e}_1 + i \vec{e}_2 = \vec{\eta} \exp(i \nu \theta)$. The generalized frequency (since $\vec{\eta}_{2\pi} = -\vec{\eta}_0$) is always equal to the half of unity independent of energy ($\nu = 0.5$).

The precession frequency in the example considered is considerably different from its value for the conventional storage ring where $\nu = \gamma q_a / q_0$ and varies continuously with an energy. Since $\nu = 0.5$, by correct selecting of betatron oscillations one can easily avoid all dangerous resonances at any energy value.

Example with two snakes

One can obtain the spin precession frequency equal to the half remaining the equilibrium polarization direction along the field in the main section. For this, let us introduce two snakes into two opposite straight sections of storage ring. The rotation axes of these two snakes should have an angle $\pi/2$. For example, in section I let's use the scheme of the snake given in Fig. 3 and in section II - the scheme of Fig. 4. (Note, that in section III with radial magnetic field (Fig. 3) the equilibrium polarization is produced along the field).

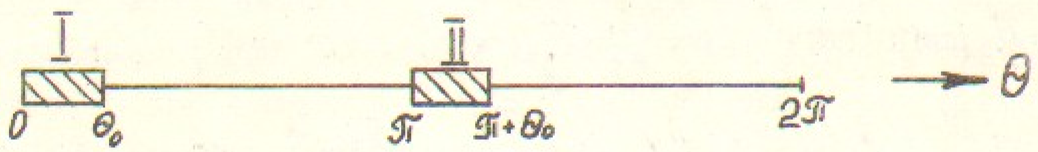

Fig. 6

Let us give the expressions for $\vec{n}_s$ and $\vec{\eta}$ in the main sections

$$\vec{n}_s(\theta_0 \div \pi) = \vec{e}_z \;, \qquad \vec{n}_s(\pi+\theta_0 \div 2\pi) = -\vec{e}_z \;,$$

$$\vec{\eta}(0) = -\vec{e}_x + i\vec{e}_y \;, \qquad \vec{\eta}(\theta_0) = \vec{e}_x + i\vec{e}_y \;,$$

$$\vec{\eta}(\theta_0 \div \pi) = (\vec{e}_x + i\vec{e}_y)\exp(-i\nu_0\tilde{\mathcal{K}}) \;,$$

$$\vec{\eta}(\pi) = (\vec{e}_x + i\vec{e}_y)\exp(-i\pi\nu_0) \;, \tag{4.10}$$

$$\vec{\eta}(\pi+\theta_0) = (\vec{e}_x - i\vec{e}_y)\exp(-i\pi\nu_0) \;,$$

$$\vec{\eta}(\pi+\theta_0 \div 2\pi) = (\vec{e}_x - i\vec{e}_y)\exp\Big[i\nu_0\Big(\int_{\pi+\theta_0}^{\theta}\mathcal{K}d\theta - \pi\Big)\Big] \;,$$

$$\vec{\eta}(2\pi) = \vec{e}_x - i\vec{e}_y$$

Since $\vec{\eta}(2\pi) = -\vec{\eta}(0)$, the precession generalized frequency as in the previous example, is equal to the half. The periodical orths $\vec{e}_1$ and $\vec{e}_2$ in the main section can be written in the form

$$\vec{e}(0) = -\vec{e}_x + i\vec{e}_y \;,$$

$$\vec{e}(\theta_0 \div \pi) = (\vec{e}_x + i\vec{e}_y)\exp\Big[-i\nu_0\int_{\theta_0}^{\theta}\mathcal{K}d\theta + i\,\theta/2\Big] \;,$$

$$\vec{e}(\theta_0+\pi) = (\vec{e}_x - i\vec{e}_y)\exp\big[-i\pi\nu_0 + i(\pi+\theta_0)/2\big] \;, \tag{4.11}$$

$$\vec{e}(\pi+\theta_0 \div 2\pi) = (\vec{e}_x - i\vec{e}_y)\exp\Big[-i\pi\nu_0 + i\nu_0\int_{\pi+\theta_0}^{\theta}\mathcal{K}d\theta + i\,\theta/2\Big] \;,$$

$$\vec{e}(2\pi) = -\vec{e}_x + i\vec{e}_y = \vec{e}(0) \;.$$

5. Finding the precession axis on the particle trajectory of deviated from the closed trajectory

For calculations of the radial polarization level and the time for its establishing one should also know the precession axis direction $\vec{n}$ on the trajectories deviated from the closed trajectory.

The angular frequency vector $\vec{W}$ at stationary conditions writted as a function for action variables I_i (amplitudes) and phases ψ_i of betatron and synchrotron oscillations has a property of the phase trajectory:

$$\vec{W}(I_i, \psi_i, \theta) = \vec{W}(I_i, \psi_i + 2\pi, \theta) = \vec{W}(I_i, \psi_i, \theta + 2\pi) \,.$$

Similarly to that as in the case of oneperiodical dependence at the motion along closed orbit there is the only possible direction for the precession axis $\vec{n}$ (excluding the spin resonance points) which posseses the property of periodicity over all phases*).

$$\vec{n}(I_i, \psi_i, \theta) = \vec{n}(I_i, \psi_i + 2\pi, \theta) = \vec{n}(I_i, \psi_i, \theta + 2\pi) . \tag{5.1}$$

Vector $\vec{n}$ being the solution of the spin motion equation (2.3) thus comprises only a fractio frequencies of an orbital motion. The spin projection $S_{\vec{n}} = \vec{S}\vec{n}$ is the exact motion integral in the storage ring field - the spin action variable (the spin quantum number). If a particle returns to the same point of a phase space (the same point and the same velocity) the precession axis direction is also repeated.

When moving along the closed orbit $\vec{n}$ is transmitted into $\vec{n}_s$. The deviation of $\vec{n}$ from $\vec{n}_s$ can be found out (far from the spin resonances) with a perturbation theory. General formulae of the linear approximation are given in Ref. /3,4/.

Let us shift to the movable system of the periodical orths $\vec{n}_s, \vec{e}_1, \vec{e}_2$ connected with the spin motion along the closed orbit. In this system an angular spin precession frequency $\vec{W}_0 = \vec{W} - \vec{W}_b$ is equal (compare with (3.4))

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*) Rigorous proof of this assertion is given in Ref. /5/.

$$\vec{S}' = [\vec{W}_0 \vec{S}] \quad , \qquad \vec{W} = \nu \vec{n}_s + \vec{w} \tag{5.2}$$

where ν is generalized precession frequency for the closed orbit determined by the formula (3.5), $\vec{w} = \vec{W} - \vec{W}_s$ is an angular velocity perturbation associated with the trajectory deviation. From the equation (5.2) at first approximation over $\vec{w}$ one can get the simple formula for calculation of $\delta \vec{n} = \vec{n} - \vec{n}_s$:

$$\delta \vec{n} = \mathcal{J}m\, \vec{\eta} \int_{-\infty}^{\theta} \vec{w} \vec{\eta}^{\,*} d\theta = \mathcal{J}m\, \vec{e}\, e^{-i\nu\theta} \int_{-\infty}^{\theta} \vec{w} \vec{e}^{\,*} e^{i\nu\theta} d\theta \tag{5.3}$$

Here, according to definition of $\vec{n}$ the integration constant is selected in such a way the spectrum of $\vec{n}$ should comprise only the perturbation frequencies. Formally, this is equivalent to integrating from $-\infty$ with a negative small imaginary adding to ν .

Sometimes it is reasonable to present the formula (5.3) in the form of expansion over the perturbation Fourier harmonics of $\vec{w}\vec{e}$. Let

$$\vec{w} \vec{e}^{\,*} = \sum_k w_k \exp(-i \psi_k)$$

where ψ_k is an integer combination of the orbital motion phases ($\psi_k = \sum k_i \psi_i$). From formula (5.3) one can get:

$$\delta \vec{n} = \mathcal{J}m\, \vec{e} \sum_k \frac{w_k}{i(\nu - \nu_k)} e^{-i\psi_k} ,$$

where $\nu_k = \sum k_i \psi_i'$ is a combination of the orbital frequencies. One can see that deviation of $\vec{n}$ is very sensitive to perturbations near the spin resonances

$$\nu \simeq \nu_k .$$

Having deviation $\delta \vec{n}$ known it is easy to calculate the spin-orbital coupling parameter $\gamma \partial \vec{n} / \partial \gamma$ through which the degree and time of relaxation for radiative polarization are expressed. The derivative $\gamma \partial \vec{n} / \partial \gamma$ is obtained at fixed transversal deviations x, z and velocities x', z'. At the presence of a coupling between the transversal motion and the phase motion,

not only the forced part proportional to ψ - function is changed but also the amplitudes of betatron oscillations.

The formulae for the equilibrium polarization degree ζ and relaxation time τ in any nonresonant situation*) is expressed through the sum α_+ and difference α_- of spin transition probabilities per unit time for a two level spin system:

$$\alpha_+ = \frac{5\sqrt{3}}{8} \hbar q_0^2 \gamma^5 \langle |\vec{v}|^3 [1 - \tfrac{2}{9} (\vec{n}_s \vec{v})^2 + \tfrac{11}{18} (\gamma \tfrac{\partial \vec{n}}{\partial \gamma})^2] \rangle ,$$

$$\alpha_- = -\hbar q_0^2 \gamma^5 \langle |\vec{v}|^2 [\vec{v} \dot{\vec{v}}] (\vec{n}_s - \gamma \partial \vec{n} / \partial \gamma) \rangle , \tag{5.4}$$

$$\zeta = \alpha_- / \alpha_+ \qquad , \qquad \tau = \alpha_+^{-1} ,$$

where brackets $\langle ... \rangle$ mean an averaging over the azimuth θ .

6. Formulae for perturbation $\vec{w}$ in linear approximation

The general formula for $\vec{w} = \vec{W} - \vec{W}_s$ can be obtained with (2.2) and (2.3). It is reasonable to give the real expressions in a linear approximation on the particle trajectory deviation.

Radius vector of the $\vec{r}$ particle in the accelerating orths system is written in form

$$\vec{r} = \vec{r}_0(\theta) + x\, \vec{e}_x(\theta) + z\, \vec{e}_z(\theta) , \tag{6.1}$$

where x is the radial and z is the vertical deviations of a particle from an ideal closed orbit $\vec{r}_0(\theta)$. The periodical orths $\vec{e}_z$, $\vec{e}_x$, $\vec{e}_y = R^{-1} d\vec{r}_0 / d\theta$ ($2\pi R$ is a length of an ideal orbit) satisfy the following relations:

$$\vec{e}_x{}' = \mathcal{K}_z \vec{e}_y \quad , \quad \vec{e}_y{}' = -\mathcal{K}_z \vec{e}_x + \mathcal{K}_x \vec{e}_z \quad , \quad \vec{e}_z{}' = -\mathcal{K}_x \vec{e}_y . \tag{6.2}$$

Here

*) Formulae (5.4) are applicable for the cases when the length of radiation $(q_0 H)^{-1}$ is much shorter than the characteristic period of magnetic field ω_H^{-1} variation. In the opposite case, $q_0 H \ll \omega_H$ the formulae for the level and time are obtained in Ref. /21/.

$$\mathcal{K}_z = -\frac{q_0 R}{\gamma_s v} H_z = \frac{H_z}{\langle H_z \rangle} \quad , \quad \mathcal{K}_x = -\frac{q_0 R}{\gamma_s v} H_x = \frac{H_x}{\langle H_z \rangle}$$

are the dimensionless components of the orbit curvature:

$$\int_0^{2\pi} \mathcal{K}_z d\theta = 2\pi \quad , \qquad \int_0^{2\pi} \mathcal{K}_x d\theta = 0 \, .$$

The spin angular velocity vector $\vec{W}$ can be presented in this system of accelerating orths, however the components are much simpler in the orth system associated with an exact direction of the velocity $\vec{v}$. Therefore the spin motion is conveniently described in the orth system

$$\vec{a} = \frac{[\vec{v}\,\vec{e}_z]}{|[\vec{v}\,\vec{e}_z]|} \quad , \qquad \vec{\tau} = \frac{\vec{v}}{v} \quad , \qquad \vec{b} = [\vec{a}\,\vec{\tau}] \, , \qquad (6.3)$$

slightly different (because of a small velocity spread in a beam) from the accelerating system (6.2). In the system (6.3) the components of the velocity precession in the directions $\vec{a}$ and $\vec{b}$ are proportional to an anomalous moment q_a and the component along $\vec{\tau}$ is slightly depend on the moment q_a ($q_a \ll q_0$). Substracting the value of an angular velocity of motion of the basis

$$\vec{W}_b = \left(\mathcal{K}_x + \frac{z''}{R}\right)\vec{a} + \left(\mathcal{K}_z - \frac{x''}{R}\right)\vec{b} + \left(\mathcal{K}_x \frac{x'}{R} + \mathcal{K}_z \frac{z'}{R}\right)\vec{\tau}$$

in the linear approximation on the trajectory deviations from the ideal one get the following simple expressions for angular velocity one of the spin rotation in the system (6.3):

$$W_a = \nu_0 \mathcal{K}_x + \nu_0 \left(\frac{\Delta\gamma}{\gamma}\mathcal{K}_x + \frac{z''}{R}\right),$$

$$W_b = \nu_0 \mathcal{K}_z + \nu_0 \left(\frac{\Delta\gamma}{\gamma}\mathcal{K}_z - \frac{x''}{R}\right), \qquad (6.4)$$

$$W_\tau = \mathcal{K}_y - \frac{\Delta\gamma}{\gamma}\mathcal{K}_y + \mathcal{K}_z' \frac{z}{R} + \mathcal{K}_x' \frac{x}{R} \, .$$

where $\Delta\gamma = \gamma - \gamma_s$, $\mathcal{K}_y = H_y / \langle H_z \rangle$.

Consequently, the following spin perturbation is occured when the orbit deviates from the closed one:

$$\vec{w} = \nu_0 \left(\frac{\Delta\gamma}{\gamma}\mathcal{K}_x + \frac{z''}{R}\right)\vec{a} + \nu_0 \left(\frac{\Delta\gamma}{\gamma}\mathcal{K}_z - \frac{x''}{R}\right)\vec{b} + \qquad (6.5)$$
$$+ \left(\mathcal{K}_z' \frac{z}{R} + \mathcal{K}_x' \frac{x}{R} - \frac{\Delta\gamma}{\gamma}\mathcal{K}_y\right)\vec{\tau} \, .$$

For compliteness let us give the known equations describing the transversal deviations of the orbit for a general case:

$$z'' + g_z z = g_{zx} x - \mathcal{K}_y x' - \frac{\Delta\gamma}{\gamma} R \mathcal{K}_x \, , \qquad (6.6)$$

$$x'' + g_x x = g_{xz} z + \mathcal{K}_y z' + \frac{\Delta\gamma}{\gamma} R \mathcal{K}_z \, ,$$

where

$$g_z = -\frac{R}{\langle H_z \rangle}\frac{\partial H_x}{\partial z} + \mathcal{K}_x^2 \quad , \quad g_{zx} = \frac{R}{\langle H_z \rangle}\frac{\partial H_x}{\partial x} + \mathcal{K}_x \mathcal{K}_z$$

$$g_x = \frac{R}{\langle H_z \rangle}\frac{\partial H_z}{\partial x} + \mathcal{K}_z^2 \quad , \quad g_{xz} = -\frac{R}{\langle H_z \rangle}\frac{\partial H_z}{\partial z} + \mathcal{K}_x \mathcal{K}_z$$

The relations between the field derivatives come from the equations $\operatorname{div} \vec{H} = 0$, $\operatorname{rot} \vec{H} = 0$:

$$\frac{\partial H_x}{\partial \theta} = R \frac{\partial H_\theta}{\partial x} + \mathcal{K}_z H_\theta \quad , \quad \frac{\partial H_z}{\partial \theta} = R \frac{\partial H_\theta}{\partial z} - \mathcal{K}_x H_\theta$$

$$\frac{\partial H_x}{\partial z} = \frac{\partial H_z}{\partial x} \quad , \quad \frac{\partial H_x}{\partial x} + \frac{\partial H_z}{\partial z} + \frac{1}{R}\frac{\partial H_\theta}{\partial \theta} = 0 \, . \qquad (6.7)$$

From the expression (6.5) for the spin perturbation occured at the orbit deviations one can see that at high energies ($\nu_0 \gg 1$) the only two components are separated along $\vec{a}$ and $\vec{b}$. The component w_τ only slightly corrects the spin orbital coupling parameter and further we neglect it (though it is easy to take into account this correction in concrete calculations).

7. Examples

Let us get the expressions for the precession direction axis the radiative polarization degree and relaxation time in the given above simple examples.

An ideal storage ring

In this case $\vec{n}_s = \vec{e}_z$ and the orthogonal solutions are rotating around z -axis with an instantaneous frequency (see 4.1). In an ideal storage ring

$$\mathcal{K}_x = \mathcal{K}_y = 0 \quad , \quad \mathcal{K}_z = \mathcal{K} \quad , \quad g_{zx} = g_{xz} = 0 \; .$$

Equations for x and z oscillations

$$z'' + g_z z = 0$$
$$x'' + g_x x = \frac{\Delta\gamma}{\gamma} R \mathcal{K} \, , \tag{7.1}$$

From the formulae (6.4) and (6.5) one get the expressions for an angular velocity of the spin rotation

$$\vec{W}_s = \nu_o \mathcal{K} \vec{b} \, , \tag{7.2}$$

$$w_a = \nu_o \frac{z''}{R} = -\nu_o g_z \frac{z}{R} \quad , \quad w_b = \nu_o \left(\frac{\Delta\gamma}{\gamma} \mathcal{K} - \frac{x''}{R} \right) = \nu_o g_x \frac{x}{R} \, .$$

With the help of the formula (5.3) for variation $\delta\vec{n}$ one has*)

$$\delta\vec{n} = -\mathcal{J}m \left\{ (\vec{e}_x + i\vec{e}_y) e^{-i\nu_o\tilde{\tau}} \int\limits_{-\infty}^{\theta} \nu_o g_z \frac{z}{R} e^{i\nu_o\tilde{\tau}} d\theta \right\} \, . \tag{7.3}$$

Consequently in an ideal storage ring a variation of is only associated with vertical deviation from the motion plane. This is an obvious fact since only vertical deviations lead to appearance of the radial component of magnetic field, and the particle radial deviations remain the magnetic field direction vertical.

Let us write down (with formula (7.3) the expressions first for the smoothed focusing and circular storage ring ($\mathcal{K}$ = I). In this case

*) Rigorously speaking, the formula (7.3) calculates $\vec{n}$ in a system of $\vec{a}, \vec{\tau}, \vec{b}$. Therefore, in linear approximation, since $\vec{b} \simeq \vec{e}_z - \frac{z'}{R}\vec{e}_y$, $\vec{n} \simeq \vec{e}_z - \frac{z'}{R}\vec{e}_y + \delta\vec{n}$, $-\frac{z'}{R}\vec{e}_y$ connected with the particle velocity angle relatively equilibrium, is non-resonant and it cannot be stored.

$$z/R = a e^{i\nu_z\theta} + a^* e^{-i\nu_z\theta} \, ,$$

where a = const is an amplitude of vertical oscillations. Substituting this expression into (7.3) one can get

$$\delta\vec{n} = -\nu_o\nu_z^2 \mathcal{J}m \Big\{ (\vec{e}_x + i\vec{e}_y) e^{-i\nu_o\theta} \int\limits_{-\infty}^{\theta} \left(a e^{i(\nu_o+\nu_z)\theta} + a^* e^{i(\nu_o-\nu_z)\theta} \right) d\theta =$$

$$= -\nu_o\nu_z^2 \mathcal{J}m \Big\{ (\vec{e}_x + i\vec{e}_y) \left[\frac{a e^{i\nu_z\theta}}{i(\nu_o+\nu_z)} + \frac{a^* e^{-i\nu_z\theta}}{i(\nu_o-\nu_z)} \right] \Big\}$$

Whence $\delta\vec{n}$ is equal to

$$\delta\vec{n} = \frac{\nu_o\nu_z^2}{(\nu_o^2 - \nu_z^2)} \left(\nu_z^2 \frac{z}{R} + \nu_o \frac{z'}{R} \right) . \tag{7.4}$$

From the latter expressions one can see that the precession axis deviation comprises only frequencies of vertical motion $\pm\nu_z$.

In the vicinity of the spin resonances $\nu_o \approx \nu_z$ the variation is strongly depend on the orbital parameter deviations.

The formula (7.4) can easily be generalized for any case. The betatron oscillations are described with the Floquet solution f_z :

$$\frac{z}{R} = a f_z(\theta) + a^* f_z^*(\theta) \tag{7.5}$$

where a = const is an amplitude,
is the Floquet solution for the vertical oscillation equation:

$$f_z'' + g_z f_z = 0 \tag{7.6}$$

The normalizing condition is the following: $\mathcal{J}m\, f_z' f_z^* = 1$
For the smoothed focusing $f_z = \nu_z^{-1/2} exp(i\nu_z\theta)$ Shifting to summation over turns in the integral of the formula (7.3) we have

$$\delta\vec{n} = -\mathcal{J}m (\vec{e}_x + i\vec{e}_y) e^{-i\nu_o\tilde{\tau}} \left\{ \frac{a \int\limits_{\theta-2\pi/p}^{\theta} \nu_o g_z f_z e^{i\nu_o\tilde{\tau}} d\theta}{1 - exp[-i\frac{2\pi}{p}(\nu_o+\nu_z)]} + \frac{a^* \int\limits_{\theta-2\pi/p}^{\theta} \nu_o g_z f_z^* e^{i\nu_o\tilde{\tau}} d\theta}{1 - exp[-i\frac{2\pi}{p}(\nu_o-\nu_z)]} \right\} , \tag{7.7}$$

where P is the number of superperiods of the storage ring. From the relation $d = (z' f_z^* - z f_z^{*\prime})/2i$ one can express $\delta \vec{n}$ through z and z'. From the expression for $\delta \vec{n}$ in a hard focusing storage ring the following resonances are possible

$$\nu_0 \simeq \pm \nu_z + KP$$

where K is an integer number.

Calculation of a deviation $\vec{n}_s - \vec{e}_z$ in the storage ring with the small radial field

The formula (7.3) is also applicable to calculations of a small deviation of $\vec{n}_s$ from $\vec{e}_z$ at a slight inconsistency of the storage ring to its ideal model. Let, for example, there is small radial field $\mathcal{K}_x$ on the orbit. The field causes the vertical distortion z_s of the closed orbit ($z_s'' + g_z z_s = R\mathcal{K}_x$)

$$z_s = f_z \int_{-\infty}^{\theta} \mathcal{K}_x f_z^* \frac{d\theta/2i}{R} + c.c. = \frac{f_z \int_{\theta-2\pi}^{\theta} \mathcal{K}_x f_z^* d\theta/R}{2i[1-\exp(-i2\pi\nu_z)]} + c.c. \tag{7.8}$$

Therefore for the deviation of $\vec{n}_s$ from $\vec{e}_z$ one can have

$$\vec{n}_s - \vec{e}_z = \nu_0 \, \mathrm{Im}\,(\vec{e}_x + i\vec{e}_y) e^{-i\nu_0 \tilde{\chi}} \frac{\int_{\theta-2\pi}^{\theta} z_s'' e^{i\nu_0 \tilde{\chi}} d\theta}{[1-\exp(-i2\pi\nu_0)]} \tag{7.9}$$

where z_s is vertical distortion of (7.8). Sometimes instead of summing on turns the underintegral expression is splitted over harmonics

$$\vec{w}\vec{e}^* \simeq \nu_0 \frac{z_s''}{R} e^{i\nu_0(\tilde{\chi}-\theta)} = \sum_K w_K e^{-iK\theta} \tag{7.10}$$

Then the expression equivalent to (7.9) can be written in the form

$$\vec{n}_s - \vec{e}_z \simeq \mathrm{Im}\left\{(\vec{e}_x + i\vec{e}_y) e^{-i\nu_0(\tilde{\chi}-\theta)} \sum_K \frac{w_K e^{-iK\theta}}{i(\nu_0 - K)}\right\}, \tag{7.11}$$

Let us give the real expressions for the spin resonance powers . By integrating over all parts of (7.10) we have

$$w_K = \frac{\nu_0}{2\pi R} \int_0^{2\pi} z_s'' e^{i\nu_0(\tilde{\chi}-\theta)} e^{iK\theta} d\theta =$$

$$= \frac{\nu_0}{2\pi} \int_0^{2\pi} \mathcal{K}_x(\theta)\, F_K(\theta) e^{iK\theta} d\theta , \tag{7.12}$$

where the response periodical function $F_K(\theta) = F_K(\theta + 2\pi)$ is determined by the vertical f_z - function of the storage ring:

$$F_K = \frac{\nu_0}{2} e^{-iK\theta} \left\{ \frac{f_z^* \int_{\theta-2\pi/P}^{\theta} \mathcal{K} f_z' e^{i\nu_0(\tilde{\chi}-\theta)+iK\theta} d\theta}{1-\exp[-i\frac{2\pi}{P}(K+\nu_z)]} - \frac{f_z \int_{\theta-2\pi/P}^{\theta} \mathcal{K} f_z^{*\prime} e^{i\nu_0(\tilde{\chi}-\theta)+iK\theta} d\theta}{1-\exp[-i\frac{2\pi}{P}(K-\nu_z)]} \right\} \tag{7.13}$$

For a circular storage ring ($\mathcal{K}$ = 1) and homogeneous focusing the response function is equal to

$$F_K = \frac{K\nu_0}{K^2 - \nu_z^2} .$$

If the β_z - function ($\beta_z = R|f_z|^2$) depends on azimuth, the value as a function of azimuth is approcimately proportional to $\sqrt{\beta_z}$.

If the radial field is introduced at a point $\theta = \theta_1$ of an orbit (in the form of δ - function), we have

$$w_K = \nu_0 \mathcal{K}_x(\theta_1)\, \eta_1\, F_K(\theta_1) e^{iK\theta_1} \tag{7.14}$$

where $\eta_1 = \ell_1/2\pi R$ is an orbit fraction occupied with the radial field $\mathcal{K}_x$.

Calculation of the degree and time of relaxation for radiative polarization in the storage ring model close to an ideal storage ring

Let us calculate the degree ζ and time of relaxation τ for the radiative polarization in an ideal storage ring. From the relations (see (7.5)):

$$\frac{z}{R} = a f_z + a^* f_z^* , \qquad \frac{z'}{R} = a f_z' + a^* f_z^{*\prime}$$

because of continuity of z and z':

$$\frac{\partial a}{\partial \gamma} f_z + \frac{\partial a^*}{\partial \gamma} f_z^* = 0 \quad , \quad \frac{\partial a}{\partial \gamma} f_z' + \frac{\partial a^*}{\partial \gamma} f_z^{*\prime} = 0$$

whence $\gamma\, \partial a/\partial\gamma = 0$ and consequently $\gamma\, \partial\vec{n}/\partial\gamma = 0$ (see (7.7)). Therefore from general formulae (5.4) we get

$$\zeta = \frac{8}{5\sqrt{3}} \simeq 92\% \quad , \quad \tau^{-1} = \frac{5\sqrt{3}}{8} \frac{\lambda_c r_e}{R^3} \langle |\mathcal{K}|^3 \rangle , \qquad (7.15)$$

where $\lambda_c = \hbar/m$, $r_e = e^2/m$. Since in an ideal storage ring the only effects are significant of the direct interaction of the spin and radiation ($\gamma\, \partial\vec{n}/\partial\gamma = 0$) the latter result can be obtained on the spin flip probabilities directly in acts irradiation. In this way the result (7.15) has been first obtained for a homogeneous magnetic field ($\mathcal{K}$ = 1) in Ref. /22/ and that for an inhomogeneous magnetic fields in Refs. /23,24/.

Only with the magnetic field deviations from the ideal the precession axis $\vec{n}$ becomes dependent of energy. The deviations can be associated, for example, with the turn of quadrupoles around an orbit direction causing the z-x motions coupling. Another reason, more substantial, at very high energies ($\nu_0 \gg 1$) can be radial fields of various kind. One can calculate an accuracy for an accelerator design to provide an existence of radiative polarization. For some more details one can refer to Ref. /9/. Here we shall demonstrate a calculation of introducing the radial field into an orbit by a simple example at $\theta = \theta_1$. Let, for example, radial field generated by a vertical shift Δz_Q of one quadrupole.

Then $\mathcal{K}_x = g_z \Delta z_Q / R$. Such a field, of course, varies the whole closed orbit. Total influence of this field on the spin is described by the response function $F_\kappa(\theta)$. From (7.11) and (7.14) we have*)

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*) Total change of the spin precession frequency is described by the perturbation (6.5) $w_b \simeq \nu_0 \left(\frac{\Delta\gamma}{\gamma} \mathcal{K} - \frac{x''}{R} \right)$. In the formula (7.16) we have confined ourselves by the main (at high energies) effect - the precession mean frequency $\langle w_b \rangle = \nu_0 \Delta\gamma/\gamma$.

$$\gamma \frac{\partial \vec{n}}{\partial \gamma} \vec{e}_z = 0 \; ,$$

$$\frac{\langle |\mathcal{K}|^3 (\gamma \frac{\partial \vec{n}}{\partial \gamma})^2 \rangle}{\langle |\mathcal{K}|^3 \rangle} \simeq \langle | \gamma \frac{\partial}{\partial \gamma} \sum_k \frac{w_k e^{-ik\theta}}{\nu_0 - k} |^2 \rangle \simeq \qquad (7.16)$$

$$\simeq \langle | \sum_k \frac{w_k e^{-ik\theta}}{(\nu_0 - k)^2} | \rangle = \sum_k \frac{|w_k|^2}{(\nu_0 - k)^4} .$$

where $|w_k|^2 = \nu_0^2 g_z^2 \eta_Q^2 (\Delta z_Q)^2 / R^2 \, |F_k(\theta_1)|^2$, $(\gamma\, \partial\nu_0/\partial\gamma = 1)$.

Thus, say, the polarization degree ζ is equal to

$$\zeta = \frac{8}{5\sqrt{3}} \left[1 + \frac{11}{18} \sum_k \frac{|w_k|^2}{(\nu_0 - k)^4} \right]^{-1} . \qquad (7.17)$$

The formula (7.17) enables one to calculate a depolarizing effect at a lense shift by a value of Δz_Q.

An ideal storage ring with the fields introduced into the straight section restoring the output orbital and spin motions.

Let it be an ideal storage ring with the straight sections. In such a storage ring the level of polarization is of 92% and polarization time is defined by the Sokolov-Ternov (7.15). Let us introduce into a straight section an insert with radial fields (see Fig. 2)). Let us calculate the spin-orbital coupling which means calculations of the level and time of relaxation for radiative polarization in this simple case. The spin perturbations (6.5) take the following form:

$$w_a = -\nu_0 g_z z/R \quad , \quad w_b = \nu_0 g_x x/R ,$$

where $g_z = \mathcal{K}_x^2 - \frac{R}{\langle H_z \rangle} \partial H_z/\partial x$, $g_x = \mathcal{K}_z^2 + \frac{R}{\langle H_z \rangle} \frac{\partial H_z}{\partial x}$ are the motion equation parameters

$$z'' + g_z z = -\frac{\Delta\gamma}{\gamma} R \mathcal{K}_x$$
$$x'' + g_x x = \frac{\Delta\gamma}{\gamma} R \mathcal{K}_z \qquad (7.18)$$

since there is no quadrupoles in the section with radial fields $g_x = 0$, $g_z = \mathcal{K}_x^2$. From the formula (5.3) the variation of

$\delta\vec{n}$ is equal to

$$\delta\vec{n} = \mathrm{Im}\,\vec{\eta}\left[\int_{-\infty}^{\theta} w_x \eta_x^* d\theta + \int_{-\infty}^{\theta} w_z \eta_z^* d\theta\right]. \qquad (7.19)$$

Since η_z (see (4.6)) is equal to zero on the main part of the storage ring $\theta_0<\theta<2\pi$ the product $w_z \eta_z^* = 0$ at any azimuth θ. Therefore we have

$$\delta\vec{n} = -\nu_0 \mathrm{Im}\,\vec{\eta}\int_{-\infty}^{\theta} g_z \frac{z}{R}\eta_x^* d\theta = \nu_0 \mathrm{Im}\,\vec{\eta}\int_{-\infty}^{\theta}\left(\frac{\Delta\gamma}{\gamma}\mathcal{K}_x + \frac{z''}{R}\right)\eta_x^* d\theta \qquad (7.20)$$

The equation solution for z-motion we write down in the form

$$\frac{z}{R} = \frac{\Delta\gamma}{\gamma}\psi_z + a_z f_z + a_z^* f_z^*, \qquad (7.21)$$

where ψ_z is the dispersion function of the closed orbits, a_z is an amplitude of betatron oscillations. Substituting the latter into the formula (7.20) we get

$$\delta\vec{n} = -\nu_0 \mathrm{Im}\,\vec{\eta}\left[\frac{\Delta\gamma}{\gamma}\int_{-\infty}^{\theta} g_z\psi_z\eta_x^* d\theta + a_z\int_{-\infty}^{\theta} g_z f_z\eta_x^* d\theta + a_z^*\int_{-\infty}^{\theta} g_z f_z^*\eta_x^* d\theta\right]$$

The derivative $\gamma\partial a_z/\partial\gamma$ is obtained from the relation

$$a_z = \frac{1}{2i}\left[\left(\frac{z'}{R} - \frac{\Delta\gamma}{\gamma}\psi_z'\right)f_z - \left(\frac{z}{R} - \frac{\Delta\gamma}{\gamma}\psi_z\right)f_z'\right]^*$$

and it is equal to

$$\gamma\frac{\partial a_z}{\partial\gamma} = -\frac{1}{2i}\int_{-\infty}^{\theta}\mathcal{K}_x f_z^* d\theta = (\psi_z f_z' - \psi_z' f_z)^*/2i$$

If the value g_z in the region $0<\theta<\theta_0$ were equal to zero, the value $\gamma\partial\vec{n}/\partial\gamma$ on the main part of the storage ring would also be equal to zero. Really, in this case, the dispersion function ψ_z on the main part $\theta_0<\theta<2\pi$ would remain to be zero: $\psi_z(\theta_0\div2\pi)=0$, and therefore, $\gamma\partial a_z/\partial\gamma(\theta_0\div2\pi)=0$. The value of g_z is not equal to zero, really $(g_z(0\div\theta_0)=\mathcal{K}_x^2\neq0)$. Therefore not any scheme conserve the zeroth ψ_z function on the main part. Though the correction due to $\mathcal{K}_x^2$ at $\nu_0>>1$ is rather small $(|\gamma\partial\vec{n}/\partial\gamma|^2\lesssim\nu_0^{-2})$ and can neglected in our considerations

given below.

The value of $\gamma\partial\vec{n}/\partial\gamma$ exists only at the section with the fields introduced and it is equal to

$$\gamma\frac{\partial\vec{n}}{\partial\gamma} = \nu_0\mathrm{Im}\,\vec{\eta}\int_0^\theta \mathcal{K}_x\left[1-e^{i\nu_0\theta}F_{\nu_0}(\theta)\right]d\theta \qquad (7.22)$$

where $F_{\nu_0}(\theta)$ is a periodic response function of (7.3) where "K" is replaced by "ν_0". In particular, for the circular storage ring and the smoothed focusing we have (see 4.6):

$$\left(\gamma\frac{\partial\vec{n}}{\partial\gamma}\right)_{0\div\theta_0} = \frac{\nu_z^2}{\nu_z^2-\nu_0^2}\varphi_\theta\left(\cos\varphi_\theta\,\vec{e}_y - \sin\varphi_\theta\,\vec{e}_z\right).$$

With the value $\gamma\partial\vec{n}/\partial\gamma$ known it is easy to calculate the level and time of radiative polarization by (5.4). For example, in the scheme in Fig. 2 for the polarization level we have

$$\zeta = \frac{8}{5\sqrt{3}}\left\{1+\frac{3\pi}{\nu_0}\mathcal{K}_x^2\left[1+\frac{11\pi^2}{72}\frac{\nu_z^4}{(\nu_0^2-\nu_z^2)^2}\right]\right\}^{-1}.$$

From the formula we see that the high level polarization is feasible only in the case when the radiation in the insert field is negligible. Since the contribution into radiation is proportional to $|\mathcal{K}|^3$ one can neglect the radiation on the insert section if

$$\frac{|\mathcal{K}_x|^3\theta_0}{\int_{\theta_0}^{2\pi}|\mathcal{K}_z|^3 d\theta} \simeq \frac{3\pi}{\nu_0}\frac{\mathcal{K}_x^2}{\mathcal{K}_z^2} \simeq \left(\frac{3\pi}{\nu_0}\right)^3\frac{1}{\mathcal{K}_z^2\theta_0^2} << 1$$

this condition can be written down as the condition on the minimum possible length of the insert ℓ

$$\ell >> R\,(3\pi/\nu_0)^{3/2} \qquad (7.23)$$

The use of quadrupoles

In order to provide the high-luminosity colliding beams it is assumed to use the strong quadrupoles near the collision point. In this case, obtaining the zeroth ψ_z function on the main section is the condition insufficient making $\gamma\partial\vec{n}/\partial\gamma$ equal to zero on this section. An additional requirement is that the expression is equal to zero (see (7.19))

$$\gamma\frac{\partial}{\partial\gamma}\int_{-\infty}^{\theta} w_\theta\, \eta_z^*\, d\theta = \gamma\frac{\partial}{\partial\gamma}\int_{-\infty}^{\theta} g_x \frac{x}{R}\, \eta_z^*\, d\theta = 0$$

which is equivalent to the requirement

$$\int_0^{\theta_0} g_x f_x \eta_z^*\, d\theta = i\int_0^{\theta_0} g_x f_x \sin\varphi_\theta\, d\theta = 0 . \qquad (7.24)$$

This condition can be satisfied, for example, if the quadrupoles are situated near the collision point $\theta_{(*)}-\Delta < \theta < \theta_{(*)}+\Delta$, where $\sin\varphi_\theta = \pm\pi/2$. In this case

$$\int_0^{\theta_0} g_x f_x\, d\theta = f_x'(\theta_{(*)}-\Delta) - f_x'(\theta_{(*)}+\Delta) = 0 . \qquad (7.25)$$

Substituting the know form of f_x

$$f_x = \sqrt{\beta_x/R}\, \exp\left[i\int_{\theta_{(*)}}^{\theta}(R/\beta_x)\,d\theta\right]$$

the condition (7.25) can be rewritten in a symmetric scheme $(\beta_x(\theta_{(*)}+\Delta) = \beta_x(\theta_{(*)}-\Delta)$, $\beta_x'(\theta_{(*)}+\Delta) = \beta_x'(\theta_{(*)}-\Delta)$ in the following way

$$(\beta_x'/2R)(\theta_*+\Delta) = tg\left[\int_{\theta_{(*)}}^{\theta_{(*)}+\Delta}(R/\beta_x)\,d\theta\right].$$

The equilibrium polarization degree remains innevitably high in the scheme with quadrupoles with the condition (7.23) satisfied, when the insert section radiation can be neglected. The use of quadrupoles opens up additional possibilities for getting high degree of polarization with the violated condition (7.23). Really, if in the section $0<\theta<\theta_0$ the optimum value $|\gamma\partial\vec{n}/\partial\gamma| \sim 1$ is procided, the polarizing effect of radiation occurs also in the insert section.

One should also note the possibility of using the quadrupoles turned around the orbit which extend the possibilities for the beam control. For making $\gamma\partial\vec{n}/\partial\gamma$ equal to zero on the main section one should meet the following requirements in this case:

1) there is no coupling $z-x$ motion in the transformation matrix of betatron motion through the section $(0 \div \theta_0)$;

2) ψ_z - function remains to be equal to zero in the main section;

3) in addition to $\int_0^{\theta_0} f_x'' \sin\varphi_\theta\, d\theta = 0$ the following requirement is added

$$\int_0^{\theta_0} \psi_x'' \sin\varphi_\theta\, d\theta = 0 ,$$

which in the general case is not a consequence of the first one.

An example with one siberian snake

Introducing the strong fields into the straight section can lead not only to variation in the direction $\vec{n}_s$ but also can increase the spin orbital coupling parameter $\gamma\partial\vec{n}/\partial\gamma$. A good illustration is an example with one siberian snake. In this case, an equilibrium polarization in the main section lies in the storage ring plane and it is directed along the velocity in the opposite section at $\theta=\pi$ (4.9):

$$\vec{n}_s(\theta_0 \div 2\pi) = \vec{e}_x \sin\nu_0(\tilde{\varkappa}-\pi) + \vec{e}_y \cos\nu_0(\tilde{\varkappa}-\pi).$$

At very high energies the main depolarizing effect is the spin angle diffusion $\vec{n}$ with a velocity

$$\left(\gamma\frac{\partial\vec{n}}{\partial\gamma}\right)^2 \simeq \nu_0^2(\tilde{\varkappa}-\pi)^2.$$

Thus $\gamma\partial\vec{n}/\partial\gamma = 0$ at $\theta=\pi$ and achieves its maximum value $\pi^2\nu_0^2$ near the insert section. Since $\langle(\gamma\partial\vec{n}/\partial\gamma)^2\rangle \approx \frac{\pi^2}{3}\nu_0^2 \gg 1$ the radiative polarization degree in this case tends to zero. A polarization produced on the initial stage vanishes for a time approximately

$$\tau_{dep}^{-1} = \alpha_+^{-1} \approx \frac{8}{5\sqrt{3}}\left[\frac{\lambda_c r_e}{R^3}\gamma^5\langle|\varkappa|^3\rangle\, \frac{11\pi^2\nu_0^2}{54}\right]^{-1},$$

which is approximately by ν_0^2 times lower than the polarization time for an ideal storage ring without an insert.

An example with two snakes

If in the high energy storage ring the introducing of one snake increases significantly depolarizing effects, two snakes enable one to have the equilibrium polarization direction vertical and depolarizing effects of the same order as those in the case without snakes. More than that, the snakes completely remove the resonance diffusion of spins (the latter will be considered below).

Calculation of the spin orbital coupling $\gamma\partial\vec{n}/\partial\gamma$ with the presence of two snakes is practically the same as for calculations in the case of an ideal storage ring with the insert restoring the output orbit and direction of equilibrium polarization.

In the main sections of the storage ring $\theta_0 < \theta < \pi$, $\pi+\theta_0 < \theta < 2\pi$ the value $\gamma\partial\vec{n}/\partial\gamma$ is equal to zero if all requirements met similar to those given on p. 29:

1) in transformation matrices for betatron oscillations through the insert sections there is no coupling of $z-x$ - motions;

2) a dispersion function ψ_z remains to be equal to zero on the main sections;

3) in the insert sections the quadrupoles should be placed in such a way not to violate the following conditions for any trajectory

$$\int_0^{\theta_0}\left(\frac{\Delta\gamma}{\gamma}\mathcal{K}_z - \frac{x''}{R}\right)\eta_z d\theta = 0 \;, \qquad \int_{\pi}^{\pi+\theta_0}\left(\frac{\Delta\gamma}{\gamma}\mathcal{K}_z - \frac{x''}{R}\right)\eta_z d\theta = 0 \,.$$

In particular, all three conditions turn out to be satisfied if there is no focusing elements in the insert sections (in this case, $\frac{\Delta\gamma}{\gamma}R\mathcal{K}_z - x'' = g_x x \simeq 0$).

If the conditions (7.23) are satisfied, radiation in the sections themselves can be neglected. With the presence of two snakes in a symmetric storage ring there is no polarizing effects under conditions (7.23) satisfied because of the spin

re-orientation on the beam orbit. Though, polarization can be provided either with the wigglers (sections with a large value of $\langle\mathcal{K}^3\rangle$ or with the laser /21,25,26/. The role of a snake can be served, in particular, by the spin turn sections (conditions (7.3) in this case are violated). Note that in this case it is not necessary to have the field value larger in the places where $\vec{n}_s$ is directed along the field (in this case, a degree ζ tends to 92%). There are many possibilities for getting the high degree of polarization at any angle between the field and $\vec{n}_s$ directions because of arrangement of the required vector $\gamma\partial\vec{n}/\partial\gamma$ (see general formula (5.4) for ζ). So, the extremum $\zeta \simeq 95\% > 92\%$ is achieved in the section with a very high value of $\langle\mathcal{K}^3\rangle$ where we have

$$\vec{n}_s = -\sqrt{\frac{7}{11}}\frac{[\vec{v}\vec{v}']}{|\vec{v}'|} \pm \sqrt{\frac{4}{11}}\vec{v} \;, \qquad \gamma\frac{\partial\vec{n}}{\partial\gamma} = \frac{2\sqrt{7}}{11}\left[\sqrt{\frac{4}{11}}\frac{[\vec{v}\vec{v}']}{|\vec{v}'|} \mp \sqrt{\frac{7}{11}}\vec{v}\right].$$

Even in the case when the direction $\vec{n}_s$ is perpendicular to the field the arrangement $\gamma\frac{\partial\vec{n}}{\partial\gamma} \simeq \pm\sqrt{\frac{18}{11}}\frac{[\vec{v}\vec{v}']}{|\vec{v}'|}$ enables one to get the degree of $\simeq$ 60%.

In conclusion of this section let us note the following. Even at two spin overturn along the orbit the spin resonant diffusion is removed (which is quite substantial at $q_a\Delta\gamma/q_0 \gtrsim 1$) as well as the resonance dependence of polarization degree on energy. Introducing 2M snakes enables one decrease at least by M^2 times an influence of nonresonance depolarizing factors $((\gamma\partial\vec{n}/\partial\gamma)^2 \sim M^{-2}$) and therefore to attenuate by M times the design accuracy requirements for the storage ring magnetic system. This method of an increase in spin stability one compare with the use of strong focusing instead of the soft one for betatron oscillations of a particle.

8. Spin resonance diffusion

Depolarizing influence of energy fluctuations increases sharply with an approach to the spin resonances because of an increase in the spin orbital coupling parameter $\gamma\partial\vec{n}/\partial\gamma$. In order to avoid the high power resonances one should select the storage ring magnetic system parameters and particle energy in appropriate way. Though, there always exist resonance harmonics of sufficiently high order and one should aware how the weak resonances could still influence on the polarization degree. In addition, there are some situations when it is impossible to avoide the powerful resonances. As an example could serve the conventional storage rings at sufficiently high energies such that precession frequency spread in a besm becomes on the order of the resonance distances.

Another important example is the colliding beam depolarizing effect.

Let us clear up the physical meaning of the resonance diffusion. Let w_k be the spin resonance power, Λ is a decrement of radiative damping, $\sigma_\nu^2 = \langle \frac{d}{dt}[\delta(\nu-\nu_k)^2] \rangle / \Lambda$ is an established spread of the spin resonance position where the diffusion rate caused by the energy quantum fluctuations. At very high energies $\sigma_\nu \simeq \nu_0 \frac{\Delta\gamma}{\gamma} = \frac{q_e}{q_0} \Delta\gamma$ where $\Delta\gamma$ is a beam energy spread.

Let us first evaluate the diffusion rate in the resonance region for the case when $\sigma_\nu \gg w_k$ and $\sigma_\nu \gg \Lambda$. In this case, at every instant in the resonance region where the spin diffusion rate is maximum quite a small portion of particles appeared. Of course, any particle comes to the resonance region because of the diffusion and damping processes. Each particle comes to the resonance region on average times per time unit. If the resonance is sufficiently powerful $(w_k^2 \gtrsim \Lambda \sigma_\nu)$ and resonance crossing (because of diffusion and damping processes) proceeds slowly then even at single in resonance a strong random deviation of a particle spin is occured. For a time Λ^{-1} when in the resonance region nearly all particles have been the entire disappearance of polarization took place. Thus, the characteristic time of depolarization at $w_k^2 \gtrsim \Lambda \sigma_\nu$ is equal to

$$\tau_{dep} \sim \Lambda^{-1}.$$

At $w_k^2 \ll \sigma_\nu \Lambda$ polarization disappears for the multiple fast resonance crossing with average velocity $\sigma_\nu \Lambda$. During the time Λ^{-1} approximately one crossing at average will occur with a velocity $\approx \sigma_\nu \Lambda$:

$$\delta S_R \approx w_k / \sqrt{\sigma_\nu \Lambda} .$$

By mean quadratically adding the passing results one can get that the depolarizing rate τ_{dep}^{-1} is approximately equal to

$$\tau_{dep}^{-1} \approx \frac{w_k^2}{\sigma_\nu \Lambda} \Lambda = \frac{w_k^2}{\sigma_\nu} .$$

In this case, an exact formula gives

$$\tau_{dep}^{-1} = \pi \sum_k \langle |w_k|^2 \delta(\nu - \nu_k) \rangle , \qquad (8.1)$$

where $\delta(\nu-\nu_k)$ is a delta function and brackets $\langle \dots \rangle$ mean an averaging over the particle distribution in a beam.

At calculation of the equilibrium polarization degree one should add the rate τ_{dep}^{-1} found out by formula (8.1) to α_+ (see (5.4)). It is important that in the radiative polarization region when $\tau_{dep} \sim \tau_{pol}$ there always takes place the case of the fast uncorrelated origins where the formula (8.1) is valid. It is associated with that the damping decrement Λ is much in excess of τ_{pol}^{-1}. The calculation result does not depend on the certain character of the diffusion and damping source, it is only determined by equilibrium distribution over particle energies and amplitudes of betatron oscillations.

This formula describes the cases of superhigh energy and depolarizing effect of a colliding beam. For completeness let us give the rate of a resonance depolarization for another case when there is a small spread at $\sigma_\nu \ll \Lambda$. At*) $w_k \gg \Lambda$ polarization vanishes for a time w_k^{-1} just because of power spread w_k associated with a particle deviation from the equilibrium orbit. Since $\Lambda \gg \tau_{pol}^{-1}$ this is the case when there is no radiation polarization. At $w_k \ll \Lambda$ the depolarization rate is evaluated in the following way. The time for an eventual variation of the main perturbation direction $\vec{w}_k$ is on the order of Λ^{-1}. Therefore one should mean quadratically add the results of the value variation for time $\sim \Lambda^{-1}$ $(\delta S_R \sim w_k/\Lambda)$.

*) Generally speaking, here Λ is an inverse characteristic time for perturbation direction $\vec{w}_k$ because of the diffusion and damping processes. This inverse time can be in excess of a radiative damping for the spin resonances of higher orders, for example, for the resonances of synchrotron oscillations (with large numbers).

Whence we get

$$\tau_{dep}^{-1} \approx \frac{w_k^2}{\Lambda^2}\Lambda = w_k^2/\Lambda .$$

This is a maximum feasible depolarization velocity in an exact resonance at small σ_γ .

The resonance diffusion studies in more detail are considered in Refs. /4,6,7,9,27/, where also considered are the examples of calculations of resonance diffusion at very high energies with a presence of colliding beam in conventional storage rings.

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РАДИАЦИОННАЯ ПОЛЯРИЗАЦИЯ В НАКОПИТЕЛЯХ

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