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ASYMPTOTIC BEHAVIOUR OF EXCLUSIVE PROCESSES IN QCD

6. SU(3)-SYMMETRY BREAKING EFFECTS
7. WAVE FUNCTIONS OF THE MESONS WHICT CONTAIN C and B QUARKS

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CONTENT

6. SU(3)-SYMMETRY BREAKING EFFECTS

It is the purpose of this chapter to find the wave functions of the mesons which contain S-quark and to investigate the properties of the SU(3)-symmetry breaking effects in exclusive processes.

The wave functions we consider are the leading twist wave functions of K, $K^*_{\lambda=0}(890) \equiv K^*_L$, $K^*_{|\lambda|=1} \equiv K^*_\perp$ and $\varphi_{L,\perp}$ mesons. They are defined analogously to those of π and $\rho_{\lambda=0}$, $\rho_{|\lambda|=1}$ mesons:

$$\langle 0|\bar{s}(z)\hat{z}\gamma_5 u(-z)|K^+(p)\rangle = i f_K p_\mu \int_{-1}^{1} d\xi\, e^{iz(\xi p)} \varphi_K(\xi), \quad z^2=0,$$

$$\langle 0|\bar{s}(z)\hat{z}\, u(-z)|K_L^{*+}(p)\rangle = f^V_{K^*} p_\mu \int_{-1}^{1} d\xi\, e^{iz(\xi p)} \varphi^V_{K^*}(\xi), \qquad (6.1)$$

$$\langle 0|\bar{s}(z)\sigma_{\mu\nu} u(-z)|K^*_\perp(p)\rangle = f^T_{K^*}(\epsilon^\perp_\mu p_\nu - \epsilon^\perp_\nu p_\mu)\int_{-1}^{1} d\xi\, e^{iz(\xi p)} \varphi^T_{K^*}(\xi),$$

$$\int_{-1}^{1} d\xi\, \varphi_i(\xi) = 1,$$

where $\epsilon^\perp_\mu$ is the $K^*_\perp$ meson polarization vector. Each of these wave functions can be represented in the form:

$$\varphi_i(\xi) \equiv \varphi_i^{(+)}(\xi) + \varphi_i^{(-)}(\xi), \quad \varphi_i^{(\pm)}(\xi) = \pm\varphi_i^{(\pm)}(-\xi).$$

The SU(3)-symmetry breaking leads to a number of effects:

a) $f_K \neq f_\pi$, $f^V_{K^*} \neq f^V_\rho$, $f^T_{K^*} \neq f^T_\rho$, i.e. the values of the wave functions at the origin differ;

b) $\varphi^+_K(\xi) \neq \varphi_\pi(\xi)$, $\varphi^{V,T(+)}_{K^*} \neq \varphi^{V,T}_\rho(\xi)$, i.e. the wave function components which are symmetric under a quark momenta interchange, differ also;

c) $\varphi_i^-(\xi) \neq 0$, i.e. there appear the wave function components which are anti-symmetric under a quark momenta interchange.

We investigate below all these effects and consider a number of applications.

6.1 SYMMETRIC COMPONENTS OF WAVE FUNCTIONS /4.2/

6.1.1 Sum rules and the moment values

Let us start from the investigation of the K meson wave function $\varphi_K^+(\xi)$ and consider with this purpose the correlator:

$$T^n = i\int dx\, e^{iqx} \langle 0|T\{\bar{s}(x)\hat{z}\gamma_5 (iz\overleftrightarrow{D})^n u(x),\ \bar{u}(0)\hat{z}\gamma_5 s(0)\}|0\rangle =$$

$$= (zq)^{n+2}\, I^n(q^2), \quad z^2=0. \tag{6.2}$$

The corresponding sum rules have the form:

$$\frac{1}{\pi M^2}\int_0^{\sigma} \mathrm{Im}\, I^n(s)\, e^{-s/M^2} ds = \frac{1}{4\pi^2}\frac{3}{(n+1)(n+3)} + \frac{1}{12M^4}\langle \frac{\alpha_s}{\pi}G^2\rangle$$

$$+\frac{16}{9}\pi \frac{\langle\sqrt{\alpha_s}\bar{u}u\rangle\langle\sqrt{\alpha_s}\bar{s}s\rangle}{M^6} + \frac{16}{81}\pi(2n+1)\frac{\langle\sqrt{\alpha_s}\bar{u}u\rangle^2 + \langle\sqrt{\alpha_s}\bar{s}s\rangle^2}{M^6} +$$

$$\frac{m_s\langle\bar{s}s\rangle}{M^4} - \frac{n}{6}\frac{m_s}{M^6}\langle\bar{s}\, ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a s\rangle + \ldots \tag{6.3}$$

The spectral density is taken, as usually, in the form:

$$\frac{1}{\pi}\mathrm{Im}\, I^n(s) = f_K^2\langle\xi^n\rangle_K\, \delta(s-m_K^2) + \frac{3}{4\pi^2(n+1)(n+3)}\theta(s-S_K^n). \tag{6.4}$$

As far as we are interested here in the SU(3)-symmetry breaking effects, we substact from (6.3) the corresponding sum rules for the pion. Then one obtains :

$$\frac{f_K^2}{f_\pi^2}e^{-m_K^2/M^2} = 1 + \frac{M^2}{4\pi^2 f_\pi^2}\left\{e^{-S_\pi^0/M^2} - e^{-S_K^0/M^2}\right\} + \frac{m_s\langle\bar{s}s\rangle}{f_\pi^2 M^2}, \tag{6.5}$$

$$\frac{f_K^2}{f_\pi^2}\frac{\langle\xi^2\rangle_K}{\langle\xi^2\rangle_\pi}e^{-m_K^2/M^2} = 1 + \frac{M^2}{20\pi^2 f_\pi^2\langle\xi^2\rangle_\pi}\left\{e^{-S_\pi^2/M^2} - e^{-S_K^2/M^2}\right\} +$$

$$+\frac{m_s\langle\bar{s}s\rangle}{f_\pi^2\langle\xi^2\rangle_\pi M^2} - \frac{1}{3}\cdot\frac{m_s\langle\bar{s}\, ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a s\rangle}{f_\pi^2\langle\xi^2\rangle_\pi\, M^4}, \tag{6.6}$$

$$\frac{f_K^4}{f_\pi^4}\frac{\langle\xi^4\rangle_K}{\langle\xi^4\rangle_\pi}e^{-m_K^2/M^2} = 1 + \frac{3M^2}{140\pi^2 f_\pi^2\langle\xi^4\rangle_\pi}\left\{e^{-S_\pi^4/M^2} - e^{-S_K^4/M^2}\right\} +$$

$$+\frac{m_s\langle\bar{s}s\rangle}{f_\pi^2\langle\xi^4\rangle_\pi M^2} - \frac{2}{3}\frac{m_s}{M^4}\frac{\langle\bar{s}\, ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a s\rangle}{f_\pi^2\langle\xi^4\rangle_\pi}. \tag{6.7}$$

The corresponding correction terms entering the r.h.s. (6.5)-(6.7) are due to figs.6.1-6.3 diagram contributions*. We use at the treatment of (6.5)-(6.7): $m_s \simeq 150\ \mathrm{MeV}$, $\langle\bar{u}u\rangle \simeq -(0.25\ \mathrm{GeV})^3$, $\langle 0|\bar{u}\, ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a u|0\rangle \simeq 1.5\ \mathrm{GeV}^2\langle 0|\bar{u}u|0\rangle$ [1.49]; $\langle\bar{s}s\rangle \simeq (0.75\div 0.8)\langle\bar{u}u\rangle$, $\langle\bar{s}\, ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a s\rangle \simeq (0.75\div 0.8)\langle\bar{u}\, ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a u\rangle$ (see below).

The standard treatment gives then:

$$\Delta_0 = \left(\frac{f_K^2}{f_\pi^2}-1\right) = 0.4\pm 0.08, \quad f_K = (1.20\pm 0.04) f_\pi, \tag{6.8a}$$

$$\Delta_2 = \left(\frac{f_K^2\langle\xi^2\rangle_K}{f_\pi^2\langle\xi^2\rangle_\pi}-1\right) = 0.10\pm 0.02, \quad \frac{\langle\xi^2\rangle_K}{\langle\xi^2\rangle_\pi} \simeq 0.8\pm 0.02, \tag{6.8b}$$

$$\Delta_3 = \left(\frac{f_K^2\langle\xi^4\rangle_K}{f_\pi^2\langle\xi^4\rangle_\pi}-1\right) = 0.03\pm 0.03, \quad \frac{\langle\xi^4\rangle_K}{\langle\xi^4\rangle_\pi} = 0.7\pm 0.03. \tag{6.8c}$$

It is seen from (6.8b),(6.8c) that <u>the K meson wave function $\varphi_K(\xi)$ is narrower than $\varphi_\pi(\xi)$.</u>

The result (6.8a): $f_K = (1.2\pm 0.04) f_\pi$ agrees with the experimental value $f_K = (1.2\div 1.25) f_\pi$, known from the decay $K\to\mu\nu$. It is worth noting that assuming $\langle\bar{s}s\rangle = \langle\bar{u}u\rangle$ one obtains from (6.5): $f_K = 1.09 f_\pi$, i.e. too small value.

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* As will be shown below, $\langle\bar{u}u - \bar{s}s\rangle \simeq 0.2\langle\bar{u}u\rangle$, and so this contribution can be neglected here.

Proceeding in an analogous way, one obtains for the vector meson wave functions (the accuracy is the same as in (6.8)):

$$\left(\frac{f_{K^*}^V}{f_\rho^V}\right)^2 = 1.1\,,\quad \left(\frac{f_\varphi^V}{f_\rho^V}\right)^2 = 1.3,\quad \frac{\langle\xi^2\rangle_{K^*}^V}{\langle\xi^2\rangle_\rho^V} = 0.85\,,\quad \frac{\langle\xi^2\rangle_\varphi^V}{\langle\xi^2\rangle_\rho^V} = 0.7, \qquad (6.9)$$

$$\left(\frac{f_{K^*}^T}{f_\rho^T}\right)^2 = 1.1\,,\quad \left(\frac{f_\varphi^T}{f_\rho^T}\right)^2 = 1.3,\quad \frac{\langle\xi^2\rangle_{K^*}^T}{\langle\xi^2\rangle_\rho^T} = 0.75\quad \frac{\langle\xi^2\rangle_\varphi^T}{\langle\xi^2\rangle_\rho^T} = 0.6. \qquad (6.10)$$

It is seen from (6.8)-(6.10) that there is strict regularity: the larger is the value of the wave function at the origin, the more narrow is the distribution of quarks over the longitudinal momentum in this state, i.e.:

$$f_\varphi > f_{K^*} > f_\rho > f_K > f_\pi\,,$$

while

$$\left(\langle\xi^2\rangle_\varphi^V\right) < \left(\langle\xi^2\rangle_{K^*}^V\right) < \left(\langle\xi^2\rangle_\rho^V\right) < \left(\langle\xi^2\rangle_K\right) < \left(\langle\xi^2\rangle_\pi\right),$$

$$\left(\langle\xi^2\rangle_\varphi^T\right) < \left(\langle\xi^2\rangle_{K^*}^T\right) < \left(\langle\xi^2\rangle_\rho^T\right).$$

Various exclusive amplitudes have the form of the product of f_i and $I(\varphi_i)$, where $I(\varphi_i)$ are the corresponding integrals of the wave functions (for instance, $I(\varphi) = \int_{-1} d\xi\, \varphi(\xi)/(1-\xi)$). Therefore, these two effects work in opposite directions: if $f_i > f_j$, then $I(\varphi_i) < I(\varphi_j)$, and they have a tendency to compensate each other. Which of these two effects will be more significant, depends on the process under consideration (see below).

6.1.2 Model wave functions and applications

The "standard model form" (4.19a) is used for the leading twist wave functions: $\varphi_i(\xi) = (1-\xi^2)[A_i\xi^2 + B_i]$, $\int_{-1} d\xi\, \varphi_i(\xi) = 1$. Using the π and ρ_L meson wave functions (4.10) and (4.19), one has then from (6.8)-(6.10):

$$\varphi_K^+(\xi) = \frac{15}{4}(1-\xi^2)(0.6\xi^2 + 0.08) \qquad (6.11)$$

$$\varphi_{K^*}^{V(\parallel)}(\xi) = \frac{15}{4}(1-\xi^2)(0.1\xi^2 + 0.18)\,,$$

$$\varphi_\varphi^V(\xi) = \frac{15}{4}(1-\xi^2)(-0.05\xi^2 + 0.21)\,. \qquad (6.12)$$

It is seen from (6.12) that $\varphi_{K^*}^V(\xi)$ and $\varphi_\varphi^V(\xi)$ coincide really with the asymptotic wave function $\varphi_{as.}(\xi) = \frac{3}{4}(1-\xi^2)$.

Let us describe now some applications. The formulae (5.4) and (5.11) describe the branching ratios for $\chi_0 \to M_1^{\lambda=0} M_2^{\lambda=0}$ and $\chi_2 \to M_1^{\lambda=0} M_2^{\lambda=0}$ decays, where M_1 and M_2 are two mesons with zero helicities. Using (6.11), one obtains then:*

$$\frac{Br(\chi_0 \to KK)}{Br(\chi_0 \to \pi\pi)} \simeq \frac{Br(\chi_2 \to KK)}{Br(\chi_2 \to \pi\pi)} = 0.70 - 0.75. \qquad (6.13)$$

The experimental data are /2.6/:

$$Br(\chi_0 \to \pi^+\pi^-) = (0.9 \pm 0.2)\%\,;\quad Br(\chi_2 \to \pi^+\pi^-) = (0.20 \pm 0.11)\%,$$

$$Br(\chi_0 \to K^+K^-) = (0.8 \pm 0.2)\%\,;\quad Br(\chi_2 \to K^+K^-) = (0.16 \pm 0.12)\%. \qquad (6.1$$

Although the experimental uncertainties are large, the data (6.14) show that the K^+K^--mode does not exceed the $\pi^+\pi^-$ mode and this agrees with our predictions (6.13). At the same time, the naive estimates a là S.Brodsky and P.Lepage /2.2/:

$$\frac{Br(\chi_{0,2} \to K^+K^-)}{Br(\chi_{0,2} \to \pi^+\pi^-)} \simeq \left(\frac{f_K}{f_\pi}\right)^4 \simeq 2$$

contradict the experiment. The reason is clear: the K meson wave function is narrower than the pion one, and in this case this overcompensates the effect due to $(f_K/f_\pi) > 1$.

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* The anti-symmetric part of the K meson wave function, (see the sect.6.2) gives negligible contributions into these decays.

One has also for the K^+ meson form factor in the region $Q^2 \simeq 10\ GeV^2$ (see (1.8)):

$$\frac{F_K(q^2)}{F_\pi(q^2)} = \frac{f_K^2}{f_\pi^2}\frac{I_K}{I_\pi}\,, \qquad I_\pi = \left|\int_{-1}^{1}\frac{d\xi\,\varphi_\pi(\xi)}{1-\xi^2}\right|^2 = (2.5)^2\,,$$

$$I_K = \frac{2}{3}\left|\int_{-1}^{1}\frac{d\xi}{1-\xi^2}\left(\varphi_K^+(\xi) - \xi\varphi_K^-(\xi)\right)\right|^2 + \frac{1}{3}\left|\int_{-1}^{1}\frac{d\xi}{1-\xi^2}\left(\varphi_K^+(\xi) + \xi\varphi_K^-(\xi)\right)\right|^2. \quad (6.15a)$$

Using the wave functions $\varphi_K^+(\xi)$ (6.11) and $\varphi_K^-(\xi)$ (6.29), one obtains:

$$\frac{F_K(q^2)}{F_\pi(q^2)} \simeq 0.9\,, \qquad \frac{I_K}{I_\pi} \simeq \frac{2}{3}\,. \qquad (6.15b)$$

The contribution of the photon exchange diagram ,fig.6.6, into the decays $\psi \to K^+K^-$, $\psi \to \pi^+\pi^-$ gives then:

$$\left.\frac{Br(\psi\to K^+K^-)}{Br(\psi\to\pi^+\pi^-)}\right|_{photon} = \frac{F_K^2(M_\psi^2)}{F_\pi^2(M_\psi^2)} \simeq 0.8$$

The experimental data are /2.6/:

$$Br(\psi\to\pi^+\pi^-) = (0.011 \pm 0.005)\%\,, \quad Br(\psi\to K^+K^-) = (0.022 \pm 0.008)\%.$$

Of course, the experimental uncertainties are very large, but it seems that the K^+K^- mode exceeds that of $\pi^+\pi^-$ considerably. This may be due to the contribution of the fig.6.5 diagram into the $\psi \to K^+K^-$ decay. This contribution is zero in the exact SU(3)-symmetry limit and is, therefore, proportional to the product of $\varphi_K^+(\xi)$ and $\varphi_K^-(\xi)$ wave functions. Although it is the strong interaction contribution, it includes additional small factors due to the loop ($\sim \alpha_s/\pi \simeq 10^{-1}$) and due to the SU(3)-symmetry breaking ($\sim 0.1 \div 0.2$) which suppress this contribution. The rough estimate shows that as a result, this contribution is of the same order as the electromagnetic

contribution, fig.6.6. Therefore, just this additional strong interaction contribution into the $\psi \to K^+K^-$ decay may be responsible for the possible large difference between $\psi\to\pi^+\pi^-$ and $\psi \to K^+K^-$ (This contribution has been calculated in terms of the K meson wave function $\varphi_K(\xi)$ in the paper /6.6/, but it is difficult to use this result).

6.2 ANTI-SYMMETRIC COMPONENTS OF WAVE FUNCTIONS /6.1/

6.2.1 Qualitative discussion and estimates. The values of the quark condensates $\langle\bar u u\rangle, \langle\bar d d\rangle, \langle\bar s s\rangle$.

The values of the φ_K^- moments are determined, evidently, by the matrix elements of operators with an odd number of derivatives, see (6.1):

$$\langle 0|\bar u(0)\hat z\gamma_5(i z\overset{\leftrightarrow}{D})^{2n+1}s(0)|K(p)\rangle = i f_K (zp)^{2n+2}\langle \xi^{2n+1}\rangle_K\,, \quad \xi = x_s - x_u \qquad (6.16)$$

(and analogously for the wave functions $\varphi_\cdot^-(\xi)$ in (6.1)).

Let us obtain firstly some estimates for the characteristic quantities under consideration. Consider with this purpose the following correlators:

$$T_P = i\int dx\, e^{iqx}\langle 0|T\{\bar u(x)\hat z\gamma_5(i z\overset{\leftrightarrow}{D})^{2n+1}s(x),\ \bar s(0)\gamma_5 u(0)\}|0\rangle =$$

$$= (zq)^{2n+2}\, I_P^{2n+1}(q^2)\,, \quad z^2 = 0\,, \quad n = 0,1,2\ldots \qquad (6.17)$$

$$T_A = i\int dx\, e^{iqx}\langle 0|T\{\bar u(x)\hat z\gamma_5(i z\overset{\leftrightarrow}{D})^{2n+1}s(x),\ \bar s(0)\hat z\gamma_5 u(0)\}|0\rangle =$$

$$= (zq)^{2n+3}\, I_A^{2n+1}\,. \qquad (6.18)$$

The K meson contributions into the corresponding spectral densities are:

$$\frac{1}{\pi}\mathrm{Im}\, I_P^{2n+1}(s) = \left[i f_K\langle\xi^{2n+1}\rangle_K\right]\left[-i f_K\frac{m_K^2}{m_s+m_u}\right]\delta(s-m_K^2) + \ldots$$

$$\frac{1}{\pi}\, \mathrm{Im}\, I_A^{2n+1}(s) = \left[-if_K\right]\left[+if_K \langle \xi^{2n+1} \rangle_K\right] \delta(s - m_K^2) + \dots \qquad (6.19)$$

Take now the scale $M_0^2 \simeq M_{K^*}^2 \simeq 0.8\,\mathrm{GeV}^2$ in the corresponding sum rules. The experience shows that it is sufficient, as a rule, to retain at this scale the K meson contribution only at l.h.s. and the first leading contributions at r.h.s. of sum rules, neglected terms are then ≃ 20-30% of the retained ones. We expect, therefore, that the estimates below have the accuracy $\lesssim$ 20-30%.

One has from (6.17)-(6.19), n=0, in this approximation (the terms at r.h.s. (6.20),(6.21) are the contributions of the fig. 6.1 and fig.6.2 diagrams, the later being dominant really):

$$e^{-m_K^2/M_0^2} f_K^2 \frac{m_K^2}{m_s + m_u} \langle X_s - X_u \rangle_K \simeq \frac{m_s - m_u}{8\pi^2} M_0^2 + \frac{\langle \bar{s}s - \bar{u}u \rangle}{1} + \dots, \qquad (6.20)$$

$$e^{-m_K^2/M_0^2} f_K^2 \langle X_s - X_u \rangle_K \simeq \frac{m_s^2 - m_u^2}{4\pi^2} + \frac{m_u \langle \bar{u}u \rangle - m_s \langle \bar{s}s \rangle}{M_0^2} + \dots \qquad (6.21)$$

As a result ($m_s = 150\,\mathrm{MeV}$, $m_u = 4\,\mathrm{MeV}$, $\langle \bar{u}u \rangle = -(250\,\mathrm{MeV})^3$):

$$\frac{\langle \bar{u}u \rangle}{\langle \bar{s}s \rangle} \simeq \left[1 + \frac{m_K^2}{M_{K^*}^2}\left(\frac{m_s - m_u}{m_s + m_u}\right)\right] \simeq 1.3, \quad \frac{\langle \bar{u}u - \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq 0.23, \qquad (6.22)$$

$$\langle X_s - X_u \rangle_K \simeq -\frac{m_s \langle \bar{s}s \rangle}{f_K^2 M_{K^*}^2}\, e^{m_K^2/M_{K^*}^2} \simeq 0.12. \qquad (6.23)$$

The analogous relations for the pion have the form* ($m_d = 7\,\mathrm{MeV}$):

$$f_\pi^2 \frac{m_\pi^2}{m_u + m_d} \langle X_d - X_u \rangle_\pi \simeq \frac{m_d - m_u}{8\pi^2} M_\rho^2 + \langle \bar{d}d - \bar{u}u \rangle, \qquad (6.24a)$$

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* Besides, there is the electromagnetic contribution into $\langle X_d - X_u \rangle_\pi$.

$$f_\pi^2 \langle X_d - X_u \rangle_\pi \simeq \frac{m_d^2 - m_u^2}{4\pi^2} + \frac{m_u \langle \bar{u}u \rangle - m_d \langle \bar{d}d \rangle}{M_\rho^2}, \qquad (6.24b)$$

$$\frac{\langle \bar{u}u \rangle}{\langle \bar{d}d \rangle} = \left[1 + \frac{m_\pi^2}{M_\rho^2}\left(\frac{m_d - m_u}{m_d + m_u}\right)\right] \simeq 1.009, \qquad (6.24c)$$

$$\langle X_d - X_u \rangle_\pi \simeq \frac{\langle \bar{u}u - \bar{d}d \rangle}{\langle \bar{u}u + \bar{d}d \rangle} \simeq 0.4 \cdot 10^{-2}. \qquad (6.24d)$$

The result (6.22) agrees with those obtained in /6.2-6.4,4.3/ by different methods, and the result (6.24c) agrees with that obtained in /6.5/.

It follows from (6.23) and (6.24) that the S-quark in K-meson and d-quark in the π meson carry larger longitudinal momentum fractions as compared with the U-quark. This result were trivial if the S-quark mass be much larger than the inverse confinement radius, but it is highly non-trivial for the light S- and d-quarks.

The experience with sum rules shows that the properties of the second resonance in the spectral density are, as a rule, opposite to those of the lowest one. For instance, while the pion wave function is wider than the asymptotic wave function, $\varphi_{as}(\xi)$, the A_1-meson wave function is narrower than $\varphi_{as}(\xi)$, etc. Such behaviour seems natural, taking into account the duality relations. While the properties of the true spectral density are, on the average, the same as in the pert.th. (i.e. $\bar{\varphi} \simeq \varphi_{as}$), some redistribution of the properties takes place really, so that the contributions of the separate resonances lie above or below the average. Therefore, while $\langle X_s - X_u \rangle > \frac{m_s^2 - m_u^2}{8\pi^2} \cdot \frac{M_{K^*}^2}{m_K^2 f_K^2}$ for the K meson and $\langle X_d - X_u \rangle > \frac{m_d^2 - m_u^2}{8\pi^2} \cdot \frac{m_\rho^2}{m_\pi^2 f_\pi^2}$ for the pion, these differences can have opposite signs for the corresponding next resonances.

It is seen from (6.22) and (6.24c) that <u>the heavier is the quark, the smaller is the absolute value of its vacuum condensate</u>. This is also highly non-trivial for the light S- and d-quarks. (As for the connection with the chiral pert.th., see the Appendix D).

Let us discuss now in short the situation with the higher moments $\langle \xi^3 \rangle$, $\langle \xi^5 \rangle$ which characterize the width of the wave function $\varphi^-(\xi)$. As it has been pointed above, the main contribution into the sum rules gives the fig.6.2 diagram. Since the whole momentum is carried by one quark and the other one is a "wee" in this diagram, this contribution corresponds to the wave function of the type: $[\delta(1-\xi) - \delta(1+\xi)]$. Just for this reason this contribution does not decreas when n increases.

It is evident beforehand that the true wave function is narrower, so that the true values of its moments fall off with n. The decrease of the moment values is ensured by next non-perturbative corrections in the sum rules. Indeed, the next power correction in the correlator (6.18) is described by the fig.6.3 diagram and is proportional to: $-n\, m_s \langle 0|\bar{s} ig\sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a s|0\rangle / M^2$. This contribution has the sign opposite to the fig.6.2 diagram contribution and its absolute value grows with n. Therefore, it ensures the decrease of the moment values $\langle \xi^{2n+1} \rangle$ with an increase of n.

Analogously, the next power correction in the correlator (6.17) should also ensure the decrease of the moment values with n. In this case the next power correction is proportional to:
$-n : [\langle 0|\bar{s}\sigma_{\mu\nu} ig G_{\mu\nu} s|0\rangle - \langle 0|\bar{u} ig \sigma_{\mu\nu} G_{\mu\nu} u|0\rangle]$ Therefore, this contribution should have the sign opposite to $[\langle \bar{s}s\rangle - \langle \bar{u}u\rangle]$, i.e.

$$[\langle 0|\bar{s}\sigma_{\mu\nu} ig G^a_{\mu\nu} \lambda^a s|0\rangle - \langle 0|\bar{u} ig \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a u|0\rangle] > 0. \quad (6.25)$$

(The condition (6.25) is required also for the self-consistency of the results which can be obtained from the correlators (6.17), (6.18). Because $[\langle \bar{u}\sigma_{\mu\nu} ig G_{\mu\nu} u\rangle] < 0$, <u>the absolute value of the vacuum condensate</u> $\langle 0|\bar{\psi}\sigma_{\mu\nu} ig G_{\mu\nu}\psi|0\rangle$ <u>also decreases as the quark mass encreases</u> (at least, in the region $m_q \lesssim m_s$).

Moreover, it seems natural to suppose that the presence of the additional gluon field $G_{\mu\nu}$ does not influence the dependence of the matrix element on the quark mass for the light S, d-quarks*. In this case:

$$\frac{\langle 0|\bar{s} ig \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a s|0\rangle}{\langle 0|\bar{u} ig \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a u|0\rangle} \simeq \frac{\langle \bar{s}s\rangle}{\langle \bar{u}u\rangle} \simeq 0.75 \div 0.8 . \quad (6.26)$$

6.2.2 Quantitative analyses of sum rules

Let us return to the correlator (6.17). The complete form of the corresponding sum rule is:

$$\frac{f_K^2 m_K^2}{m_s + m_u} \langle (x_s - x_u)^n \rangle_K \, e^{-m_K^2/M^2} \doteq \frac{3}{8\pi^2} \frac{m_s - m_u}{n+2} M^2 \left[1 - e^{-S_n/M^2}\right] +$$
$$+ \langle \bar{s}s - \bar{u}u\rangle - \frac{1}{3} n \frac{1}{M^2} \left[\langle \bar{s} ig \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a s\rangle - \langle \bar{u} ig \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a u\rangle\right]. \quad (6.27)$$

Using (6.22), (6.26) for the matrix elements in (6.27), one has after the standard fitting procedure:

$$\langle \xi \rangle_K \equiv \langle x_s - x_u \rangle = 0.10 \pm 0.02 ,$$
$$\frac{\langle \xi^3 \rangle_K}{\langle \xi \rangle_K} = 0.55 \div 0.60 . \quad (6.28a)$$

---

*If this matrix element is dominated by the instanton contribution, this is the case.

The results for K^* meson obtained from the analogous sum rules look as follows:

$$\langle x_s - x_u \rangle^V_{K^*} \simeq \langle x_s - x_u \rangle^T_{K^*} = 0.15 \div 0.20 ,$$

$$\frac{\langle (x_s - x_u)^3 \rangle^V_{K^*}}{\langle x_s - x_u \rangle^V_{K^*}} \simeq \frac{\langle (x_s - x_u)^3 \rangle^T_{K^*}}{\langle x_s - x_u \rangle^T_{K^*}} \simeq 0.50 \div 0.55 . \qquad (6.28b)$$

The asymptotic form of the leading twist wave function $\varphi_i^-(\xi, \mu \to \infty)$ is: $\varphi_{as}^-(\xi) = \frac{15}{4}\left[\langle x_s - x_u \rangle_i\right] \xi (1-\xi^2)$, and for this wave function: $\left[\langle \xi^3 \rangle / \langle \xi \rangle\right] = \frac{3}{7} \simeq 0.43$. Therefore, the wave functions $\varphi_i^-(\xi, \mu \sim 1 GeV)$ are somewhat wider than $\varphi_{as}^-(\xi)$. Chosing, as usually, the model wave function in the form: $\varphi_i^-(\xi) = \xi(1-\xi^2)[A\xi^2 + B]$ and using the results (6.28), one obtains:

$$\varphi_i^-(\xi) = \frac{35}{4} \xi^3 (1-\xi^2) \langle x_s - x_u \rangle_i \quad , \quad i = K, K^*_V, K^*_T . \qquad (6.29)$$

For this wave function: $\left[\langle \xi^3 \rangle / \langle \xi \rangle\right] = \frac{5}{9} \simeq 0.56$ (compare with (6.28a), (6.28b)).

The K-meson wave functions $\varphi_K^+(\xi)$ (6.11), $\varphi_K^-(\xi)$ (6.29) and the total wave function $\varphi_K(\xi) = \varphi_K^+(\xi) + \varphi_K^-(\xi)$ are shown at fig.6.4. The ratio $\left|\varphi_K^-(\xi)/\varphi_K^+(\xi)\right| \simeq 0.2 \div 0.3$ at the characteristic values of ξ : $0.6 \lesssim |\xi| \lesssim 0.8$, and this seems reasonable.

6.2.3 Applications: $F_{K^0}(q^2)$, $F^*_{K^0}(q^2)$, $\eta_c \to K^* K$.

The asymptotic behaviour of the K^0 or $K^{*0}_{|\mu|=0|}$ form factor has the form:

$$\langle K^0(P_2) | J_\mu(0) | K^0(P_1) \rangle = (P_1 + P_2)_\mu F_{K^0}(q^2) \, , \; q = P_2 - P_1 \quad F_{\bar{K}^0} = -F_{K^0} ,$$

$$F_{K^0}(q^2) \to \frac{32\pi\bar{\alpha}_s}{9q^2} \cdot \frac{4}{3} |f_K|^2 I_K^+ I_K^- , \qquad (6.30)$$

$$I_K^+ = \int_{-1}^{1} \frac{d\xi}{1-\xi^2} \varphi_K^+(\xi) \, , \quad I_K^- = \int_{-1}^{1} \frac{d\xi}{1-\xi^2} \cdot \xi \varphi_K^-(\xi) .$$

Using the wave functions (6.11), (6.12) and (6.29), one obtains in the region $|q^2| \simeq 10 \div 15$ GeV^2 ($\alpha_s = 0.35$):

$$F_{K^0}(q^2) \simeq 0.1 \, GeV^2 / q^2 \; ; \quad F^*_{K^0}(q^2) \simeq (0.2 \div 0.28) GeV^2 / q^2 . \qquad (6.31)$$

Because $F_{\pi^+}(q^2) \simeq F_{K^+}(q^2) \simeq (0.5 \div 0.6) \frac{GeV^2}{(-q^2)}$ in this region, the K^0 form factor is ~ 5 times smaller and has the opposite sign as compared with $F_{\pi^+}(q^2)$. The cross section of the process $e^+e^- \to \bar{K}^0 K^0$ is (out of resonances):

$$\sigma(e^+e^- \to K^0 \bar{K}^0) = \frac{1}{4} |F_{K^0}(q^2)|^2 \sigma(e^+e^- \to \mu^+\mu^-) \simeq 2 \cdot 10^{-37} cm^2 ,$$
$$(q^2 \simeq 10 \, GeV^2).$$

Let us note also that while the strong interaction contribution, fig.6.5, and the electromagnetic contribution, fig.6.6, are, it seems, close in magnitude for the $\psi \to K^+ K^-$ decay, the fig.6.5 contribution will then be the leading one in the $\psi \to K^0 \bar{K}^0$ decay, because $\left[(\psi \to K^0\bar{K}^0)/(\psi \to K^+K^-)\right]_{fig\, 6.6} = \left[\frac{F_{K^0}(q^2 = M_\psi^2)}{F_{K^+}(q^2 = M_\psi^2)}\right]^2 \simeq \left(\frac{1}{5}\right)^2 \simeq 4 \cdot 10^{-2}$.

The second example is the charmonium ground state decay /2.1, 2.4/: $\eta_c(2980) \to K^* K$:

$$Br(\eta_c \to K^* K) = (4\pi\bar{\alpha}_s)^2 \frac{4}{9} \left(\frac{f_K f^V_{K^*}}{M^2_{\eta_c}}\right)^2 \cdot I^2 ,$$

$$I = \int_{-1}^{1} \frac{d\xi_1}{1-\xi_1^2} \varphi_K(\xi_1) \int_{-1}^{1} \frac{d\xi_2}{1-\xi_2^2} \varphi^V_{K^*}(\xi_2) \frac{(\xi_1 - \xi_2)}{1 - \xi_1 \xi_2} , \qquad (6.32)$$

Numerically /6.1/:

$$Br(\eta_c \to K^* K) \simeq 2 \cdot 10^{-2} \% \, , \quad I \simeq 1.25 .$$

6.3 CONCLUSIONS

1. Sumiltaneous self-consistent treatment of a set of sum rules allows one to investigate the properties of the SU(3)-symmetry breaking effects in the meson wave functions and in the

quark vacuum condensates. We have found a number of regularities:

a) the larger is the wave function at the origin, the narrower is the distribution of quarks in the longitudinal momentum;

b) the heavy is the quark, the smaller are the values of its vacuum condensates $\langle 0|\bar{\psi}\psi|0\rangle$ and $\langle 0|\bar{\psi}ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a\psi|0\rangle$.

c) the heavy is the quark, the larger part of the meson longitudinal momentum it carries (for the lightest mesons in the given channels).

The most interesting, all these points are true even for the light d- and S-quarks.

2. The typical magnitudes of the SU(3)-symmetry breaking effects are $\simeq$ 20–30%:

$$\frac{\langle \bar{u}u-\bar{s}s\rangle}{\langle\bar{u}u\rangle}\simeq\frac{\langle \bar{u}ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a u-\bar{s}ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a s\rangle}{\langle\bar{u}ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a u\rangle}\simeq 0.20\div 0.25,$$

$$\left[\frac{\langle x_s-x_u\rangle}{\langle x_s\rangle}\right]_K\simeq 0.2\,,\quad \left[\frac{\langle x_s-x_u\rangle}{\langle x_s\rangle}\right]_{K^*}\simeq 0.3\,,$$

while these of the SU(2)-isotopic symmetry are $\simeq$ 1%:

$$\frac{\langle \bar{u}u-\bar{d}d\rangle}{\langle\bar{u}u\rangle}\simeq 0.9\cdot 10^{-2}\,,\quad \left[\frac{\langle x_d-x_u\rangle}{\langle x_d\rangle}\right]_{\pi}\simeq 0.8\cdot 10^{-2}.$$

The characteristic branching ratio for the charmonium two-particle decay is $\simeq$0.1–1%. Therefore, the characteristic branching ratio for those decays which are non-zero only due to SU(3)-symmetry breaking effects is: $(0.2\div 0.3)^2(0.1\div 1)\%\simeq (4\div 9)(0.1\div 1)\cdot 10^{-2}\,\%$. For instance,

$$Br\,(\eta_c(2980)\to K^*K)\simeq 2\cdot 10^{-2}\%.$$

7. WAVE FUNCTIONS OF THE MESONS WHICH CONTAIN c, b QUARKS

We consider in this chapter the wave functions of the mesons with an open flavour, i.e. $Q\bar{q}$, where Q is the heavy quark c or b and q is the light quark u,d,s. These wave functions are of interest for the following reason. The amplitudes include usually the integrals of the form: $\int_0^1 dx\,\varphi(x)/1-x$, where $\varphi(x)$ is the meson wave function. The largest part of the $Q\bar{q}$-meson momentum is carried, of course, by the heavy quark Q. Therefore, the wave function $\varphi(x)$ has the strong extremum at $(1-x)\ll 1$, and this enhances amplitudes. It is the goal of this chapter to investigate the properties of the $Q\bar{q}$-meson wave functions in more details and to elucidate the characteristic properties of the processes which contain such mesons /6.1/.

7.1. GENERAL DISCUSSION AND ESTIMATES.

Let us denote the longitudinal momentum fractions carried by the light and by the heavy quarks (at $P_z\to\infty$) as $\langle x_q\rangle$ and $\langle x_Q\rangle$ correspondingly. We have for the nonrelativistic bound state: $\langle x_q\rangle/\langle x_Q\rangle\simeq m_q/M_Q\ll 1,\ \langle x_q\rangle\ll 1, \langle x_Q\rangle\simeq 1$, i.e. the mean momentum fractions are determined by the mass values mainly, while the interaction effects can be neglected. The corresponding estimate for the bound state of one relativistic quark and one heavy non-relativistic quark has the form:

$\langle x_q\rangle/\langle x_Q\rangle\simeq M_0/M_Q\ll 1$, where $M_0\simeq k_\perp\simeq$(350–400) MeV is the characteristic QCD scale. This gives for the B-meson ($M_b\simeq$ 4.75GeV): $\langle x_q\rangle\simeq 0.07$–$0.08$, $\langle x_b\rangle\simeq 0.92$–$0.93$ and for the D-meson ($M_c\simeq 1.5$ GeV): $\langle x_q\rangle\simeq 0.20$, $\langle x_c\rangle\simeq 0.80$.

Let us compare this with the K-meson wave function (see (6.11) and (6.29)):

$$\varphi_K^A(\xi)=\varphi_K^{A(+)}+\varphi_K^{A(-)}\simeq\frac{15}{4}(1-\xi^2)\left[0.6\xi^2+0.25\xi^3+0.08\right],\ \xi=x_s-x_u.$$

As an illustration, the expected characteristic form of D- and B-meson wave functions is presented at fig.7.1, and the K-meson wave function is also shown here for comparison. The D-meson wave function has one strong extremum at $x_c \simeq 0.8$, $x_u \simeq 0.2$, while the K-meson wave function has two extrema at $x_s \simeq 0.8$, $x_u \simeq 0.2$ and $x_s \simeq 0.2$, $x_u \simeq 0.8$, each about two times smaller than for D. We expect, therefore, that <u>the processes with D-mesons are not, in general, enhanced as compared to that of K- (or π)-mesons.</u> For instance, we expect (at $f_D \simeq f_K \simeq 165$ MeV, see below):

$$\frac{\Upsilon({}^3P_{0,2}) \to D^+D^-}{\Upsilon({}^3P_{0,2}) \to K^+K^-} \simeq O(1). \tag{7.1}$$

Therefore, our viewpoint here is opposite to those expressed in the paper /6.6/.

At the same time, the ratio D/K can be large if the K-meson amplitude is suppressed for some reason. This is just the case for the ratio:

$$\frac{\Upsilon({}^3S_1) \to D^+D^-}{\Upsilon({}^3S_1) \to K^+K^-} \simeq \left(\frac{1}{0.2}\right)^2 \gg 1. \tag{7.2}$$

This ratio is large not because D^+D^--decay is enhanced, but because K^+K^--decay is suppressed. The reason is as follows. The diagram at fig.6.5 gives the main contribution into the decays $\Upsilon({}^3S_1) \to \bar{D}D$. These decays are zero in the SU(4)-symmetry limit, but the SU(4)-symmetry is badly broken ($\simeq 100\%$, see the wave function $\varphi_D(x)$ at fig.7.1, this wave function is symmetric under $x \leftrightarrow (1-x)$ in the exact symmetry limit). Hence, there is no real suppression in this case. At the same time, the contribution of this diagram into the $\Upsilon({}^3S_1) \to \bar{K}K$ decays is indeed suppressed (by the factor $\simeq 1/5$), because it is zero in the SU(3)-

symmetry limit, and the SU(3)-symmetry breaking effects are small ($\simeq 20\%$, see the wave function $\varphi_K(x)$ at fig.7.1).

7.2. <u>SUM RULES AND PROPERTIES OF THE D(1870), F⁺(2010) AND B(5200)-MESON WAVE FUNCTIONS</u>

The $Q\bar{q}$-meson wave function of the leading twist is defined as usually:

$$\langle 0|\bar{q}(z)\gamma_\mu\gamma_5 \exp\left\{ig\int_{-z}^{z} d\sigma B(\sigma)\right\} Q(-z)|M(p)\rangle = i f_Q P_\mu \int_{-1}^{1} d\xi\, e^{i\xi(zp)} \varphi_M^A(\xi),$$
$$\int_{-1}^{1} d\xi\, \varphi_M^A(\xi) = 1, \quad \xi = x_Q - x_q. \tag{7.3}$$

In order to find the constants f_D, f_F and f_B and the values of the moments $\langle x_q^n \rangle$, let us consider the correlator:

$$T_{\mu\nu}^n = i\int dx\, e^{iqx} \langle 0|T\,\bar{q}(x)\gamma_\mu\gamma_5 Q(x)\,\bar{Q}(0)\gamma_\nu\gamma_5 (iz\vec{D})^n q(0)|0\rangle =$$
$$(zq)^n \left[q_\mu q_\nu T_L^n(q^2) + (q_\mu q_\nu - g_{\mu\nu}q^2) T_\perp^n(q^2)\right], \quad z^2 = 0. \tag{7.4}$$

The pseudoscalar mesons we are interested in, contribute into the spectral density $\mathrm{Im}\, T_L^n$. We use below the technique proposed by E.V.Shurayk in /1.52/, i.e. the energy E is used instead of q^2: $q^2 = (M_q + E)^2$, $E \ll M_Q$. But in contrast with /1.52/, we put $(q^2)^n \simeq (M_Q^2 + 2EM_Q)^n$ and keep all the terms $\sim (2EM_Q/M_Q^2)^k$ when calculating the pert. th. contribution, fig.7.2. At the same time, one can neglect the energy E in comparison with M_Q when calculating the non-perturbative contributions, figs.7.3, 7.4.

The pert. th. contribution, fig.7.2, can easily be calculated by using the dispersion relation. For instance, at n=0:

$$\mathrm{Im}\, T_L^{pert}(s) = \frac{3}{8\pi} \frac{M_Q^2 (s - M_Q^2)^2}{s^3} \simeq \frac{3}{2\pi} \frac{M_Q^4 E^2}{(M_Q^2 + 2M_Q E)^3},$$
$$T_L^{pert}(E_0) = \frac{1}{\pi}\int_0^\infty \frac{dE}{E - E_0} \mathrm{Im}\, T_L^{pert}(E), \quad s = (M_Q + E)^2, \tag{7.5}$$

and after the "nonrelativistic borelezation" the pert. th. contribution takes the form ($\beta = 2M/M_Q$):

$$\frac{1}{\pi}\int_0^\infty dE\, e^{-E/M}\, \mathrm{Im}\, T_L^{pert.}(E) = \frac{3}{2\pi^2}\frac{M^3}{M_Q^3}\int_0^\infty \frac{dx\, x^2 e^{-x}}{(1+\beta x)^3}\,. \tag{7.6}$$

The non-perturbative contributions, figs. 7.3 and 7.4, can be calculated in a standard way. The lowest resonance contribution into the spectral density is:

$$\frac{1}{\pi}\mathrm{Im}\, T_L^{n=0}(s) = f_Q^2\,\delta(s-m_R^2)+\ldots \simeq f_Q^2\frac{1}{2M_Q}\delta(E-E_R)+\ldots, \tag{7.7}$$

where E_R is the resonance energy and m_R is its mass, $m_R = M_Q + E_R$. As a result, the sum rule for a determination of the constant f_Q has the form:

$$f_Q^2 M_Q e^{-E_R/M} + \Omega_0(\beta)\frac{6}{\pi^2}M^3 e^{-E_0/M}\left(1+\frac{E_0}{M}+\frac{1}{2}\frac{E_0^2}{M^2}\right) =$$

$$\Omega_0(\beta)\frac{6}{\pi^2}M^3 - \frac{\langle\sqrt{\alpha_s}\,\bar{u}u\rangle}{\bar{\alpha}_s} + \frac{1}{32M^2}\langle 0|\bar{u}\sigma_{\mu\nu}\, ig\, G^a_{\mu\nu}\lambda^a u|0\rangle +$$

$$\frac{\pi}{162}\frac{1}{M^3}\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2 + \ldots, \tag{7.8}$$

$$\Omega_\ell(\beta) = \frac{1}{\Gamma(k+3)}\int_0^\infty \frac{dx\, x^\ell e^{-x}}{(1+\beta x)^\ell}\,,\quad \Omega_\ell(\beta=0)=1,\quad \beta = 2M/M_Q.$$

The sum rule for $\langle X_q\rangle$ can be obtained analogously and has the form:

$$f_Q^2 M_Q\left(M_Q\langle X_q\rangle\right)e^{-E_R/M} + \Omega_1(\beta)\frac{24}{\pi^2}M^4 e^{-E_1/M}\left(1+\frac{E_1}{M}+\frac{1}{2}\frac{E_1^2}{M^2}+\frac{1}{3!}\frac{E_1^3}{M^3}\right) =$$

$$\Omega_1(\beta)\frac{24}{\pi^2}M^4 + \frac{1}{12M}\langle 0|\bar{u}\sigma_{\mu\nu}\, ig\, G^a_{\mu\nu}\lambda^a u|0\rangle + \tag{7.9}$$

$$\frac{\pi}{144}\frac{1}{M^2}\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2 + \ldots$$

Here: E_0 and E_1 are the corresponding duality intervals, $\bar{E}_R \simeq$ 400 MeV is the energy of the lowest resonance,

$$f_Q^2 M_Q \to \text{const} \quad \text{and} \quad \langle X_q\rangle M_Q \to \text{const} \quad \text{at } M_Q \to \infty,$$

$$\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle \simeq -1.35\cdot 10^{-2}\,\mathrm{GeV}^3,\quad \langle 0|\bar{u}\sigma_{\mu\nu}\, ig\, G^a_{\mu\nu}\lambda^a u|0\rangle \simeq -2.35\cdot 10^{-2}\,\mathrm{GeV}^5.$$

Let us point out here some characteristic features of the sum rules (7.8) and (7.9).

a) Each derivative $\vec{D}$ in the correlator (7.4) introduces the factor $\sim (M/M_Q)$. Therefore, the me momentum fraction of the light quark is: $\langle X_q\rangle \simeq (\bar{M}/M_Q) \ll 1$. This agrees, of course, with our estimate presented above, but taking now into account non-perturbative corrections we can determine more precisely the value of $\langle X_q\rangle$.

b) The power corrections in the correlator $T^n_{\mu\nu}$ start from the operator with the dimensionality (n+3). The pure gluonic corrections like $\langle\frac{\alpha_s}{\pi}G^2\rangle$, etc., give however small contributions and can be neglected. Therefore, the main power corrections are determined by the operators: $\langle\bar{u}u\rangle$ -for f_Q^2, $\langle\bar{u}\sigma ig G\lambda u\rangle$ -for $f_Q^2\langle X_q\rangle$, $\langle\bar{u}g^2 GGu\rangle$ - for $f_Q^2\langle X_q^2\rangle$, etc.

c) The main power correction $\langle\bar{u}\sigma ig G\lambda u\rangle$ in the sum rule (7.9) for $\langle X_q\rangle$ has the sign opposite to the pert. th. contribution, fig. 7.2, and increases $\langle X_q\rangle$. Indeed, in the fig. 7.4 diagram which gives this power correction, the light quark is a "wee". Therefore, this power correction tends to diminish a role of such configurations where the light quark is a "wee", i.e. it increases $\langle X_q\rangle$.

The fit has been performed for (7.8) and (7.9) in the region 0.4 GeV $\lesssim$ M $\lesssim$ 0.8 GeV. As a result, it was obtained from (7.8) for

$\overline{M}_c \simeq 1.5\,\mathrm{GeV}$, $\overline{M}_b \simeq 4.75\,\mathrm{GeV}$, $\overline{E}_R \simeq 400\,\mathrm{MeV}$, $0.3 \lesssim \bar{\alpha}_s \lesssim 0.4$: *

$$f_B \simeq (90-100)\ \mathrm{MeV}, \quad f_D \simeq (160-175)\ \mathrm{MeV}, \quad \overline{E}_0 \simeq 0.8\ \mathrm{GeV}, \qquad (7.10)$$

and from (7.9)

$$\langle X_q \rangle_B = (0.10 \pm 0.02), \quad \overline{E}_1 \simeq 1.6\ \mathrm{GeV}. \qquad (7.11)$$

It is impossible to obtain the value of $\langle X_q \rangle_D$ from (7.9), because there is no region where the contribution of the term $\langle \bar{u} \sigma i g G \lambda u \rangle$ is small. The results of other authors for f_B and f_D can be found in /1.52a, b 7.1-7.4/ and they do not differ greatly from (7.10), except for /1.52b/.

When D-meson is replaced by F^+-meson, the u-quark is replaced by s-quark in (7.8). Taking into account that $\langle \bar{s}s \rangle \simeq 0.8 \langle \bar{u}u \rangle$ (see ch.6) and $E_F \simeq 550$ MeV for $F^+(2020)$-meson, one finds that the whole effect due to $\langle \bar{s}s \rangle \neq \langle \bar{u}u \rangle$ is compensated by $E_F \neq E_D$, while the constant f_F remains nearly equal to f_D:

$$f_F \simeq f_D \simeq (160-175)\ \mathrm{MeV}. \qquad (7.12)$$

One has from (7.12):

$$\frac{\Gamma(F^+ \to \tau^+ \nu)}{\Gamma(\pi^+ \to \mu^+ \nu)} = \frac{f_F^2\, m_\tau^2\, M_F \left(1 - m_\tau^2/M_F^2\right)^2}{f_\pi^2\, m_\mu^2\, M_\pi \left(1 - m_\mu^2/M_\pi^2\right)^2} \simeq 1.6 \cdot 10^3, \qquad (7.13)$$

$$\tau(F^+ \to \tau^+ \nu) = \tau(\pi^+ \to \mu^+ \nu) / 1.6 \cdot 10^3 \simeq 1.6 \cdot 10^{-11}\ \mathrm{sec},$$

$$Br(F^+ \to \tau^+ \nu) = \frac{\tau_{tot}(F^+)}{\tau(F^+ \to \tau^+ \nu)} \simeq 1.2\% \ \text{for}\ \tau_{tot}(F^+) \simeq 2 \cdot 10^{-13}\ \mathrm{sec}.$$

---

* Because $(q^2 - M_Q^2) \ll M_Q^2$, the quarks are near the "mass shell". For this reason, the "constituent masses" $M_c \simeq 1.5\,\mathrm{GeV}$ and $M_b \simeq 4.75\,\mathrm{GeV}$ are used instead of the "euclidian masses" $M_c(q^2 = -M_c^2) \simeq 1.25\,\mathrm{GeV}$, $M_b(q^2 = -M_b^2) \simeq 4.3\,\mathrm{GeV}$.

In conclusion of this chapter let us note the following. The estimates obtained above for B- and D-mesons look like: $\langle X_q \rangle_B \simeq 0.08$, $\langle X_q \rangle_D \simeq 0.20$. More accurate approach using the sum rules shows that non-perturbative effects tend to increase slightly the value of $\langle X_q \rangle$. Therefore, there is every reason to expect that $\langle X_q \rangle_D \gtrsim 0.20$. This confirms the qualitative conclusion made above, that the processes with D-mesons are not enhanced as compared with those with K- or π -mesons. This can be checked in the following way.

Consider the inclusive reaction: $e^+e^- \to M(p) + X$, where M is the meson with the momentum p. The missing mass is: $M_X^2 = (1-z)Q^2$, $p = z\,Q/2$, and so the process is quasiexclusive at $z \to 1$, because $M_X^2 \ll Q^2$. The above considerations allow one to expect that the D- and K, π production cross sections are roughly the same at large Q and $z \to 1$:

$$\frac{d\sigma(e^+e^- \to D^+ + X)/dz}{d\sigma(e^+e^- \to K^+ + X)/dz} \simeq O(1) \quad \text{at } z \to 1.$$

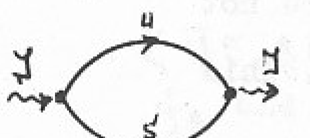

fig. 6.1

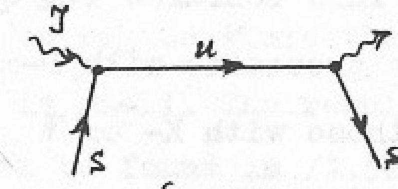

fig 6.2

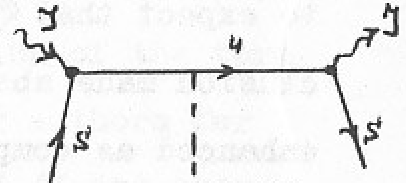

fig 6.3

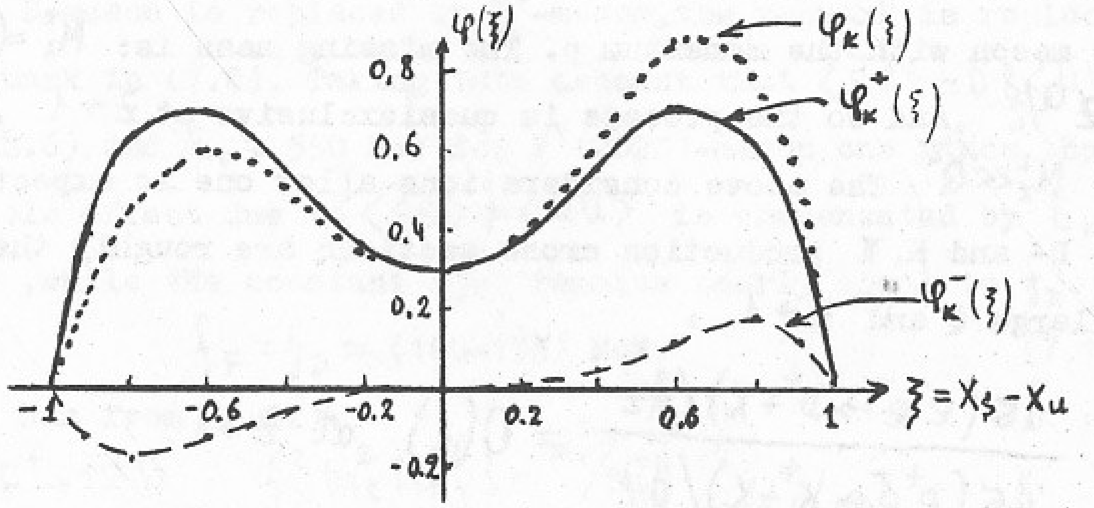

fig. 6.4

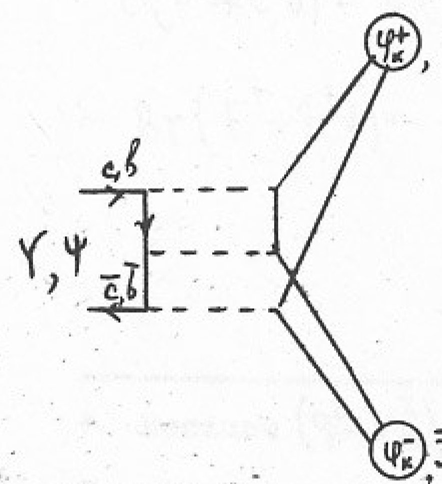

fig. 6.5

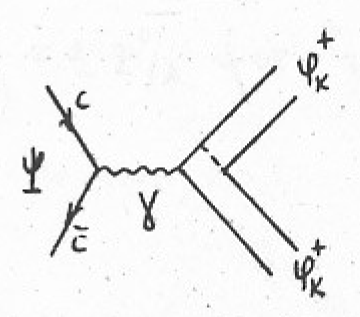

fig 6.6

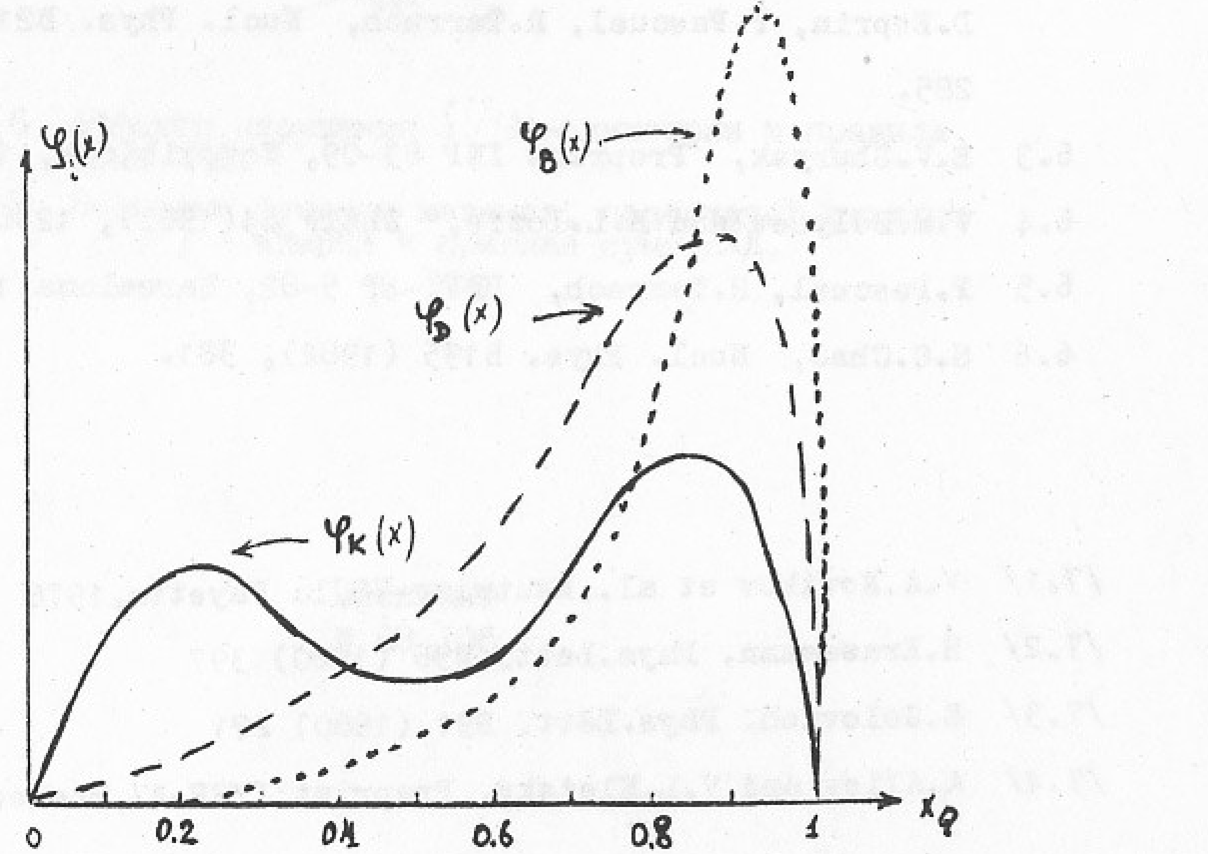

fig. 7.1

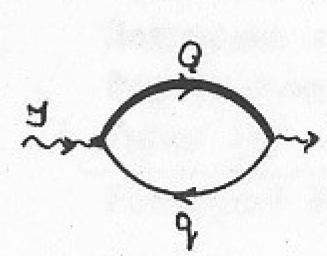

fig. 7.2

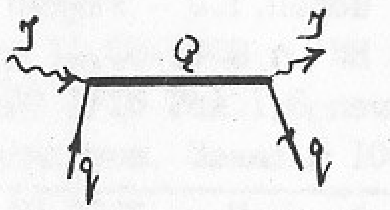

fig. 7.3

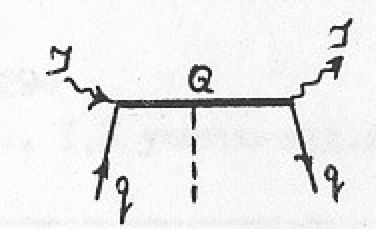

fig. 7.4

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АСИМПТОТИЧЕСКОЕ ПОВЕДЕНИЕ ЭКСКЛЮЗИВНЫХ ПРОЦЕССОВ В КХД

6. Эффекты нарушения $SU(3)$-симметрии и правила сумм КХД.
7. Волновые функции мезонов, содержащих тяжелые c , b - кварки и правила сумм КХД.

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