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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
СО АН СССР

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THE EFFECT OF BOUND STATES OF HEAVY QUARKS ON THE CORRECTIONS TO INTERMEDIATE VECTOR BOSON MASSES

Preprint 82-30

Новосибирск

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A b s t r a c t

The corrections to the masses of intermediate vector bosons of weak interaction due to bound states of heavy quarks are found in nonrelativistic approximation. It turns out that taking into account bound states of quarks changes the values of Z - and W - boson masses negligibly.

Recently one-loop corrections to Z - and W - boson masses occuring due to the loops with Higgs and intermediate vector bosons and also with fermions (quarks and leptons) have been calculated by several groups [1-5]. It turned out that the biggest corrections come from the light fermions whose masses are much less than intermediate boson masses. Taking into account all the fermions leads to the enhancement of boson masses by approximately 3 GeV [1]. Accounting strong interaction in first order in α_S leads to the additional enhancement of boson masses by several hundreds MeV due to the light fermions. First order α_S corrections provided by (b,t) -doublet have peculiarity (fracture) as functions of m_t and are negative when

$$m_t = M_Z/2 \quad \text{or} \quad m_t = M_W - m_b \tag{1}$$

due to Coulomb interaction[1]. The possibility for t -quark mass to be sufficiently large to satisfy one of the relationships (1) which means that bound quark state masses are in the vicinity of the appropriate vector boson mass is not excluded by some authors (see, e.g., [7]). So it is of interest to take into account the effect of bound quark states on intermediate vector boson mass corrections.

In zero approximation W - and Z - masses are given by formulas:

$$M_W^{(0)} = \frac{1}{\sin\theta_W}\left(\frac{\pi\alpha}{G_F\sqrt{2}}\right)^{1/2}, \quad M_Z^{(0)} = \frac{1}{\cos\theta_W \sin\theta_W}\left(\frac{\pi\alpha}{G_F\sqrt{2}}\right)^{1/2}. \tag{2}$$

When taking account of vacuum polarization corrections to the squared W - and Z - masses arise [2]:

$$\delta M_W^2 = \mathrm{Re}\,\Pi^{WW}(M_W^2) - \left[\Pi^{WW}(0) + M_W^2\left(\frac{c}{s}\frac{\Pi^{\gamma Z}(q^2)}{q^2} + \frac{\Pi^{\gamma\gamma}(q^2)}{q^2}\right)_{q^2=0}\right], \tag{3}$$

$$\delta M_Z^2 = \mathrm{Re}\,\Pi^{ZZ}(M_Z^2) - \left[\Pi^{WW}(0)/\cos^2\theta_W + M_Z^2\left(\frac{c^2-s^2}{cs}\frac{\Pi^{\gamma Z}(q^2)}{q^2} + \frac{\Pi^{\gamma\gamma}(q^2)}{q^2}\right)_{q^2=0}\right],$$

$c = \cos\theta_W$, $s = \sin\theta_W$

where the terms in square brackets are responsible for the renormalization of low energy parameters in formulas (2). Here $\Pi_{\mu\nu}^{ij} = g_{\mu\nu}\Pi^{ij} + q_\mu q_\nu \Pi'^{ij}$ - vacuum polarization tensor,

1) It worth noting that the singularity in polarization operator found in [6] doesn't mean an infinite boson mass correction because this correction is defined by the position of the poles of boson propagator and is always finite. In the present case the formulas of 6 lead to small ($\sim$10 MeV) decreases of boson masses (in more detail see below).

$i, j = \gamma, Z, W$.

We shall confine ourself to the nonrelativistic problem. Then $\Pi^{ij}(q^2) = 3a^i a^j 2G_k(0,0)$, where a^i is the coefficient in the quark-i-boson interaction vertex $\Gamma^i_\mu = \gamma_\mu (a^i + b^i \gamma_5)$. $G_k(0,0)$ is the value of the Green function of two-quark system when $\vec r = \vec r\,' = 0$ (more exactly, it is $\lim_{\vec r, \vec r\,' \to 0} [G_k(\vec r, \vec r\,') - G_{k_0}(\vec r, \vec r\,')]$ at some renormalization point k_0); $k^2/2\mu = \varepsilon - m_1 - m_2$, $\varepsilon = \sqrt{q^2}$, μ is the reduced mass of two quarks m_1, m_2, coefficient 2 is connected with accounting quark polarizations, 3 is the colour factor. We have: $a^W = g/2\sqrt2$, $a^Z(Q_q) = (Q_q/|Q_q|)(1 - 4|Q_q|\sin^2\theta_W) g/4\cos\theta_W$, $a^\gamma(Q_q) = Q_q g \sin\theta_W$, Q_q - quark charge in units of e.

The effective potential between quarks at short distances has the form $U = -\tilde\alpha_s/r$, where $\tilde\alpha_s = (4/3)\alpha_s$, $\alpha_s = \alpha_s(r)$ - running coupling constant at a distance r, i.e. at a transmitted momentum $\sim 1/r$:

$$\alpha_s(r) = \frac{2\pi}{[11 - (2/3)N_f(1/r)]\ln(1/r\Lambda)} \qquad (4)$$

Here $N_f(1/r)$ - the number of flavours of the quarks whose masses are smaller than $1/r$, $\Lambda \sim 100$ MeV. For bound quark states the typical distance at which α_s is taken is $1/\varkappa_n$ where $\varkappa_n = |k_n| = \mu\tilde\alpha_s/n$, $n = 1, 2, \ldots$ is the number of level, $k_n = i\varkappa_n$. Thus, α_s is determined by self-consistency condition $\alpha_s(n/\mu\tilde\alpha_s) = \alpha_s$. We shall take Λ = 70 MeV. Let intermediate boson mass be in proximity to corresponding bound state masses, i.e., t - quark mass satisfies approximately one of the conditions (1). Then it may be accepted for the number of the first levels of $t\bar t$ that α_s = 0.2 and for the first $b\bar t$-levels - α_s = 0.25.

On the other hand, the effective potential is essentially modified at the distances $\sim a$ where $\alpha_s(1/a)$ = 1. For Λ = = 70 MeV $1/a$ = 140 MeV. The effect of these distances on $G_k(0,0)$ is described by power corrections $\sim 1/(\varkappa a)^n$ [8] where $k = i\varkappa$ in the region of bound states. Thus, in the region of first levels where $\varkappa_n = \mu\tilde\alpha_s/n$, n = 1,2,..., for $b\bar t$- and especially for $t\bar t$- system this effect is small enough to enable us to give the quantitative estimate of $G_{i\varkappa}(0,0)$

in the Coulomb field approximation. The expression for $G_{i\varkappa}$ in Coulomb field is taken from [8]. It can be written in the following form:

$$G_k(0,0) = -i\mu\frac{k}{2\pi} + \frac{\mu^2\tilde\alpha_s}{\pi}\ln\frac{k}{const} + \frac{\mu^2\tilde\alpha_s}{\pi}\left[\psi\left(1 + \frac{\mu\tilde\alpha_s}{2k}\right) - \psi(1)\right], \quad \psi(z) = \frac{d}{dz}\ln\Gamma(z). \qquad (5)$$

Here two first terms are nonrelativistic analogs of the contributions to the vacuum polarization $\Pi^{ij} = 3a^i a^j 2G_k(0,0)$ of the zero and first order in α_s correspondingly. In fact, they were calculated in [1-6]. The third term is the sum of the convergent nonrelativistic self-energy diagrams of all the orders in α_s from the second. Thus, correction δM_i^2 can be represented in the form $\delta M_i^2 = \delta(M_i^{(0)})^2 + \delta(M_i^{(1)})^2 + \delta\tilde M_i^2$ where $i = Z, W$; $\delta(M_i^{(0)})^2$, $\delta(M_i^{(1)})^2$ are the zero and first order α_s contributions; $\delta\tilde M_i^2$ is found from the formulas (3) where $\tilde\Pi^{ij} = 3a^i a^j 2\tilde G_k(0,0)$ is substituted for Π^{ij}; $\tilde G_k(0,0) = (\mu^2\tilde\alpha_s/\pi)[\psi(1 + \mu\tilde\alpha_s/2k) - \psi(1)]$. The last expression has the poles when $-ik = \varkappa = \varkappa_n$ = $= \mu\tilde\alpha_s/n$, i.e., when Z- or W-mass coincides with the appropriate bound quark state mass the corrections $\delta\tilde M_i^2$ calculated with the help of (3) is infinite. This case corresponds to the degenerate perturbation theory when the mass correction is determined by the matrix element for the transition $Z \to t\bar t$ or $W \to b\bar t$ which is proportional to the first power of g, the semiweak interaction constant. Because of this one could expect considerable effect in comparison with the ordinary perturbation theory with noncoincident masses. In the present case M_i^2 is found from the equations $M_i^2 - (M_i^{(0)})^2 - \delta M_i^2(M_i^2) = 0$, $i = Z, W$, or $M_i^2 - (M_i^{(2)})^2 - \delta\tilde M_i^2(M_i^2) = 0$, where the zero and first order α_s corrections calculated in [1-6] are included in $M_i^{(2)}$: $M_i^{(2)} = M_i^{(0)} + \delta M_i^{(0)} + \delta M_i^{(1)}$. We shall take the following values for masses: $m_b \approx 5$ GeV, $M_Z^{(2)} \approx 90$ GeV, $M_W^{(2)} \approx 80$ GeV. In fig. 1,2 the dependences $\delta\tilde M_Z(m_t)$, $\delta\tilde M_W(m_t)$ are represented. They consist of a set of peaks whose widths are of the order of their heights. Also shown are the dependences $\delta M_i^{(1)}(m_t)$ taken from [6]. When $m_1 + m_2 \to M_i$ there is a singularity of the form $\tilde\Pi \sim -3a^i a^j 2\frac{\mu^2\tilde\alpha_s}{\pi}\ln\sqrt{m_1 + m_2 - M_i}$

resulting in the peak $\sim$10 MeV in the dependence $\delta\tilde{M}_i(m_t)$. This singularity cancel the coulomb singularity when $m_1+m_2 \to M_i$ in polarization operator in the first order in α_S [6]. In accordance with this mentioned above the decrease $\sim$ 10 MeV in $\delta M_i^{(2)}(m_t)$ when $m_1+m_2 \to M_i$ provided by the first order correction $\delta M_i^{(1)}(m_t)$ is compensated by the increase of $\delta\tilde{M}_i(m_t)$ then.

In the region of highly excited levels considerable role is played by the corrections connected with the deviation of the potential from the Coulomb one at large distances. So there is no sense in calculating the mass shifts $\delta\tilde{M}_i$ in this region with great accuracy. Nevertheless one can say that they will be quite small in comparison with the mass corrections $\delta\tilde{M}_i$ in the region of first levels. Thus, occuring bound states in the annihilation channel reveal itself only in the narrow (of the order of 100 MeV in t-quark mass) regions in the vicinity of thresholds for these states. Corresponding corrections to Z - and W- masses are tens of MeV for the set of first levels. As for the zero and first order α_S corrections to Z - and W - masses [1-6] due to (b,t) - doublet, they are $\sim$100 MeV in this region of altering m_t. Thus, if $M_{Z,W}$ coincides with the bound quark states corrections to Z - and W - masses increase negligibly in comparison with the perturbation theory. The reason for this consists in smallness of α_S because the matrix element for the transition $Z \to t\bar{t}$ $(W \to b\bar{t})$ is proportional to the value of Ψ - function of two-quark system in the centre: $|\Psi_n(0)| \sim [(\mu/n)(4/3)\alpha_S]^{3/2}$.

The author is indebted to Dr. I.B.Hriplovich for stating the problem and valuable discussions and considering the work and Dr. A.I.Vainshtein for considering the work.

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Figure captions

Fig. 1. $\delta M_Z^{(1)}$ - Z - mass shift in the first order in α_S [6], $\delta\tilde{M}_Z$ - the additional mass shift for account of bound states $t\bar{t}$.

Fig. 2. $\delta M_W^{(1)}$ - W - mass shift in the first order in α_S [6], $\delta\tilde{M}_W$ - the additional mass shift for account of bound states $b\bar{t}$.

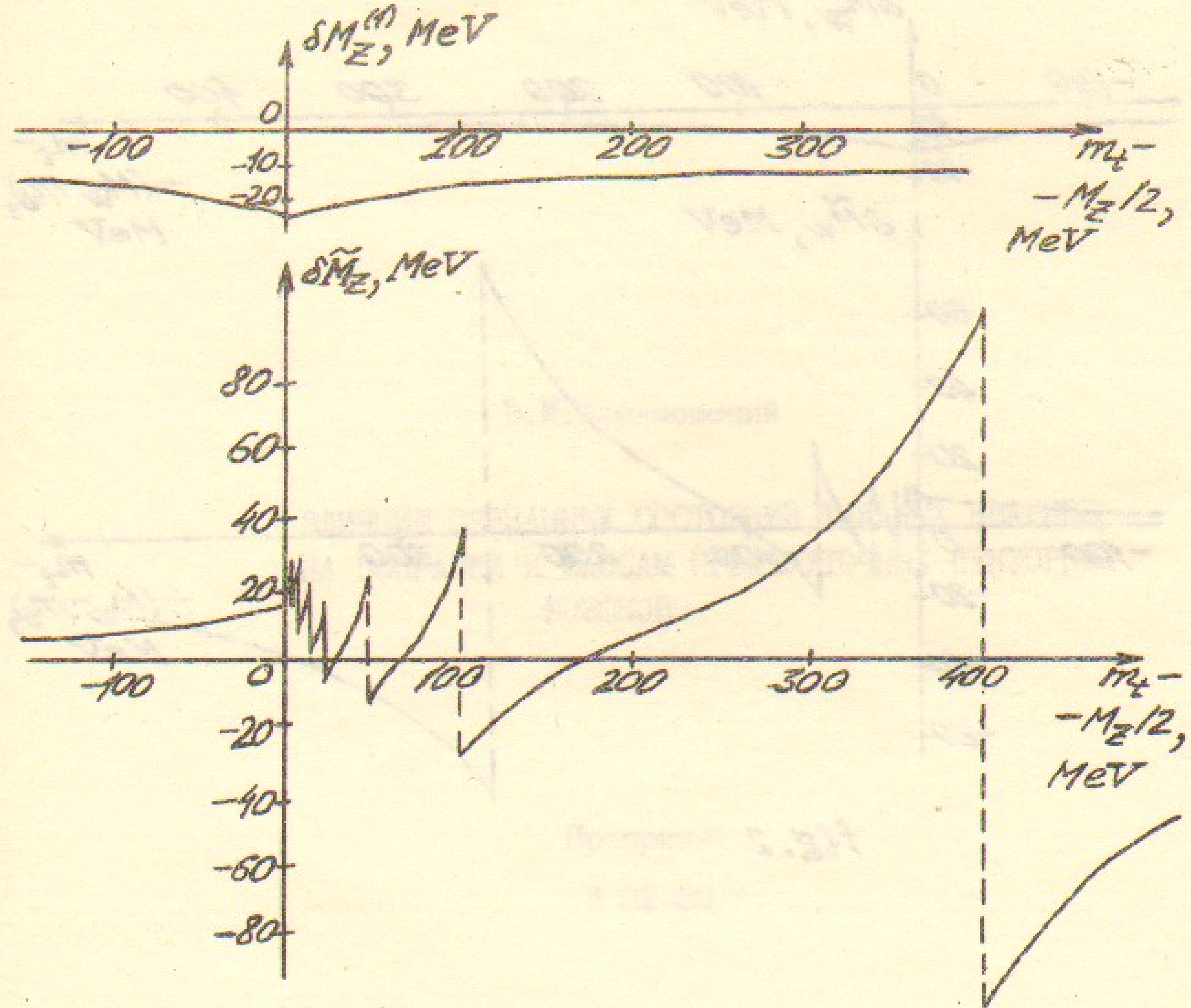

fig. 1

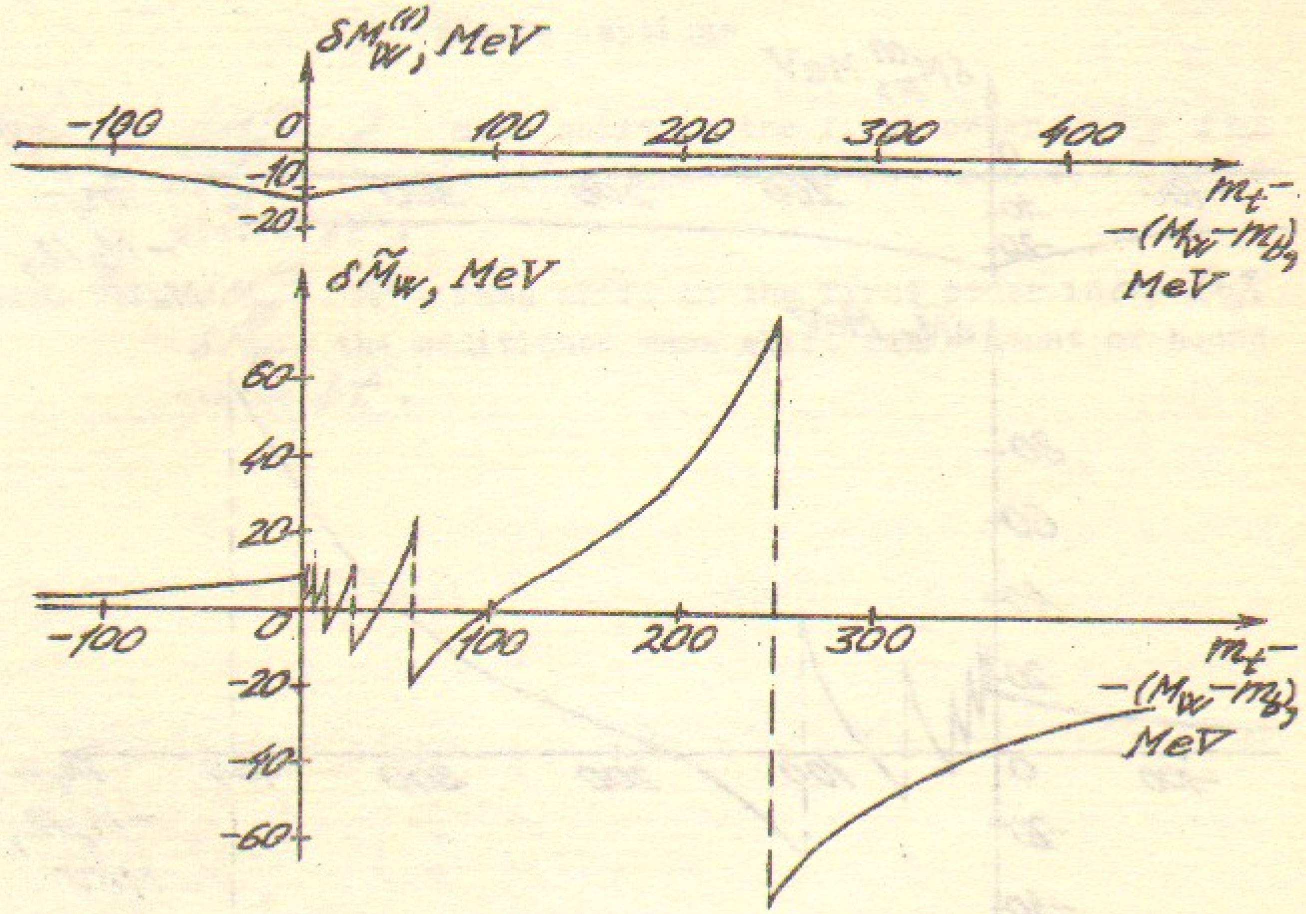

fig. 2

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ВЛИЯНИЕ СВЯЗАННЫХ СОСТОЯНИЙ ТЯЖЕЛЫХ КВАРКОВ НА ПОПРАВКИ К МАССАМ ПРОМЕЖУТОЧНЫХ ВЕКТОРНЫХ БОЗОНОВ

Препринт

№ 82-30

Работа поступила - 22 января 1982 г.

Ответственный за выпуск - С.Г.Попов
Подписано в печать 15.03-1982 г. МН 03153
Формат бумаги 60x90 1/16. Усл. 0,5 печ.л., 0,4 учетно-изд.л.
Тираж 290 экз. Бесплатно. Заказ № 30.

Ротапринт ИЯФ СО АН СССР, г.Новосибирск, 90