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ASYMPTOTIC BEHAVIOUR OF EXCLUSIVE PROCESSES IN QCD

1. INTRODUCTION
2. POWER BEHAVIOUR

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НОВОСИБИРСК

A B S T R A C T

The main ideas, methods and results in the investigation of the exclusive processes asymptotic behaviour are reviewed. We discuuss the power behaviour and its dependence on hadron quantum numbers, logarithmic corrections, properties of nonperturbative hadronic wave functions. Applications to the meson and baryon form factors, strong, electromagnetic and weak decays of heavy mesons, elastic scattering, threshold behaviour of inclusive structure functions, etc., are described. Comparison of theoretical predictions with experimental data is made whanever possible.

The review can be of interest to theoreticians, experimentalists and students specializing in elementary particle physics. The experts in this field can also find new results (nonleading logarithms, higher twist processes, novel applications, etc.).

GENERAL CONTENT

1. INTRODUCTION

1.1 Preface

It seems that there are no doubts at present that the quantum chromodynamics (QCD) /1.1/ is the right fundamental theory of the strong interactions. Because in QCD (unlike QED) the charge "antiscreening" takes place /1.2/ , the effective charge decreases at small distances (the "asymptotic freedom"). This gives the possibility to calculate the interactions reliably at small distances with the aid of a perturbation theory. In particular, there is a considerable progress in our understanding of inclusive process properties (see, for instance, the reviews /1.3/).

At the same time, there are strong nonperturbative effects in QCD, for instance, the famous instanton fields /1.4/ . Because the nonperturbative fields and interactions are nonsingular at small distances, their role at the large momentum transfers is very small. At the large (~ 1 fm) distances, however, the nonperturbative effects are dominant and responsible, in particular, for the confinement of coloured states and a formation of hadrons.

Because the observed hadrons are always on the mass-shell, even in the hard processes with the large momentum transfers, the properties of the hadronic amplitudes depend essentially both on the small and large distance interactions. In other words, they depend both on the perturbative interactions, which ensure the large momentum transfers and on the nonperturbative ones which are responsible for the hadron formation out of quarks. Therefore, in order to calculate not only the energy dependence, but also the absolute values of the amplitudes, one should be able to calculate both the hard perturbative part of the amplitude and the nonperturbative hadronic wave functions. As it will be shown in detail below, the properties of the exclusive processes are tightly connected with the properties of hadronic wave functions.

The investigation of the asymptotic behaviour of the exclusive processes includes: the meson and baryon form factors, the large angle scattering, the exclusive electroproduction, the strong and electromagnetic decays of the heavy quarkonia, the weak decays of the heavy mesons with an "open flavour" (D(1865), F(2015), B(5020) ...),etc. And there was a great progress in the last years in our understanding of the exclusive process properties.

Investigation of the exclusive asymptotic amplitude behaviour, in particular, the form factors, has of course, a long history. Various simple models have been considered, and the main conclusion which was drawn about the asymptotic behaviour of the composite state form factor was the following. The asymptotic behaviour depends essentially on many factors: the number of constituents, the values of their spins and their angular moments, the used interaction Lagrangian, etc. (see, for instance, |1.5 - 1.8|).

Using the simple dimensional considerations in 1973 in the papers |1.9, 1.10| were proposed the famous "dimensional counting rules", which connect the asymptotic behaviour with the minimal number of constituents. The predictions for the "dimensional counting rules" agree well, on the whole, with the experimental data on the pion and nucleon form factors and various large angle elastic scattering cross-sections. This agreement served as a stimulus for further theoretical investigations |1.11 - 1.17|. The summary of the theoretical and experimental investigations in this region up to 1975 has been given in the reviews |1.18, 1.19| (the later papers are |1.20 - 1.23|).

The modern approach to investigation of the asymptotic behaviour of the exclusive amplitudes starts from 1977 |1.24-1.26| . From 1977 until now there has been seen a rebelling activity in this region (see |1.27 - 1.30| , for other references see below in the text), and a large number of theoretical and experimental results has been accumulated here at present. It is the purpose of this paper to review these results.

The detailed description of the methods developed for a calculation of the loop logarithmic corrections to the Born's exclusive amplitudes can be found, for instance, in the reviews |1.31, 1.32| which may be recommended to those readers who are interested in these problems in the first place. These problems are studied well at present, and in ch.3 we present a short description of the main methods and results obtained in this region.

It is a conviction of the authors of this paper based on the experience in concrete calculations, that the higher order perturbation theory logarithmic corrections play a very modest role in a description of the exclusive processes at present energies ($Q^2 \sim (10 - 100)$ GeV^2).

The situation here is similar with that of the deep inelastic scattering. The logarithmic corrections become really important at very large Q^2 alone. For instance, the asymptotic form of the nucleon structure function $F_2(x,Q^2)$ is $F_2(x,Q^2) \sim \delta(x)$ at $Q^2 \to \infty$. However, $F_2(x,Q^2)$ for $Q_0^2 \sim 1$ GeV^2 differs strongly from $\delta(x)$, the logarithmic evolution with increasing Q^2 is very slow and therefore, even at Q^2 10^3 GeV^2 the structure function $F_2(x,Q^2)$ still differs greatly from $\delta(x)$.

Moreover, unlike the inclusive structure functions, the exclusive amplitudes decrease like the powers of Q^2 at large Q^2. Therefore, it will be impossible to measure experimentally the exclusive processes at very large Q^2 because the corresponding probabilities will be extremely small. As a result, the role of the logarithmic corrections in the experimentally accessible region of Q^2 is very mild.

Therefore, we put the main attention to investigation of the power behaviour of the exclusive amplitudes (chs.2, 5, 8-10), to the properties of the hadronic wave functions (chs. 4, 6, 7-10) and to the concrete calculation of various exclusive processes (chs. 5, 7-10). It is worth noting that the knowledge of the hadronic wave functions allows us to calculate the absolute values of the exclusive amplitudes, not only their energy dependence.

Below in sects. 1,3 and 1.4 the main ideas and methods are presented on which the modern description of the exclusive processes is based.

1.2 Dimensional counting rules

Let us begin our consideration with the simplest exclusive process - the elastic electron-hadron scattering, Fig. 1.1. The hadronic part of the total amplitude is given by the form factor, i.e. by the matrix element $\langle P_2 | J_\mu(0) | P_1 \rangle$. We will be interested in the form factor asymptotic behaviour at the large momentum transfer,

$|q^2| = |(P_2-P_1)^2| \gg$ 1 GeV^2. In the center of the mass system (c.m.s.): $\vec{P}_1+\vec{P}_2=0$, $\vec{q}=\vec{P}_2-\vec{P}_1=-2\vec{P}_1$, $q^2\simeq-\vec{q}^{\,2}$, and the process looks as at Fig. 1.2. The initial hadron which moves with the large momentum $\vec{P}_1$ along the Z-axis, absorbs the photon with the momentum $\vec{q}=-2\vec{P}_1$ and moves afterwards in the opposite direction with the momentum $\vec{P}_2=-\vec{P}_1$.

If we wish to describe this process in the QCD framework, we should use the description in terms of quarks and gluons. Hence, let us imagine the hadron (the meson for simplicity) as being composed of the two quarks. In the meson rest frame each quark has its momentum $|\vec{K}|\sim\mu_0$, the energy $E\sim\mu_0$ and the virtuality $\delta\sim\mu_0^2$, where μ_0 is the characteristic scale, $\mu_0\simeq(300\text{-}400)$ MeV. In the c.m.c. each of these two quarks has the transverse momentum $k_\perp\sim\mu_0$, the virtuality $\delta\sim\mu_0^2$ and the longitudinal momentum $x_i\vec{P}_1$, $x_1+x_2=1$. One of the initial quarks absorbs the photon and moves then with the momentum $(x_1\vec{P}_1+\vec{q})=-(2-x_1)\vec{P}_1$ in the opposite direction, so that just after this the system looks like that shown in Fig. 1.3. It is clear that such a state is very unlike the final meson state, for which two quarks move with large and nearly parallel momenta directed opposite to the Z - axis. Indeed, the two quark system shown in Fig. 1.3 will transform certainly into the two opposite side jets. We are interested, however, in the rare process when there are only one meson in the final state. For this, the quarks have to interact hardly with each other in order to turn over the spectator. The amplitude of this hard interaction, Fig. 1.4 is small, because the exchange of the gluon with the virtuality $\sim\vec{q}^{\,2}$ is needed (the amplitude for the gluon exchange is $\sim 1/\vec{q}^{\,2}$) and, besides, the intermediate quark with the momentum $-(2-x_1)\vec{P}_1$

should also have the virtuality $\sim \vec{q}^2$ (the amplitude for the quark exchange is $\sim 1/|\vec{q}|$). Hence, the price for the spectator turn over is $\sim 1/|\vec{q}|^3$. How can we find the behaviour of the meson form factor? The simplest recipe is that of the "dimensional counting rules". One should count for the dimensionalities of the external quark lines in Fig. 1.4. Substituting the free spinor $u(x_1\vec{p}_1) \sim \sqrt{x_1|\vec{p}_1|}\, u_0$, $u_0 \sim 1$ for each external quark line, one has the additional factor $\sim (\sqrt{|\vec{q}|^4}) \sim \vec{q}^2$. On the whole, the final result looks like

$$\left|\langle p_2 | J_\mu | p_1 \rangle\right| \sim \frac{\text{const}}{|\vec{q}|} \sim \left|(p_2+p_1)_\mu \frac{\text{const}}{q^2}\right|. \tag{1.1}$$

The asymptotic behaviour of the baryon form factor can be found in similar way. Each quark which turns over its momentum gives in addition: the spinor ($\sim 1/|\vec{q}|$) and the gluon ($\sim 1/\vec{q}^2$) propagators and the pair of external spinors ($\sim |\vec{q}|$), i.e. $\sim$ ($1/\vec{q}^2$) on the whole. Therefore

$$\left|\langle B(p_2) | J_\mu | B(p_1) \rangle\right| \sim \frac{\text{const}}{|\vec{q}|^3} \sim \left|\bar{B}_2 \gamma_\mu B_1 \frac{\text{const}}{q^4}\right|, \tag{1.2}$$

where B_i is the baryon spinor.

The behaviour

$$F_n(q^2) \sim \frac{\text{const}}{(q^2)^{n-1}} \tag{1.3}$$

is predicted in this way for any hadronic form factor /1.9 , 1.10 /, where n is the minimal number of the constituents ($n=2$ for mesons, $n=3$ for baryons). In analogy, one can obtain for the large angle scattering:

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2}\left(\frac{M_0^2}{s}\right)^{\sum_i n_i - 2} f\left(\frac{s}{t}\right), \quad s \sim |t| \gg M_0^2, \tag{1.4}$$

where $\sum_i n_i$ is the minimal total number of constituents. For instance: $(d\sigma/dt)_{pp} \sim 1/s^{10}$, $(d\sigma/dt)_{\pi p} \sim 1/s^8$, $(d\sigma/dt)_{\gamma p - \pi p} \sim 1/s^7$,etc. These results are well known (see the review /1.19/) and we don't dwell upon these questions any more. Let us note only that the experimental data on the pion /1.33/ and the nucleon /1.34/ form factors and on the large angle scattering cross sections /1.34-1.36, 1.19/ agree well with the predictions of the "dimensional counting rules". It is important also that the agreement begins with the momentum transfers $\sim$ few $(\text{GeV})^2$.

The predictions of the "Dimensional counting rules" are based finally on the absence of any dimensional scale except for the momentum transfers themselves. Therefore, they will be true (neglecting the loop logarithmic corrections) for any Lagrangian with the dimensionless coupling constants (in four dimensions), and there is nothing specific for the QCD.

Moreover, at more serious level of investigation, one should be able to answer the following questions:

1. What is the dependence of the asymptotic behaviour on the hadronic quantum numbers? Would the asymptotic behaviour change when the pion is replaced by the scalar, vector or tensor mesons? Or would the asymptotic behaviour change when the hadron spin, helicity, P, C -parity, etc. is changed?
2. What is the absolute normalization of each given hadronic exclusive amplitude? (i.e., what are the values of the "const." in (1.1) - (1.3)?).

3. How does the asymptotic behaviour change when the higher loop corrections are taken into account? Is this change power-like or logarithmic only? If it is logarithmic, to what function would these logarithms sum?

4. How can one calculate the power corrections to the leading behaviour? And so on.

The "dimensional counting rules" give no answer to any of these questions.

The whole further content of this paper are the answers to the above questions and the applications of the developed methods to the concrete calculations of exclusive processes.

1.3 Operator expansions and hadronic wave functions

The method used in this paper to investigate the exclusive process asymptotic behaviour is based on obtaining the corresponding operator expansions. The main idea and the scheme for obtaining and using such operator expansions in the QCD have been proposed in /1.24/.

The operator expansion method has been introduced into the particle physics by K. Wilson /1.37/, and later was widely used for: the deep-inelastic scattering /1.38/, the processes with heavy quarks /1.39, 1.40/, the calculation of the "strong corrections to the weak interaction" /1.41/, the calculation of the light resonance parameters /1.42/, etc.

In most cases the operator expansions are used in a small distance region, $\bar{x}_i \ll 1/\mu_0$, and this smallness is ensured by the "internal reasons" - by the W -boson mass, the heavy c, b quark masses, etc. All the components of the external hadronic momenta are small at the same time: $\bar{p}_i \sim \mu_0$, $\bar{x}_i \bar{p}_j \ll 1$. In this case the main contribution to the opera-

tor expansion give the operators with a minimal dimensionality.

When considering the asymptotic behaviour of exclusive processes, we deal with the situation where the smallness of relative distances is ensured by the "external reasons", say, the large momentum transfers between the hadrons. In this case: $\bar{x}_i \bar{p}_j \sim 1$, and the parameter which determines the role of each given operator is the twist (i.e. the dimensionality minus the spin), not the dimensionality.

The method for obtaining the operator expansions for exclusive processes doesn't follow directly from the Wilson's expansion of local operator products. Below in this section we describe the main idea and the simplified scheme for obtaining corresponding operator expansions in the QCD (see the next Chapters for details).

Small and large distance interactions are determined in the QCD by the essentially different physics. It is natural, therefore, to separate out the contributions into the amplitude, which are due to small and large distance interactions.

Let us turn to the meson form factor, Fig. 1.6, and consider more closely the properties of each line in this diagram. As for the quark and gluon propagators $\langle 0|\bar{\Psi}(z_1)\Psi(0)|0\rangle$ and $\langle 0|B_\nu(z_1)B_\lambda(z_2)|0\rangle$, their virtuality is: $\Delta^2 \sim k^2 \sim q^2$, i.e. $(z_1-0)^2 \sim (z_2-0)^2 \sim (z_2-z_1)^2 \sim 1/q^2$, so that it is justified to use for them the lowest order perturbation theory expressions (remind about the asymptotic freedom in the QCD):

$$\langle 0|\bar{\Psi}(z_1)\Psi(0)|0\rangle \simeq S_0(z_1-0) \sim \frac{\hat{z}_1}{z_1^4}\,;\langle 0|B_\nu(z_1)B_\lambda(z_2)|0\rangle \simeq D^0_{\nu\lambda} \sim \frac{g_{\nu\lambda}}{(z_2-z_1)^2}\,.$$

At the same time, the use of the free spinors for the external quark lines is evidently wrong. Two final quarks which were produced nearly at the same point, $(z_2-z_1) \sim 1/q$, move then for a long time at the relative distance $\sim (1/K_\perp) \sim 1/\mu_0$ With nearly parallel momenta. It is evident that these two quarks will interact strongly with each other, and this strong interaction can't be neglected.

It is clear now at what points we would improve the previous calculations (Sect. 1.2) in order to obtain the right results.

1. Those lines in the diagram, Fig. 1.6 , which describe the particle propagation at small distances (i.e. the lines with the large virtuality) can be replaced, in the first approximation, by the free propagators. In other words, only large $(\sim q)$ Fourier-components of the operators $\Psi(z)$ and $B_\mu(z)$ are essential here, and these parts of the field operators can be substituted by the free operators. (On account for the nonperturbative effects, there will appear the power corrections like $\sim \langle 0|G^2_{\mu\nu}|0\rangle/q^4$, etc. The logarithmic loop corrections will be accounted for in the ch.3).

2. At the same time, just the soft Fourier-components of the field operators are essential for those lines, which have small virtuality and enter the matrix elements with small momentum transfers. The interaction in this momentum region is indeed strong and mostly nonperturbative and, since we don't calculate it explcidly, such lines should remain the Heisenberg operators acting in the small virtuality and small momentum transfer $(\sim \mu_0)$ subspace.

Let us write, therefore, the meson form factor in the form:

$$\langle P_2|J_\mu|P_1\rangle \to \int dz_1 dz_2 \langle P_2|\left\{\overline{\Psi}_\alpha(z_1)\exp\left(ig\int_{z_2}^{z_1} d\sigma_\mu B_\mu(\sigma)\right)\Psi_\beta(z_2)\right\}_{ij}|0\rangle_\mu \times$$

$$C^{ijk\ell}_{\alpha\beta\gamma\delta} \langle 0|\left\{\overline{\Psi}_\gamma(z_2)\exp\left(ig\int_0^{z_2} d\sigma_\nu B_\nu(\sigma)\right)\Psi_\delta(0)\right\}_{k\ell}|P_1\rangle_\mu , \qquad (1.5)$$

$$C^{ijk\ell}_{\alpha\beta\gamma\delta} = (ig)^2\left[e_q\gamma_\nu\, iS(z_1-0)\gamma_\mu\right]_{\alpha\delta}\left[-iD_{\nu\lambda}(z_1-z_2)\right](\gamma_\lambda)_{\gamma\beta}\left(\frac{\lambda^a}{2}\right)^{i\ell}\left(\frac{\lambda^a}{2}\right)^{kj}(-1) + \text{„perm"}.$$

In (1.5): $\alpha, \beta, \gamma, \delta$ are the spinor indices, i, j, k, ℓ and a are the color indices, e_q is the electric charge of the quark ($e_u = 2/3$, $e_d = -1/3$), Ψ and B_ν are the Heisenberg fieldoperators, $S(z)$ and $D_{\nu\lambda}(z)$ are the free quark and gluon propagators, "permut" denotes the contributions of three analogous diagrams and the factor (-1) in (1.5) is due to the anticommutativity of the quark field operators.

The expression (1.5) has been obtained as follows.

1. As it was explained above, the propagators with large virtualities are taken as free ones.

2. If one replaces in (1.5) $\exp(\ldots.)$ by unity and considers Ψ_i as the free quark operators, then (1.5) coincides with the usual Born diagram, Fig. 1.6.

3. On accounting the contributions like those shown in Fig.1.7, the free quark field operators are replaced by the Heisenberg ones.

4. On accounting the gluonic radiation into the initial state during the quark motion along the path ("0") - ("Z_1")-("∞")-("Z_2"), there appears (in the Abelian case, for the sake of simplicity) the factor $\exp\left\{ig\int_0^{z_2} d\sigma_\nu B_\nu(\sigma)\right\}$ /1.43/.

5. In analogy, due to radiation into the final state along the path ("Z_1") - ("0") - ("$-\infty$") - ("Z_2") there appears the factor $\exp\left\{ig\int_{z_2}^{z_1} d\sigma_\mu B_\mu(\sigma)\right\}$.

The matrix element of the bilocal operator in (1.5) can be interpreted as the amplitude for the meson break-up into a pair of the "soft" quarks, i.e. as the two-quark component of the total meson wave function (which has, naturally, many-particle components as well). The index "μ" denotes that the integration over internal momenta in this matrix element is truncated from above by "μ" (i.e. the virtualities or transverse momenta don't exceed "μ"). If the inverse confinement radius is $R_0^{-1} \simeq$ (200) MeV and the corresponding interaction falls off rapidly at smaller distances, then the cut-off at $\mu \simeq 1$ GeV doesn't affect really the value of the matrix element which is determined mainly by the interaction at larger distances.

Let us consider now for definiteness the π^+-meson and write:

$$\langle 0|\{\bar{d}_\gamma(z_2) e^{ig\int_{z_1}^{z_2}} u_\delta(z_1)\}_{ij}|\pi(p)\rangle = \frac{\delta^{ij}}{3}\left\{-\frac{1}{4}(\gamma_\rho\gamma_5)_{\delta\gamma}\langle 0|\bar{d}\, e^{ig\int}\gamma_\rho\gamma_5 u|\pi\rangle + \dots \right. \tag{1.6}$$

$$\langle 0|\bar{d}(z_2) e^{ig\int}\gamma_\rho\gamma_5 u(z_1)|\pi(p)\rangle = i p_\rho \int dx_1 dx_2\, \delta(1-x_1-x_2)\, e^{-ix_1(z_1p)-ix_2(z_2p)}\varphi_\pi(x_{1,2}) + \dots \tag{1.7}$$

(and analogously for the final pion). We have put in (1.7) $(z_2-z_1)^2$ equal to zero, because $(z_2-z_1)^2 \sim 1/q^2$, and we'll neglect here the logarithmic corrections. Substituting (1.6), (1.7) into (1.5), we have for the leading contribution:

$$\langle \pi^+(p_2)|J_\mu(0)|\pi^+(p_1)\rangle = (p_2+p_1)_\mu F_\pi(q^2), \quad F_\pi(q^2) \to \frac{32\pi\alpha_s}{9}\,\frac{I_\pi}{-q^2},$$

$$I_\pi = \int_0^1 d_2x\, d_2y\, \varphi_\pi^A(y)\left[\frac{e_u}{x_2y_2} - \frac{e_d}{x_1y_1}\right]\varphi_\pi^A(x), \quad d_2x = dx_1 dx_2\, \delta(1-x_1-x_2). \tag{1.8}$$

As a result, the form of the small distance ($\sim 1/q$) interaction, i.e. the function C in (1.5) has determined completely all the dependence on the asymptotic variable q^2. All the information about the strong nonperturbative interactions at large ($\sim 1/\mu_0$) distances is separated out and concentrated into the pion wave function $\varphi_\pi(x_{1,2})$. The form of

the wave function determines the absolute normalization of the whole amplitude.

The physical meaning of the variables x_1 and x_2 in (1.7) can be easily clarified. Let us compare for this purpose the expression (1.7) with the corresponding model expression, where the pion is replaced with the quark and antiquark with the momenta: $k_1 = x_1 p$, $k_2 = x_2 p$, $x_1 + x_2 = 1$, $0 \le x_{1,2} \le 1$, $p_z \to \infty$. In this case the matrix element of the bilocal operator in (1.7) is proportional to:

$$\exp\{-ik_1z_1 - ik_2z_2\} = \exp\{-ix_1(z_1p) - ix_2(z_2p)\}.$$

Therefore, x_1 and x_2 have the meaning of the longitudinal momentum fractions carried by quark and antiquark, and the wave function $\varphi_\pi(x_1, x_2)$ describes their distribution in the longitudinal momenta inside the pion (at $p_z \to \infty$).

It may seem surprising that at $|\vec{q}| \to \infty$ the form factor includes not the wave function at the origin (i.e. the quantity $\langle 0|\bar{d}(0)\gamma_\rho\gamma_5 u(0)|\pi(p)\rangle = ip_\rho\int dx_1 dx_2\, \delta(1-x_1-x_2)\varphi_\pi(x_1,x_2)$, but the nontrivial integral over $x_{1,2}$. The reason is as follows. In the c.m.s. the photon wavelength is: $\lambda \sim 1/|\vec{q}|$, and hence it "feels" indeed only the distances $\sim (1/|\vec{q}|)$. However, the hadron longitudinal size is also $\sim (\mu_0/\gamma) \sim 1/|\vec{q}|$ due to the Lorentz contraction (γ - is the Lorentz factor, $\gamma \sim |\vec{q}|/\mu_0$). Therefore, the photon "feels" the distribution of quarks in longitudinal momenta inside the hadron. On the other hand, the transverse size of the hadron remains $\sim 1/\mu_0$, and therefore the distribution of quarks over transverse momenta is not resolved.

One of the essential difficulties in early investigations of the composite state form factor was the following. One has tried, on the first stage, to simulate in some way the hadron state of the constituents and, at the second stage, to find the behaviour of the form factor of this model state. There arise then the questions: a) what are the interpolation operators for each given hadron?, b) in what way the answer for the form factor depends on the chosen interpolation operator?

We approach the problem from the opposite side, i.e. from the small distance region. The expression (1.5), as becomes clear from its derivation is the mesonic matrix element of the second term in the operator expansion:

$$J(0) \to [\bar{\Psi}(0)\Psi(0)]_\mu + g^2 \oint dz_1 dz_2 [\bar{\Psi}\Psi]_\mu |0\rangle C_1 \langle 0| [\bar{\Psi}\Psi]_\mu +$$
$$g^3 \oint dz_1 dz_2 dz_3 [\bar{\Psi}\Psi G]_\mu |0\rangle C_2 \langle 0| [\bar{\Psi}\Psi]_\mu + \qquad (1.9)$$
$$g^4 \oint dz_1 dz_2 dz_3 dz_4 [\bar{\Psi}\bar{\Psi}\bar{\Psi}]_\mu |0\rangle C_3 \langle 0| [\Psi\Psi\Psi]_\mu + \ldots.$$

(G is the field strenght tensor). Each term in the expression (1.9) has the following meaning (after taking the matrix element between the initial and the final hadron states). Some quantum fluctuation takes place in the small vicinity of the point "0" with the result that photon transforms into two independent systems of "soft" quarks (and gluons) with virtualities up to " μ^2 ". The amplitudes for the photon transition into each given state are given in (1.9) by the functions C_i which are determined by the small distance interactions (the $\oint$ notation means that the integration is performed over the small ($\sim 1/q^4$) space-time volume around the point "0"). Both produced clusters of soft partons move then in the opposite directions of the Z -axis, and the interaction within

each of these two clusters transform them into the initial and final hadron states. The "softness" means that the partons within the cluster move with large and nearly parallel momenta, the transverse momenta and virtualities are small and the total invariant mass of the cluster is small ($M_i^2 \ll |q^2|$).It is clear from the dimensional considerations that the larger is the number of valence constituents, the more suppressed (by additional powers of $1/q$) is the contribution into the asymptotic behaviour.

It should be pointed that we have described just the scheme for obtaining the corresponding operator expansions. Below the concrete calculations for various exclusive processes are presented which show this scheme at work.

1.4 The QCD sum rules

To be able to calculate the absolute values of the exclusive amplitudes, one needs to know the hadronic wave functions $\varphi(x_i,\mu)$. These wave functions are the fundamental objects of the theory and at $\mu \sim 1$ GeV are determined mainly by the large distance nonperturbative interactions. At $\mu \gtrsim 1$ GeV the slow evolution of $\varphi(x_i,\mu)$ with increasing of μ is caused mainly by the logarithmic perturbation theory loop corrections and is described by the renormalization group (or by the evolution equations a la Lipatov-Altarelli-Parisi), see ch.3.

The wave functions $\varphi(x_i,\mu)$ tend to their asymptotic form: $\varphi(x_i,\mu\to\infty)\to\varphi_{as}(x_i)$ in the formal limit $\mu\to\infty$. The form of the asymptotic wave function $\varphi_{as}(x_i)$ is pure perturbative in nature and can easily be found (see the appendix its B). Moreover, this asymptotic form of the wave function, $\varphi_{as}(x_i)$, is the universal one and doesn't depend on the form of $\varphi(x_i,\mu\approx 1\,\text{GeV}$

because at $\mu\to\infty$ $\varphi(x_i,\mu)$ "forgets" its original form * and its "parent hadron". The wave function $\varphi(x_i,\mu\simeq 1\,GeV)$ can differ greatly in its form from $\varphi_{as}(x_i)$ and therefore, because the evolution with μ is extremely slow at large μ, the true wave function $\varphi(x_i,\mu)$ will become much alike $\varphi_{as}(x_i)$ only at extremely large μ. Because in the exclusive processes the characteristic normalization point of the wave function, $\bar\mu$, is determined by the characteristic internal momentum transfers (very roughly, $\bar\mu\sim q$), in the range of experimentally accessible values of q^2 (10 GeV$^2\lesssim|q^2|\lesssim$100 GeV^2) the effects of logarithmic evolution are very mild, and the form of the wave function $\varphi(x_i,\bar\mu)$ still remains much unlike its asymptotic form, $\varphi_{as}(x_i)$ (analogously to the deep-inelastic structure function $F_2(x,Q^2)$). Therefore, we put main attention to the investigation of the properties of the wave functions $\varphi(x_i,\mu\simeq 1\,GeV)$, using the nonperturbative methods available at present.

It was proposed in /1.44/ to investigate the properties of the wave functions $\varphi(x_i,\mu\simeq 1\,GeV)$ using the method of the QCD sum rules, developed by A.I. Vainshtein, V.I. Zakharov and M.A. Shifman /1.42/. Below we describe in short the essence of this approach. The detailed description is presented in chs. , , and for various meson wave functions, and in the ch. for the nucleon wave function.

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*) The deep-inelastic structure function $F_2(x,Q^2)$ has an analogous property: $F_2(x,Q^2)\to$ const.$\delta(x)$ at $Q^2\to\infty$, independently of the target properties (only the value of the "const." depends on the target, but the form $\sim\delta(x)$ is universal).

The leading twist π -meson wave function has been determined above in (1.7). We rewrite now it in the form:

$$\langle 0|\bar d(z)\hat z\gamma_5\exp\{ig\int_{-z}^{z}d\sigma_\lambda B_\lambda(\sigma)\}u(-z)|\pi^+(p)\rangle\equiv$$

$$\sum_n\frac{i^n}{n!}\langle 0|\bar d\hat z\gamma_5(iz_\nu\overleftrightarrow{D}_\nu)^n u(0)|\pi^+(p)\rangle=i(zp)f_\pi\int_{-1}^{1}d\zeta\,e^{i\zeta(zp)}\varphi_\pi^A(\zeta)\quad(1.10)$$

$$\overleftrightarrow{D}=\overrightarrow{D}-\overleftarrow{D},\quad\overrightarrow{D}=\overrightarrow{\partial}-igB^a\frac{\lambda^a}{2}.$$

In (1.10): $\zeta=(x_1-x_2)$ is the relative longitudinal momentum fraction, $f_\pi\simeq 133$ MeV is the pion weak decay constant, which is known experimentally from the $\pi\to\mu\nu$ decay: $\langle 0|\bar d(0)\gamma_\mu\gamma_5 u(0)|\pi(p)\rangle=ip_\mu f_\pi$. Therefore, the dimensionless wave function $\varphi_\pi^A(\zeta)$ in (1.10) is normalized as follows:

$$\langle\zeta^0\rangle\equiv\int_{-1}^{1}d\zeta\,\varphi_\pi^A(\zeta)=1.\quad(1.11)$$

The matrix elements at l.h.s. of (1.10) have the form:

$$\langle 0|\bar d\hat z\gamma_5(iz\overleftrightarrow{D})^n u|\pi(p)\rangle=$$

$$z_\nu z_{\mu_1}\dots z_{\mu_n}\langle 0|\bar d\gamma_\nu\gamma_5 i\overleftrightarrow{D}_{\mu_1}\dots\overleftrightarrow{D}_{\mu_n}u|\pi\rangle=(zp)^{n+1}C_n,\quad z^2=0,\quad(1.12)$$

where C_n are some constants. Decomposing the r.h.s. of (1.10) into the z -series and comparing with (1.12), one has:

$$\langle 0|\bar d\hat z\gamma_5(iz\overleftrightarrow{D})^n u|\pi(p)\rangle=(zp)^{n+1}f_\pi\langle\zeta^n\rangle,$$

$$\langle\zeta^n\rangle=\int_{-1}^{1}d\zeta\,\zeta^n\varphi_\pi^A(\zeta)=C_n,\quad z^2=0.\quad(1.13)$$

Therefore, the evaluation of the constants C_n in (1.12) is, at the same time, the evaluation of the wave function moments, $\langle\zeta^n\rangle$. Having the information about the wave function moments,one can reconstruct the wave function itself.

The method of the QCD sum rules just allows one to calculate the values of the matrix elements like those in (1.12), i.e. the values of the constants C_n. We now show how the sum rule method works, using the wave function $\varphi_\pi^A(\zeta)$ as an example.

Let us consider the correlator$\left(\overleftrightarrow{D}=\vec{D}-\overleftarrow{D},\ \vec{D}=\vec{\partial}-igB^a\frac{\lambda^a}{2}\right)$:

$$I_{no}(z,q)=i\int dx\, e^{iqx}\langle 0|T\,O_n(x)\,O_0^+(0)|0\rangle=(zq)^{n+2}\,I_{no}(q^2),$$

$$O_n(x)=\bar d(x)\hat z\gamma_5(iz\overleftrightarrow{D})^n u(x),\quad O_0^+(0)=\bar u(0)\hat z\gamma_5 d(0),\quad z^2=0,\quad n=0,2,4\ldots \tag{1.14}$$

and calculate its asymptotic behaviour at $q^2\to-\infty$. The main contribution gives the interaction at small distances, $\bar x\sim 1/q$. The quarks (gluons) propagate, of course, not in an empty space, but through the physical vacuum, filled with the nonperturbative quark and gluon fields. However, because the nonperturbative interactions have a characteristic scale $x_0\sim 1/\mu_0$ and fall off rapidly at small distances, the leading contribution into the asymptotic behaviour of $I_{no}(q^2)$ give the usual perturbation theory, for instance, the diagram at Fig. 1.9 (plus higher order corrections $\sim\alpha_s(q^2)/\pi$). The contribution of this diagram into $I_{no}(q^2)$ has the form: (1.15)

$$I_{no}^{pert.th.}(q^2)=-\frac{\ln(-q^2)}{4\pi^2}\int_{-1}^{1}d\xi\,\xi^n\cdot\frac{3}{4}(1-\xi^2)=-\frac{\ln(-q^2)}{4\pi^2}\cdot\frac{3}{(n+1)(n+3)}\ldots$$

The interaction of quarks (gluons) with the nonperturbative vacuum fluctuations can be accounted for by means of the suitable multipole expansion in small parameter $(\bar x/x_0)\sim(\mu_0/q)$. At present the convenient method of operator expansion in the external field is developed /1.46/, which allows to calculate the nonperturbative corrections to the correlator. These corrections are expressed through the vacuum expectation values of various local operators: $\langle 0|G^2_{\mu\nu}|0\rangle$, $\langle 0|\bar\psi\psi|0\rangle$, etc,* Figs. 1.10 and 1.11. As a result, one has the asymptotic expansion of the form: $I_{no}(q^2)=\frac{1}{\pi}\int_0^\infty \mathrm{Im}\,I_{no}(s)\,ds/(s-q^2)\longrightarrow$

*) The usual perturbation theory contributions are, by definition, subtracted out of these matrix elements.

$$-\frac{\ln(-q^2)}{4\pi^2}\frac{3}{(n+1)(n+3)}+\frac{\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle}{12q^4}-\frac{32\pi}{81}(11+4n)\frac{\langle 0|\sqrt{\alpha_s}\bar uu|0\rangle}{q^6}+C_k\frac{\langle O_k\rangle}{(q^2)^k}+\ldots \tag{1.16}$$

It is convenient to do the special integral transform("borelezation") /1.42/ : $\oint\exp(-q^2/M^2)\,dq^2/2\pi i$, after which one obtains:

$$\frac{1}{\pi M^2}\int_0^\infty ds\, e^{-s/M^2}\,\mathrm{Im}\,I_{no}(s)=\frac{1}{4\pi^2}\frac{3}{(n+1)(n+3)}+\frac{\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle}{12M^4}+$$
$$\frac{16}{81}\pi(11+4n)\frac{\langle 0|\sqrt{\alpha_s}\bar uu|0\rangle}{M^6}+\ldots+C_k\frac{(-1)^k}{(k-1)!}\frac{\langle 0|O_k|0\rangle}{(M^2)^k}+\ldots \tag{1.17}$$

Therefore, the "borelezation" suppresses the contributions of the high dimension operators into the r.h.s. of (1.17) and of the intermediate states with $s>M^2$ into the l.h.s. The parameter M^2 in (1.17) replaces q^2 in (1.16) and serves as the scale parameter. The operator O_K with the dimension (mass)^{2k} has the vacuum matrix element $\langle 0|O_k|0\rangle\sim(\mu_0)^{2k}$ and therefore, the expression (1.17) is the multipole expansion in the small parameter (μ_0/M). The absolute values of the nonperturbative corrections in (1.17) show at what scale the behaviour of the true correlator (and the true spectral density $\mathrm{Im}\,I_{no}(s)$) begins to deviate noticeably from the perturbation theory behaviour.

The sum rules like (1.17) are used as follows. The asymptotic behaviour of the spectral density $\mathrm{Im}\,I_{no}(s)$ at large $s\gg\mu_0^2$ is known and coincides with the spectral density for the free quarks: $\frac{1}{\pi}\mathrm{Im}\,I_{no}(s)\to\frac{1}{\pi}\mathrm{Im}\,I_{no}^{p.th.}(s)=3/4\pi^2(n+1)(n+3)$. Therefore, such a parameter s_n (the duality interval) is introduced, that $\mathrm{Im}\,I_{no}(s)\simeq\mathrm{Im}\,I_{no}^{p.th.}(s)$ at $s\gtrsim s_n$. The low energy behaviour of $\mathrm{Im}\,I_{no}(s)$ is approximated by the contributions of one or two lowest lying resonances. Hence, the model spectral density has the form (see (1.13), (1.14)):

$$\frac{1}{\pi}\operatorname{Im} I_{no}(s)=f_\pi^2\langle\zeta^n\rangle_\pi+f_A^2\langle\zeta^n\rangle_A+\theta(s-s_n)\frac{3}{4\pi^2(n+1)(n+3)}, \quad (1.18)$$

where the pion and the A_1 -meson contributions are shown explicitly.

The values of the vacuum averages at the r.h.s. of (1.17) can't be calculated theoretically at present. Their values have been found phenomenologically from the corresponding sum rules for the charmonium /1.43/ and PCAC+SU(6)/1.45/:

$$\langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}^2|0\rangle\simeq 1.2\cdot 10^{-2}\,GeV^4, \quad \langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle\simeq 1.35\cdot 10^{-2}\,GeV^3 \quad (1.19)$$

Confining ourselves by these terms and neglecting the contributions of higher dimension operators *, we can consider the r.h.s. of (1.17) as known.

After this, the scale parameter M^2 in (1.17) is varied in such a limit that the nonperturbative power corrections give sizeable (but not very large) contribution, say, from 5% up to 35%. Then the free parameters entering the l.h.s. of (1.17) ($f_\pi^2\langle\zeta^n\rangle_\pi$, s_n, ...) should be chosen so that to obtain the best fit in this interval of the M^2 -values.** As a result, one can determine approximately the values of the wave function moments $\langle\zeta^n\rangle$ and the constants f_π ,...

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*) The "current" quark masses are very small /1.45, 1.43 / : $m_u\simeq 4$ MeV, $m_d\simeq 7$ MeV, and can be neglected therefore.

**) As a rule, we don't determine the values of the resonance masses from the sum rules, but consider them as known.

Besides, it is seen from (1.17) that at $n\gg 1$ the real expansion parameter is (M^2/n). At large n and fixed M^2 the relative values of the nonperturbation corrections (with respect to the perturbation theory contribution) increase. Therefore, the total duality interval s_n increases also and $s_n\simeq n\cdot(1\ GeV^2)$ at $n\gg 1$, and the real spectral density coincides with the asymptotic one at $s\gtrsim n\ (GeV^2)$ only, and deviate considerably from it at $s< n\ (GeV^2)$. It is evident from the physical considerations that such an enormous duality interval $\sim n\ GeV^2$, $n\gg 1$ can not be filled with a few resonances. Roughly speaking, there will be $\sim n$ broad resonance-like structures each of which fills its own finite duality interval at $n\gg 1$: $s_n^{(\pi)}\to s_\infty^{(\pi)}$ for π. Therefore, the duality relation for the pion contribution looks at large $n\gg 1$ as follows:

$$\frac{1}{\pi}\int_0^{s_\infty^{(\pi)}}ds\operatorname{Im}I_{no}(s)\simeq\frac{1}{\pi}\int_0^{s_\infty^{(\pi)}}ds\operatorname{Im}I_{no}^{pert.th.}(s),$$

$$f_\pi^2\langle\zeta^n\rangle_\pi\simeq\frac{3}{4\pi^2}\frac{s_\infty^{(\pi)}}{(n+1)(n+3)}\simeq\frac{3}{4\pi^2}\frac{s_\infty^{(\pi)}}{n^2}. \quad (1.20)$$

This shows that $\langle\zeta^n\rangle_\pi=\int_{-1}^{1}d\zeta\,\zeta^n\varphi_\pi^A(\zeta)\sim n^{-2}$ at $n\gg 1$ and thus $\varphi_\pi^A(\zeta)\sim(1-\zeta^2)$ at $|\zeta|\to 1$. Therefore, <u>the wave function $\varphi_\pi^A(\zeta)$ has at $|\zeta|\to 1$ the same behaviour as $\varphi_{as}(\zeta)$.</u> It is clear that the above considerations are general ones and the same conclusion can be made for other wave functions as well.

On the whole, we know about the pion wave function $\varphi_\pi^A(\zeta)$:

a) the overall normalization: $\langle\zeta^0\rangle=\int_{-1}^{1}d\zeta\,\varphi_\pi^A(\zeta)=1$,

b) the behaviour at the boundaries: $\varphi_\pi^A(\zeta)\sim(1-\zeta^2)$ at $|\zeta|\to 1$,

c) the values of few first moments: $\langle\zeta^2\rangle=\int_{-1}^{1}d\zeta\,\zeta^2\varphi_\pi^A(\zeta)$, $\langle\zeta^4\rangle=\int_{-1}^{1}d\zeta\,\zeta^4\varphi_\pi^A(\zeta)$, ... obtained from the sum rules.

Excluding the pathological cases, it is sufficient really to have such an information for the reconstruction of all characteristic features of the wave function itself (see ch.4 for more details).

There is, at present, a large number of papers, in which the QCD sum rules are used for the calculation of: the light meson masses and the constants f_π, f_ρ, f_{A_1}, ... /1.42, 1.47/ ; the baryon masses /1.48, 1.49/ ; the spectroscopy of the charmonium and the bottonium states /1.42, 1.50, 1.51/; the properties of the "open heavy flavour" mesons B (5200), D(1870) /1.52/ ; the three particle vertices /1.53/; the baryon magnetic moments /1.54/, etc.

The gain experience with the QCD sum rules shows unambiguously that (when properly used) they always give the right results with the accuracy no worse than about (20-30)%. It seems that there are no at present other nonperturbative method which can compete with the QCD sum rules in this respect.

Below the method of the QCD the sum rules is widely used for the investigation of the hadronic wave functions.

2. POWER BEHAVIOUR

2.1 The Dependence of the Asymptotic Behaviour on Hadronic Quantum Numbers /1.24/.

The basic idea and the scheme for obtaining the operator expansions have been described above in sect. 1.2. Let us consider now in detail the properties of some form factors.

2.1.1 General Results

It is convenient to decompose the bilocal operators in (1.5) over the operators with the definite quantum numbers*:

$$4\bar{d}_\beta u_\alpha = \left\{ 1(\bar{d}u) + \gamma_5(\bar{d}\gamma_5 u) + \gamma_\rho(\bar{d}\gamma_\rho u) - \gamma_\rho\gamma_5(\bar{d}\gamma_\rho\gamma_5 u) - \tfrac{1}{2}\sigma_{\rho\tau}(\bar{d}\sigma_{\rho\tau}u) \right\}_{\alpha\beta}. \quad (2.1)$$

The bilocal operator $\bar{d}\gamma_\rho\gamma_5 u$ (twist 2) gives the leading contribution into the π^+-meson form factor. For the scalar meson the leading contribution will give the operator $\bar{d}\gamma_\rho u$ (twist 2). Let us define the wave function $\varphi_\delta(\xi)$ of the scalar meson:

$$\langle 0|\bar{d}(z_2)\gamma_\rho u(z_1)|\delta^+(p)\rangle = e^{-i(z_1+z_2)p/2} \langle 0|\bar{d}\left(\tfrac{z_2-z_1}{2}\right)\gamma_\rho u\left(\tfrac{z_1-z_2}{2}\right)|\delta^+(p)\rangle,$$

$$\langle 0|\bar{d}(z)\gamma_\rho u(-z)|\delta^+(p)\rangle = p_\rho f_\delta \int_{-1}^{1} d\xi\, e^{i\xi(zp)} \varphi_\delta(\xi), \quad (2.2)$$

$\varphi_\delta(\xi)$ determines the distribution of the quarks in the longitudinal momentum fractions x_u, x_d at $p_z \to \infty$, $\xi = x_u - x_d$, $x_u + x_d = 1$. Using (2.2) we have from (1.5):

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*) Here and further the gluonic exponents are not shown explicitly to simplify the notation.

$$\langle \delta^+(p')|J_\mu(0)|\delta^+(p)\rangle = (p'+p)_\mu F_\delta(q^2), \quad F_\delta(q^2) \to \frac{32\pi\alpha_s}{9}\frac{|f_\delta|^2}{-q^2} I_\delta, \tag{2.3}$$

$$I_\delta = \int_{-1}^{1} d\xi_1 d\xi_2\, \varphi_\delta(\xi_2)\left[\frac{e_u}{(1-\xi_1)(1-\xi_2)} - \frac{e_d}{(1+\xi_1)(1+\xi_2)}\right]\varphi_\delta(\xi_1).$$

Comparing (2.3) with (1.8), (1.10) we see that the asymptotic expressions for both form factors coincide, only the wave functions differ. (In the isotopic symmetry limit : $\varphi_\pi(\xi) = \varphi_\pi(-\xi)$, $\varphi_\delta(\xi) = -\varphi_\delta(-\xi)$) . It will be shown below that this coincidence is not accidental, though it may seem surprising at the first glance, because the scalar meson is the P -wave state of the two quarks and one can expect that the form factor asymptotic behaviour will be suppressed /1.9/.

It is not difficult to see that the result analogous to (2.3) will be true for the form factor of any meson with the helicity $\lambda = 0$ and any other quantum numbers - spin,parity, ...etc. Indeed, let us write the general form of the matrix element:

$$\langle 0|\bar d(z)\gamma_\rho u(-z)|P,J,\lambda\rangle = P_\rho(\varepsilon^\lambda_{\mu_1\ldots\mu_J} z_{\mu_1}\cdots z_{\mu_J})(M_R)^J f_J\tilde\varphi_J(zP)+\ldots \tag{2.4}$$

where: M_R is the meson mass, J is its spin and ε^λ is the polarization tensor. Because at $P_z \to \infty$ $\varepsilon^{\lambda=0}_\mu \simeq P_\mu/M_R$ while $\varepsilon^{|\lambda|=1}_\mu \sim O(1)$, we have for the case $\lambda = 0$:

$$\langle 0|\bar d(z)\gamma_\rho u(-z)|P,J,\lambda=0\rangle = P_\rho(zP)^J f_J\varphi_J(zP)+\ldots \tag{2.5}$$

The behaviour of the wave function (2.5) is analogous to (2.2) and gives the asymptotic behaviour of the form (2.3) (the wave function $\varphi_J(zP)$ is dimensionless).

The most simple reciepe for calculation of the $\lambda_1 = \lambda_2 = 0$

form factor is the following:

a) replace the initial meson by two free quarks with the momenta $K_1 = x_1 P$, $K_2 = x_2 P$, $x_1 + x_2 = 1$ (for the final meson: $K_3 = y_1 P'$, $K_4 = y_2 P'$, $y_1 + y_2 = 1$).

b) write the expression for the Born diagram, fig. 1.6:

$$J_\mu(0) \to (ig)^2\left[\bar u\frac{\lambda^a}{2}\gamma_\nu\hat\Delta\gamma_\mu u\right]\left[\bar d\frac{\lambda^a}{2}\gamma_\nu d\right]\left(\frac{i}{\Delta^2}\right)\left(\frac{-i}{k^2}\right)e_u + \text{permut}$$

$$k = y_2P' - x_2P, \quad k^2 = x_2y_2q^2, \quad \Delta = P' - x_2P, \quad \Delta^2 = x_2q^2,$$

where u and d are the quark field operators;

c) simplify by using the equations of motion:

$$\hat P' d = \bar u\hat P' = \hat P u = \bar d\hat P = 0,$$

$$\left[\bar u\frac{\lambda^a}{2}\gamma_\nu\hat\Delta\gamma_\mu u\right]\left[\bar d\frac{\lambda^a}{2}\gamma_\nu d\right] = -2x_2P_\mu\left[\bar u\frac{\lambda^a}{2}\gamma_\nu u\right]\left[\bar d\frac{\lambda^a}{2}\gamma_\nu d\right];$$

d) introduce the wave functions:

$$\langle 0|u^i_\alpha\bar d^j_\beta|P,J_1,\lambda_1=0\rangle = -\frac{\delta^{ij}}{3}\langle 0|\bar d_\beta u_\alpha|P,J_1,\lambda_1=0\rangle \Rightarrow$$

$$-\frac{\delta^{ij}}{3}(\gamma_\rho)_{\alpha\beta}\frac{1}{4}\langle 0|\bar d\gamma_\rho u|P,J_1,\lambda_1=0\rangle \Rightarrow -\frac{\delta^{ij}}{3}\frac{1}{4}(\hat P)_{\alpha\beta}f_{J_1}\varphi_{J_1}(\xi)$$

(and analogously for the final meson, for axial particles $\bar d\gamma_\rho u \to \bar d\gamma_\rho\gamma_5 u$);

e) calculate the traces over the spinor and color indices,sum the contributions of four Born diagrams and integrate:

$$\int_{-1}^{1} d\xi_1\int_{-1}^{1} d\xi_2, \quad x_1 = (1+\xi_1)/2,\ x_2 = (1-\xi_1)/2,\ y_1 = (1+\xi_2)/2,\ y_2 = (1-\xi_2)$$

The result is the expression (2.3) with the replacement:

$$f_\delta\varphi_\delta(\xi_1) \to f_{J_1}\varphi_{J_1}(\xi_1), \quad \overset{*}{f}_\delta\overset{*}{\varphi}_\delta(\xi_2) \to \overset{*}{f}_{J_2}\overset{*}{\varphi}_{J_2}(\xi_2).$$

This scheme is applicable to the calculation of any exclusive amplitude which is determined by the leading twist operators (wave functions). For the higher twist operators the recipe is more complicated, see ch.9.

Increasing the helicity $|\lambda|$ in (2.4) by one unit we obtain the suppression in the asymptotic behaviour $\sim (M_R/q)$. Proceeding in similar way, one can obtain a general expression for the asymptotic behaviour of the meson form factor (in the reference frame $\vec{P}_1+\vec{P}_2=0$):

$$\langle P_2, J_2, \lambda_2 | J_\lambda(0) | P_1, J_1, \lambda_1 \rangle \sim \left(\frac{1}{\sqrt{|q^2|}}\right)^{|\lambda_1-\lambda_2|+2n-3}, \quad (2.6)$$

where $n=2$ for meson(n is the minimal number of constituents), the current helicity λ and the meson helicities λ_1, λ_2 are related: $\lambda=\lambda_1+\lambda_2$. Similar result will hold if one replaces the electromagnetic current by the vector or axial-vector ones.

Because the asymptotic behaviour becomes more and more suppressed with increasing of $|\lambda_{1,2}|$, i.e. we are considering now the nonleading contributions, we should take into account in the decomposition (1.9) not only the two-particle operators $\bar\Psi\Gamma\Psi$, but the three-particle $\bar\Psi\Psi G$ and other higher twist operators as well*. It is evident, however, that the behaviour (2.6) remains true. Indeed, consider, for instance, the term in (1.9) with two quark and one transverse (i.e. $|\lambda_g|=1$) gluon for the initial meson. The gluon may undertake the unit of the angular momentum projection L_z due to its helicity $|\lambda_g|=1$ and this gives the prize $\sim q$, but the twist of $\bar\Psi\Psi G$ is one unit larger than that of $\bar\Psi\Psi$ and this gives the penalty $\sim 1/q$. Therefore, the final result is the same as (2.6).

Really, the validity of the asymptotic behaviour of (2.6) implies some requirements on the behaviour of the hadronic

*) This is evident also from the gauge invariance.

wave functions. The integrals (as in (1.8), (2,3)) have the denominators $D(x_i)$ such that $D(x_i)\to 0$ at $x_i\to 0,1$. The asymptotic behaviour (2.6) implies that the hadronic wave functions $\varphi(x_i)\to 0$ at $x_i\to 0,1$ sufficiently fast, so that the integrals over x_i are convergent and this region doesn't influence the asymptotic behaviour. These questions are discussed in some details below in sect. 3.5 .

2.1.2. The Simple Model

In order to clarify the meaning of the result (2.6),let us consider the simple model. The meson (in the rest system) is replaced with two free quarks with the momenta $K_1\sim K_2\sim M_0$, $K_1+K_2=P$. The state with the meson quantum numbers is constructed in the usual way, following M. Jacob and G.Wick:

$$|JM\lambda_1\lambda_2\rangle=\int_0^{2\pi}d\varphi\int_{-1}^{1}d\cos\theta\, \overset{*}{D}{}^{J}_{M\lambda}(\theta,\varphi)\,|\vec{K},\lambda_1,\lambda_2\rangle,$$

where J is the meson spin , and M is the spin projection onto the Z -axis. Choosing the definite linear combinations in λ_1, λ_2 , one can construct the state with definite P -parity, G -parity,etc.. As a result, we have a "meson state" in the rest frame. Let us perform now the boost into the center of the mass system (c.m.s.) of two mesons.The meson form factor is equal in this model to the usual Feynman diagram, fig. 2.1, multiplied by two D-functions and integrated over the angles $\theta_{1,2}$ and $\varphi_{1,2}$. Therefore, the meson wave function in this model is proportional to the D^J -function multiplied by two free quark spinors.

The kinematics is the following. In the rest frame of the meson:

$$\vec{K}_1=-\vec{K}_2=\vec{K},\quad |\vec{K}|=\frac{1}{2}\left(M_0^2-4m^2\right)^{1/2}=\frac{M_0}{2}v, \quad (2.7)$$

$$K_z=\frac{M_0}{2}v\cos\theta,\quad \vec{K}_\perp=\frac{M_0}{2}v\sin\theta\cdot\vec{n}_\perp,\quad E=M_0/2,$$

where M_0 is the meson mass, v is the velocity, m is the quark mass, $\vec{n}_\perp$ is the ort in the transverse x,y-plane. In the c.m.s. (γ is the Lorentz factor, $\gamma \sim q/M_0 \to \infty$):

$$\tilde{P}=\gamma M_0,\quad \tilde{k}_1^z=\gamma(E+k_1^z)=\gamma\frac{M_0}{2}(1+v\cos\theta_1)\equiv x_1\tilde{P}_z,\quad \vec{\tilde{k}}_\perp=\vec{k}_\perp,$$

$$\tilde{k}_2^z=\gamma\frac{M_0}{2}(1-v\cos\theta_1)\equiv x_2\tilde{P}_z,\quad x_1+x_2=1,\quad \tilde{k}_1^+=(\tilde{E}+\tilde{k}_1^z)\simeq 2x_1\tilde{P}_z,$$

$$\tilde{k}_2^+=(\tilde{E}+\tilde{k}_2^z)\simeq 2x_2\tilde{P}_z,\quad \tilde{k}_1^-=(\tilde{E}-\tilde{k}_1^z)\simeq\frac{\vec{k}_\perp^2+m^2}{2x_1\tilde{P}_z}\sim\frac{M_0^2}{q}\sim\tilde{k}_2^-. \qquad (2.8)$$

(For the final meson:

$$k_3^\perp=-k_4^\perp=\ell_\perp,\quad \tilde{k}_3^z=-\gamma(E+k_3^z)=-\gamma\frac{M_0}{2}(1+v\cos\theta_2)\equiv-y_1\tilde{P}_z,$$

$$\tilde{k}_4^z=-\gamma\frac{M_0}{2}(1-v\cos\theta_2)\equiv-y_2\tilde{P}_z,\quad y_1+y_2=1,$$

$$\tilde{k}_3^-=(\tilde{E}-\tilde{k}_3^z)\simeq 2y_1\tilde{P}_z,\quad \tilde{k}_4^-\simeq 2y_2\tilde{P}_z,\quad \tilde{k}_3^+\sim\tilde{k}_4^+\sim M_0^2/q.$$

It may be seen from (2.8) that the longitudinal momentum fraction carried by the first quark is

$$\frac{1-v}{2}\le x_1=\frac{1}{2}(1+v\cos\theta_1)\le\frac{1+v}{2}. \qquad (2.9)$$

For the case of nonrelativistic (in the meson rest frame) quarks: $v\ll 1$ and the support of $x_{1,2}$ is localized at $x_{1,2}\simeq 1/2$. For the case of ultrarelativistic quarks: $v\to 1$ and the used above variable $(-1)\le\xi_1=(x_1-x_2)=\cos\theta_1\le 1$. The gluon propagator in fig. 2.1 has the form:

$$\frac{1}{k^2}\simeq\frac{4}{(1-\xi_1)(1-\xi_2)q^2+8\vec{k}_\perp\vec{\ell}_\perp+O(k_\perp^2,\ell_\perp^2)};$$

$$\vec{k}_\perp\vec{\ell}_\perp=k_\perp\ell_\perp\cos(\varphi_2-\varphi_1), \qquad (2.10)$$

$$\xi_1=x_1-x_2=v\cos\theta_1,\qquad \xi_2=y_1-y_2=v\cos\theta_2$$

It may be seen from (2.10) that the diagram of fig. 2.1 depends nontrivially on both $\cos\theta_1$ and $\cos\theta_2$. Therefore, the integration over $\cos\theta_1$ and $\cos\theta_2$ with D^J-functions doesn't influence the asymptotic behaviour. In other words, the asymptotic behaviour doesn't depend on meson spins. At the same time, the diagram of fig. 2.1 depends on the angles $\varphi_{1,2}$ only through the factors $\exp\{\pm i\varphi_{1,2}/2\}$ in the quark spinors and the term $(\vec{k}_\perp\vec{\ell}_\perp)$ in the gluon propagator (2.10). Let us take for simplicity $\lambda_1=\lambda$ and $\lambda_2=-\lambda$, then integration over $\varphi_{1,2}$ takes the form:

$$\int_0^{2\pi}d\varphi_1 e^{i\lambda\varphi_1}\int_0^{2\pi}d\varphi_2 e^{-i\lambda\varphi_2},$$

and this separates from the diagram of fig. 2.1 the term with the behaviour $\sim\exp\{i\lambda(\varphi_2-\varphi_1)\}$. To obtain such a term, one needs to decompose the propagator (2.10) into the $(\vec{k}_\perp\vec{\ell}_\perp/q^2)^n$ series and to separate the term $\sim(\vec{k}_\perp\vec{\ell}_\perp/q^2)^{|\lambda|}$. As a result, each unit L_z, the quark angular momentum projection, leads to the suppression $\sim(k_\perp/q)$ of the form factor asymptopia, because $L_z\neq 0$ is ensured by the term $\sim(k_\perp/q)^{|L_z|}$ *. Consider as an example the form factor $\langle\lambda_2=2|J_{\lambda=1}|\lambda_1=-1\rangle$, because this example contains all the main characteristic features. Its asymptotic behaviour is determined by the diagrams of the type shown in fig. 2.2 (the arrows show the projections of the quark spins onto the z-axis). As compared with the "normal dimensional behaviour" $\sim(1/q)$, there is the additional suppression $\sim(k_\perp/q)^2$ due to $L_{z_1}=-1$, $L_{z_2}=+1$.

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*) All these properties follow in fact, from the Lorentz covariance and will be true, therefore, when the higher orders of α perturbation theory are taken into account.

Moreover, there is additional suppression $\sim(m/q)$ due to the quark helicity flip or $\sim(k_\perp/q)$ because the quark has the small transverse momentum $k_\perp \sim \mu_0$ and its helicity doesn't coincide with its spin projection onto the z -axis (for the light u - and d -quarks just this contribution will be the most important, because $\bar{k}_\perp \gg m_u, m_d$). As a result, $\langle \lambda_2=2 | J_{\lambda=1} | \lambda_1=-1 \rangle \sim (1/q^4)$, and that agrees with (2.6). It is not difficult to see that the asymptotic behaviour (2.6) will be true for the baryon form factors as well (in this case $n=3$ in (2.6)).

Integration of (2.10) over $\cos\theta_{1,2}$ with the D^J - functions separates the term:

$$\frac{1}{(1-z_1)(1-z_2)} = \frac{1}{(1-v\cos\theta_1)(1-v\cos\theta_2)} \to (v\cos\theta_1)^{\ell_1}(v\cos\theta_2)^{\ell_2}, \qquad (2.11)$$

where ℓ is the quark angular momentum in the meson rest frame.. If the quarks are light, then $v \simeq 1$ (see (2.7)), and there is no significant numerical suppression. For the heavy nonrelativistic (in the meson rest frame) quarks $v \ll 1$ and (2.11) show that the asymptotic form of the form factor contains the numerical smallness $\sim (v)^{\ell_1+\ell_2}$, but the dependence on q is unchanged.

It may seem surprising at the first sight that there is no suppression of the asymptopia $\sim (|\vec{p}_0|/q)^{\ell_1+\ell_2} \sim (M_0 v/q)^{\ell_1+\ell_2}$ for the heavy nonrelativistic quarks ($\vec{p}_0$ is the relative quark momentum in the meson rest frame). The reason is that the suppression $\sim (M_0 v/q)^{\ell_1+\ell_2}$ is present at the true nonrelativistic limit only, i.e. at $M_0 v \ll |\vec{q}| \ll M_0$. Really, in our case we are outside this region, because $|\vec{q}| \to \infty$ and $|\vec{q}| \gg M_0$. Hence, the relative quark momentum in the c.m.s.

is:
$$\tilde{p}_0 \to (x_1-x_2)p_z = z_1 p_z = v\cos\theta_1 \cdot \frac{|\vec{q}|}{2} \gg M_0 v,$$
$$\tilde{p}_0' \to (y_1-y_2)(-p_z) = -v\cos\theta_2 \cdot |\vec{q}|/2,$$

and the suppression factors look like (compare with (2.11)):

$$\sim (\tilde{p}_0/|\vec{q}|)^{\ell_1} (\tilde{p}_0'/|\vec{q}|)^{\ell_2} \sim (v\cos\theta_1)^{\ell_1}(v\cos\theta_2)^{\ell_2}.$$

It is worth noting that, as it is clear from previous considerations, all the above obtained results for the asymptotic behaviour are applicable also to the processes of the type:

$$\langle p_2 = \sum_i p_i' | J | p_1 \rangle, \quad \langle p_2 = \sum_i p_i' ; p_1 = \sum_j p_j | J | 0 \rangle$$

where, for instance, the state $\langle p_2 = \sum_i p_i' |$ is the hadronic jet with the invariant mass $M^2 = (\sum p_i')^2 \ll |q^2|$ and all momentum transfers inside the jet $(p_i' - p_k')^2 \ll |q^2|$. The mass M may be large, $M \gg 1$ GeV , but fixed, and $|q^2| \to \infty$. (When the internal momentum transfers are large, $M^2 \sim (p_i'-p_k')^2 \gg 1$ GeV^2, the quarks can be considered as free). It is clear from fig. 2.1 that the physics is that the colourless two-quark system is produced at small distances $\sim 1/q$, and it doesn't matter to what hadrons it later transforms.

2.2 The Selection Rules

2.2.1 General Selection Rules

The following most characteristic features have been pointed above |1.24, 2.1| :

a) Because the quark transverse momentum inside the hadron is not large, $\bar{k}_\perp \sim \mu_0$, the directions of the quark and hadron momenta nearly coincide at $|p_z| \sim q \gg \mu_0$. Hence (up to corrections $\sim k_\perp/q$), the quark helicity coincides with the projection of its spin onto the direction of the hadron momentum.

b) For the hadronic states (wave functions) with $L_z \neq 0$ (L_z is the projection of the quark angular momentum onto the direction of the momentum of its hadron) the asymptotic behaviour of the exclusive amplitudes is power suppressed.

c) The QCD interaction is of the vector nature and at the quark-gluon vertex the quark helicity is conserved (up to corrections $\sim m/q$).

The following general selection rule follows evidently from the properties "a" and "b": the leading asymptotic contributions give the hadronic states (wave functions) in which the hadron helicity is equal to the sum of its constituent helicities (quarks or gluons). With account of "c" one has evidently: the sum of hadronic helicities is conserved in hard exclusive processes, $(\sum \lambda_i)_{\text{initial}} = (\sum \lambda_i)_{\text{final}}$. (The consequences of the baryon helicity conservation in the baryon form factors are considered in detail in /1.23/).

Let us give few simple consequences of general selection rules.

The hard exclusive amplitudes in which the mesons with $|\lambda| > 1$ or the baryons with $|\lambda| > 3/2$ participate, are suppressed (for the mesons composed of gluons - at $|\lambda| > 2$) .

The final states with $|\lambda| = 1/2$ dominate in the electroproduction $\overset{*}{\gamma} N \to N'$ at large $|q^2|$, and their transition form factors have the same asymptotic behaviour as the elastic nucleon form factor, while the transition form factors for the final states with $|\lambda| = 3/2$ have additional suppression $\sim 1/Q$. Therefore, for the polarized deep-inelastic electron scattering: $\left[F_1^{3/2}(x)/F_1^{1/2}(x)\right] \sim (1-x)$ at $x \to 1$.

The simple and useful consequences of the general formula (2.6) were pointed out in /2.2/ . Consider the two hadron

production in e^+e^- collisions, i.e. the electromagnetic hadron form factors at positive and large q^2 . Because the photon in $e^+e^- \to \gamma \to H_1 H_2$ has $S_z = \pm 1$ (Z is the e^+e^- collision axis) and the leading mesonic form factors are those with $\lambda_1 = \lambda_2 = 0$ and baryonic ones are with $|\lambda_1| = |\lambda_2| = 1/2$ (see, 2.6), then the angular distribution of mesons is "$\sin^2\theta$" and that of baryons is "$(1 + \cos^2\theta$). This selection rule is especially useful, because it's much easy to measure the angular distribution than the hadron polarizations.

2.2.2 The Simple Interpretation

There exist the simple and visible interpretation of the above described selection rules. Let us consider the quark - quark - photon (gluon) vertex. The quarks are on or near to the mass shell. In the two-quark c.m.s. the process of the photon (gluon) absorption looks as in fig. 2.3, where the arrows indicate the directions of momenta and spins. Because the quark helicity is conserved in the interaction with the vector particle, the photon (gluon) has the helicity $|\lambda| = 1$, i.e. it is transversely polarized. This is a well known property of the vertex, which leads, for instance, to the dominance of the transverse cross-section σ_T and the smallness of the longitudinal cross-section σ_L for the deep-inelastic ep - scattering.

This property is generalized strightforwardly to the collinear processes with the larger number of photons or gluons. For instance, it is seen from fig. 2.4 that the total angular momentum projection of the two-gluon system is equal to unity. For the collinear process this means that one gluon has $S_z = 1$ and the second one $S_z = 0$.

Analogously, replacing the quark with a transversely polarized gluon on or near to the mass shell, we have from the three-gluon vertex, fig. 2.5, that the virtual gluon has $S_z=0$, i.e. the helicity of the real gluon turns over.

Using these properties it's not difficult to obtain the selection rules. For instance, for the meson form factor, fig. 2.6, the gluon has $|S_z|=1$ and so the photon helicity is $\lambda=0$, i.e.. the longitudinal form factor dominates. For the baryon, fig. 2.7, the transverse form factor is the leading one. It is clear that the systems with an even number of quarks have the leading longitudinal form factor and those with an odd number of quarks have the transverse one /1.27/.

2.2.3 The Selection Rules for the Heavy Quarkonium Decays

It has been pointed in /2.1, 2.3/ that the above described method for the calculation of hadronic form factors may be applied to the description of heavy quarkonium exclusive decays as well. We describe here the selection rules for the quarkonium decays into two light hadrons /1.44, 2.18/.

The decay of the C -even quarkonium into two mesons is described by the diagrams like those shown in fig. 2.8, and for the C -odd one shown in fig. 2.9. It is clear from figs. 2.8 and 2.9 that the nonsuppressed decays are of two types only: a) from the initial state with $S_z=0$ (in the quarkonium rest frame, the z -axis is the direction of the meson motion);; b) from the initial state with $|S_z|=2$. For the case "a" two produced mesons have the helicities $\lambda_1=\lambda_2=0$, for the case "b" $\lambda_1=-\lambda_2=\pm 1$.

The baryon pair is produced as shown in figs. 2.10, 2.11. For the C -even quarkonium, fig. 2.10, the initial $|S_z|=1$

and the baryon helicities are $|\lambda_1=-\lambda_2|=1/2$. For the C -odd one, fig. 2.11, $|S_z|=1,3$ and $|\lambda_1=-\lambda_2|=1/2$ and $|\lambda_1=-\lambda_2|=3/2$, respectively.

There exist the useful selection rule for the two meson decays from the initial state with $J<2$, connected to the case "a" above. Let us introduce the quantum number "naturalness": $\sigma=(-1)^S P$, where P is the parity, and S is the spin. If $\sigma_{initial}\neq(\sigma_1\sigma_2)_{final}$, then the amplitude contains the unit antisymmetric tensor $e_{\mu\nu\lambda\sigma}$. Besides, because the produced mesons have $\lambda_1=\lambda_2=0$, their polarization tensors reduce to their momenta. Therefore, the amplitude is zero, because there are no quantities with which the indices of $e_{\mu\nu\lambda\sigma}$ can be contracted. As a result, we have the selection rule: the naturality is conserved for two-meson decays of the quarkonium with $J<2$. Some examples of the suppressed and allowed decay are presented below.

$$
\begin{array}{ll}
{}^1S_0 \not\to \rho\rho,\ A_1A_1,\ BB, & {}^1S_0 \to \rho B,\ \delta\pi,\ A_1A_2, \\
{}^3S_1 \not\to \rho\pi,\ \rho A_1,\ \delta B, & {}^3S_1 \to \pi B,\ \rho A_2,\ \delta\rho, \\
{}^3P_0 \not\to \pi A_2,\ \rho B,\ A_1A_2, & {}^3P_0 \to \pi\pi,\ \rho\rho,\ \pi A_1, \\
{}^3P_1 \not\to \rho\rho,\ \pi A_1,\ A_1A_1, & {}^3P_1 \to \rho B,\ \pi A_2;\ \delta A_1, \\
{}^1P_1 \not\to \delta\rho,\ \rho A_2,\ A_1B, & {}^1P_1 \to \pi\rho,\ BA_2,\ \rho A_1, \\
{}^3P_2 \not\to \pi A_2,\ \delta A_1, & {}^3P_2 \to \pi\pi,\ \rho\rho,\ \pi A_1,\ \rho B.
\end{array}
$$

For the decays into two gluonium states, one can see from figs. 2.12 and 2.13 that the initial state can only have $|S_z|=0,2$, and the meson helicities are: a) $S_z=0\to(\lambda_1=\lambda_2=0),(\lambda_1=\lambda_2=\pm2)$; b) $|S_z|=2\to$ $\lambda_1=0,\ |\lambda_2|=2$. I is seen that because the gluon helicity turns over at the three-gluon vertex, fig. 2.5, (unlike the quark helicity, fig. 2.3), the sum of the hadron helicities is not conserved, for the processes with the gluonia, fig. 2.13, /2.4, 2.18/.

It is worth noting that all the above described selection rules are obtained in the formal limit $|q^2| \to \infty$ (or $M_Q \to \infty$, where M_Q is the heavy quark mass), when one can safely neglect the power corrections. This is evidently the case in the Υ-region $M_b \simeq 5$ GeV, $Q^2 \simeq 100$ GeV^2. However, in the charmonium region $M_c \simeq 1.5$ GeV, $Q^2 \simeq 10$ GeV^2 the situation is not so evident. It is discussed below in chs. 8,9 in detail.

2.3 <u>Some Qualitative Applications</u>

2.3.1 The difficulties with quantitative calculations

1. Let us try to estimate the value of the pion electromagnetic form factor. Because in the isotopic symmetry limit the π-meson wave function is symmetric: $\varphi_\pi^A(\zeta) = \varphi_\pi^A(-\zeta)$, see (1.10), we can rewrite (1.8) in the form:

$$F_{\pi^+}(q^2) \to \frac{32\pi\bar{\alpha}_s}{9} \frac{|f_\pi|^2}{-q^2} \left| \int_{-1}^{1} \frac{d\zeta}{1-\zeta^2} \varphi_\pi^A(\zeta) \right|^2, \quad \int_{-1}^{1} d\zeta\, \varphi_\pi^A(\zeta) = 1, \tag{2.12}$$

where the constant $f_\pi \simeq 133$ MeV is known from the $\pi \to \mu\nu$ decay. Let us note first of all, that (2.12) predicts the definite sign of F_π, which coincides with the prediction of the vector dominance model (VDM)*: $F_\pi^{VDM} \simeq \frac{m_\rho^2}{m_\rho^2 - q^2} \simeq \frac{0.6\,\mathrm{GeV}^2}{-q^2}$ (2.13)

*) If the vector gluon is replaced by the (pseudo)scalar one, the sign is opposite.

Because $F_{\pi^+}(q^2=0) = 1$, it is seen from (2.12) that $F_\pi(q^2)$ can have no zeroes in the euclidean region $q^2 < 0$. The experimental data show /1.33/ that $F_\pi(q^2)$ is indeed positive in the region $0 \leq (-q^2) \lesssim 10\,\mathrm{GeV}^2$.

Let us try now to use two characteristic wave functions:

$$\varphi_{nonrel}(\zeta) = \delta(\zeta) \quad \text{and} \quad \varphi_{as}(\zeta) = \frac{3}{4}(1-\zeta^2).$$

First of them imitates the nonrelativistic wave function for which $x_1 \simeq x_2 \simeq 1/2$, $\zeta = (x_1 - x_2) \simeq 0$ (see (2.9)), i.e. each quark carries one half of the pion momentum. Substituting that into (2.12), we have at $\bar{\alpha}_s \simeq 0.3$:

$$F_\pi(q^2) \simeq \frac{0.06\,\mathrm{GeV}^2}{-q^2}.$$

The wave function $\varphi_{as}(\zeta)$ is the asymptotic form of $\varphi_\pi^A(\zeta, \mu^2)$ at $\mu^2 \to \infty$ (see, ch.3). For this wave function

$$F_\pi(q^2) \simeq \frac{0.15\,\mathrm{GeV}^2}{-q^2}.$$

The experimental data |1.33| in the interval $|q^2| \lesssim 10\,\mathrm{GeV}^2$ are described sufficiently well by the ρ-meson contribution (2.13). It is seen that both wave functions $\varphi_{nonrel}(\zeta)$ and $\varphi_{as}(\zeta)$ give too small values for $F_\pi(q^2)$.

2. The branching ratio for the decay $\chi_0(3415) \to \pi^+\pi^-$ is |2.3, 1.44| (see ch. V for details):

$$Br(\chi_0 \to \pi^+\pi^-) = \left| \pi\bar{\alpha}_s \frac{16\sqrt{2}}{27} \frac{f_\pi^2}{M_c^2} I_0 \right|^2, \tag{2.14}$$

$$I_0 = \int_{-1}^{1} \frac{d\zeta_1 \varphi_\pi^A(\zeta_1)}{1-\zeta_1^2} \int_{-1}^{1} \frac{d\zeta_2 \varphi_\pi^A(\zeta_2)}{1-\zeta_2^2} \frac{1}{1-\zeta_1\zeta_2} \left[1 + \frac{1}{4} \frac{(\zeta_1-\zeta_2)^2}{1-\zeta_1\zeta_2} \right]. \tag{2.15}$$

Using the wave functions $\varphi_{nonrel}(\zeta)$ and $\varphi_{as}(\zeta)$ and $\bar{\alpha}_s = 0.3$, $M_c \simeq 1.5$ GeV in (2.14), (2.15), one obtains for the (2.14) numbers $\simeq 4.10^{-3}$% and $\simeq 3.10^{-2}$% respectively, which are too

small as compared with the experimental value /2.5, 2.6/ :
$Br(\chi_0\to\pi^+\pi^-)=(0.9\pm0.2)\%$.

3. The properties of the nucleon electromagnetic form factors are described in detail in ch. 10. The experimental behaviour /1.34/ of the proton magnetic form factor $G_M^p(q^2)$ at $0\le(-q^2)\lesssim 30\,GeV^2$ and the neutron one $G_M^n(q^2)$ at $0\le(-q^2)\lesssim 10\,GeV^2$ is described approximately by the famous dipole formulae:

$$\frac{1}{\mu_p}G_M^p(q^2)\simeq\frac{1}{\mu_n}G_M^n(q^2)\simeq\left(1-q^2/\mu^2\right)^{-2},\quad \mu^2=0.71\,GeV^2,$$
$$G_M^p(q^2=0)=\mu_p=2.79,\quad G_M^n(q^2=0)=\mu_n\simeq-1.91, \tag{2.16}$$

where $\mu_p(\mu_n)$ is the proton (neutron) magnetic moment. The real calculation of $G_M^{p,n}(q^2)$ in terms of the nucleon wave function $\varphi_N(x_1,x_2,x_3)$, $\sum x_i=1$, fig. 2.7, is presented in ch. 10. As it was first noted in /2.7/, an attempt to use the nonrelativistic form $\varphi_{nonrel}(x_i)=f_N\,\delta(x_1-1/3)\,\delta(x_2-1/3)$ for the nucleon wave function $\varphi_N(x_i)$ gives the qualitatively wrong results: $\frac{1}{\mu_p}G_M^p(q^2)<0$, $\frac{1}{\mu_n}G_M^n(q^2)<0$, that contradicts the experiment (2.16). An attempt to use the asymptotic form of the nucleon wave function: $\varphi_{as}(x_i)=f_N\cdot 120\,x_1x_2x_3$ also gives the wrong results /2.8, 2.9/ : $\frac{1}{\mu_p}G_M^p(q^2)=0$, $\frac{1}{\mu_n}G_M^n(q^2)<0$. Moreover, the dimensional constant f_N which determines the value of the nucleon wave function at the origin (and is analogous to f_π) can be found by using the QCD sum rules: $|f_N|\simeq 0.5\cdot10^{-2}\,GeV^2$. Using this value of f_N and the wave function $\varphi_{as}(x_i)$, one obtains for the $\frac{1}{\mu_n}G_M^n(q^2)$ the result which is two orders of a magnitude smaller than (2.16) /1.49/.

We see, therefore, that all things are very bad. There may be two reasons for this. a) The region $|q^2|\simeq 10\,GeV^2$ for $F_\pi(q^2)$

and $\chi_0\to\pi\pi$ and $|q^2|\simeq(20-30)\,GeV^2$ for $G_M^{p,n}(q^2)$ is not the asymptotic region and the power corrections are very large or even dominate*, b) The leading terms are the leading ones already, the power corrections are reasonably small, but the true pion and nucleon wave functions are very unlike both the wave functions $\varphi_{nonrel}(x)$ and $\varphi_{as}(x)$.

It is argued below in ch. 4-10 that the case "b" is realized. The pion and the nucleon wave functions obtained with the help of the QCD sum rules differ greatly in their properties from both $\varphi_{nonrel}(x)$ and $\varphi_{as}(x)$. It will be shown in chs. 5, 6, 9, 10 that using the realistic models (obtained with the help of the QCD sum rules) for the hadronic wave functions, it is possible to obtain the quantitative description of a large number of various exclusive processes in agreement with the experiment. Hence, below in this section, we don't try to obtain the quantitative results and describe some applications on the qualitative level only.

2.3.2 The ρ meson Form Factors

As it was pointed earlier, for any meson with $\lambda=0$ the leading twist wave functions are determined by the bilocal operators $\bar\psi_2\gamma_\mu\psi_1$ for the natural particles $\rho,\delta,A_2,\ldots$ and $\bar\psi_2\gamma_\mu\gamma_5\psi_1$ for the unnatural ones $-\pi,A_1,B\ldots$ For the $\rho_L\equiv\rho_{\lambda=0}$ meson, $M_\rho\varepsilon_\mu^{\lambda=0}(p)=p_\mu+O(M_\rho/p)$:

$$\langle 0|\bar d(z)\gamma_\mu u(-z)|\rho_L(p)\rangle=p_\mu f_\rho\varphi_\rho^V(zp), \tag{2.17}$$
$$\varphi_\rho^V(zp)=\int_{-1}^{1}d\xi\,\exp\{i\xi(zp)\}\,\varphi_\rho^V(\xi),\quad \int_{-1}^{1}d\xi\,\varphi_\rho^V(\xi)=1.$$

The constant f_ρ in (2.17) which determines the value of the ρ_L-meson wave function at the origin is known experimentally from the decay $\rho^0\to e^+e^-$: $f_\rho\simeq 200$ MeV. In the

*For this viewpoint see, for instance, /4.5/.

isotopic symmetry limit: $\varphi_\rho^V(\zeta)=\varphi_\rho^V(-\zeta)$.

For the $\rho_\perp \equiv \rho_{|\lambda|=1}$ -meson the leading twist 2 wave function has the form:

$$\langle 0|\bar d(z)\sigma_{\mu\nu}u(-z)|\rho_\perp(p)\rangle=(\varepsilon^\perp_\mu p_\nu-\varepsilon^\perp_\nu p_\mu)f_\rho^T\varphi_\rho^T(zp), \tag{2.18}$$

$$\varphi_\rho^T(zp)=\int_{-1}^{1}d\zeta\, \exp\{i\zeta(zp)\}\varphi_\rho^T(\zeta),$$

$$\int_{-1}^{1}d\zeta\,\varphi_\rho^T(\zeta)=1,$$

$\varphi_\rho^T(\zeta)=\varphi_\rho^T(-\zeta)$ in the $SU(2)$ -limit.

The following question arises naturally. As the $\rho_\perp$ -meson wave function (2.18) has the leading twist 2, it seems that the form factor of the $\rho_\perp$ -meson will have the same behaviour as those of the ρ_L and π -mesons. At the same time, the general formula (2.6) predicts that the $\rho_\perp$ -meson form factors are suppressed (see also /1.27/):

$$\langle\rho_\perp|J_{|\lambda|=1}|\rho_L\rangle\lesssim 1/q^2\,,\qquad \langle\rho_{\lambda_2=\pm1}|J_{\lambda=0}|\rho_{\lambda_1=\mp1}\rangle\lesssim 1/q^3.$$

The reason for this suppression can easily be seen from the diagrams shown in fig. 2.14. The transition $\rho_L\leftrightarrow\rho_\perp$ requires the change of the quark spin projection S_z (suppression $\approx k_\perp/q$), and the transition $(\rho_{\lambda_2=\pm1})\leftrightarrow(\rho_{\lambda_1=\mp1})$ requires that the spin projections of both quarks should be turned over (suppression $\sim k_\perp^2/q^2$). (The transition $(\rho_{\lambda_2=\pm1})\leftrightarrow(\rho_{\lambda_1=\pm1})$ is impossible at all, because in this case $|\lambda|=|\lambda_1+\lambda_2|=2$, but the vector current can have $|\lambda|\le 1$ only. For the external source which can have $|\lambda|=2$, the decay into $\rho_\perp\rho_\perp$ is not suppressed, for instance, the decay of the tensor charmonium $\chi_2(3555)\to\rho_\perp\rho_\perp$, see ch. 5).

The behaviour of the $\langle\lambda_2=\pm1|J_{\lambda=0}|\lambda_1=\mp1\rangle$ form factor

drastically changes when the vector gluon is replaced with the (pseudo) scalar one in the diagram in fig. 2.14, because the quark helicity turns over at the (pseudo)scalar gluon vertex:

$$\langle\lambda_2=\pm1|J_{\lambda=0}|\lambda_1=\mp1\rangle\sim\begin{cases}1/q^3 & \text{vector gluon}\\ 1/q & \text{scalar gluon}\end{cases}$$

It is easy to understand that all the above described properties are not specific for the ρ -meson only, but are true for any meson with the helicity $|\lambda|=1$.

2.3.3 Two-Current Form Factors

These form factors can be measured in the reactions: $e^+e^-\to e^+e^-$ + (meson) or $e^+e^-\to\gamma$ + (meson), fig. 2.15.

Let us consider as an example the π^0 -meson /2.10/:

$$T_{\mu\nu}=i\int dx\, e^{-iq_1x}\langle\pi^0(p)|T\, J_\mu(x)J_\nu(0)|0\rangle=e_{\mu\nu\lambda\sigma}q_1^\lambda p^\sigma F_{\gamma\pi}(q_1^2,q_2^2), \tag{2.19}$$

$$F_{\gamma\pi}(0,0)=-\frac{1}{2\sqrt2\,\pi^2 f_\pi}$$

(the value $F_{\gamma\pi}(0,0)$) is determined by the axial-current anomaly /2.11/). The leading asymptotic behaviour of $F_{\gamma\pi}$ can easily be found as follows.

Replace the π^0 -meson with the free quark-antiquark pair with the longitudinal momentum fractions $x_1=\frac{1+\zeta}{2}$, $x_2=\frac{1-\zeta}{2}$, fig. 2.15 plus the crossing diagram (we use the frame: $\vec p+\vec q_2=0$). Then

$$iT_{\mu\nu}\to 2e_u^2\left(\frac{i}{\Delta^2}\right)\bar u\gamma_\mu\hat\Delta\gamma_\nu u+(u\to d)= \tag{2.20}$$

$$\frac{2e_u^2}{x_1q_1^2+x_2q_2^2}\,e_{\mu\lambda\nu\sigma}\Delta_\lambda\cdot\bar u\gamma_\sigma\gamma_5 u+(u\to d),$$

where $e_u = 2/3$, $e_d = -1/3$ are the quark charges, $\Delta = x_2 p - q_1$, $\Delta^2 = (x_1 q_1^2 + x_2 q_2^2)$ and the formula $\gamma_\mu \hat{\Delta} \gamma_\nu = -i e_{\mu\lambda\nu\sigma} \gamma_\sigma \gamma_5$ was used.

Introduce the π -meson wave function:

$$\langle \pi^0(p) | \bar{u} \gamma_\sigma \gamma_5 d | 0 \rangle \to -\frac{i f_\pi}{\sqrt{2}} p_\sigma \varphi_\pi^A(z) = -\langle \pi^0 | \bar{d} \gamma_\sigma \gamma_5 d | 0 \rangle \quad (2.21)$$

The final answer is:

$$F_{\gamma\pi}(q_1^2, q_2^2) \to \frac{2\sqrt{2}(e_u^2 - e_d^2)}{-(q_1^2 + q_2^2)} f_\pi \int_{-1}^{1} \frac{dz\, \varphi_\pi^A(z)}{1 + \omega z}, \quad \omega = \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2}. \quad (2.22)$$

At $q_1^2 = q_2^2$, $\omega = 0$, $\int dz\, \varphi_\pi^A(z) = 1$, and the answer is determined by the local limit:

$$F_{\gamma\pi}(q^2, q^2) \to \frac{\sqrt{2}}{3} \frac{f_\pi}{-q^2} = F_{\gamma\pi}(0,0) \frac{4\pi^2 f_\pi^2}{3 q^2} \simeq F_{\gamma\pi}(0,0) \frac{0.25\ \mathrm{GeV}^2}{q^2} \quad (2.23)$$

At $q_2^2 = 0$, $\omega = 1$:

$$F_{\gamma\pi}(q^2, 0) \to \frac{2\sqrt{2}}{3} \frac{f_\pi}{-q^2} \int_{-1}^{1} \frac{dz\, \varphi_\pi^A(z)}{1 - z}. \quad (2.24)$$

The form factors of other C -even neutral mesons η, η', f^0 can be obtained in the same way.

In this simple case the operator expansion begins with the terms $\sim (\alpha_s)^0$ (i.e. there is no gluon) and is, in fact, the usual Wilson expansion of the product of two local currents.

2.3.4 The Form Factors $\gamma\pi\rho$, $\gamma\pi\omega$.

The matrix element has the form:

$$\langle \pi^+(p_2) | J_\mu(0) | \rho_\perp^+(p_1) \rangle = e_{\mu\nu\lambda\sigma} p_1^\lambda p_2^\sigma \rho_\perp^\nu F_{\pi\rho}(q^2), \quad F_{\pi\rho} \sim \frac{1}{q^4}, \quad (2.25)$$

where $\rho_\perp^\nu$ is the $\rho_\perp$ -meson polarization vector. The ρ -meson has the helicity $|\lambda| = 1$ in the meson's c.m.s. and so the quark spin projection S_z turns over, fig. 2.14. Therefore, the form factor $F_{\pi\rho}$ is suppressed ($\sim k_\perp/q$) in accordance with (2.6). (This suppression has been pointed out by many authors /1.17, 1.27/). As a result, $\sigma(e^+e^- \to \pi\rho)/\sigma(e^+e^- \to \pi\pi) \sim (M^2/q^2)$ at large q^2. The calculation of the form factor $F_{\pi\rho}(q^2)$ is described in detail in ch. 9.

It is not difficult to see that $F_{\pi^\pm\rho^\mp} = F_{\pi^0\rho^0} = \frac{1}{3} F_{\pi^0\omega}$.

2.3.5 The Exclusive Electroproduction

An interesting combination of the short and large distance interactions give us the processes of small angle exclusive electroproduction: $\gamma^* N \to N\pi(\rho, \omega \ldots)$ or $\pi N \to e^+e^- N$, fig. 2.16, at $s \sim |q^2| \gg |t| \sim M^2$ /2.12, 2.1/ .

The asymptotic behaviour of these processes can be found supposing that the usual Regge description is applicable, fig. 2.17. Then the question is reduced to the calculation of the asymptotic behaviour of the vertex current-hadron-reggeon. Considering the reggeon as a particle with the spin $\alpha(t)$ which changes with the "mass" t, we can obtain the reggeon form factor as an analytic continuation of the usual form factor. Let us remind in connection with this (see (2.6)), that at fixed helicities the asymptotic behaviour of the form factor (i.e. the power $(1/q^2)^K$) does not depend both on the hadron spins and masses, which influence only the value of the coefficient in (2.6). Therefore, the reggeon form factors have the same behaviour in $1/q^2$, and only the coefficient is the function of t.

Let us write the reggeon contribution into the amplitude $\gamma^* N \to \pi N$ in the form:

$$\varepsilon_\mu^\lambda T_\mu^{(\alpha)} = \varepsilon_\mu^\lambda \beta_\mu(q,k)\left(\frac{\nu}{q^2}\right)^{\alpha(t)} \beta_N(t), \quad \nu=(s-u)/4,$$

where ε_μ^λ is the photon polarization vector. Then:

$$\varepsilon_\mu^\lambda \beta_\mu^{(-)}(q,k) \sim \varepsilon_\mu^\lambda \left(k_\mu \frac{\beta^{(-)}(t)}{q^2}\right)$$

for the Regge-trajectoties with $P\bar{\sigma} = -1$ ($\pi, A_1, B \ldots$, $\bar{\sigma}$ is the signature and P is the parity). Therefore (in the s -channel c.m.s.):

$$T^{(-)}_{\lambda=0}(\nu,Q^2,t) \sim Q^{-1} f_L(\omega,t), \quad T^{(-)}_{\parallel} \sim Q^{-2} f_\parallel(\omega,t), \quad T^{(-)}_\perp = 0,$$

$$\omega = (2\nu/Q^2) \sim \mathrm{const}, \quad t \sim \mathrm{const}, \quad Q^2 = -q^2 \to \infty,$$

where $\lambda_\parallel$ denotes the transverse photon polarization parallel to the reaction plane and $\lambda_\perp$ – orthogonal to the reaction plane. Analogously, the trajectories with $P\bar{\sigma} = +1$ (ρ, ω, A_2) give:

$$T^{(+)}_{|\lambda|=1} \sim Q^{-2}\varphi_1(\omega,t), \quad T^{(+)}_{\lambda=0} \sim Q^{-3}\varphi_0(\omega,t).$$

Therefore, the production of the pions by the longitudinally polarized photons dominates:

$$\frac{d\sigma_L}{dt} = Q^{-6} F_L(\omega,t), \quad \frac{d\sigma_T}{dt} = Q^{-8} F_T(\omega,t), \quad |t| \sim \mu^2, \ Q^2 \to \infty.$$

These formulae describe the production of ρ, ω mesons as well, the only difference if that now the trajectories with $P\bar{\sigma} = +1$ give the main contribution at large Q^2.

The cross-section σ_T is the dominant one for the backward production, fig. 2.17:

$$\frac{d\sigma_T}{du} = Q^{-10}\Phi_T(\omega,u), \quad \frac{d\sigma_L}{du} = Q^{-12}\Phi_L(\omega,u), \quad |u| \sim \mu^2, \ Q^2 \to \infty.$$

It is interesting that the behaviour of the cross-section changes at high energies, because for $\gamma^* N \to \pi N$:

$$T_{\lambda=0} \sim \left(Q^{-1}\omega^{0}\beta_\pi(t) + Q^{-3}\omega^{1/2}\beta_\rho(t)\right)\beta_N(t)$$

and therefore:

$$\frac{d\sigma_L(\nu,Q^2)}{d\sigma_L(\nu,0)}\bigg|_{\gamma^* N \to \pi N} \sim \begin{cases} \nu^{-1} Q^{-2} & \text{at } \nu < Q^6/\mu^4 \\ Q^{-8} & \text{at } \nu > Q^6/\mu^4 \end{cases}$$

Unfortunately, the available experimental data are too poor and have too low Q^2 values.

2.3.6 The Threshold Behaviour of the Inclusive Structure Functions

Knowing the asymptotic behaviour of exclusive processes, we can determine the behaviour of inclusive cross-sections in the threshold region where the "inclusive mass" M_x is not very large. Some examples of inclusive electroproduction are presented in fig. 2.18 and the corresponding threshold regions are shown. It is evident that in the threshold region the process is "quasi-inclusive" and "quasi-exclusive" simultaneously, so that both descriptions should agree here (the duality, the correspondence principle) /2.13, 2.14/ .

Let us consider some examples.

a) The deep-inelastic scattering, fig. 2.18a.

The structure function $F_1(x, Q^2)$ obeys the scaling behaviour when the photon has the transverse polarization:

$$F_\perp(x,Q^2)\to F_\perp(x),\quad x=Q^2/2\nu,\quad M_x^2=\frac{1-x}{x}Q^2,\quad Q^2=-q^2\to\infty.$$

In the threshold region $Q^2 \gg M_x^2 \gtrsim M^2$:

$$F_\perp(x)\sim(1-x)^K\sim\left(M_x^2/Q^2\right)^K.$$

In order to determine the value of " K " for the nucleon target, consider the diagrams like that in fig. 2.19 (the arrows show the quark helicities, $\perp$ and L are the photon or gluon polarizations). The propagators in Fig. 2.19 have the virtualities $\sim\left(K_\perp^2 Q^2/M_x^2\right)\gg M^2$ at $Q^2\gg M_x^2$, and so the perturbation theory is applicable to them. The nucleon wave function φ_N is introduced in the usual way for the target quarks in Fig. 2.19. One can introduce the resonance wave function φ_N^* for the quarks on the right of fig. 2.19 at $M_x\sim 1\,GeV$, or one can consider these quarks as being free at $M_x\gg 1\,GeV$. Therefore, the diagram in fig. 2.19 describes, in fact the form factor for the $\perp$ -photon. This is the leading form factor which gives the behaviour $\sim 1/Q^3$ for the amplitude in fig. 2.19. (The quark, gluon and photon polarizations shown in figs. 2.19, 2.21 are in accordance with the selection rules, sect. 2.2). The structure function is proportional to the amplitude square. Therefore*:

$$F_\perp^N(x)\sim(1-x)^{K_\perp^N}\sim\left(1/Q^2\right)^{K_\perp^N}\sim 1/Q^6,\quad K_\perp^N=3. \tag{2.26}$$

The $\perp$ - form factor (i.e..for the transverse photon polarization) is suppressed for the case of the pion target (the final state at fig. 2.20 have $|\lambda|=1,\ \rho_\perp,\ \omega_\perp$, see sect. 2.3.5), the amplitude has the behaviour $\sim 1/Q^2$ and as a result

*) For simplicity we don't distinguish M_x and M .

/1.12, 1.17, 1.27/:

$$F_\perp^\pi(x)\sim(1-x)^{K_\perp^\pi}\sim\left(1/Q^2\right)^{K_\perp^\pi}\sim 1/Q^4,\quad K_\perp^\pi=2. \tag{2.27}$$

The longitudinal structure function $F_L(x,Q^2)$, as is well known, has no scaling behaviour in the leading order in α_s : $F_L(x,Q^2)\to(1/Q^2)f_L(x)$. The threshold behaviour of $f_L(x)$ at $x\to 1$ is also determined by the diagrams in figs. 2.19, 2.20, but now the nucleon form factor is suppressed, while the pion one is not. Hence,

$$F_L^N(x,Q^2)\sim\frac{1}{Q^2}f_L^N(x)\sim\frac{1}{Q^2}(1-x)^{K_L^N}\sim\left(1/Q^2\right)^{K_L^N+1}\sim 1/Q^8,\quad K_L^N=3,$$

$$F_L^\pi(x,Q^2)\sim\frac{1}{Q^2}f_L^\pi(x)\sim\frac{1}{Q^2}(1-x)^{K_L^\pi}\sim\left(1/Q^2\right)^{K_L^\pi+1}\sim 1/Q^2,\quad K_L^\pi=0. \tag{2.28}$$

It is known (see, for instance, /2.15/) that $F_L(x,Q^2)$ acquires the scaling behaviour in the next order in α_s due to the contributions shown in figs. 2.21 and 2.22:

$$\Delta F_L^{\pi,N}(x,Q^2)\sim\alpha_s\bar{f}_L^{\pi,N}(x)\sim\alpha_s(1-x)^{\rho_L^{\pi,N}}. \tag{2.29}$$

It is not difficult to see that in the form factor region the amplitudes corresponding to figs. 2.21 and 2.22 diagrams have the behaviour $\sim 1/Q^5$ and $\sim 1/Q^4$, respectively.

Therefore:

$$\Delta F_L^N(x,Q^2)\sim\alpha_s(1-x)^5,\quad \Delta F_L^\pi(x,Q^2)\sim\alpha_s(1-x)^4. \tag{2.30}$$

It is clear from the diagrams shown in figs. 2.21 and 2.22 that the additional quark-antiquark pair (i.e. "the sea-pair") can have any flavour. Therefore, the admixture of the antiquarks, s -quarks, etc. in the nucleon also has the behaviour $\sim d_s(1-x)^5$ for $F_L^N(x,Q^2)$. For the case of the structure function $F_\perp(x,Q^2)$ the diagrams shown in figs. 2.21 and 2.22 have the behaviour $\sim 1/Q^6$ and $\sim 1/Q^3$ for the nucleon and pion target, respectively. Therefore, the admixture of nonvalence quarks in $F_\perp(x,Q^2)$ is $\sim d_s(1-x)^6$ for the nucleon and $\sim d_s(1-x)^3$ for the pion.

b) The process $e^+e^- \to p' + X$, fig. 218b is the crossing one with respect to the deep-inelastic scattering $ep \to e + X$, fig. 2.18a. Hence, the threshold behaviour of these inclusive cross-sections at $M_x^2 << q^2$ can be found in a complete analogy with the above given discussion (see, for instance, /2.16/).

The process shown in fig. 2.18c coincides in the region of not too large M_x with the exclusive electroproduction. Hence, the results described in sect. 2.3..3 can be used for the determination of the threshold behaviour. For instance, the cross section of inclusive pion electroproduction on the nucleon target has the behaviour in the threshold region:

$$\frac{d\sigma_\perp^{incl}}{dt\, dM_x^2} \sim \left(\frac{\mu^2}{Q^2}\right)^2 \left(\frac{M_x^2}{Q^2}\right)^K f\left(\frac{s}{Q^2}, t\right), \quad Q^2 \sim s = (q+p)^2 \gg M_x^2 \gtrsim \mu^2.$$

Using the results obtained in sect. 2.3.3, one finds: $K=2$. In the parton model language it means that the threshold behaviour of the pion $\to$ quark fragmentation function $F_{q/\pi}$ and of the quark $\to$ pion one $F_{\pi/q}$ is the same. (In other cases one also obtains the same threshold behaviour: $F_{a/b} \sim F_{b/a}$).

The above described approach is, evidently, too rough. The main unanswered question is: what is the absolute normalization of the inclusive processes in the threshold region, i.e. what is the value of the constant C_0 in $F(x,Q^2) \to C_0(1-x)^K$? Is it possible to calculate the value of C_0 knowing the target wa-

ve function $\varphi(y_i)$? It seems at the first glance that there is every prospect of success as the virtualities of all the propagators in figs. 2.19 and 2.20 are large $(\gg \mu^2)$ at $x \to 1$. Unfortunately, it is impossible at present to calculate C_0 unambiguously, and this is clear from the dimensional consideration. The structure function $F(x,Q^2)$ and the constant C_0 are dimensionless, while the expressions for the diagrams shown in figs. 2.19 and 2.20 include in the numerators the dimensional target wave function (the leading twist nucleon wave function φ_N has the dimensionality μ^2, the pion one, $\varphi_\pi \sim \mu^1$). This means that the integration over the phase space of the final free quarks (see figs. 2.19 and 2.20) contains at $M_x^2 \gg \mu_0^2$ the power infrared divergences. These infrared divergences ensure the appearance of the infrared cut off μ_0 in the denominator in order to make the answer dimensionless. Therefore, the result is highly sensitive to the infrared cut-off, i.e.. to the large distance dynamics. (At the same time, if there is no scaling behaviour, say, $F_L^\pi(x,Q^2) \to C_0^\pi/Q^2$, the constant C_0^π has the dimensionlaity μ^2 and can be expressed through the integral of the leading twist pion wave function $f_\pi \varphi_\pi^A$).

It has been proposed in the papers /2.17/ to use the process "$e^+e^- \to$(two jets) + (meson)" to obtain from the experiment the information about the properties of the meson wave function $\varphi_M(x)$. The idea is as follows, fig. 2.23. Select only such the events in which the meson is well separated out of all other hadrons in the phase space (i.e. the angle θ is not too small at high energies). Then the cross section of such process can be expressed through the integral of the meson wave function:

$$\int_0^1 \frac{dx\,\varphi_M(x)\left(1+z-zx\right)}{(1-x)(1-zx)} ,$$

where: $E_0 = q_0/2$ is the initial energy, E_M is the meson energy, $z = E_M/E_0$ is the energy fraction. Therefore, by measuring the cross-section as a function of "z" one can, in principle, obtain information about the properties of the meson wave function $\varphi_M(x)$. The needed kinematical restrictions, the expressions for the cross sections, etc. are described in details in |2.18|.

2.3.7 The Scattering

The above described approach can be used, of course, for the claculation of those contributions to the elastic scattering amplitudes which are caused by the hard interaction at small distances. However, there has been no noticeable progress in this region up to now. It seems that one of the main reasons for this is the enormous number of the Born diagrams.

Various diagrams give the different angular dependence, in particular, in the region $\cos\theta \to 1$, i.e. $s \gg |t| \gg \mu^2$. Consider, for instance, the elastic pp -scattering. The diagrams like those shown in fig. 2.24 describing the quark exchange between the protons (i.e. the gluon exchanges between the protons are absent) give the contribution into the scattering amplitude: $M_q \lesssim s^{-1} t^{-3}$ |1.19|. The diagrams shown in fig. 2.25 describing the exchanges both the quarks and gluons between the protons and those shown in fig. 2.26, in which only the gluons are exchanged between the protons, give the contributions: $M_{qg} \sim M_g \lesssim s^0 t^{-4}$.

In the scattering (unlike the form factors) there are

other contributions into the scattering amplitude, in addition to the usual hard contributions shown in figs. 2.24 - 2.26. It has been shown in |1.11| that the independent scattering of quarks on each other, fig. 2.27, give the contribution that exceeds the prediction of "dimensional counting rules". Unlike the hard scattering mechanism, figs. 2.24 - 2.26, where nearly "ready-made" hadrons (i.e. the colourless clusters of quarks with parallel momenta within each cluster) are formed in the small vicinity of the point "0", fig. 2.28, for the scattering mechanism shown in fig. 2.27 the quarks which were produced at the points "0_i" far away from each other, get together into the given hadron, fig. 2.29. The calculation shows |2.19| that for the pp -scattering this mechanism gives the contribution:

$$\left(\frac{d\sigma}{dt}\right)_{pp} \sim \frac{1}{t^8}\,\varphi\!\left(\frac{s}{t}\right), \quad \varphi\!\left(\frac{s}{t}\right) \sim \text{const at } s \gg |t| \qquad (2.31)$$

while the Born contributions like those shown in fig. 2.24 give: $(d\sigma/dt)_{pp} \lesssim s^{-2} t^{-8}$. Therefore, one would expect that the contributions shown in fig. 2.27 will be dominant in the forward region $s \gg |t| \gg \mu^2$. Indeed, the pp -scattering data from ISR ($\sqrt{s} \simeq 50$ GeV) and FNAL ($P_L \simeq 400$ GeV) at 5 GeV$^2 \lesssim |t| \lesssim$ 14 GeV^2 |2.20| are well described by the formula (2.31). Unfortunately, the present theory is not able to predict the absolute values of the contributions shown in fig. 2.27. (For more details, see |1.29, 2.19|). One can expect |1.29| that the figs 2.24 and 2.25 contributions dominate in the region $s \sim |t| \gg \mu^2$ (due to the much larger number of the diagrams), while those shown in figs. 2.27 and 2.26 are the dominant ones in the region $s \gg |t| \gg \mu^2$ *. The existing experimental data support this assumption.

Let us not also that the experimental data /1.34, 2.20/ at $p_L \simeq 20$ GeV, $\theta \simeq 90^\circ$ show the large spin assymetry. There is a number of possible explanations of this assymetry in the literature / 2.21 /.

2.4 CONCLUSIONS

The four main questions have been formulated above in sect. 1.2, the answers on which this review is devoted to. This chapter answers mainly the question number one - the dependence of the asymptotic behaviour on the hadron quantum numbers. Our understanding of exclusive processes properties is much deeper now as compared with the level of the "dimensional counting rules". At present we understand sufficiently completely the dynamics and main characteristic properties of the exclusive processes, the origin and the applicability region of the "dimensional counting rules". Moreover, the above described method of operator expansions gives the possibility to calculate also the absolute values of exclusive amplitudes, if the information about the properties of hadronic wave functions is available. The absolute values of amplitudes are, as a rule, very sensitive to the form of hadronic wave functions and, as was shown above, the attempts to use the simplest wave functions don't meet with the success. More realistic models of hadronic wave functions will be obtained below in chs. 4, 6, 9 and 10 by

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*) On account of the loop corrections, fig. 2.26 diagrams give the contributions into the scattering amplitude $\sim i s^1 t^{-5}$.

using the QCD sum rules, and used for calculation of various exclusive processes. It will be shown that it is possible to obtain the predictions for a large number of exclusive processes which are, on the whole, in agreement with the experiment.

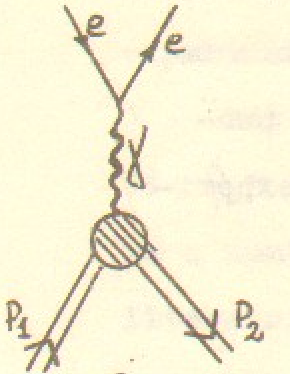


fig. 1.1

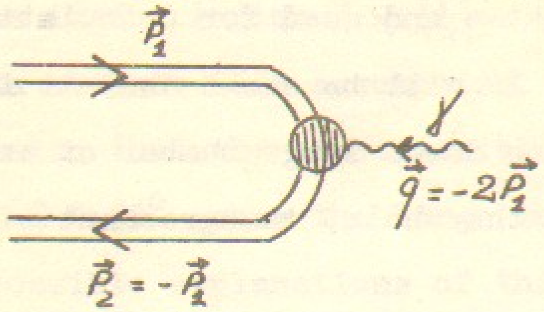


fig 1.2

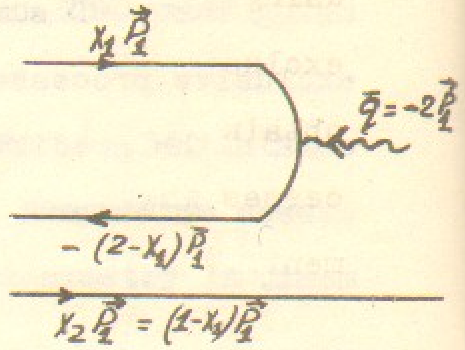


fig. 1.3

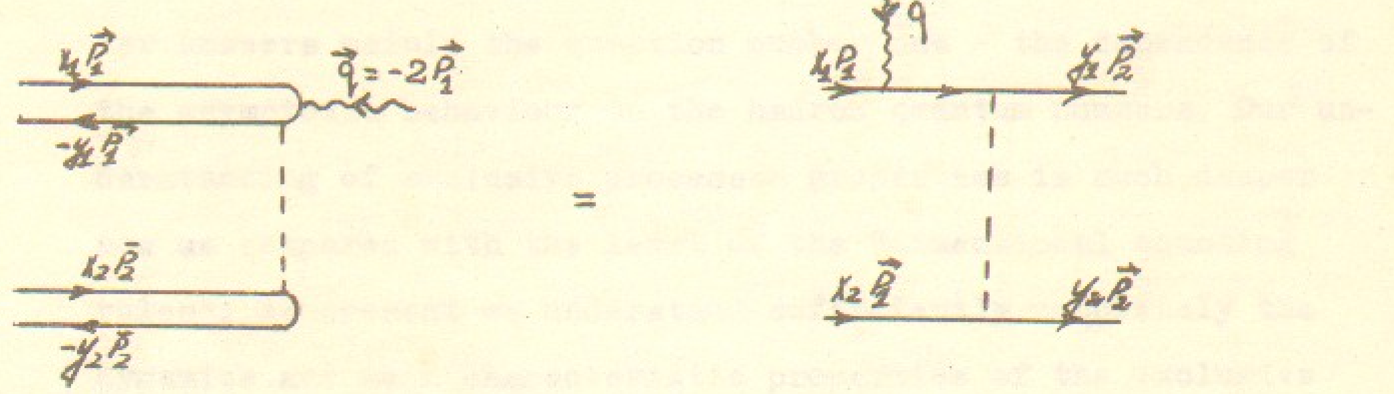


fig. 1.4

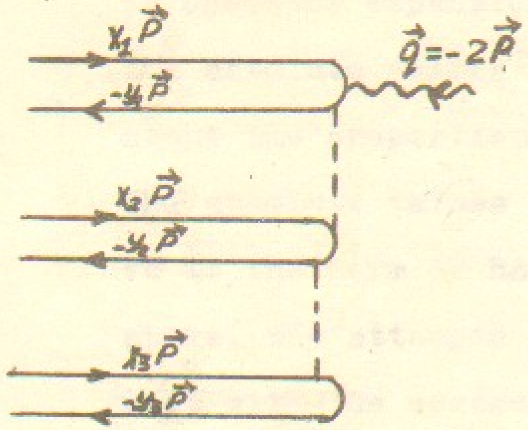
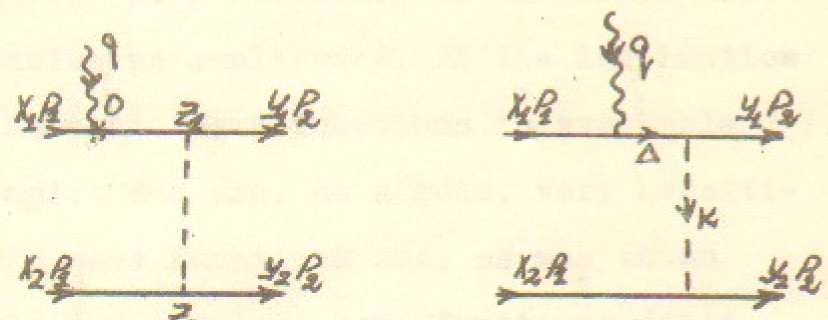


fig 1.5



$$\Delta = P_2 - x_2 P_1 , \quad \Delta^2 \simeq x_2 q^2$$

$$\kappa = y_2 P_2 - x_1 P_1 , \quad \kappa^2 \simeq x_2 y_2 q^2$$

fig. 1.6

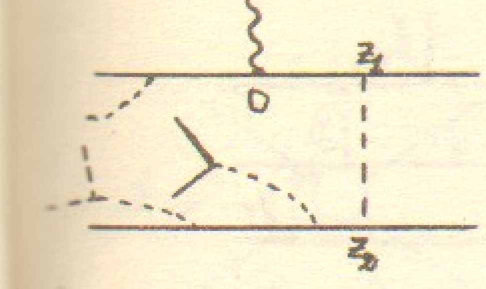


fig. 1.7

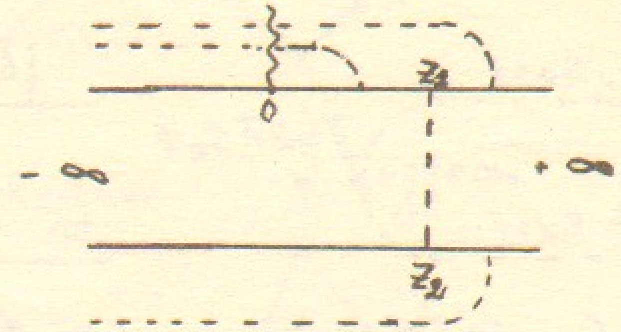


fig. 1.8

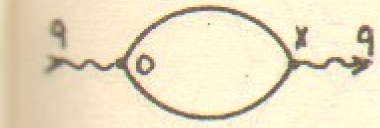


fig1.9

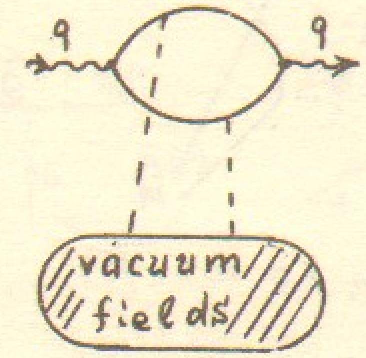


fig 1.10

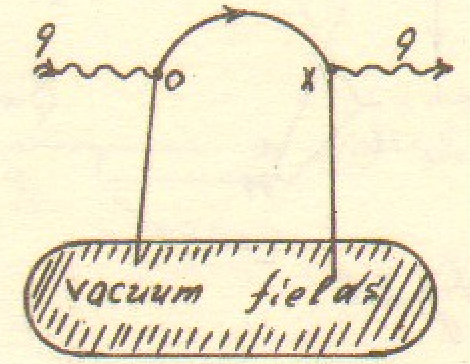


fig. 1.11

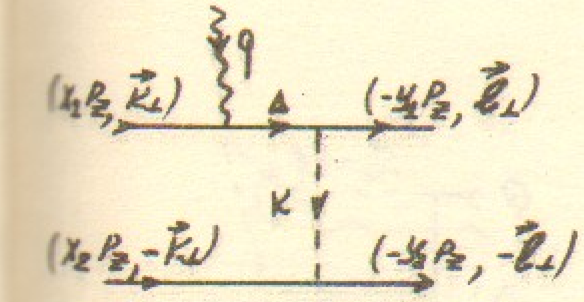


fig. 2.1

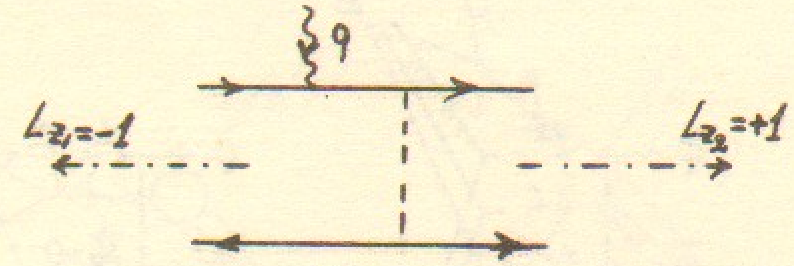


fig. 2.2

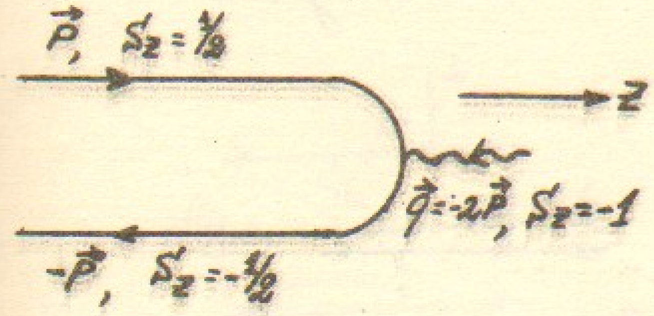


fig. 2.3

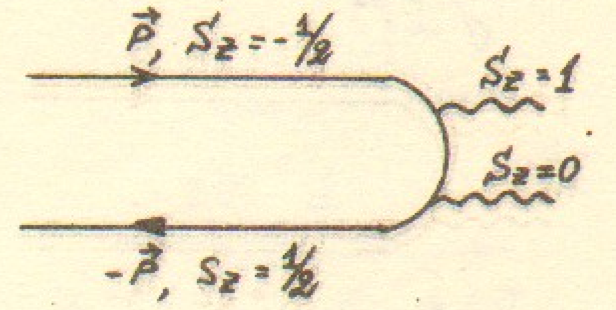


fig. 2.4

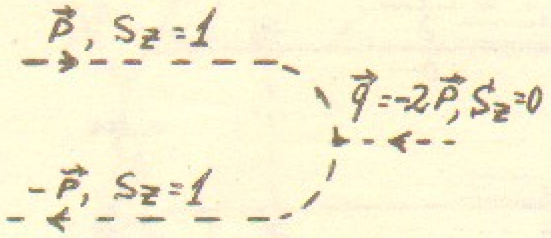


fig. 2.5

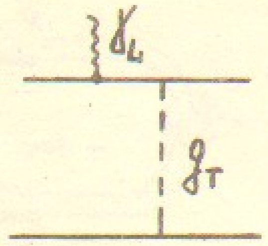


fig. 2.6

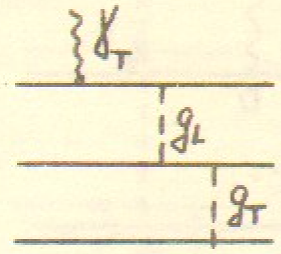


fig. 2.7

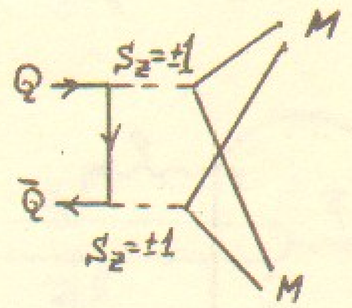


fig. 2.8

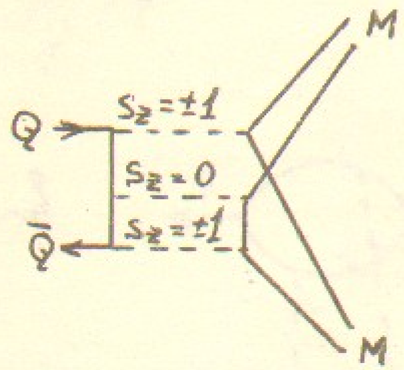


fig. 2.9

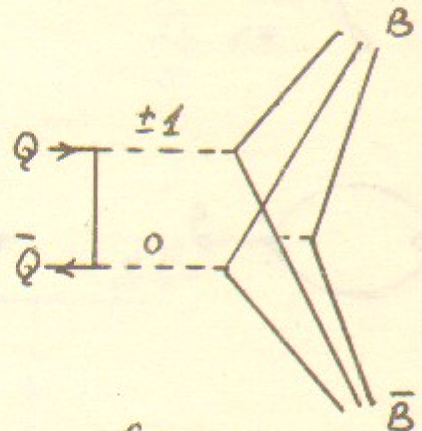


fig. 2.10

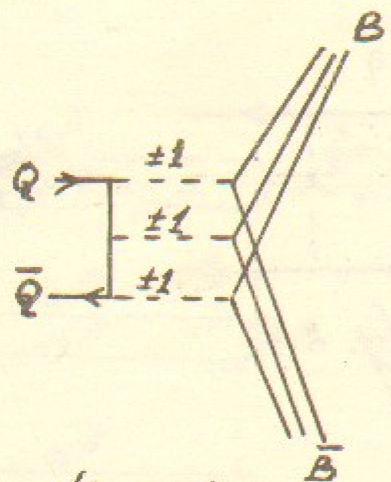


fig. 2.11

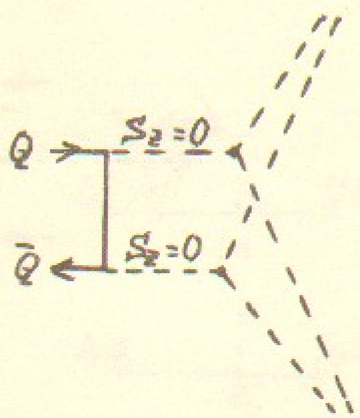


fig. 2.12

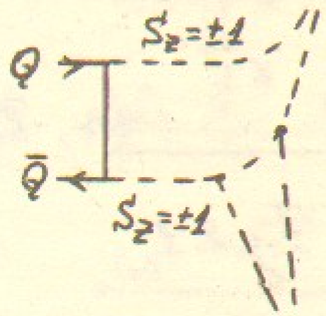


fig. 2.13

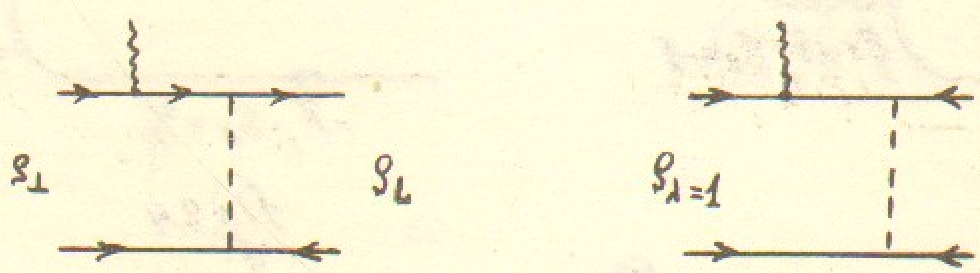


fig. 2.14

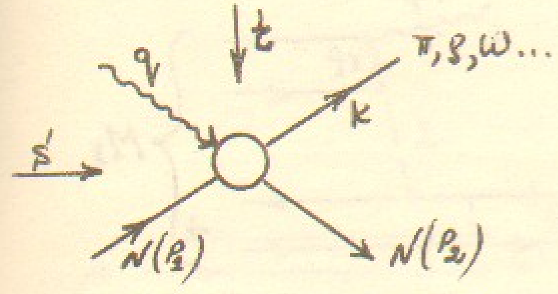


fig. 2.16

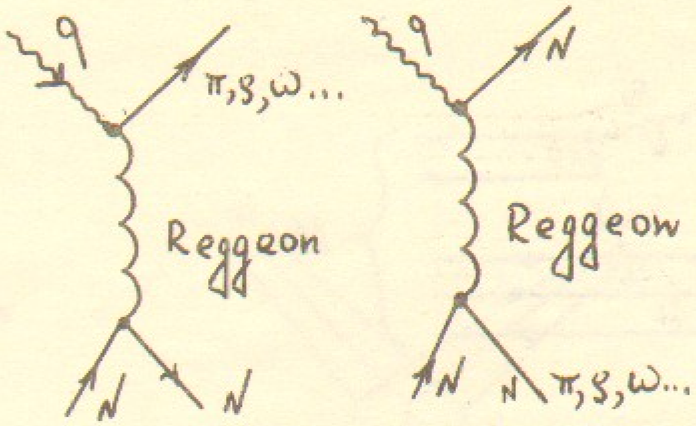


fig. 2.17

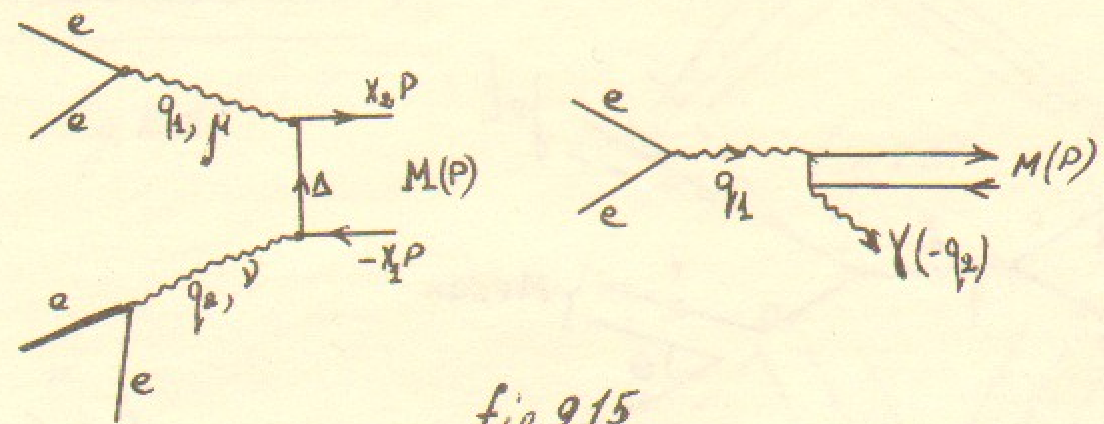
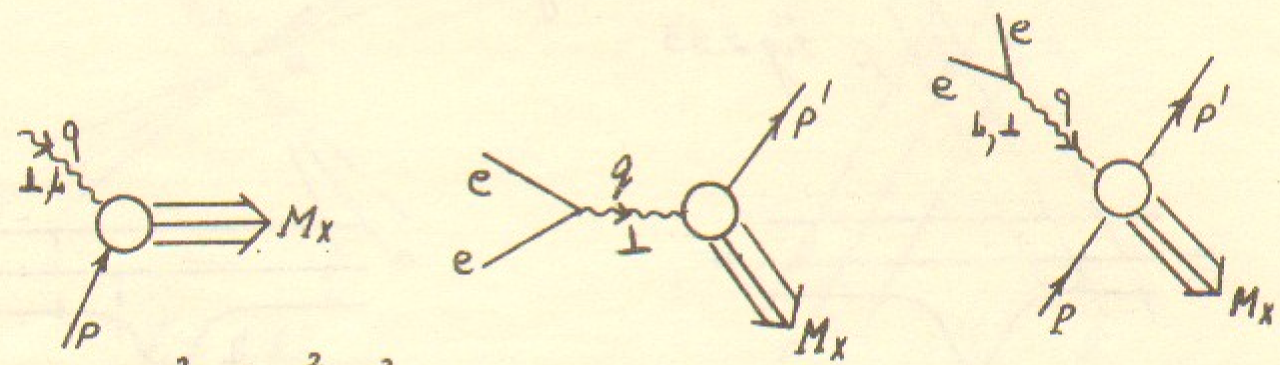


fig. 2.15



$|q^2| \gg M_X^2 = (q+p)^2 \gtrsim \mu^2$

a. b. c.

fig. 2.18

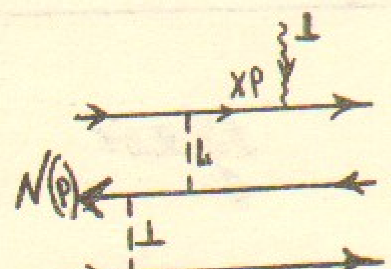


fig. 2.19

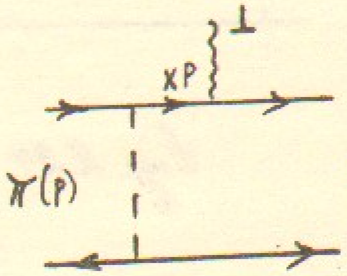


fig. 2.20

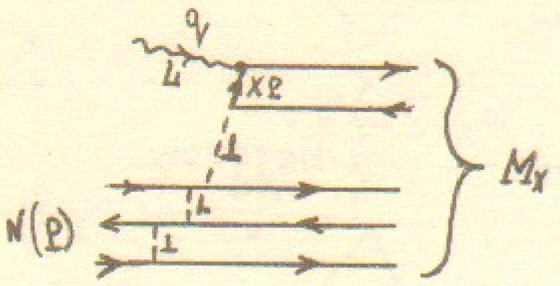


fig. 2.21

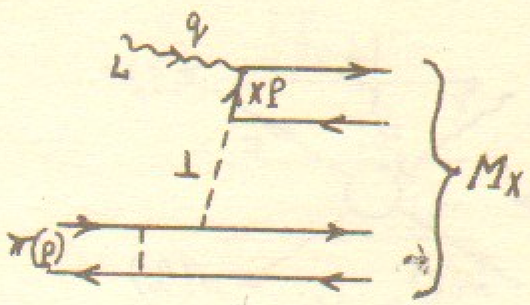


fig. 2.22

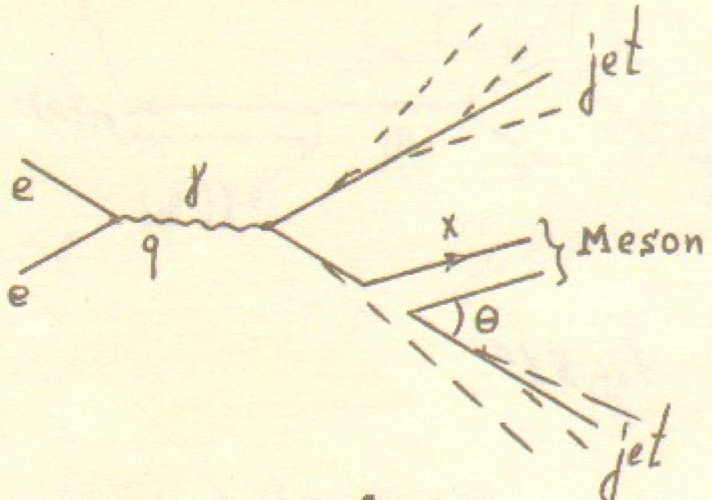


fig 2.23

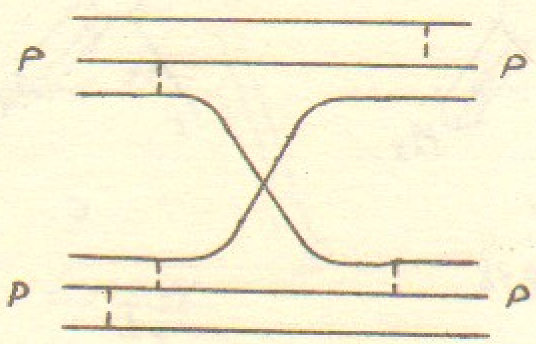


fig. 2.24

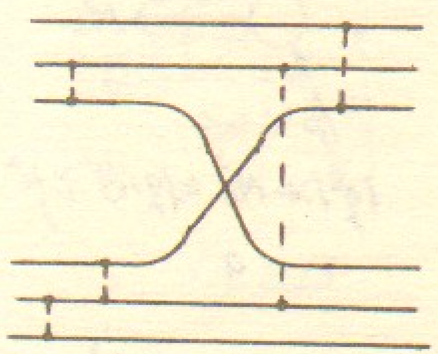


fig. 2.25

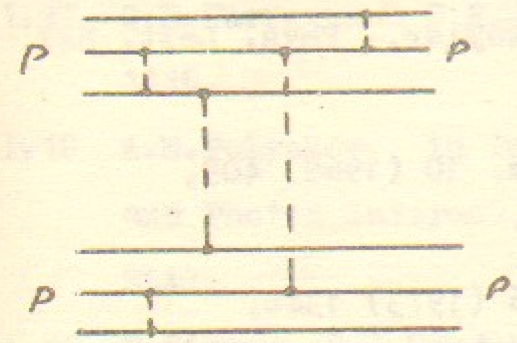


fig. 2.26

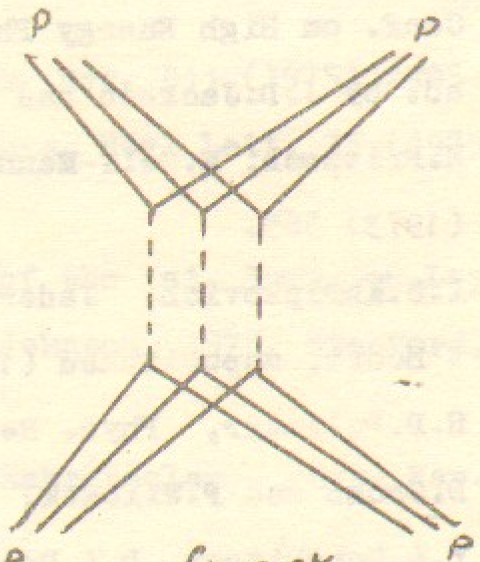


fig. 2.27

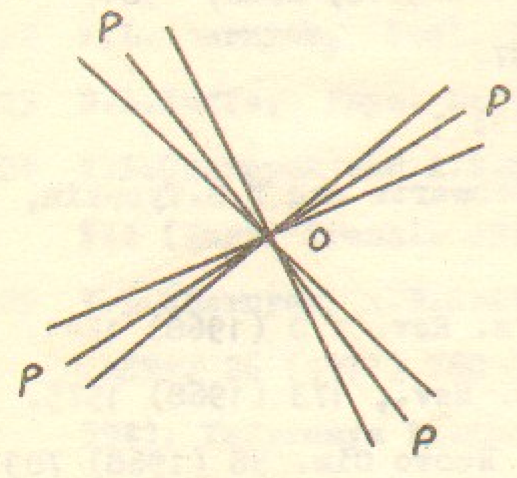


fig. 2.28

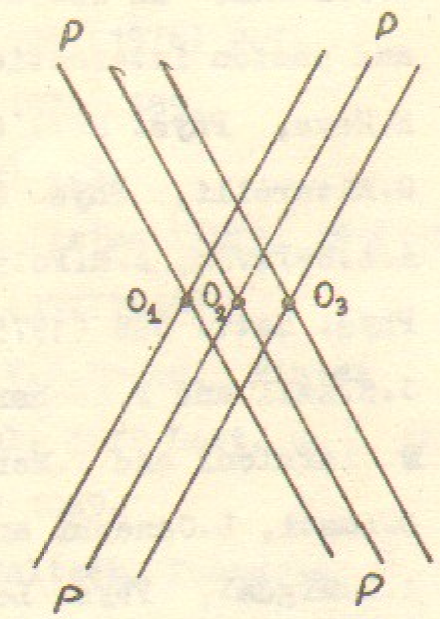


fig. 2.29

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АСИМПТОТИЧЕСКОЕ ПОВЕДЕНИЕ ЭКСКЛЮЗИВНЫХ ПРОЦЕССОВ В КХД

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Препринт
№ 83-103

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