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# THE CALCULATION OF THE DEPOLARIZING INELUENCE OF MAGNETIC STRUCTURE´S IMPERFECTIONS IN ELECTRON-POSITRON STORAGE RINGS

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# THE CALCULATION OF THE DEPOLARIZING INFLUENCE OF MAGNETIC STRUCTURE'S IMPERFECTIONS IN ELECTRON-POSITRON STORAGE RINGS

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A b s t r a c t

Radiative kinetics of polarization for magnetic structures of the real storage rings at high - VEPP-4 (5 GeV) and ultra-high - LEP (50 GeV) energies is calculated taking into account a limited accuracy of focusing elements' mounting. Methods of the spin-orbital coupling parameter calculation are presented in a comfortable form for using of computers.

## 1. Introduction

There is no a depolarizing influence of energy fluctuations in the ideal storage ring. Radiative kinetics of polarization in this case is determined by only spin-flip radiation, and electrons get polarized along the magnetic field direction with the radiative polarization degree $P_0$ = 0,92 and relaxation time [1]:

$$\tau_0\,[hours] = \frac{2{,}74\cdot 10^{-8}\,R^3[cm]}{\langle|\mathcal{K}|^3\rangle\,E^5[GeV]}$$

where E is the particle energy, R - a mean radius of a storage ring, $\mathcal{K}$ - dimensionless orbit curvature (in units of $R^{-1}$), brackets $\langle\ldots\rangle$ mean averaging over an azimuth $\theta$ from zero to $2\pi$.

In real storage rings there are some restrictions on the accuracy of magnetic system elements' mounting. Owing to this reason vertical closed orbit distortions and coupling between radial and vertical betatron oscillations appear, i.e. the orbital motion of electrons becomes not plane. It results in the dependence of particle's spin motion on orbital one*).

The total theory of radiative kinetics of polarization in storage rings was built in Ref. [2,3]. The fundamental conception of this theory is the spin-orbital coupling parameter $\vec{d}$, the physical meaning of which one may explane in a such way.

In the motion of a particle along the closed trajectory there exists a periodical solution for spin direction vector $\vec{n}(\theta) = \vec{n}(\theta+2\pi)$ **) [4], around of which a spin precesses with the frequency $\nu_0$ (in the ideal storage ring $\nu_0 = \nu = E[MeV]/440{,}6$ in units of revolution frequency). The equilibrium particle's spin projection on the unit vector $\vec{n}$ is an integral of motion: $S_n = \vec{S}\cdot\vec{n}$ = const. Let

*) The return influence is negligible

**) If one remember, that rotations in the three-dimensional space form the group, this fact would be almost evident.

the particle's energy jump on the magnitude $\frac{\delta\gamma}{\gamma}$ at some azimuth $\theta_0$ ( $\gamma$ is a relativistic factor). We assume the spin direction to be not change at this moment. When the particle moves along the nonequilibrium trajectory, its spin is deflected from the equilibrium direction $\vec{n}$ by supplementary fields, determined by this trajectory. During the radiative damping time $\lambda^{-1}$ orbital motion relaxates to equilibrium one but the spin projection $S_n$ isn't recovered. The resultant change $\delta S_n$ is determined by the spin-orbital coupling parameter $\vec{d}(\theta)$ :

$$\delta S_n = \vec{S}(\theta_0)\cdot\vec{d}(\theta_0)\frac{\delta\gamma}{\gamma} \qquad (1.1)$$

The value $\vec{d}$ is a periodical function of an azimuth $\theta$ as well as vector $\vec{n}$ . Total methods of calculation of $\vec{d}$ in nonideal storage rings are contained in Ref.[2]. In our considerations when the deviation of $\vec{n}$ from vertical is small, the equilibrium polarization degree P and relaxation time $\tau$ in accordance with Ref. [2] are

$$P = G\cdot P_0 \qquad (1.2)$$
$$\tau = G\cdot\tau_0$$

where

$$G = \frac{\langle |\mathcal{K}|^3\rangle}{\langle |\mathcal{K}|^3[1+\frac{11}{18}|\vec{d}|^2]\rangle}$$

Factor G is sharply decreased near by spin resonances $\nu = \nu_\ell = \ell,\ \ell \pm \nu_x,\ \ell \pm \nu_z$, where $\nu_x$ and $\nu_z$ are radial and vertical betatron numbers, $\ell$ is integer. In consequence of the energy spread in a beam the spin precession frequency is modulated by the law:

$$\nu = \bar{\nu} + \Delta\cos\nu_\gamma\theta$$

Here $\bar{\nu}$ - unperturbed frequency, $\Delta = \bar{\nu}\frac{\delta\gamma}{\gamma}$, $\nu_\gamma$ - the frequency of synchrotron oscillations. Below we don't take into account the influence of synchrotron oscillation on spin motion. Such approximation is valid within regions of tuning out of spin resonances $|\nu - \nu_\ell|$ much greater than max $\{\sigma_\nu, \nu_\gamma\}$ ( $\sigma_\nu$ is the spread of $\Delta$ in a beam, determined by the energy spread)[5].



The aim of this work is to investigate depolarizing effects due to imperfections in real magnetic structures, which may be more complicated than simple analytical models [5]. For instance, the presence in a storage ring of a few number of strong quadrupoles (in fact in mini-$\beta$ sections), depolarizing influence of which is compared or more greater than action of all other elements, doesn't allow to applicate statistical model's results in a smooth approximation. The rigorous calculation of the spin-orbital coupling parameter is also necessary for systems using wigglers (say, for hastening of polarization rate), as the radiative kinetics of polarization in this case may be essentially dependent on the values of $\vec{d}$ at places of wigglers' localization. In such situations it's impossible to calculate the spin-orbital coupling parameter as well as parameters of orbital motion without using of computers. So all expressions for $\vec{d}(\theta)$ in this work are comfortable for programming.

## 2. The method of the spin-orbital coupling parameter calculation

Let the particle's energy jump at some azimuth $\theta_0$, then after N revolutions of the particle in a storage ring the change of spin projection on $\vec{n}$ is ($|\vec{S}| = 1$):

$$\delta S_n(\theta_0+2\pi N) = -\frac{\vec{S}(\theta_0+2\pi N)\cdot\delta\vec{S}_\perp(\theta_0+2\pi N)}{S_n} \qquad (2.1)$$

where $\delta\vec{S}_\perp$ is the change of spin component transversal to $\vec{n}$ as a result of the supplementary fields' action on the nonequilibrium particle's spin. Let's define

$$\delta\vec{S}_\perp^{(K)} = \delta\vec{S}_\perp(\theta_0+2\pi K) - \delta\vec{S}_\perp(\theta_0+2\pi(K-1))$$

So as perturbations are small, one may write

$$\delta\vec{S}_\perp^{(K)}(\theta) = \vec{a}_K(\theta)\, S_n \frac{\delta\gamma}{\gamma} e^{-2\pi K\lambda} \qquad (2.2)$$

after that the total change of $\delta\vec{S}_\perp$ is defined by the sum of all $\delta\vec{S}_\perp^{(K)}$ taking into account phases of their precession around $\vec{n}$ with the spin rotation frequency $\nu_0$ :

$$\delta\vec{S}_\perp(\theta_0+2\pi N) = \sum_{K=1}^{N}[\delta\vec{S}_\perp^{(K)}\cos(\phi_N-\phi_K) + \vec{n}\times\delta\vec{S}_\perp^{(K)}\sin(\phi_N-\phi_K)]$$

$(\phi_N = 2\pi N\nu_0,\ \phi_K = 2\pi K\nu_0)$.

One can rewrite the last expression in more detailed form:

$$\delta\vec{S}_\perp(\theta_0+2\pi N) = S_n\frac{\delta\gamma}{\gamma}[\vec{p}\cos\phi_N - \vec{q}\sin\phi_N + \vec{n}\times(\vec{p}\sin\phi_N + \vec{q}\cos\phi_N)] \qquad (2.3)$$

where

$$\vec{p}(\theta) = \mathrm{Re}\left\{\sum_{K=1}^{N}\vec{a}_K(\theta)\, e^{-i2\pi K(\nu_0-i\lambda)}\right\} \qquad (2.4)$$



$$\vec{q}(\theta) = \mathrm{Im}\left\{\sum_{K=1}^{N}\vec{a}_K(\theta)\, e^{-i2\pi K(\nu_0-i\lambda)}\right\} \qquad (2.4)$$

(if $2\pi N\lambda \gg 1$ then $\vec{p}$ and $\vec{q}$ don't depend on N).

Taking into account, that

$$\vec{S}_\perp(\theta_0+2\pi N) = \vec{S}_\perp(\theta_0)\cos\phi_N + \vec{n}\times\vec{S}_\perp(\theta_0)\sin\phi_N$$

and substituting (2.3) into (2.1) one may write:

$$\delta S_n = -\vec{S}\,(\vec{p} + \vec{n}\times\vec{q})\frac{\delta\gamma}{\gamma}$$

From the last relation and definition of $\vec{d}$ (1.1) it follows that

$$\vec{d}(\theta) = -(\vec{p} + \vec{n}\times\vec{q}) \qquad (2.5)$$

Thus, there are three steps for the computation of $\vec{d}(\theta)$ with using of the form (2.5):

i) to find $\vec{n}(\theta)$ by means of integration of spin motion equations for an equilibrium particle;

ii) to find the vector $\vec{a}_K$ by means of integration of spin motion equations for a nonequilibrium particle within limits from 0 to $2\pi$ at k-th revolution (see formula (2.2));

iii) to calculate the sum (2.4).

Let's consider these points in applying to our case.

It is comportable to describe a spin motion in a coordinate base of unit vectors $\vec{e}_{1,2,3}$, connected with an exact direction of particle's velocity $\vec{v}$ [5]. Such base differs negligibly from the conventional system of unit vectors $\vec{e}_{x,y,z}$, used for calculation of orbital motion in storage rings [6,7], as the velocity spread in a beam is small. These two systems are related so:

$$\vec{e}_1 = \frac{\vec{v}\times\vec{e}_z}{|\vec{v}|}, \quad \vec{e}_2 = \frac{\vec{v}}{|\vec{v}|}, \quad \vec{e}_3 = \vec{e}_1\times\vec{e}_2$$

The periodical solution for a spin in a storage ring satisfies the equation

$$\frac{d\vec{n}}{d\theta} = \vec{\omega}_0 \times \vec{n} \tag{2.6}$$

where $\vec{\omega}_0$ is an instantaneous frequency of an equilibrium particle's spin precession.

Let $Z_0$ be the vertical closed orbit distortions, normalized on the mean radius R*). Then in a linear approximation with respect to deviations of an orbit from an ideal one we have a simple expression for the frequency $\vec{\omega}_0$ in the base $\vec{e}_{1,2,3}$ [5]:

$$\begin{aligned} \omega_{01} &= \nu Z_0'' \\ \omega_{02} &= 0 \\ \omega_{03} &= \nu \mathcal{K} \end{aligned} \tag{2.7}$$

$( Z_0'' = d^2 Z_0 / d\theta^2 )$. The expression (2.7) is written for a case $\nu \gg 1$. In the first order of perturbation theory one can obtain the periodical solution of equation (2.6) taking into account (2.7) ($n_z \simeq 1$):

$$n_x(\theta_0) + i n_y(\theta_0) \approx n_1 + i n_2 = \frac{i\nu e^{i\nu\phi}}{(1 - e^{i2\pi\nu})} \int_{\theta_0}^{\theta_0 + 2\pi} Z_0'' e^{-i\nu\phi} d\theta \tag{2.8}$$

where $\phi(\theta_0) = \int_0^{\theta_0} \mathcal{K} d\theta$ is the equilibrium particle's rotation angle in a guide field.

*) We don't consider radial closed orbit distortions, as they cause only the coherent shift of the spin precession frequency and have no influence on radiative kinetics of polarization in our approaching.



Quantum fluctuations of the particle's energy excite betatron oscillations with radial and vertical deviations $X$ and $Z$ (in units of R) from the equilibrium orbit. In this case the linear with respect to $\frac{\delta\gamma}{\gamma}$, $X$, $Z$ and $Z_0$ expression for the instantaneous precession frequency $\vec{\omega}$ may be written ( $\nu \gg 1$) [2,5]:

$$\begin{aligned} \omega_1 &= \nu\left(1 + \frac{\delta\gamma}{\gamma}\right) Z_0'' + \nu Z'' \\ \omega_2 &= 0 \\ \omega_3 &= \nu\mathcal{K}\left(1 + \frac{\delta\gamma}{\gamma}\right) - \nu X'' \end{aligned} \tag{2.9}$$

By means of integration of spin motion equation for a nonequilibrium particle

$$\frac{d\vec{S}}{d\theta} = \vec{\omega} \times \vec{S}$$

within limits from $\theta_1 = \theta_0 + 2\pi(K-1)$ to $\theta_2 = \theta_0 + 2\pi K$ ( $K$ = 1,2 ... is a number of a revolution) one can find the solution for $\vec{a}_K = ( a_x^{(K)}, a_y^{(K)}, 0 )$ in the first order of perturbation theory*):

$$\Delta_K(\theta_0) = ( a_x^{(K)} + i a_y^{(K)} ) \frac{\delta\gamma}{\gamma} e^{-2\pi K\lambda} =$$

$$= i\nu\left[2\pi \frac{\delta\gamma}{\gamma} e^{-2\pi K\lambda} - X'(\theta_2) + X'(\theta_1)\right] \times \left[n_x(\theta_0) + i n_y(\theta_0)\right] e^{i2\pi\nu} -$$

$$- \nu^2 \frac{\delta\gamma}{\gamma} e^{-2\pi K\lambda + i\nu(2\pi + \phi)} \left[\int_{\theta_1}^{\theta_2} Z_0'' \phi e^{-i\nu\phi} d\theta - \phi \int_{\theta_1}^{\theta_2} Z_0'' e^{-i\nu\phi} d\theta\right] + \tag{2.10}$$

*) While obtaining the expression (2.10) we've restricted the expansion of $\exp i[\nu\Phi(1 + \delta\gamma/\gamma) - \nu X']$ by linear terms that'll lead (see below) to the account of only linear resonances $\nu = \ell$ and $\nu \pm \nu_x = \ell$. In the general case owing to the spin precession frequency modulation by radial betatron oscillations there are also nonlinear spin resonances $\nu \pm m\nu_x = \ell$ ( $m$ - integer), but they can be ignored, because their widths are very small.

$$+\nu^2 e^{i\nu(2\pi+\phi)}\int_{\theta_1}^{\theta_2} Z_0'' X' e^{-i\nu\phi}\, d\theta - i\nu e^{i\nu(2\pi+\phi)}\int_{\theta_1}^{\theta_2} Z'' e^{-i\nu\phi}\, d\theta$$

In (2.10) we don't include the term $-i\nu\frac{\delta\gamma}{\gamma} e^{i[2\pi(\nu+i\kappa\lambda)+\nu\phi]} \times \int_{\theta_1}^{\theta_2} Z_0'' e^{-i\nu\phi} d\theta$, as the parameter $\nu$ is supposed to be very big. From (2.4) and (2.5) the formula for $\vec{d}$ is derived

$$(d_x + i d_y)\frac{\delta\gamma}{\gamma} = -\sum_{K=1}^{\infty} \Delta_K e^{-i2\pi K\nu} \qquad (2.11)$$

and $d_z = 0$, because the deviations of $\vec{n}$ from vertical are small.

Further, using (2.10) and (2.11) we'll calculate the partial depolarizing influence of quadrupoles' rotations and vertical closed orbit distortions.

## 3. Depolarizing influence of the quadrupoles' rotations

Let there is an imperfection in the form of radial gradient of radial field $\partial H_x/\partial X$, which is connected with the normal gradient $\partial H_z/\partial X$ by the relation:

$$\partial H_x/\partial X \approx \alpha\, \partial H_z/\partial X$$

where $\alpha$ is the rotation angle of the quadrupole around the orbit direction ( $\alpha \ll 1$). According to (2.9) supplementary fields, acting on the nonequilibrium particle's spin are defined by orbital motion which is described by equations:

$$X'' + g_x X = \mathcal{K}\frac{\delta\gamma}{\gamma} \qquad (3.1)$$

$$Z'' + g_z Z = \varkappa X$$



where $\varkappa(\theta) = \partial H_x/\partial X$ is in units of the mean guide field, $g_x, g_z$ are the focusing functions. The contribution of the member $\varkappa Z$ into X-motion (see (3.1)) is neglected, that's true if the vertical beam size isn't excited very much (for example, in the case, when the working point is far from linear coupling resonance of X - and Z - oscillations, or when this resonance is compensated by special skew quadrupoles).

By means of variable constant method one can derive from (3.1):

$$Z = A_z f_z + \overline{A_z f_z}$$

$$A_z' = \frac{1}{2i}\varkappa X \overline{f_z}$$

$$X = A_x f_x + \overline{A_x f_x} \qquad (3.2)$$

$$A_x' = \frac{1}{2i}\mathcal{K}\frac{\delta\gamma}{\gamma}\overline{f_x}$$

Here $f_{x,z}$ are the solutions of (3.1) with zero right part. Up-line means the complex conjugation. Let's define $f_{x,z}$ so, that $f_{x,z}\exp(-i\nu_{x,z}\theta)$ would be the usual Floquet functions. Besides (3.2) it is convenient to use the following description of X-motion, involving Twiss parameters [6,7]:

$$X = \frac{\delta\gamma}{\gamma}(C f_x + \overline{C f_x}) + \psi_x\frac{\delta\gamma}{\gamma} \qquad (3.3)$$

where $\psi_x(\theta)$ is the dispersion function. A complex constant C may be easily found by using initial conditions at the moment of a photon emission at azimuth $\theta_0$ ( $X = X' = 0$) [7]:

$$C = \frac{1}{2}\sqrt{\mathcal{H}}\, e^{i\varphi}$$

$$\mathcal{H}(\theta_0) = \frac{\psi_x^2 + (\alpha_x\psi_x + \beta_x\psi_x')^2}{\beta_x} \qquad (3.4)$$

$$\varphi(\theta_0) = \arccos\left(-\psi_x/\sqrt{\mathcal{H}\beta_x}\right) - \mu_x$$

where $\beta_x = |f_x|^2$, $\alpha_x = -\beta_x'/2$, $\mu_x(\theta_0) = \int_0^{\theta_0} \frac{d\theta}{\beta_x}$.

Let $Z_0'' = 0$. Then, using (2.10) and (2.11), one can write:

$$\Delta_K = -i\nu e^{i\nu(2\pi+\phi)} \int_{\theta_1}^{\theta_2} Z'' e^{-i\nu\phi} d\theta$$

Sum of all $\Delta_K$ from (2.11) may be found, if one use the conditions $Z'(\theta_0) = Z'(\infty) = 0$:

$$(d_x + i d_y)\frac{\delta\gamma}{\gamma} = -\nu^2 e^{i\nu\phi} \int_{\theta_0}^{\infty} Z' \mathcal{K} e^{-i\nu\phi} d\theta \qquad (3.5)$$

So as (see (3.2))

$$A_z' f_z + \overline{A_z' f_z} \equiv 0$$

then integration of (3.5) by parts gives:

$$(d_x + i d_y)\frac{\delta\gamma}{\gamma} = \frac{i\nu^2}{2} e^{i\nu\phi} \int_{\theta_0}^{\infty} \varkappa x \Big[ f_z \int_{-\infty}^{\theta} \bar{f}_z' \mathcal{K} e^{-i\nu\phi} d\theta' - \bar{f}_z \int_{-\infty}^{\theta} f_z' \mathcal{K} e^{-i\nu\phi} d\theta' \Big] d\theta \qquad (3.6)$$

The lower limit in the integrals $\int_{-\infty}^{\theta} f_z' \mathcal{K} e^{-i\nu\phi} d\theta'$ and $\int_{-\infty}^{\theta} \bar{f}_z' \mathcal{K} e^{-i\nu\phi} d\theta'$, which are determined with an accuracy of an additive constant, is chosen $-\infty$. Integration must be performed with adding of a small term to the exponent index, which doesn't change the subintegral functions on the physically essential interval of the variable $\theta$. It's convenient to define the periodical function $F^\nu(\theta)$ [5]:

$$F^\nu(\theta) = \frac{\nu}{2} e^{i\nu\theta} \Big[ f_z \int_{-\infty}^{\theta} \bar{f}_z' \mathcal{K} e^{-i\nu\phi} d\theta' - \bar{f}_z \int_{-\infty}^{\theta} f_z' \mathcal{K} e^{-i\nu\phi} d\theta' \Big] =$$

$$= \frac{\nu}{2} e^{i\nu\theta} \Big\{ \big[1 - e^{i\frac{2\pi}{m}(\nu+\nu_z)}\big]^{-1} f_z \int_{\theta-2\pi/m}^{\theta} \bar{f}_z' \mathcal{K} e^{-i\nu\phi} d\theta' - \qquad (3.7)$$

$$- \big[1 - e^{i\frac{2\pi}{m}(\nu-\nu_z)}\big]^{-1} \bar{f}_z \int_{\theta-2\pi/m}^{\theta} f_z' \mathcal{K} e^{-i\nu\phi} d\theta' \Big\}$$



(3.7)

Here $m$ is the number of superperiods of the given storage ring magnetic structure. Then (3.6) may be rewritten in more simple form:

$$(d_x + i d_y)\frac{\delta\gamma}{\gamma} = i\nu e^{i\nu\phi} \int_{\theta_0}^{\infty} \varkappa x F^\nu e^{-i\nu\theta} d\theta \qquad (3.8)$$

In such way $F^\nu$-function characterises the spin motion response on perturbations $\varkappa(\theta)$ (about the calculation of $F^\nu$ - function see Appendix).

Substituting (3.3) into (3.8) and integrating by parts, one may find:

$$d_x + i d_y = \frac{i\nu}{2} e^{i\nu\phi} \Big[ C[1 + i\,\mathrm{ctg}\,\pi(\nu_x - \nu)] \int_{\theta_0}^{\theta_0+2\pi} \varkappa \beta_x^{1/2} F^\nu e^{i(\mu_x - \nu\theta)} d\theta +$$

$$(3.9)$$

$$+ \bar{C}[1 - i\,\mathrm{ctg}\,\pi(\nu_x + \nu)] \int_{\theta_0}^{\theta_0+2\pi} \varkappa \beta_x^{1/2} F^\nu e^{-i(\mu_x + \nu\theta)} d\theta +$$

$$+ (1 - i\,\mathrm{ctg}\,\pi\nu) \int_{\theta_0}^{\theta_0+2\pi} \varkappa \psi_x F^\nu e^{-i\nu\theta} d\theta \Big]$$

For speeding up of a numerical integration in (3.9) it's convenient to use the same way as in (3.7) (see Appendix).

It is evident from (3.7) and (3.9) that the contribution of $\varkappa(\theta)$ - perturbations into a value of spin-orbital coupling parameter is more significant near by resonances $\nu = \ell$, $\nu \pm \nu_x = \ell$, $\nu \pm \nu_z = m \cdot \ell$ ($\ell = 0, \pm 1, \pm 2 \ldots$).

Formulae (3.9) and (1.2) were used for calculation of depolarizing effects of quadrupoles' rotations in the storage rings VEPP-4 (m = 1) and LEP (m = 4)*).

Fig. 2a represents the dependence of the factor $G = P/P_0 = \tau/\tau_0$ on $\nu(\gamma)$ for some random rotations of all VEPP-4 quadrupoles. The energy region is near by the $\gamma$ - - resonances. For comparison Fig. 2b illustrates the effect of the rotation of one strong quadrupole, which is situated in mini-$\beta$ section of VEPP-4. The working point of VEPP-4 is near by the coupling resonance $1+\nu_x=\nu_z$, , so spin resonances $\nu \pm \nu_x = \ell$ and $\nu \pm \nu_z = \ell$ are not distinguished on the figures.

Fig. 3 shows the depolarizing effects of all LEP quadrupoles' random rotations within the energy region from 44 up to 47 GeV.

## 4. Depolarizing effects due to vertical closed orbit distortions

An especially hard influence on a spin is caused by radial fields in a storage ring. Radial fields appear mainly by reason of the vertical displacements $\delta Z$ of quadrupoles from their ideal positions. The presence in the median plane of the radial field $H_x = \delta Z \cdot \partial H_z / \partial X$ leads to vertical closed orbit distortions $Z_0(\theta)$. The summary action of radial fields on a spin is characterized by means of a value $Z_0''$ (see (2.7) and (2.9)), which is determined by equation:

$$Z_0'' = -g_z Z_0 + H_x \tag{4.1}$$

($H_x$ is in units of the mean guide field). Using the same way, as in preceeding section, one can find the contribution of vertical closed orbit distortions into value of spin-orbital coupling parameter $\vec{d}$ :

$$d_x + i d_y = \frac{\nu^2}{2} e^{i\nu\phi} \Big\{ -\pi(1+\mathrm{ctg}^2\pi\nu) \int_{\theta_0}^{\theta_0+2\pi} Z_0'' e^{-i\nu\phi} d\theta + \tag{4.2}$$

*) All LEP parameters are taken from "Blue Book" [8].



$$+ (1 - i\,\mathrm{ctg}\,\pi\nu)\Big[ \int_{\theta_0}^{\theta_0+2\pi} Z_0'' \phi\, e^{-i\nu\phi} d\theta - \phi \int_{\theta_0}^{\theta_0+2\pi} Z_0'' e^{-i\nu\phi} d\theta \Big] -$$

$$- C\,[1 + i\,\mathrm{ctg}\,\pi(\nu_x - \nu)] \int_{\theta_0}^{\theta_0+2\pi} Z_0'' \beta_x^{-1/2} (i - \alpha_x)\, e^{i(\mu_x - \nu\phi)} d\theta +$$

$$+ \bar{C}\,[1 - i\,\mathrm{ctg}\,\pi(\nu_x + \nu)] \int_{\theta_0}^{\theta_0+2\pi} Z_0'' \beta_x^{-1/2} (i + \alpha_x)\, e^{-i(\mu_x + \nu\phi)} d\theta - \tag{4.2}$$

$$- (1 - i\,\mathrm{ctg}\,\pi\nu) \int_{\theta_0}^{\theta_0+2\pi} Z_0'' \psi_x' e^{-i\nu\phi} d\theta \Big\}.$$

The two first members in (4.2) are dominant. They may be obtained by the direct differentiation on an energy of expression for $\vec{n}(\theta)$ (2.8). The origin of other members in (4.2) is connected with a modulation of the spin precession frequency by radial betatron oscillations. Numerical integration in (4.2) one can make in the same way as in (3.7) (see Appendix). One must be careful that the step of integration should be correct, especially on the area places of "strong" quadrupoles, where subintegral expressions in betatron terms (see (4.2)) may be sign-variable functions of an azimuth. One can see from (4.2) that radial fields bring in an especially large spin-orbital coupling near by the resonances $\nu = \ell$ and $\nu \pm \nu_x = \ell$ ($\ell = 0, \pm 1, \pm 2 \ldots$).

Figures 4a,b illustrate the depolarizing influence of vertical displacements of quadrupoles in VEPP-4. Fig. 5 represents the dependence of $G$ on $\nu(\gamma)$ for random vertical displacements of the LEP-quadrupoles. This dependence is plotted only for half-integer values of $\nu$ ($\nu = \ell + 1/2$), that are at most disposed of integer spin resonances, which are rather strong in the present case.

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## Appendix

It's convenient to rewrite integrals in (3.7) in such form (let m = 1 for simplicity):

$$\left[1-e^{i2\pi(\nu+\nu_z)}\right]^{-1}\int\limits_{\theta-2\pi}^{\theta}\overline{f_z'}\,\mathcal{K}\,e^{-i\nu\phi}d\theta' = \left[1-e^{i2\pi(\nu+\nu_z)}\right]^{-1}\int\limits_{-2\pi}^{0}\overline{f_z'}\,\mathcal{K}\,e^{-i\nu\phi}d\theta + \int\limits_{0}^{\theta}\overline{f_z'}\,\mathcal{K}\,e^{-i\nu\phi}d\theta'$$

$$\left[1-e^{i2\pi(\nu-\nu_z)}\right]^{-1}\int\limits_{\theta-2\pi}^{\theta}f_z'\,\mathcal{K}\,e^{-i\nu\phi}d\theta' = \left[1-e^{i2\pi(\nu-\nu_z)}\right]^{-1}\int\limits_{-2\pi}^{0}f_z'\,\mathcal{K}\,e^{-i\nu\phi}d\theta + \int\limits_{0}^{\theta}f_z'\,\mathcal{K}\,e^{-i\nu\phi}d\theta'$$

It allows to speed up the numerical integrating in $M/2$ times, where M is the number of points in a storage ring, in which one must calculate $F^\nu$. Let's introduce such notations:

$$v_1(\theta) = -\int\limits_0^\theta \mathcal{K}\beta_z^{-1/2}\left[-\alpha_z\cos(\mu_z+\nu\phi)-\sin(\mu_z+\nu\phi)\right]d\theta'$$

$$v_2(\theta) = -\int\limits_0^\theta \mathcal{K}\beta_z^{-1/2}\left[\cos(\mu_z+\nu\phi)-\alpha_z\sin(\mu_z+\nu\phi)\right]d\theta'$$

$$v_3(\theta) = -\int\limits_0^\theta \mathcal{K}\beta_z^{-1/2}\left[-\alpha_z\cos(\mu_z-\nu\phi)-\sin(\mu_z-\nu\phi)\right]d\theta'$$

$$v_4(\theta) = -\int\limits_0^\theta \mathcal{K}\beta_z^{-1/2}\left[\cos(\mu_z-\nu\phi)-\alpha_z\sin(\mu_z-\nu\phi)\right]$$



$$v_j^\circ = v_j(-2\pi)\ ,\quad j=1,2,3,4$$

$$V_1(\theta) = \frac{1}{2}\left[v_1^\circ + v_2^\circ\,\mathrm{ctg}\,\pi(\nu+\nu_z)\right] - v_1$$

$$V_2(\theta) = \frac{1}{2}\left[v_1^\circ\,\mathrm{ctg}\,\pi(\nu+\nu_z) - v_2^\circ\right] + v_2$$

$$V_3(\theta) = \frac{1}{2}\left[v_3^\circ - v_4^\circ\,\mathrm{ctg}\,\pi(\nu-\nu_z)\right] - v_3$$

$$V_4(\theta) = \frac{1}{2}\left[v_3^\circ\,\mathrm{ctg}\,\pi(\nu-\nu_z) + v_4^\circ\right] - v_4$$

$$\mathrm{Re}\{F^\nu(\theta)\} = \frac{\nu}{2}\beta_z^{1/2}\left[V_1\cos(\mu_z+\nu\theta)-V_2\sin(\mu_z+\nu\theta)-V_3\cos(\mu_z-\nu\theta)-V_4\sin(\mu_z-\nu\theta)\right]$$

Then

$$\mathrm{Im}\{F^\nu(\theta)\} = \frac{\nu}{2}\beta_z^{1/2}\left[V_2\cos(\mu_z+\nu\theta)+V_1\sin(\mu_z+\nu\theta)+V_3\sin(\mu_z-\nu\theta)-V_4\cos(\mu_z-\nu\theta)\right]$$

Because of the periodicity of the $F^\nu$-function over azimuth it's enough to determine it over a single superperiod of a storage ring magnetic structure. Numerical integration in expressions above must be performed with the step $\Delta\theta \ll \min\left\{\frac{2\pi}{\nu}, \frac{2\pi}{\nu_z}\right\}$

Fig. 5 demonstrates the function $|F^\nu(\theta)|$ for VEPP-4 (m = 1) at two different energies of the storage ring.

## Figure Captions

Fig. 1. $G(\nu)$ for VEPP-4 ( $\nu_z$ = 9,57, $\nu_x$ = 8,60)

a) random rotations of all quadrupoles with the angle spread $\alpha \sim 10^{-3}$

b) the single "strong" quadrupole is rotated through the angle $\alpha = 5\cdot 10^{-4}$. Dotted lines mark $\gamma$ - - resonances.

Fig. 2. The dependence $G(\nu)$ for random rotations of all LEP quadrupoles with the angle spread $\alpha \sim 5\cdot 10^{-4}$.

Fig. 3. $G(\nu)$ for VEPP-4 ( $\nu_z$ = 9,57, $\nu_x$ = 8,60)

a) random vertical displacements of all quadrupoles with the spread $\delta z \sim 0{,}02$ cm (r.m.s. value of vertical closed orbit distortions $\sim$ 1 mm)

b) the single "strong" quadrupole is displaced on $\delta z = 0{,}05$ cm (rms value of distortions $\sim$ 2 mm).

Fig. 4. Depolarizing influence of uncontrollable vertical displacements of all LEP - quadrupoles with the spread $\delta z \sim 0{,}005$ cm (r.m.s. value of vertical closed orbit distortions $\sim$ 3 mm).

Fig. 5. Function $|F^{\nu}(\theta)|$ for VEPP-4 ( $\nu_z$ = 9,57)

a) E = 5 GeV ( $\nu$ = 11,36)

b) E = 4.75 GeV ( $\nu$ = 10.73).



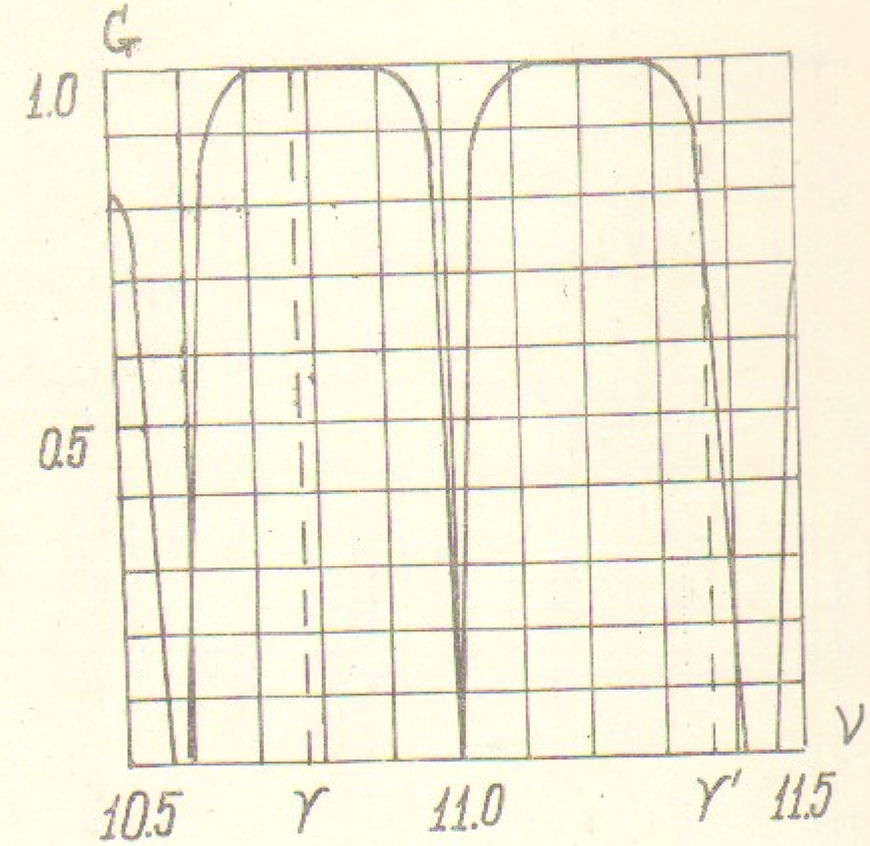


Fig 1 (a)

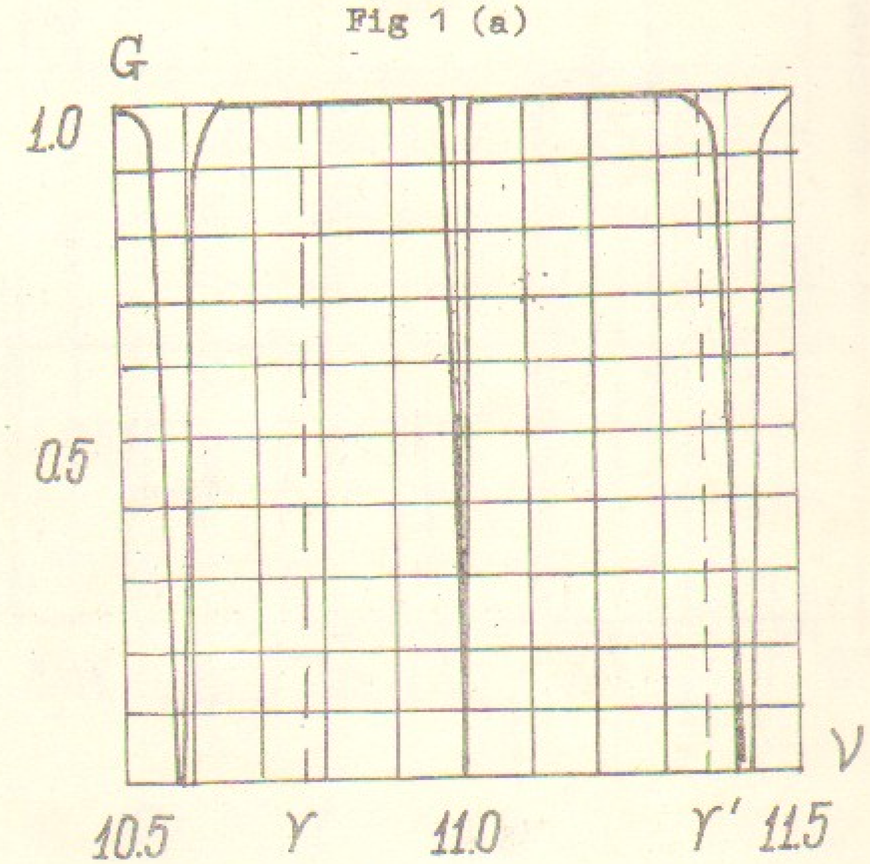


Fig 1 (b)

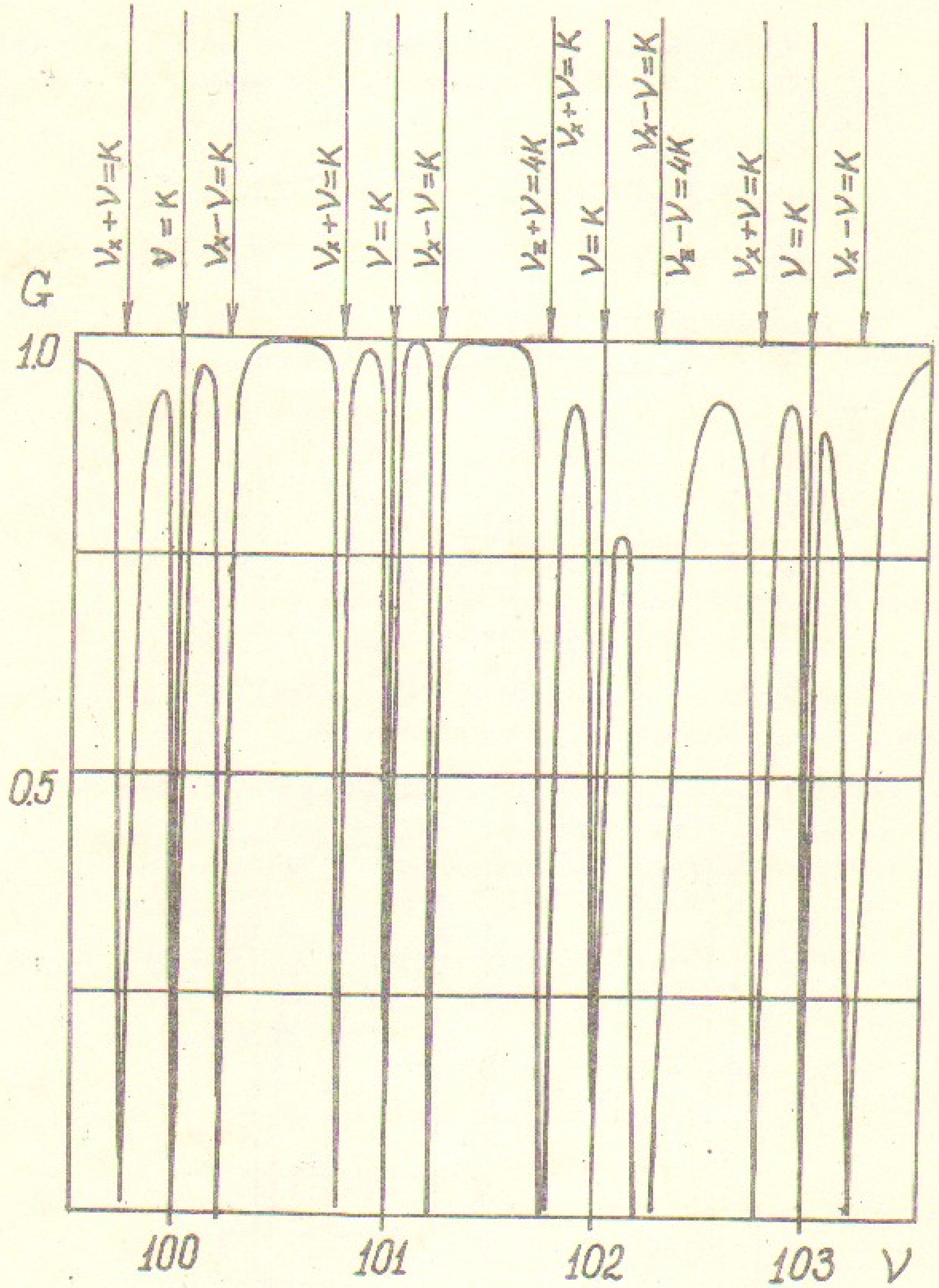


Fig 2



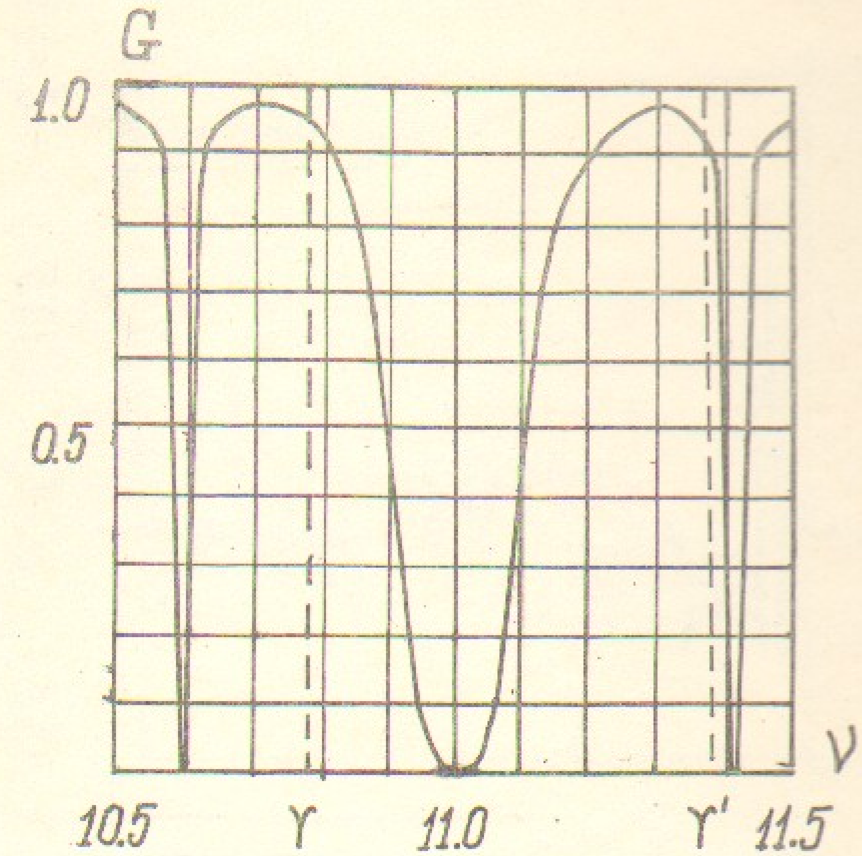


Fig 3 (a)

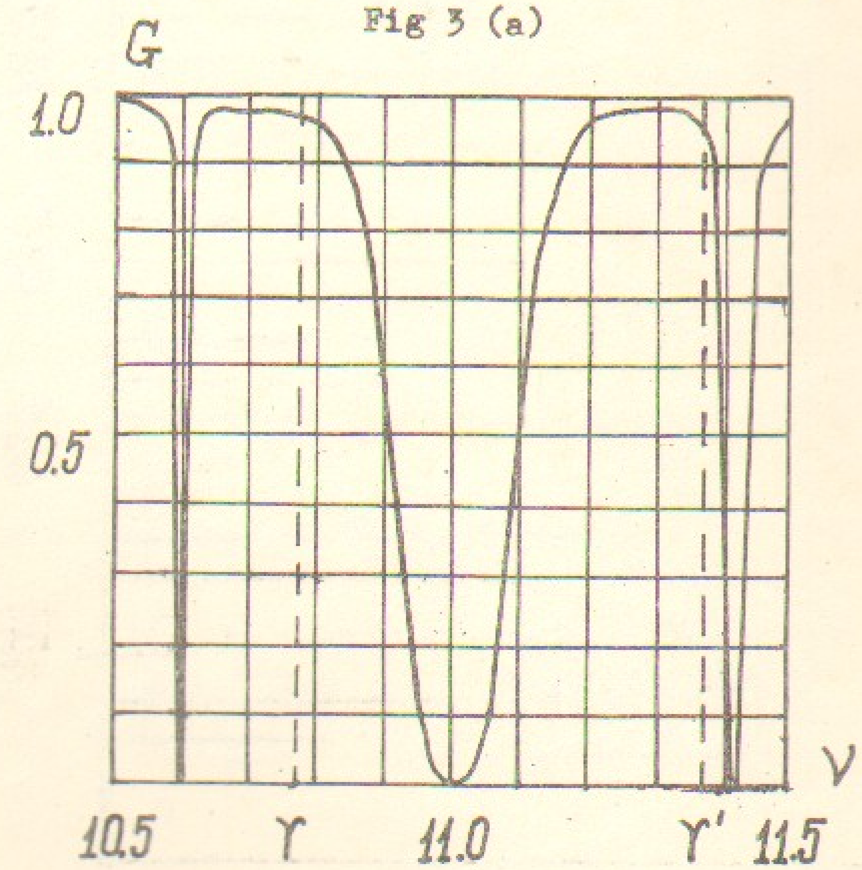


Fig 3 (b)

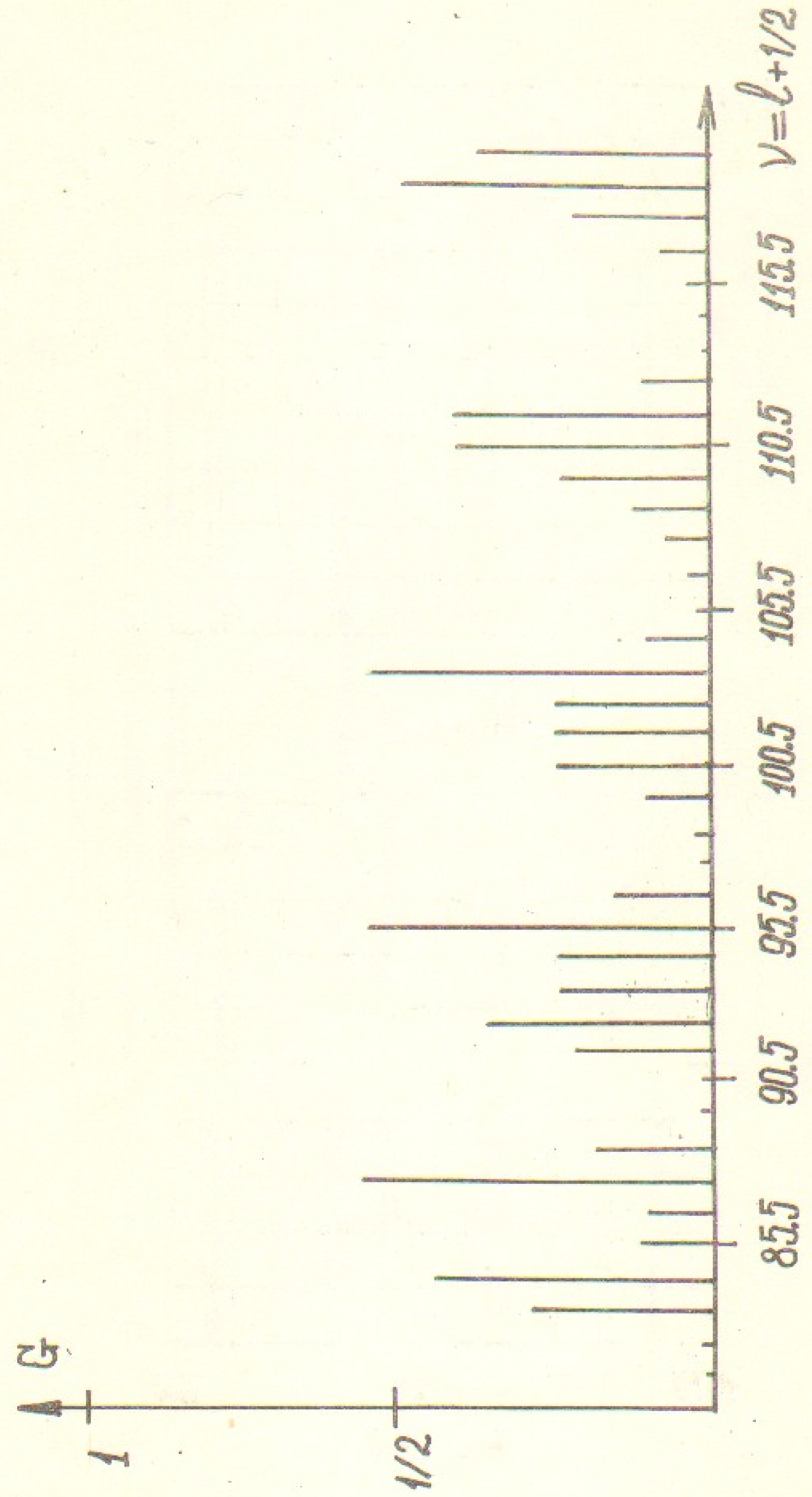


Fig 4



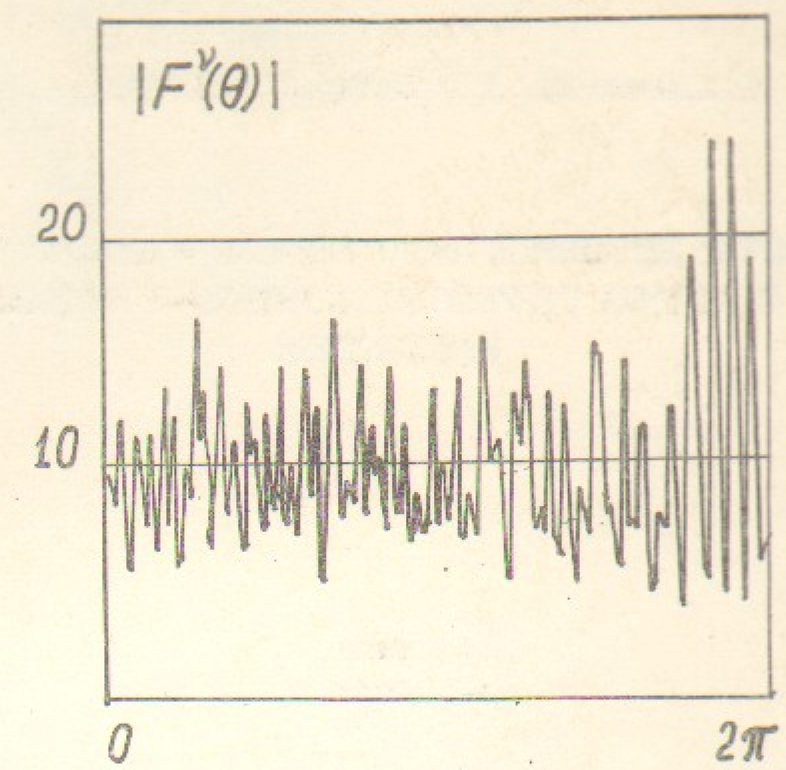


Fig 5 (a)

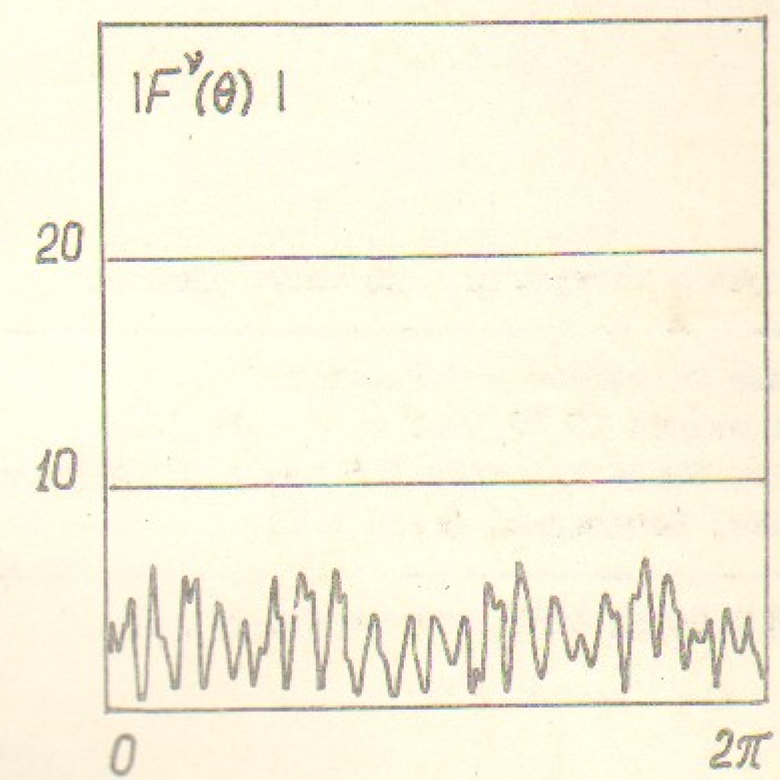


Fig 5 (b)

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РАСЧЕТ ДЕПОЛЯРИЗУЮЩЕГО ВЛИЯНИЯ ПОГРЕШНОСТЕЙ
В МАГНИТНЫХ СТРУКТУРАХ ЭЛЕКТРОН-ПОЗИТРОННЫХ
НАКОПИТЕЛЕЙ

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