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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
СО АН СССР

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THE ROLE OF INSTANTONS IN QUANTUM CROMODYNAMICS III. QUARK - GLUON PLASMA

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Abstract

With the increase in density and/or temperature of the matter, being in the quark-gluon plasma phase, the suppression of the instanton-induced effects takes place. At some critical parameters the chiral symmetry is restored. In this first order transition the massive quasipartic les - valons - are substituted by nearly massless quarks and gluons, while the "instanton liquid" dissociates into "instanton molecules" with zero topological charge.

1. Introduction

This is the third in the series of papers devoted to instanton-induced effects in QCD. In the first two we have discussed the structure of physical vacuum [1] and hadrons [2]. In this paper we consider the finite density and/or temperature matter. We show below that at low density (comparable to that of ordinary hadrons) the matter can be considered in first approximation as consisting of quasiparticles with the instanton-induced effective mass. With further increase in matter density the phase transition takes place in which the chiral symmetry is restored. In this point the "instanton liquid" is breaking into "molecules" consisting of instanton-antiinstanton bound pair. At still larger density the instanton-induced effects rapidly decrease and become unimportant.

This scenario is not, of course, completely new, so some discussion of earlier works is in order here. The idea that asymptotically dense matter is nearly ideal gas of quarks and gluons was suggested soon after the discovery of asymptotic freedom by Collins and Perry [3]. The perturbative calculations of its properties were done, see e.g. references in reviews [4,5]. In [9,4] it was suggested that instantons are suppressed in matter and that this phenomenon is important in the transition region. More detailed investigations were done in papers [5,10-12].

Quite separate branch of investigations is connected with confinement effects studied in lattice approximation. It was predicted in [6] that at some temperature T_{conf} this property of the QCD vacuum is completely lost. Recent Monte-Carlo calculations [7] have indeed observed such transition. The connection between the confinement effects and the instanton physics remains unclear. To large extent the existence of two separate scales, suggested in [8] and discussed in detailes in [1,2], allows one to consider these types of effects separately. For exampl., this idea provides the natural explanation of the approximate additivity of "valons" inside the ordinary hadrons. Presumably the matter of comparable density can be considered in similar way.

However, the chiral symmetry breaking (discussed in [1]) is the collective effect of the "instanton liquid" and it is connected not only with the properties of the individual instanton (ρ_c) but with instanton separation ($R \sim n_c^{-1/4}$)as well, so in fact the "large scale physics" enters here. Nevertheless, encouraged by the reasonable results obtained in [1], we consider below the restoration of the chiral symmetry which takes place at some line on density-temperature plane. The question can be asked about the relation of relevant parameters T_{chir}, n_{chir} to that of the deconfinement transition.

Although little is known about confinement, these two transitions seem to be different for they depend on quark masses in different way, say only one transition remains in gluodynamics. The order of the transition seems to be different too, we find the first order transition and deconfinement is probably the second order transition [6,7] .

We are not certain which transition happens at larger density. If $n_{chir} > n_{conf}$ which we consider as more natural possibility, there are the following three phases of matter (in the order of increasing density): the hadronic gas, the plasma of "valons", the plasma of "perturbative" quarks and gluons.

Let us note now one difference between this paper and [1,2]. There we have considered effects previously much discussed by other authors (on the one-instanton level), so we have omitted

the derivation of relevant formulae, considering only averaging over instantons and applications. However, in the case of the nonzero density and/or temperature the results of different authors are contradictory. Let us make therefore some introductory comments here.

The qualitative effect of the matter on the instantons was first considered in [9] (see also [4]). The conclusion was that the Debye screening makes the charge to be never large, so the large scale instantons (divergent in vacuum) are not dangerous. More detailed investigations were performed by Abrikosov [10], Corvalho [11] , Baluni [12] and Gross,Pisarsky and Yaffe[5] . They all have observed the instanton suppression, however at more quantitative level there are essential differences between their results.

In the case of zero temperature degenerate quark gas with chemical potential μ the following set of results are suggested:

$1 - \frac{9}{128}\rho^2\mu^2 N_f + O(\mu^4)$	$\rho\mu \ll 1$	[11]	(1)
$\exp(-N_f\rho^2\mu^2)$	$\rho\mu \gg 1$	[11]	(2)
$\exp(-2N_f\rho^2\mu^2)$	all $\rho\mu$	[12]	(3)
$1 + O(\alpha_s(\rho))$	– " –	[10]	(4)

Where the given factors should be incerted in the ordinary instanton density.

As we show below, none of these results is valid in strict sense. The method of calculation leading to (1) is correct but some errors seem to be present in the original work, so the coefficient is different. Other calculations (2-4) are done for fermion determinant in the instanton field. The problem itself has no direct physical meaning because the nonzero density or temperature obviously affects the instanton itself, so the calculations with the original Belavin Polyakov-Schwartz-Tyupkin solution can not be justified.

Still it is interesting to have such result, which can be considered at least qualitatively. The methodical part of such calculation is rather instr uctive too. In section 2 such calculation is made, it is based on the evaluation of the scattering amplitude in forward direction for quark and gluon on the instanton. The method of calculation is much more simple and general than those used by other authors. In the limiting case considered previously it agrees with (2) while in (3) the explicit mistake is found: the zero modes are erroneously omitted. So, this question is more or less settled. It is gratifying to see that indeed all forms of matter suppress instantons. Again we remind, that such calculation is not selfconsistent. Still usially the account for feedback conserves the sign of the effect.

This statement is demonstrated in section 3, where the low density and/or temperature plasma is considered just by expansion in powers of μ or T . The very interesting observation here is that these two quite different calculations and that made in [1] lead to close values of the constituent quark mass, being 200 Mev with 20% accuracy. Analogous calculation for " constituent gluon" have produced zero mass, indicating once more that they do not exist.

In section 4 we come to discussion of the phase transition

in which the chiral symmetry is restored. The basis of our discussion is the nonlinear equation of consistency for the value of the quark condensate. This equation was alreády considered in [1] for the physical vacuum, and now we discuss what happens with its solutions as far as the instanton dnesity is suppressed by matter. Another possible approach is from the superdense phase side, it is based on the consideration of the instanton-induced attraction in scalar quark- antiquark channel leading to the instability at certain instanton density. Both approaches agrees that the transition takes place if the instanton density is reduced by the factor of 2, but we are not able at the moment to say at which exactly parameters of matter it takes place. The estimates based on the extrapolation of (24) and naive picture of valon overlap in space point toward density of the order of $1 \div 3$ fermi$^{-3} = 7 \div 20\ n_{nuclear}$.

Effects connected with instantons in the superdense phase are discussed in section 5 . The absense of quark condensate makes (for massless quarks) the existence of separate instantons impossible. However, the instanton-antiinstanton molecules with zero topological charge can still exist, and we estimate their contribution. Asymptotically it falls as $n^{-14/3}$, or very strongly.

Some discussion of the results is made in section 6. In particular, we briefly consider the question whether it is possible to observe the phase transition in question in experiments with heavy-ion high-energy collisions.

2. Instanton determinants in matter

We start this section with brief introduction into the general formalism, describing the field theory at finite density and/or temperature. More detailed discussion, in particular of the diagram matic methods of the pertubation theory, can be found in reviews [4,5] for the QCD case.

The statistical sum Z can be defined by the general functional integral

$$Z = \int \mathcal{D}A^a_\mu \, \mathcal{D}\bar{\Psi} \mathcal{D}\Psi \exp(S_{YM} + S_F) \tag{5}$$

$$S_{YM} = -\frac{1}{4}\int_0^{1/T} dx_4 \int d^3x \, (G^a_{\mu\nu})^2 \; ; \; S_F = \int_0^{1/T} dx_4 \int d^3x \, \bar{\Psi}(\hat{\mathcal{D}} + \mu\gamma_4 + im)\Psi$$

where $\hat{\mathcal{D}} = (\partial_\alpha - ig\frac{t^a}{2}A^a_\alpha)\gamma_\alpha$, μ and T are the chemical potential and temperature. The integration is done over fields A_μ (or $\bar{\Psi}, \Psi$) which are periodic (or antiperiodic) on the imaginary finite "time" interval $[0, 1/T]$. Note that at $\mu, T \to 0$ we obtain the ordinary field theory in Euclidean space-time. For definiteness, let us give here our notations, namely the connection between Euclidean (E) and Minkovsky (M) fields

$$x^E_4 = i x^M_0 \; , \; x^E_m = x^M_m \; ; \; A^E_4 = -i A^M_0 \; ; \; A^E_m = -A^M_m$$
$$\gamma^E_4 = \gamma^M_0 \; ; \; i\gamma^E_m = \gamma^M_m \; ; \; \{\gamma_\mu \gamma_\nu\} = -2\delta_{\mu\nu} \tag{6}$$
$$\psi^M = \psi^E \; ; \; \bar{\psi}^E = i\bar{\psi}^M$$

Note that the opposite sign of A_μ as compared to coordinates is needed in order not to change the sign of g in $\mathcal{D}_\mu$.

Integration over the fermions leads to standard Mattew-Salam expression

$$Z = \int \mathcal{D}A_\mu \exp(S_{YM} + S_{eff})$$
$$S_{eff} = \ln \det_T (\hat{\mathcal{D}} + \mu\gamma_4 + im) \tag{7}$$

We remind that the temperature enter S_{eff} via the boundary conditions, and the field A_μ is present in the covariant derivative. It is useful to separate the vacuum effects from those of matter as follows

$$\Delta S_{eff} = S_{eff}(T,\mu) - S_{eff}(T=0,\mu=0) = \ln \det_{T\mu}[\hat{\mathcal{D}} + \mu\gamma_4 + im] - \ln \det[\hat{\mathcal{D}} + im]$$

We remind that ΔS_{eff} contains all the effect of the matter only at zero temperature, at the nonzero one the matter contains also gluons which are accounted by the further integration over A_μ.

The standard perturbative calculation of the determinant is the summation of loop diagrams as follows

$$S_{eff} = \sum_n \frac{(ig)^n}{n} \int \Pi^{a_1 \dots a_n}_{\mu_1 \dots \mu_n} \delta^4(\Sigma K_i)(2\pi)^4 \prod_{i=1}^n A^{a_i}_{\mu_i}(K_i) \frac{d^4K_i}{(2\pi)^4} \tag{8}$$

Here the factor $1/n$ is shown explicit ly, all the rest factors for the n-point polarization operator $\Pi_{\mu_1 \dots \mu_n}$ are given by ordinary Feynman rules. We also remind that at nonzero μ the integration over p_4 is done over the line $p_4 - i\mu$, and that the nonzero T changes this integration into summation over the discrete Matsubara frequences $(p_4)_n = \pi T(2n+1)$.

The integration over momentum loop in $\Pi_{\mu_1 \dots \mu_n}$ can be done as follows. There are n poles with identical residue, so the factor 1/n is canceled and the whole series can be absorbed into the full Green function in the field A^a_μ as follows ($E_p^2 \equiv m_q^2 + p^2$)

$$\Delta S_{eff} = -\int \frac{d^3p\, n(E_p)}{(2\pi)^3\, 2E_p} \mathrm{Tr}_{\left(\substack{spin\\ colour\\ flavour}\right)} T^{(q)}(p,p) \; ; \tag{9}$$

$$n(E) = \left(\exp\left(\frac{E-\mu}{T}\right)+1\right)^{-1} + \left(\exp\left(\frac{E+\mu}{T}\right)+1\right)^{-1}$$

This expression has very transparant physical meaning: one sums up all quarks and antiquarks with their thermodynamical weights, multiplied by the forward scattering amplitude over the external field A_μ defined as

$$T^{(q)}_{\lambda\lambda'}(pp') = \int d^4x\, d^4y\, \bar{\chi}_{p\lambda}(x)\hat{A}(x) S(x,y) \hat{A}(y) \chi_{p'\lambda'}(y) \tag{10}$$

Here $\chi_{p\lambda}(x)$ are the ordinary Dirac spinors with momentum p and polarization λ normalized as $\mathrm{Tr}\,\bar{\chi}\chi = \hat{p}+m$. Using the equation of motion

$$(i\hat{\partial} + \hat{A} - m)\, S(x,y) = \delta(x,y) \tag{11}$$

the amplitude can be further simplified :

$$\mathrm{Tr}\, T^{(q)}(p,p) = \int dx\, dy\, \mathrm{Tr}\left[(\hat{p}+m)(i\hat{\overleftarrow{\partial}}_x - m) S (i\hat{\partial}_y - m)\right] \tag{12}$$

Of course, this expression can also be derived directly from reduction formulae.

Our discussion above was based on the perturbation theory, but the results look so general that they can be generalized to fields of arbitrary intensity. The new nonperturbative phenomenon that should be added here is the appearance of bound states in strong enough fields. They should be added to the sum over states in (9) as follows

$$\sum_n n(E_n)\ldots + \int \frac{d^3p}{(2\pi)^3} n(E_p)\ldots \tag{13}$$

and, of course, to be included in the Green function. In the case of the instanton field, to which we now proceed, such bound states are the t'Hooft zero modes. Their contribution was ignored in [10,12], which is incorrect.

It should also be noted, that the transparent expressions (9)-(12) correspond, of course, to Minkovsky momenta p_μ. The calculations with instantons are, on the contrary, done in Euclidean space. In order to overcome this difficulty Baluni [12] has used rather complicated Penrose' method to continue the instanton formulae to ordinary Minkovsky space. We think that this continuation is not in fact needed, for it is easier to work from the beginning in Euclidean one.

For massless quarks the explicit Green function in the instanton field is known from the work [15] .

$$S(x,y) = -\frac{\psi_0(x)\,\psi_0^+(y)}{m} + S'(x,y) + m\int S'(x,z)\, S'(z,y)\, dz + O(m^2) \tag{14}$$

$$S' = \overrightarrow{\hat{\mathcal{D}}}_x \Delta(x,y)\left(\frac{1+\gamma_5}{2}\right) + \Delta(x,y)\overleftarrow{\hat{\mathcal{D}}}_y\left(\frac{1-\gamma_5}{2}\right)$$

$$\Delta(x,y) = \frac{(x-y)^{-2} + \rho^2(x^2+y^2+i\bar{\eta}^a_{\mu\nu}x_\mu y_\nu \tau^a)/2x^2y^2(x-y)^2 - \rho^2/2x^2y^2}{4\pi^2\left[(1+\rho^2/x^2)(1+\rho^2/y^2)\right]^{1/2}} \tag{14}$$

$$\psi_0 = \frac{\sqrt{2}\rho}{\pi}\frac{\hat{x}}{(x^2+\rho^2)^{1/2}(x^2)^{1/2}}\chi \quad ; \quad \sum_\alpha \chi_\alpha \bar{\chi}_\alpha = \frac{1}{2}(1-\gamma_5) \; ; \; \chi^*\chi = 1$$

Here Δ is the Green function of the scalar particle, and ψ_0 is the t'Hooft zero mode. All expressions here correspond to the so called singular gauge in which the field A_μ fall at $x\to\infty$ rapidly enough so that the scattering amplitude makes sense.

Now let us substitute these expressions into (12) and calculate the scattering amplitude of the quark on the instanton.

The largest term is o(1/m) due to zero modes. It looks singular in the chiral limit m→0 but one should remember that the averaging over the instanton density should also be done, and it is itself proportional to m^{N_f}, where N_f is the number of flavours. So, for $N_f=1$ the result is finite and it is just the effective interaction found by t'Hooft in his classical paper. However, this term gives zero contribution to (9) for the quark chirality is flipped, while the determinant is connected with the diagonal (in quark polarization) part of the scattering. Evidently that this amplitude should contain one extra power of m , and the total effect vanishes for massless quarks.

This conclusion is, in some sense, the trivial one: for single instanton in massless theory zero modes lead to zero contribution in the "empty" perturbative vacuum as

well as in the quark-gluon plasma. However, as discussed in [1] in greater details, instantons are not separated in the physical vacuum and due to their interaction with antiinstantons the chiral symmetry is spontaneously broken. In this case the role of the quark mass is played by some effective mass proportional to the quark condensate, being nonzero in the chiral limit. It is then clear that the same should be true for quark-gluon plasma of small density, while for large density plasma (in which the chiral symmetry is restored) the separated instantons can not exist indeed. These phenomena we discuss below.

Let us now return to scattering amplitude on the instanton which is diagonal in chirality. It is physically clear that after the averaging over instantons is made no Lorentz frame is preferable and therefore the amplitude can not depend on the quark momentum p_μ but its square p^2. In Minkovsky space on the mass shell of the massless quark $p^2=0$. In Euclidean space it corresponds to only one point, the origin, and in fact it is the only one we need to obtain the result. The calculations become than very simple. With the use of

$$\psi_0(p) \equiv \int d^4x\, e^{ipx}\psi_0(x) \xrightarrow[p\to 0]{} 2i\sqrt{2}\rho\pi\frac{\hat{p}}{p^2}\chi \tag{15}$$

$$\Delta(p,p') \equiv \int d^4x\, d^4y\, e^{ipx-ip'y}\Delta(x,y) \xrightarrow[p=p'\to 0]{} 4\pi^2\rho^2/p^2$$

we find the scattering amplitude in question

$$\underset{\substack{\text{spin}\\ \text{colour}\\ \text{flavour}}}{\mathrm{Tr}} T^{(q)}(p,p) = 8\pi^2\rho^2 N_f(-1+2) = 8\pi^2\rho^2 N_f \tag{16}$$

$$\Delta S_{eff} = -N_f\rho^2\mu^2$$

Two terms in this expression correspond to the contribution of zero and nonzero modes, respectively. Note that they just reflect the number of chiral components entering into the calculation.

The zero modes were erroneously omitted by Baluni [12], and his result coincides with that of nonzero ones. Note, that our result also corresponds to that of Corvalho [11] (see (2)), but it is obtained only for $\rho\mu \gg 1$ and the resulting coefficient of $N_f \rho^2 \mu^2$(equal to unity!) was in fact expressed as rather complicated integral evaluated numerically. In our notations, the method of Corvalho corresponds to direct analysis of expression (10) in momentum representation. It was shown by V.F. Dmitriev that in coordinate representation this cal culation is more simple and the result can be obtained analytically, the result is the same.

In the nonzero termperature case there are physical gluons, which also scatter over the instantons. Formally this effect comes from the integration over A^a_μ by the saddle-point method which also produces the determinant of all fluctuations around the instantons. The formula (9) is then generalized in obvious way: one should add gluons with their thermodynamical weight and scattering amplitude. We do not discuss this in details and give the result for the amplitude

$$\underset{\binom{\text{spin}}{\text{colour}}}{\mathrm{Tr}}\, T^{(g)} = 16\pi^2 \rho^2 N_c \tag{17}$$

and for complete effective action

$$\Delta S_{eff} = - a(T,\mu)\,\rho^2 \tag{18}$$

$$a(T,\mu) = \int \frac{d^3p}{(2\pi)^3 2p}\left[\frac{8\pi^2 N_f}{\exp\left(\frac{p-\mu}{T}\right)+1} + \frac{8\pi^2 N_f}{\exp\left(\frac{p+\mu}{T}\right)+1} + \frac{16\pi^2 N_c}{\exp\left(\frac{p}{T}\right)-1}\right] \tag{18}$$

The three terms here are, of course, the contribution of quarks, antiquarks and gluons of the matter. The comparison of this result with that of [5] will be done below. So, we have completed the calculation of the instanton determinants in matter.

However, as already discussed in the introduction, this problem is strictly speaking unphysical, for matter also affect the instantons and this point was not taken into account. Formally there are no reason to consider the original Belavin-Polyakov-Schwartz-Tyupkin solution in matter. What one should do is to find the effective Lagrangian (7) for any field A_μ and than to look for the saddle point of $S_{YM} + S_{eff}$, not S_{YM} alone. In other terms, in the equations of the fields

$$D_\mu G_{\mu\nu} = j_\nu \tag{19}$$

one should include also the current j_ν induced in matter by the field of the fluctuation. So far this problem is solved only in the limit of low density matter (to be considered in the next section) and n=0,T nonzero (see below).

Let us also note that the results of Corvalho [11] imply that in the case of very dense matter (or $\mu\rho \gg 1$, more accurately) this current can be written in linear approximation

$$j_\mu(x) = \int dy\, \Pi_{\mu\nu}(x,y)\, A_\nu(y) \tag{20}$$

where $\Pi_{\mu\nu}$ is the ordinary polarization operator calculated in perturbative theory, see e.g. [4,5]. However, even in this case the problem is difficult to solve.

Fortunately, in the case of zero μ but large T one can go somehow further. Harrington and Shepard [14] have found the instanton-type solution with periodic boundary conditions called "caloron". It looks like follows $(\theta \equiv \pi T)$

$$A^a_\mu = \bar{\eta}^a_{\mu\nu}\,\Pi(-i\partial_\nu \Pi^{-1}) \tag{21}$$

$$\Pi(t,z) = 1 + \frac{\theta\rho'^2}{z} sh(2\theta z)/(ch 2\theta z - \cos 2\theta t)$$

At $zT \ll 1$ it becomes the ordinary instanton but with the radius

$$\rho^2 = \rho'^2/(1+\theta^2\rho'^2/3) \tag{22}$$

The further calculation of determinants in this field was resently performed by Gross, Pisarsky and Yaffe [5] so that quantum fluctuations around the caloron are also known. The result looks as follows

$$dn(\rho') = dn(\rho')_{T=0}\, \exp\left[-\theta^2\rho'^2\left(\frac{2N_c}{3}+\frac{N_f}{3}\right)\right] \times \tag{23}$$

$$\times \exp\left[-12 A(\theta\rho')(1+N_c/6 - N_f/6)\right]$$

$$A(\theta\rho') \equiv \frac{1}{12}\left[\int \frac{d^3x}{(4\pi)^2}\int_{-1/2T}^{1/2T} dx_4 \left(\frac{\partial\Pi}{\Pi}\right)^4 - \int \frac{d^4x}{(4\pi)^2}\left(\frac{\partial\Pi}{\Pi}\right)^4_{T=0}\right] \simeq$$

$$\simeq -\frac{1}{12}\ln(1+\theta^2\rho'^2/3) + \frac{0.01289}{[1+0.15858(\theta\rho')^{-3/2}]^8}$$

Note that the given explicit expression for $A(\theta\rho')$ is just numerical extrapolation with high enough accuracy, $O(10^{-5})$ [5].

Note also that the part not connected with $A(\theta\rho')$ coinsides with that found above (18), and that proportional to $A(\theta\rho')$ is the effect of the instanton deformation on quantum level. The classical effect of the deformation is represented by the change in parameter ρ (22) to be important in what follows. We discuss in more details the limits of small and large T below, and here only comment that the instanton deformation by matter can not be neglected compared with effects given by (18). Still the effect is qualitatively the same: instantons are suppressed by matter but with different demping factor.

3.Low density and/or temperature plasma

The words "low density" in this section mean the matter with energy density of the order of that inside ordinary hadrons. As discussed in details in [2], it is in some sense low as compared to that of vacuum fluctuations. There exist arguments that the instanton-type fluctuations are not strongly affected inside hadrons, and therefore it should be so in the infinite matter of comparable density. If so, the instanton-induced effects in this case can be considered by expansion in powers of matter parameters, μ and T.

Of course, in this density region we are near the deconfinement transition. Still it was argued in [2] that even for the masses of hadrons these effects are not very important, although they are of course responsible for the very existence of hadrons. Presumably this is also true for the energy of the matter with comparable density, although confinement effects are more important near the transition point in further derivatives, say compressibility or heat capacity.

Let us start with the case of zero density and low temperature, in which the results of [5] can be used. Expanding (23) with the account of difference in caloron and instanton radii (22) one finds the following correction factor

$$1 - \pi^2 T^2 \rho^2 N_f/3 + O(T^4 \rho^4) \tag{24}$$

Interesting phenomenon is the cansellation of terms connected with gluons, while the quark part is the same as for undeformed instantons (see (18)).

With our delta-function approximation for the instanton density we can easily find the following instanton-induced correction to the thermodynamical potential of the matter

$$\Delta \Omega = \frac{2}{3} N_f \pi^2 T^2 \rho_c^2 n_c \tag{25}$$

We remind that Ω =-pV where p is pressure and V is volume, so instantons in matter reduce its pressure.

Similar calculation in the case of zero temperature and low density can be made by the direct expansion of the general expression (5) in powers of μ , as it was suggested by Corvalho [11] . It is easy to see that the first derivative vanishes and the second is equal to

$$\left.\frac{\partial^2 \Omega}{\partial \mu^2}\right|_{\mu=0} = \langle \int dx\, dy\, \Pi_{00}(x,y) \rangle \tag{26}$$

$$\Pi_{\mu\nu} = \mathrm{Tr}\left[\gamma_\mu S(x,y) \gamma_\nu S(y,x) \right]$$

Note that these calculations are done at μ=0, so in contrast with calculations of section 2 the undeformed instanton is used.

Here the averaging means over the vacuum state and the propagator $S(x,y)$ is the exact one. In the instanton approximation this can be calculated explicitly using the results of [16] for $\Pi_{\mu\nu}$. (note that they should be multiplied by 2/3 in order to have the true result, see discussion in [2]). One more comment is that zero modes are crucial for such calculations, without them one finds completely unreasonable results with the non-conserved external vector current (say, electric charge). The following expression is useful here [16]:

$$\int d\left(\frac{x+y}{2}\right) \Pi_{\mu\mu}(x-y=\Delta) = \frac{12 \rho^4}{\pi^2 \Delta^2} \frac{\partial}{\partial \Delta^2} \left[\frac{1}{\Delta \sqrt{\Delta^2 + 4\rho^2}} \ln\left(\frac{\sqrt{\Delta^2+4\rho^2}+\Delta}{\sqrt{\Delta^2+4\rho^2}-\Delta} \right) \right] \tag{27}$$

which leads to the following correction to the thermodynamical potential

$$\Delta \Omega = \frac{3}{2} \rho_c^2 \mu^2 N_f n_c \tag{28}$$

First, this result disagrees with that by Corvalho (1)even corrected to 2/3 mentioned above. Second, the result also disagrees with (18) by the factor 3/4. Again, the deformation of the instantons changes the result quantitatively but not qualitatively.

Now, some more general discussion of these results is in order. We have argued in [2] that the structure of ordinary hadrons in terms of quasiparticles, the valons, is reasonable. These valons seems to enter in additive way in many hadronic properties like masses, cross sections etc. Is it also reasonable to describe the low density and/or temperature matter in such terms ?

In order to unswer this question we should compare the results obtained above for the instanton-induced corrections with the model considering the matter as some plasma of massive constituents. Expanding in power of masses up to the first order in m^2 one finds:

$$\Omega(\mu=0,T)=-\frac{\pi^2T^4(N_c^2-1)}{45}-\frac{7\pi^2T^4N_cN_f}{180}+\frac{m_g^2T^2(N_c^2-1)}{12}+\frac{m_{qT}^2T^2N_cN_f}{12}+\ldots \quad (29)$$

$$\Omega(\mu,T=0)=-\frac{\mu^4N_cN_f}{12\pi^2}+\frac{m_{q\mu}^2\mu^2N_cN_f}{4\pi^2}+\ldots$$

By the identification of the mass corrections with (25,28) one can obtain the following values for the "effective masses"

$$m_{qT}=2\pi\rho_c(2n_c/N_c)^{1/2} \qquad m_g=0$$

$$m_{q\mu}=2\pi\rho_c(3n_c/2N_c)^{1/2} \quad (30)$$

In this connection let us remind also another value of the effective quark mass which enters the instanton density and was found in [1], to be equal to $M_{eff}=2\pi\rho_c(n_c/N_c)^{1/2}$. It is very interesting that all three expressions, although different, are all numerically close: with our values for n_c, ρ_c they are equal to 170, 210, 240 Mev respectively for $M_{eff}, m_{q\mu}, m_{qT}$. It shows that the model of additive (noninteracting) constituent quarks makes sense as some approximation, although it is not exact.

Another interesting point here is zero value of the "constituent gluon" mass. It does non mean, of course, that gluoniums are very light. On the contrary, as it was shown in [2] our model for QCD vacuum leads to rather heavy gluonic states due to strong instanton-induced gluon-gluon interaction. The meaning of this result is that " constituent gluon" picture is wrong.

4.Restoration of the chiral symmetry

We have discussed in [1] that instanton-antiinstanton interaction leads to spontaneous breakdown of chiral symmetry if their density is large enough. As far as instantons are suppressed in matter, the chiral symmetry is restored.

Let us start with the low density end. In this phase our main object is the equation for the selfconsistent value of the quark condensate $\langle\bar\Psi\Psi\rangle$. This equation was considered in [1] where we have estimated the value of $\langle\bar\Psi\Psi\rangle$ in vacuum. Now our problem is to make more detailed analysis of its dependense on the instanton density.

We remind that the quark condensate determines the instanton density because of zero fermion modes, due to which it is proportional to ($M_{eff}\equiv-2\pi^2\langle\bar\Psi\Psi\rangle\rho^2/3$):

$$(M_{eff}\rho)^{N_f} \qquad\qquad (M_{eff}\rho\ll 1) \quad (31)$$

In the opposite limit $M_{eff}\rho\gg 1$ the fermionic determinant approach unity, while the dependense at $M_{eff}\rho\sim 1$ is not accurately known. As far as it is not very essential, at Fig.1 we have plotted this determinant as $th(M_{eff}\rho)$, with both limits being correct. Let us write down the instanton density in the following way:

$$n(T,\mu)=n_c\cdot F(T,\mu)\cdot\prod_{i=1}^{N_f}\frac{th(m_i\rho_c+M_{eff}\rho_c)}{th(m_i\rho_c+M_c\rho_c)} \quad (32)$$

where we have included the factor $F(T,\mu)$ describing the instanton suppression by matter and M_c is M_{eff} in vacuum.

The selfconsistency equation is the following one:

$$-\langle\bar{\Psi}\Psi\rangle = 2n(T,\mu)/M_{eff} \qquad (33)$$

At Fig.1 we have plotted the dependence of both r.h.s. and l.h.s. of this equation on $M_{eff}\,\rho$. The curves (1) and (2) correspond to suppression factor values F=1 (or QCD vacuum) and F_c=1/2. In the former case there exist three solutions for the equation (33): the trivial one $\langle\bar{\Psi}\Psi\rangle=0$, and two nontrivial ones corresponding to the ground state (the true vacuum) and another unstable one. At F_c only two solutions remain and with further suppression of the instantons only trivial solution is possible. This is the typical picture of the mean field approximation known for many systems in physics.

Interesting result is that the transition turns out to be of the first order with finite jump of the $\langle\bar{\Psi}\Psi\rangle$ at n_{chir}. Moreover, instead of decreasing its value even increases as compared with the vacuum value. This is because the role of the condensate in the instanton density is very great at realistic number of flavours N_f=3.

The equation (33) suggests that the case N_f=1 is singular for in this case the condensate decreases proportional to the instanton number. This is natural because in this case there is no phase transition for there is no symmetry other than U(1) to be violated, and this symmetry is violated at any density of the matter. So, with N_f=1 we are always in the "valon" phase with nonsero mass, decreasing with increasing n,T.

Now a few words about the approach to the transition from the chirally symmetric phase. As discussed in [1] (see references therein) the symmetry breaking manifest itself as the tachion pole developing in the Green function for the scalar $\bar{q}q$ pair. Our numerical estimate for the kernel of the corresponding Bethe-Solpeter equation gave $\sum(p) \simeq 2\exp(-2p\rho_c)$ where the instability condition looks as $\sum(p)>1$. Again one can see that reduction in the instanton density by the factor of two makes the instability impossible at all pair momenta p.

Now, what are the numerical values of the critical parameters? The simple-minded picture of geometrical valon overlap leads to prediction that the critical energy density should be about one order of magnitude larger than inside hadrons. The application of the demping factors (24) and (23) give, respectively

$$n_{chir} \simeq 1\ \mathrm{fm}^{-3}\ (\text{ if } T=0\ );\quad T_{chir} \simeq 160\ \mathrm{Mev}\ \ (\text{if } n=0) \qquad (34)$$

We think that further account for instanton-instanton interaction will lead to some clustering of instantons and matter so that they can shere the space more effectively and transition parameters be shifted to somehow larger values.

5. High density and/or temperature plasma

This section is devoted to discussion of the instanton-induced effects in the asymptotically hot or dense matter, where they are essentially reduced in magnitude compared with these in physical vacuum. In this case instantons with dimensions smaller than ρ_c become the main ones, with radii of the order of $1/T$ or $1/\mu$. This is, for example, evident from (22). As a result, the barrier penetration factor $\exp(-8\pi^2/g^2)$ contains $g(1/T)$ and not $g(\rho)$, as suggested in [9,4].

Starting with pure gluodynamics at high T we may explicitly calculate the instanton contribution based on the expression (23)

$$\Delta\Omega = 2\int dn(\rho', T=0)\exp\left[-\theta^2\rho'^2\left(\frac{2N_c}{3}+\frac{N_f}{3}\right)+\ln\left(1+\frac{\theta^2\rho'^2}{3}\right)\right] \propto \left(\frac{\Lambda}{T}\right)^{\frac{11N}{3}} T^4 \qquad (35)$$

At high enough temperature the main contribution is connected with small instantons so that their interaction can be neglected.

The situation is different in the case of more than one massless quark flavour, in which the absense of chiral symmetry breaking in high density phase leads to zero density of separated instantons. We remind that it is due to different chirality of zero modes $\bar{\Psi}_0, \Psi_0$ due to which the instanton density is proportional to the product of quark masses, explicitly violating the chiral symmetry.

The nonperturbative effects are then connected with instanton-antiinstanton pairs with zero topological (and axial) charge. In this case N_f quark pairs emitted by the instanton are absorbed by the antiinstanton. The corresponding amplitude is proportional to

$$A(z,\bar{z}) \xrightarrow[(z-\bar{z})^2 \gg \rho^2\bar{\rho}^2]{} |S(z,\bar{z})|^2 \sim 1/(z-\bar{z})^6 \qquad (36)$$

in power N_f. So, the density of such pair can be written as follows:

$$dn_{pair} = dn_0(\rho, z)\, dn_0(\bar{\rho}, \bar{z}) \exp\left(S_{int}(\rho, \bar{\rho}, z, \bar{z})\right)$$

$$S_{int} \xrightarrow[(z-\bar{z})^2 \gg \rho^2\bar{\rho}^2]{} (-3N_f)\ln(z-\bar{z})^2 \qquad (37)$$

where dn_0 and other notations are as in [1] . The integral over $(z-\bar{z})^2$ is strongly cut-off. One can call this phenomenon the confinement of the topological charge by massless fermions.

Note that such ideas in somehow other context were discussed in the work [13] .

The contribution of small $(z-\bar{z})^2$ is uncertain because of "core" type effects, see [1] , so it is rather difficult to estimate the coefficient. Still, by dimensional reasons, the final result looks as follows

$$\Delta\Omega_{pair} \simeq const\cdot\left(\frac{\Lambda}{T}\right)^{\frac{22}{3}N_c - \frac{4}{3}N_f} T^4 \qquad (T \gg \mu, 1/\rho_c) \qquad (38)$$

and similarly in the case $T=0$, $\mu \neq 0$ by the substitution $T \to \mu$. Summarizing, the instanton effects above the point of the restoration of chiral simmetry are dropped very rapidly.

6. Summary and discussion

The main conclusion of the present work is essentially the same as in [9,4], namely: instantons are suppressed by high density and/or temperature matter. This phenomenon is now understood in somehow greater details, although some questions remain open.

The second important point follows directly from the central idea of the present series of papers, which in short can be summarised in such way: instantons in vacuum are small. Correspondingly, their suppression is shifted towart higher temperature and density region than, for example, suggested in [5] .

We also suggest that the suppression of instantons results in separate phase transition (inconnected with the deconfinement one) in which the chiral symmetry is restored. The unexpected result is that this transition seems to be of the first, not of the second order.

The contribution of the instantons both in low and high density limits are estimated, so the qualitative picture is to some extent completed. Still more theoretical calculations are needed to make it more quantitative.

The last question we are going to discuss is whether these phenomena can also be observed experimentally, in high-energy collisions of heavy ions in particular. The relevant energy densities are of the order of $10 \div 20$ of that inside nuclei. So, with energy (in Gev) per nucleon in CM system of comparable order we have (due to Lorentz contraction) the nesessary energy density. The question however remains what part of this energy can be transfered to the thermal one in the collision. This in turn depends on the intensity of the interaction between quarks and gluons.

Previously (see e.g.[4]) this question was considered at perturbative level. However, as shown in the present work, the main effects which produce leading correction to the asymptotic freedom (=free propagation of quarks and gluons) are the nonperturbative effects of the instanton nature. The key parameter here is ρ_c , the typical instanton dimension, and $\rho_c \simeq 1/600$ Mev. However, in some channels numerical enhansement factors lead to essentially larger scale of the scale-breaking effects. In particular, for gluons in 0^+ and 0^- channels we have shown in [2] that the asyptotic freedom is violated at $(p_1+p_2)^2 \simeq 10$ Gev^2. At such invariant collision energy of the gluon pair they can be effectively scattered, at larger ones they just "go through" each other (or scatter at small angles). This consideration pose some limit on the highest temperature available in laboratory to be about 400-500 Mev, which is higher than the transition parameters considered above. Another consequence is that Landau model for hadronic collisions should not be valid above the ISR energies [4]. Experiments with $\bar{p}p$ collisions at SPS will soon put this prediction under experimental test.

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Figure caption

Grafical solution of the equation (33) for the selfconsistent value of the quark condensate, the solid and the dashed curves correspond to its r.h.s., while the streight line represent the l.h.s. So, the crossing points are the solutions. The solid curve correspond to the critical value of the instanton damping factor (about 0.5) and the point marked by "C" is the critical value of the condensate. The dashed curve corresponds to the vacuum value of the instanton density, the relevant solution is marked by "V". For smaller damping factors than 0.5 the chiral symmetry is restored.

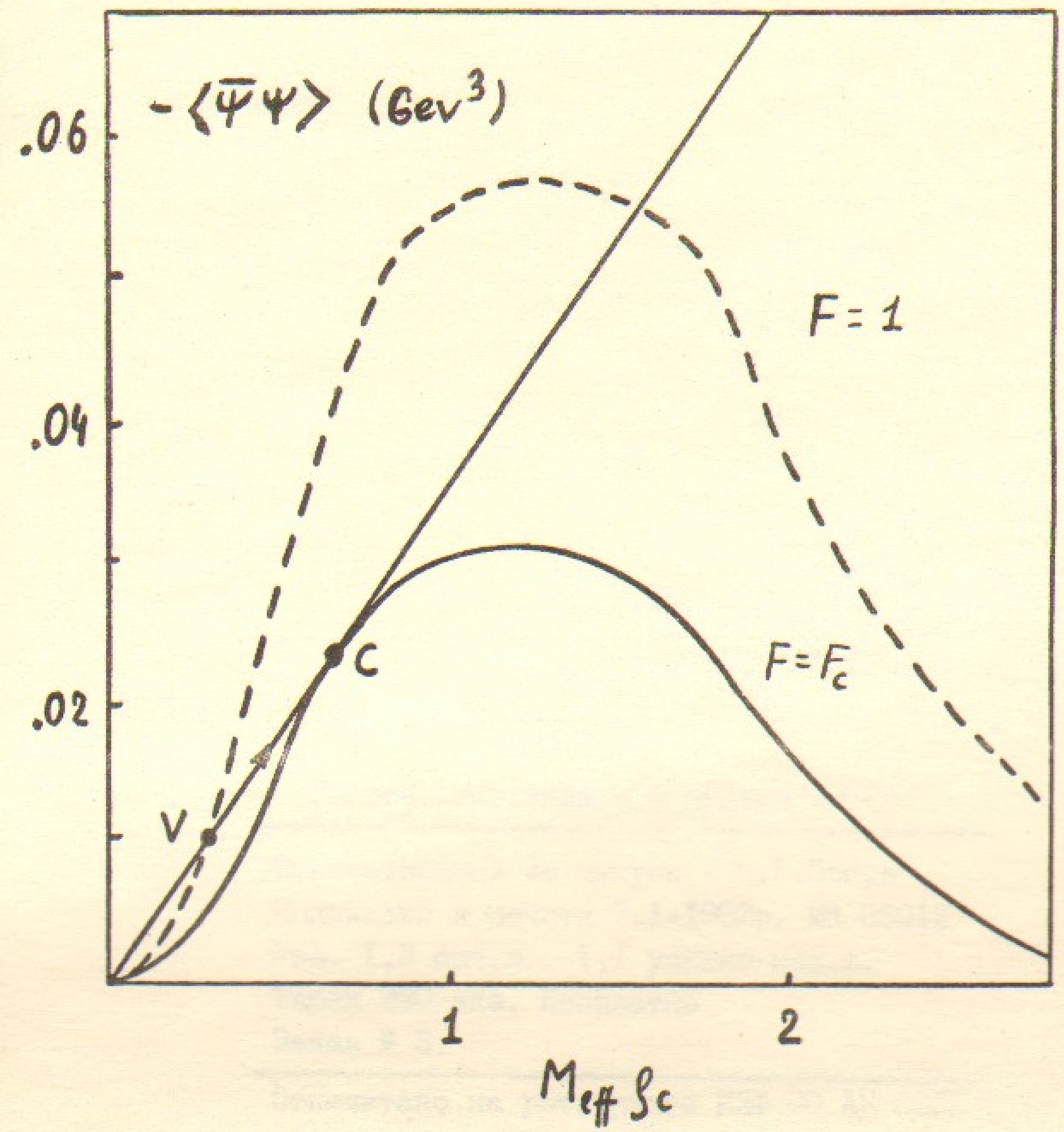

Fig. 1.

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