ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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ASYMPTOTIC BEHAVIOUR OF EXCLUSIVE PROCESSES IN QCD

8. QUARK TRANSVERSE MOMENTA INSIDE THE HADRONS. ROLE OF POWER CORRECTIONS (a qualitative description and numerical estimates)
9. HIGHER TWIST PROCESSES AND POWER CORRECTIONS

PREPRINT 83-107

НОВОСИБИРСК

C O N T E N T

8. MEAN VALUE OF A QUARK TRANSVERSE MOMENTUM. ROLE OF POWER CORRECTIONS (A QUALITATIVE DESCRIPTION AND NUMERICAL ESTIMATES)

We calculate in the sect.8.1 the mean value of a quark transverse momentum in the pion, $\langle K_\perp^2 \rangle$. The value of the "primordial" transverse momentum is of great interest in many respects. In particular, it characterizes mean values of the quark energy and momentum in hadrons and, besides, it is tightly connected with a scale of power corrections to hard processes.

The qualitative properties of power corrections in exclusive processes are discused in the sects. 8.2-8.4. The region $Q^2 \simeq$ 10 GeV^2 is chosen as a characteristic one for the following reasons.

a) This is a charmonium region and a lot of experimental data is available at present in this region,

b) The value $Q \simeq 3$ GeV is large enough for the leading terms to be dominant in exclusive amplitudes (if there are no additional suppressions for some reasons),

c) The value $Q \simeq 3$ GeV is not very large, however, so that "normal" power corrections are $\lesssim 30\%$ of the leading terms. Therefore, nonleading twist contributions can give the effects $\simeq 100\%$ or even become dominant, if the leading contribution is suppressed for some reason. A number of examples where non-leading contributions are of great importance will be considered below in this chapter. An analysis of these processes give us a deeper insight into a strong interaction dynamics and hadron properties.

8.1 THE VALUE OF THE QUARK TRANSVERSE MOMENTUM IN THE PION /4.1/

Consider first the twist 3 wave function $\varphi_\pi^P = \langle 0 | \bar{d}(z)\, i \gamma_5 u(-z) | \pi \rangle$

and the matrix element

$$\langle 0|\bar d(0) i\gamma_5 (i\vec D_\mu i\vec D_\nu) u(0)|\pi(q)\rangle = f_\pi \frac{m_\pi^2}{m_u+m_d}\left(q_\mu q_\nu A + g_{\mu\nu} B\right). \tag{8.1}$$

It is evident (take $\mu=\nu=z$, $q_z\to\infty$ in (8.1)) that $A=\langle x_u^2\rangle$, i.e. the coefficient A determines a mean square of the quark longitudinal momentum fraction, x_u ,in the pion. It is natural to determine a mean square of the quark transverse momentum in an analogous way:

$$\langle 0|\bar d(0) i\gamma_5 (i\vec D_\perp)^2 u(0)|\pi(q)\rangle \equiv f_\pi \frac{m_\pi^2}{m_u+m_d}\langle \vec k_\perp^2\rangle_\pi^P = f_\pi^2 \frac{m_\pi^2}{m_u+m_d}(-2B). \tag{8.2}$$

To find the value of coefficient B (in the chiral limit $m_u=m_d=m_\pi^2=0$),let us use equations of a motion and PCAC. Then we have from (8.1) $\left(f_\pi m_\pi^2/m_u+m_d \simeq -2\langle 0|\bar u u|0\rangle/f_\pi\right)$:

$$\langle 0|\bar d i\gamma_5 (i\vec D_\mu)^2 u|\pi(q)\rangle = f_\pi^2 \frac{m_\pi^2}{m_u+m_d} 4B =$$
$$\frac{1}{2}\langle 0|\bar d\gamma_5\sigma_{\mu\nu} g G^a_{\mu\nu}\frac{\lambda^a}{2}|\pi(q)\rangle \simeq \frac{1}{f_\pi}\langle 0|\bar u\sigma_{\mu\nu} i g G^a_{\mu\nu}\frac{\lambda^a}{2} u|0\rangle. \tag{8.3}$$

Therefore,

$$\langle \vec k_\perp^2\rangle_\pi^P \simeq \frac{1}{8}\,\frac{\langle 0|\bar u\sigma_{\mu\nu} i g G^a_{\mu\nu}\lambda^a u|0\rangle}{\langle 0|\bar u u|0\rangle}. \tag{8.4}$$

It is seen from (8.4) that $\langle \vec k_\perp^2\rangle \neq 0$ in the chiral limit due to an interaction only (a presence of gG). This is natural,because $k_\perp\to 0$ at $m_\pi\to 0$ for the case of free quarks. The value of the vacuum matrix elements ratio at r.h.s. (8.4) has been found phenomenologically in /1.49/,when investigating the baryon mass spectrum with a help of QCD sum rules. Using this result from /1.49/,one has from (8.4):

$$\langle \vec k_\perp^2\rangle_\pi^P \simeq \frac{1}{8}\left(1.5\,\mathrm{GeV}^2\right) \simeq \left(430\,\mathrm{MeV}\right)^2. \tag{8.5}$$

Let us calculate now the value $\langle \vec k_\perp^2\rangle_\pi^A$ for the leading twist pion wave function φ_π^A. Consider with this purpose the matrix element:

$$\langle 0|\bar d(0)\gamma_\mu\gamma_5 \frac{1}{2}\left[i\vec D_\nu i\vec D_\lambda + i\vec D_\lambda i\vec D_\nu\right] u(0)|\pi(q)\rangle = \tag{8.6}$$
$$i f_\pi\left\{ q_\mu q_\nu q_\lambda\langle x_u^2\rangle + \left(g_{\mu\nu}q_\lambda + g_{\mu\lambda}q_\nu\right)A + g_{\nu\lambda}q_\mu B\right\}.$$

Multiplying (8.6) by $g_{\mu\nu}$ and using equations of the motion, one has (in the chiral limit): 5A+B=0. Hence,

$$\langle 0|\bar d\gamma_\mu\gamma_5 (i\vec D_\nu)^2 u|\pi\rangle = i f_\pi q_\mu \frac{18}{5}B = -\frac{1}{2}\langle 0|\bar d\gamma_\mu\gamma_5\sigma_{\alpha\beta} i g G^a_{\alpha\beta}\frac{\lambda^a}{2} u|\pi\rangle. \tag{8.7}$$

We define $\langle \vec k_\perp^2\rangle_\pi^A$ as follows:

$$\langle 0|\bar d\gamma_\mu\gamma_5 (i\vec D_\perp)^2 u|\pi\rangle \equiv i f_\pi q_\mu\langle \vec k_\perp^2\rangle_\pi^A = -2 i f_\pi q_\mu B. \tag{8.8}$$

Therefore,one has now from (8.7),(8.8):

$$\langle 0|\bar d\gamma_\mu\gamma_5\sigma_{\alpha\beta} i g G^a_{\alpha\beta}\frac{\lambda^a}{2} u|\pi(q)\rangle = i f_\pi q_\mu \frac{18}{5}\langle \vec k_\perp^2\rangle_\pi^A,$$
$$\langle 0|\bar d g \tilde G^a_{\mu\rho}\frac{\lambda^a}{2}\gamma_\rho u|\pi(q)\rangle = i f_\pi q_\mu \frac{9}{5}\langle \vec k_\perp^2\rangle_\pi^A, \tag{8.9}$$
$$\tilde G_{\mu\rho} = \frac{1}{2}e_{\mu\rho\alpha\beta}G_{\alpha\beta}.$$

To find the value of the matrix element entering (8.9),let us consider the correlator

$$T_\mu = i\int dx\, e^{iqx}\langle 0|T\,\bar d(x)\gamma_\mu\gamma_5\sigma_{\alpha\beta} g G^a_{\alpha\beta}\frac{\lambda^a}{2}u(x)\,\bar u(0) i\gamma_5 d(0)|0\rangle = q_\mu T(q^2) \tag{8.10}$$

The leading at $|q^2|\to\infty$ contribution into T_μ gives in the chiral limit the fig.8.1 diagram:

$$T_\mu \to q_\mu\langle 0|\bar u\sigma_{\alpha\beta} i g G^a_{\alpha\beta}\frac{\lambda^a}{2}u|0\rangle/q^2. \tag{8.11}$$

Using the dispersion relation for $T(q^2)$,one has from (8.11):

$$\frac{1}{\pi}\int_0^\infty ds\, \mathrm{Im}\, T(s) = -\langle 0|\bar{u}\sigma_{\alpha\beta}\, ig\, G^a_{\alpha\beta}\frac{\lambda^a}{2}u|0\rangle. \tag{8.12}$$

The spectral density has the form:

$$\frac{1}{\pi}\mathrm{Im}\, T(s) = \left(f_\pi \frac{18}{5}\langle \vec{k}_\perp^2\rangle_\pi^A\right)\left(f_\pi \frac{m_\pi^2}{m_u+m_d}\right)\delta(s) + \ldots \simeq \tag{8.13}$$

$$-\frac{36}{5}\langle 0|\bar{u}u|0\rangle\langle \vec{k}_\perp^2\rangle_\pi^A\,\delta(s) + \ldots$$

The pion contribution is shown explicitly in (8.13). It seems reasonable to retain the pion contribution only at l.h.s. (8.12), because the spectral density $\mathrm{Im}\, T(s)$ falls off quickly at large S (the pert.th. contribution equals zero in the chiral symmetry limit). One has in this approximation from (8.12), (8.13)*:

$$\langle 0|\bar{d}\, g\, \tilde{G}_{\mu\nu}\gamma_\nu\frac{\lambda^a}{2}u|\pi(q)\rangle \simeq \frac{1}{8}\, i f_\pi q_\mu \frac{\langle 0|\bar{u}\sigma_{\alpha\beta}\, ig\, G^a_{\alpha\beta}\lambda^a u|0\rangle}{\langle 0|\bar{u}u|0\rangle}, \tag{8.14}$$

$$\langle \vec{k}_\perp^2\rangle_\pi^A \simeq \frac{5}{72}\frac{\langle 0|\bar{u}\sigma_{\alpha\beta}\, ig\, G_{\alpha\beta}\lambda^a u|0\rangle}{\langle 0|\bar{u}u|0\rangle} \simeq \frac{5}{9}\langle \vec{k}_\perp^2\rangle_\pi^\ell .$$

As a result:

$$\langle \vec{k}_\perp^2\rangle_\pi^A \simeq \frac{5}{72}\left(1.5\,\mathrm{GeV}^2\right) \simeq \left(323\,\mathrm{MeV}\right)^2. \tag{8.15}$$

It is seen from a comparison of (8.5) and (8.15) that the values $\langle \vec{k}_\perp^2\rangle_i$ differ somewhat for various wave functions, but the difference is not large. It seems natural that $\left(\langle \vec{k}_\perp^2\rangle_\pi^\ell\right) > \left(\langle \vec{k}_\perp^2\rangle_\pi^A\right)$, because a scale of the matrix elements of spin zero operators $(\bar{d}\gamma_5 u, \bar{d}u, \ldots)$ is enhanced /5.1/.

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* This matrix element has been calculated also in / 8.1 / using the QCD sum rules for different correlators. The result agrees well with our result / 4.1 / (see (8.14)).

It is natural to expect that the mean quark transverse momentum inside the other mesons ($K, \rho, K^* \ldots$) is close to that in the pion: $\langle \vec{k}_\perp^2\rangle_i \simeq \left(300-400\,\mathrm{MeV}\right)^2$.

8.2. NUMERICAL ESTIMATES OF THE WAVE FUNCTION VALUES AT THE ORIGIN /4.1/

Let us begin with the well known constants f_π, f_ρ ,which determine the values of the leading twist wave functions at the origin. The contribution of the lowest resonance ($\pi, \rho \ldots$) into the sum rule has the form (after "borelization): $f_i^2 \exp\{-m_i^2/M^2\}$, where m_i is the resonance mass, M is the scale parameter. The pert.th. contribution,fig.8.2, is: $M^2/4\pi^2$ (one loop). The usual scale of non-perturbative corrections is such that at $M \simeq m_\rho$ they are $\simeq 20\%$ of the Born contribution. Therefore,we can estimate the constants f_i using the relation:

$$f_i^2 \exp\{-m_i^2/m_\rho^2\} \simeq m_\rho^2/4\pi^2.$$

This gives for the pion: $f_\pi \simeq m_\rho/2\pi \simeq 123$ MeV(the experimental value is $f_\pi \simeq 133$ MeV),and for the ρ meson: $f_\rho \equiv f_\rho^V \simeq \sqrt{e}\, f_\pi \simeq \sqrt{e}\, m_\rho/2\pi \simeq 200$ MeV(the experimental value is $\simeq 200$ MeV). The situation with the constant f_ρ^T (see (4.13)) is the same as for f_ρ^V ,so: $f_\rho^T \simeq 200$ MeV.

The constant $f_{3\rho}^A$ determines the value of the three-particle $\rho_\perp \equiv \rho_{|\lambda|=1}$ meson wave function (twist 3) at the origin:

$$\langle 0|\bar{d}(0)\gamma_\lambda\gamma_5\, g\, G^a_{\mu\nu}\frac{\lambda^a}{2}u(0)|\rho_\perp(q)\rangle = q_\lambda e_{\mu\nu\alpha\beta}\, q_\alpha \varepsilon_\beta^\perp f_{3\rho}^A . \tag{8.16}$$

The pert.th. contribution,fig.8.3,is: $\bar{\alpha}_s M^4/720\pi^3$ (two loops). <u>Therefore,the estimate looks like*: $(f_{3\rho}^A)^2 e^{-1} \simeq \bar{\alpha}_s m_\rho^4/720\pi^3$</u>,

* Using suitable non-diagonal correlators,it is possible to determine relative signs of various constants. For instance, $\left(f_{3\rho}^A/f_\rho^T\right) > 0$, $\left(f_\rho^T/f_\rho^V\right) > 0, \ldots$

$f_{3\rho}^A \simeq 0.4\cdot 10^{-2}\,GeV^2$ at $\alpha_s \simeq 0.35$.
(More precise treatment of corresponding sum rules gives: $f_{3\rho}^A \simeq 0.6\cdot 10^{-2}\,GeV^2$, see ch.9).

For the nucleon wave function (twist 3)

$$\langle 0|\varepsilon^{ijk}(u^i C\gamma_\mu u^j)\, d^k_\alpha |N(p)\rangle = p_\mu(\gamma_5 N)_\alpha \cdot f_N$$

(C is the charge conjugation matrix, N is the nucleon spinor), the pert.th. contribution, fig.8.4, is: $M^4/960\pi^4$ (two loops). Hence, the estimate is: $|f_N|^2 \exp\{-m_N^2/M^2\} \simeq m_\rho^4/960\pi^4$, $|f_N| \simeq 0.4\cdot 10^{-2}\,GeV^2$. (More precise treatment of corresponding sum rules gives: $|f_N| \simeq 0.5\cdot 10^{-2}\,GeV^2$, see ch.10).

The pert.th. contribution, fig.8.2, for the two-particle meson wave function (twist 3)

$$\langle 0|\bar d(0)\, i\overleftrightarrow{D}_\mu u(0)|\rho_L(q)\rangle = f_\rho^s\, \varepsilon_\mu^{\lambda=0} m_\rho \simeq f_\rho^s q_\mu, \quad q_z\to\infty, \quad (8.17)$$

is: $M^4/8\pi^2$ (one loop). Hence, the estimate is*: $|f_\rho^s|^2 e^{-1} \simeq m_\rho^4/8\pi^2$, $|f_\rho^s| \simeq 0.11\,GeV^2 \simeq 25 f_{3\rho}^A$. The strong unequality $|f_\rho^s| \gg |f_{3\rho}^A|$ is mainly due to a smallness of the three-particle phase space, fig.8.3, as compared with the two-particle one, fig.8.2. This is, evidently, a general property.

In connection with above given estimates, let us note the following. The mesonic two-particle wave functions of the twist 3:

$$\langle 0|\bar d(0)\gamma_\mu^\perp u(0)|\rho_\perp(q)\rangle = f_\rho^V \varepsilon_\mu^\perp m_\rho,$$

$$\langle 0|\bar d(0)\, i\overleftrightarrow{D}_\mu u(0)|\rho_L(q)\rangle = f_\rho^s q_\mu, \quad q_z\to\infty, \quad (8.18)$$

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* Using equations of the motion, one can obtain the exact relation: $f_\rho^s = -f_\rho^T m_\rho$, where m_ρ is the ρ meson mass and the constant f_ρ^T is defined by (4.26). Hence, $f_\rho^s \simeq -0.15\,GeV^2$.

$$\langle 0|\bar d(0)\, i\gamma_5 u(0)|\pi(q)\rangle = f_\pi \frac{m_\pi^2}{m_u+m_d} \simeq -\frac{\langle 0|\bar u u+\bar d d|0\rangle}{f_\pi}, \quad (8.18)$$

$$f_\rho^V m_\rho \simeq 0.15\,GeV^2, \quad f_\rho^s \simeq -0.15\,GeV^2, \quad f_\pi \frac{m_\pi^2}{m_u+m_d} \simeq 0.22\,GeV^2,$$

determine the values of the power corrections to the leading twist contributions in exclusive processes. As it is seen from (8.18), the relative correction is $(1-2) f_\rho^s m_\rho / f_\rho^V Q \simeq (1-2) m_\rho/Q$, and not $\sim f_\rho^V/Q$, as one can naively expect. Moreover, it is seen from (8.18) that the scale of the pion and ρ meson wave functions is nearly the same, and this allows one to expect the same scale of power corrections in the exclusive amplitudes in which π or ρ mesons participate and, besides, the same scale for other mesons also.

It may seems at the first sight that the power corrections in the mesonic exclusive processes are determined mainly by the two-particle contributions, because the scale of the three-particle wave functions is smaller by one order of magnitude. This is not the case, in general, because there are also other factors working in an opposite direction (see ch.9 for details).

8.3 WHAT DO $\Psi(3100)$ MESON DECAYS TEACH US ABOUT?

8.3.1. Hadronic decays.

As it has been shown in ch.2, the decay $\Psi\to\rho\pi$ is suppressed in the limit $M_c\to\infty$, while the decays $\Psi\to B(1235)\pi$, $\Psi\to\rho A_2$ are not. The experimental data are /2.6/: $Br(\Psi\to\rho\pi) = (1.22\pm 0.12)\%$, $Br(\Psi\to B\pi) = (0.44\pm 0.11)\%$, $Br(\Psi\to\rho A_2) = (0.84\pm 0.45)\%$. Therefore, the asymptotic selection rule (see ch.2) doesn't work yet. In this case the reason is the following. The meson wave functions of the leading twist 2 are nesessarily two-particle ones. Therefore, the leading twist contributions into the decays $\Psi\to$ "two mesons" include nesessarily a loop, fig.8.7, and this gives the additional small factor $\simeq \frac{\alpha_s}{\pi} \simeq 0.1$. At the same time, the main contribution into the amplitude $\Psi \to \rho\pi$ give the three-particle wave functions, fig.8.8, and there is no smallness due to a loop.* We see, therefore, that suppressed and non-suppressed decays $\Psi \to M_1 M_2$ are of the same order because there is an additional suppression of leading twist contributions.

The leading twist contributions have no additional suppression in the C-even charmonium level decays: $\bar{c}c(^3P_J) \to M_1 M_2$, $(\chi_0 \to \pi\pi,\ \chi_2 \to \pi\pi)$, because there are now only two gluons in the intermediate state. We expect, therefore, that the leading terms are indeed leading here and that the asymptotic selection rules work already.

8.3.2 The decay $\Psi \to \pi^0 \gamma$.

The calculation of this decay was described in detail in ch.5. The leading twist contributions into this decay also have additional suppressions either due to two loops, fig.5.15, or due to the additional photon (the smallness $4\pi\alpha \simeq 1/10$), fig. 5.14. As a result, the VDM contribution, fig.5.16, which is a power correction $\sim M^2/M_c^2$ at $M_c \to \infty$, has really the same value as the leading twist term. This shows once more that at $Q^2 \simeq 10\ \mathrm{GeV}^2$ non-leading twist contributions play a role only if leading terms are suppressed for some reason.

8.3.3 The decays $\Psi \to \gamma\eta,\ \gamma\eta'$.

The decay probabilities for these processes were calculated

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* The calculation of the $\Psi \to \rho\pi$ decay is described in detail in ch.9, and the result agrees with the experiment.

for the first time in the paper /8.2/ with the help of the effective Lagrangian. The agreement with the experiment has been obtained, in spite of that the contribution considered in /8.2/ is the higher twist contribution (twist 4). The reason is that the leading twist contribution is zero in the lowest order in α_s /2.4, 8.3 /, fig.8.10. It appears at the one loop level only, fig.8.11, and is strongly suppressed numerically (see /2.4/). At the same time, the scale of the twist 4 two-gluon components of the η and η' meson wave functions is large (see /8.2/).

8.3.4 The decays $\Psi \to \gamma\sigma(0^{++}),\ \gamma f^0(2^{++})$.

Unlike to the previous case (see 8.3.3), the leading twist contributions are nonzero at the Born level in this case, fig. 8.10, /2.4, 8.3 /. However, the QCD sum rules for the correlators of pure gluonic currents have a number of specific properties, as compared with the currents made off light quarks /5.1/. For this reason, even qualitative properties of the gluonic wave functions are unknown at present. It is possible that higher twist contributions are more important here, in comparison with the quark wave functions. The experiment indicates that the scale $Q^2 \simeq 10\ \mathrm{GeV}^2$ is not large enough for the leading twist contributions to be the dominant ones in the gluonic channels. The decay $\Psi \to \gamma f^0_{\lambda=0}$ is the leading one asymptotically at $M_c \to \infty$ /2.4/, while the experimental data are /8.4/:

$$\left[\frac{Br(\Psi \to \gamma f^0_{|\lambda|=1})}{Br(\Psi \to \gamma f^0_{\lambda=0})}\right]^{1/2} = (0.88 \pm 0.13),\quad \left[\frac{Br(\Psi \to \gamma f^0_{|\lambda|=2})}{Br(\Psi \to \gamma f^0_{\lambda=0})}\right]^{1/2} = (0.04 \pm 0.019) \tag{8.19}$$

8.4 WHAT $\Psi'(3685)$ MESON IS MADE OFF?

If $\Psi'(3685)$ meson is a pure 2^3S_1 $\bar{c}c$-state, all the formulae for its decay amplitudes can be obtained in complete analogy with $\Psi(3100)$ meson. We argue below that there are, however, serious enough reasons to believe that $\Psi'(3685)$ is not a pure 2^3S_1- state.

The first indication for this is the decay $\Psi' \to \gamma\eta_c(2980)$. The experiment gives for the decay width /2.6/: $\simeq 0.7$ KeV, while the theoretical prediction (assuming that Ψ' is a pure 2^3S_1-state) is: $\simeq 4\cdot 10^{-2}$ KeV /8.5/, i.e. considerably smaller. It was pointed also in this paper (see /8.5/) that the large admixture of the $\bar{C}Cg$-component in Ψ' state is required to obtain an agreement with the experiment.

The next example is the $\Psi' \to \pi^+\pi^-$ decay. It has been shown in ch.5 that the $\bar{C}C(2^3S_1) \to \pi^+\pi^-$ decay amplitude can be expressed (with an accuracy $\lesssim 10\%$) through the pion electromagnetic form factor. The experimental data for the $\Psi(3100) \to \pi^+\pi^-$ decay are, therefore, equivalent to: $|M_\Psi^2 F_\pi(M_\Psi^2)| = (0.7 \pm 0.3)\,GeV^2$, and this number agrees with our calculation of the pion form factor and with the VDM prediction: $|M^2 F_\pi(M^2)| \simeq m_\rho^2 \simeq 0.6\ GeV^2$ (see ch.5). At the same time, the experimental data for the $\Psi' \to \pi^+\pi^-$ decay /2.6/, when interpreted in this way, give the value for the pion form factor which is three times larger:

$$Br(\Psi' \to \pi^+\pi^-) = (0.8 \pm 0.5)\cdot 10^{-4},\quad Br(\Psi' \to e^+e^-) = (0.9 \pm 0.1)\cdot 10^{-2},$$

$$\frac{Br(\Psi' \to \pi^+\pi^-)}{Br(\Psi' \to e^+e^-)} = (0.9 \pm 0.6)\cdot 10^{-2} \overset{?}{=} \frac{1}{4} F_\pi(M_{\Psi'}^2) \to \tag{8.20}$$

$$M_{\Psi'}^2 F_\pi(M_{\Psi'}^2) = (2.6 \pm 1.3)\,GeV^2 \quad ?$$

This is clearly impossible.* This example also **indicates** that Ψ' meson is not a pure $\bar{C}C$-state.

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* If one assumes that the pion form factor has the behaviour $F_\pi(M^2) \sim 1/M^4$ in this region, the discrepancy will be even larger.

Moreover, new experimental results become available recently /8.6/:

$$Br(\Psi' \to \bar{p}p) / Br(\Psi \to \bar{p}p) = (0.1 \pm 0.02), \tag{8.21a}$$

$$\frac{Br(\Psi' \to \rho^0\pi^0)}{Br(\Psi \to \rho^0\pi^0)} < 0.009,\quad \frac{Br(\Psi' \to \rho^\pm\pi^\mp)}{Br(\Psi \to \rho^\pm\pi^\mp)} < 0.015, \tag{8.21b}$$

while it seems reasonalbe to expect that all the ratios in (8.21) should not differ greatly from each other and from the ratio: $Br(\Psi' \to e^+e^-) / Br(\Psi \to e^+e^-) = (0.12 \pm 0.02)$. (Based on the dimensional counting, one can expect ruther that $(\Psi' \to \bar{p}p / \Psi \to \bar{p}p) < (\Psi' \to \rho\pi / \Psi \to \rho\pi)$).

All these facts indicate, from our point of view, that $\Psi'(3685)$ meson is not a pure 2^3S_1-charmonium state (unlike $\Psi(3100)$ meson). The reason may be the admixture of the three-particle $\bar{C}Cg$-component or the admixture of the quasi-molecular $\bar{D}D$-state (because $\Psi'(3685)$ is near the open charm threshold). Let us remind that analogous anomalies are observed in $\Upsilon'' \to \Upsilon\pi^+\pi^-$ decays, and $\Upsilon''(10350)$ is also not far from the open beauty threshold.

8.5 CONCLUSIONS

Let us enumerate in short the main results of this chapter.

1. The mean square value of the quark transverse momentum in the pion is: $\langle \vec{K}_\perp^2 \rangle \simeq (300\text{-}400\,MeV)^2$, and this value characterizes the energies and momenta of light quarks in hadrons.
2. The (n+1)-particle wave function of the leading twist "n+1" has the dimensionality $[M^n]$ and its characteristic scale is: $\sim (f_\pi - f_\rho)^n \sim (150\,MeV)^n$. A smallness of this scale is mainly due to a smallness of the many-particle phase space.

The (n+1)-particle wave function of the non-leading twist "n+k+1" has the dimensionality $[M^{n+k}]$ and its characteristic scale is: $(f_\pi - f_\rho)^n (M_\rho - 1\,\text{GeV})^k$.

3. The experiment indicates that power corrections are $\lesssim 30\%$ of the leading terms in the charmonium region $Q^2 \simeq 10\ \text{GeV}^2$, if these last are not suppressed for some reasons. There is a number of examples where non-leading terms are not small or even dominant, but the leading terms are suppressed for various reasons in all these cases (in the quark channels, in the gluon channels the situation is less clear).

4. There is a number of the examples which indicate that $\Psi'(3685)$ meson is not a pure 2^3S_1-quarkonium state (unlike to $\Psi(3100)$ meson). It well may be that $\Psi'(3685)$ state has a noticeable admixtures of the $\bar{c}cg$-component and of the quasi-molecular $\bar{D}D$-state, because $\Psi'(3685)$ is not far from the open charm threshold (similar anomalies shows also $\Upsilon''(10350)$ state which is also not far from the open beauty threshold).

9. NONLEADING TWIST PROCESSES AND POWER CORRECTIONS

9.1 THE DECAY $\Psi \to \rho\pi$ *

It was pointed in the sect.2.2.3 that the decay $\Psi \to \rho^-\pi^+$ is suppressed at $M_c \to \infty$, i.e. its amplitude has the behaviour $\sim 1/M_c^3$, while analogous non-suppressed amplitudes are $\sim 1/M_c^2$. There are two types of the diagrams, fig.9.1 and 9.2, which contribute into the $\Psi \to \rho\pi$ decay.

The ρ-meson helicity is $|\lambda| = 1$ in the $\pi\rho$ c.m.s., while the pion helicity is, of course, zero. Hence, this leads to the suppressions: a) $\sim (m_u + m_d)/M_c$ due to the quark helicity turn over, where m_u and m_d are the current quark masses; b) $\sim k_\perp/M$ because the quark helicity doesn't coincides precisely with its spin projection onto the meson momentum (see ch.2). Because $(m_u + m_d) \simeq 10\text{-}15\ \text{MeV}$ while $k_\perp \simeq 300\text{-}400\ \text{MeV}$, the contributions of the type "b" are the dominant ones (we neglect below the quark masses). The additional suppression of the amplitude means that the fig.9.1 contribution includes the product of two two-particle wave functions of the twists 2 and 3.

Because the decay amplitude is suppressed, the three-particle π and $\rho_\perp$ meson wave functions (twist 3) "come into a game", fig.9.2. At the fig.9.2a diagram the ρ meson helicity $|\lambda_\rho| = 1$ is carried by the gluon, while the quark's spins are opposite. At the fig.9.2b diagram the quarks have parallel spins and the pion zero helicity results after the quark and gluon helicities add to zero. The contributions of this type include the product of two-particle (twist 2) and three-particle (twist 3) wave functions.

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* This section is written in collaboration with I.R.Zhitnitsky

Let us begin our calculations from the diagrams like that shown at fig.9.2b. Their contributions into the decay amplitude can simply be found as follows.

1. One replaces $\rho_\perp$ meson (in the Ψ meson rest frame, ρ meson momentum is directed along the Z-axis, the angular distribution is $\sim(1+\cos^2\theta)$) by two free quarks with the longitudinal momentum fractions X_1 and X_2, $X_1+X_2=1$, while the pion is replaced by two quarks and transversally polarized gluon, $B^\perp_\mu$, with the longitudinal momentum fractions Y_1, Y_2 and Y_3 respectively, $Y_1+Y_2+Y_3=1$.

2. Because the suppression of the decay amplitude $(\sim 1/M_c)$ is ensured by a presence of three constituents of the pion, the transverse momenta of all quarks and the gluon can be neglected. For the same reason it is sufficient to retain for $\rho_\perp$ meson the leading twist wave function $\varphi^T_\rho(\xi)$ only (see (4.13)):

$$\langle\rho^-_\perp(p)|\bar{u}\,\sigma_{\mu\nu}\,d|0\rangle \to -(\rho^\perp_\mu p_\nu - \rho^\perp_\nu p_\mu) f^T_\rho\, \varphi^T_\rho(X_{1,2}),$$

where $\rho^\perp_\mu$ is the $\rho_\perp$-meson polarization vector.

3. The three-particle pion wave function $\varphi_{3\pi}(y_1,y_2,y_3)$ is introduced as follows $\left(d_3y = dy_1 dy_2 dy_3\,\delta(1-\sum_i y_i)\right)$:

$$\langle\pi^+(p')|\bar{u}_\beta\, g B^{\perp,a}_\mu \frac{\lambda^a}{2} d_\alpha|0\rangle \to \frac{i}{4}(\sigma_{\rho\nu}\gamma_5)_{\alpha\beta} f_{3\pi}\, p'_\rho\left(g_{\nu\mu} - \frac{2p_\nu p'_\mu}{Q^2}\right), \qquad (9.1)$$

$$\varphi_{3\pi}(y_1,y_2,y_3)=\varphi_{3\pi}(y_2,y_1,y_3), \quad \int_0^1 d_3y\,\varphi_{3\pi}(y)=1, \quad Q^2=M^2_\Psi=(p'+p)^2.$$

The expression (9.1) is equivalent to the following gauge-invariant definition of $\varphi_{3\pi}(y)$:

$$\langle\pi^+(p')|\bar{u}\,\sigma_{\mu\nu}\gamma_5\, g G^a_{\rho\lambda}\frac{\lambda^a}{2} d|0\rangle \to \qquad (9.2)$$

$$\left[(p'_\mu g_{\nu\lambda} - p'_\nu g_{\mu\lambda})p'_\rho - (p'_\mu g_{\nu\rho} - p'_\nu g_{\mu\rho})p'_\lambda\right] f_{3\pi}\,\varphi_{3\pi}(y).$$

The equivalence of (9.1) and (9.2) can easily be checked taking account that one of two indices of $G_{\rho\lambda}$ in (9.2) is longitudinal while the second one is transverse, and so there is the relation: $B^\perp_\lambda \to -\left(\frac{2i p_\mu}{y_3 Q^2}\right) G_{\mu\lambda}$.

The rest steps in the calculation of the fig.9.2b diagram contribution are standard ones.

The suppression $\sim 1/M_c$ is ensured by a presence of the three-particle $\rho_\perp$ meson wave function in the fig.9.2a diagram and so, it is sufficient to retain only the leading twist wave function $\varphi^A_\pi(\xi)$ for the pion. There are two three-particle $\rho_\perp$- meson wave functions, which are introduced as follows:

$$\langle\rho^-_\perp(p)|\bar{d}\gamma_\mu\gamma_5\, g B^{\perp,a}_\lambda \frac{\lambda^a}{2} u|0\rangle \to -i\frac{2p_\mu}{Q^2} e_{\tau\lambda\alpha\beta}\, p'_\tau p_\alpha \rho^\perp_\beta f_{3A}\frac{\varphi_{3A}(x)}{x_3},$$

$$\langle\rho^-_\perp(p)|\bar{d}\gamma_\mu\, g B^\perp_\lambda \frac{\lambda^a}{2} u|0\rangle \to p_\mu \rho^\perp_\lambda f_{3V}\frac{\varphi_{3V}(x)}{x_3}, \qquad (9.3)$$

$$\varphi_{3A}(x_1,x_2,x_3)=\varphi_{3A}(x_2,x_1,x_3), \quad \varphi_{3V}(x_1,x_2,x_3)=-\varphi_{3V}(x_2,x_1,x_3),$$

$$\int_0^1 d_3x\,\varphi_{3A}(x)=1, \quad \int_0^1 d_3x\,(x_u-x_d)\,\varphi_{3V}(x_u,x_d,x_3)=1.$$

The equivalent gauge-invariant definition is:

$$\langle\rho^-_\perp(p)|\bar{d}\gamma_\mu\gamma_5\, g G^a_{\nu\lambda}\frac{\lambda^a}{2}u|0\rangle \to p_\mu e_{\nu\lambda\alpha\beta}\, p_\alpha \rho^\perp_\beta f_{3A}\varphi_{3A}(x),$$

$$\langle\rho^-_\perp(p)|\bar{d}\gamma_\mu\, g G^a_{\nu\lambda}\frac{\lambda^a}{2}u|0\rangle \to i p_\mu(p_\nu\rho^\perp_\lambda - p_\lambda\rho^\perp_\nu) f_{3V}\varphi_{3V}(x), \qquad (9.4)$$

$$B^\perp_\lambda \to \frac{2i p'_\mu}{x_3 Q^2} G_{\mu\lambda}.$$

As a result, the total contribution of the diagrams like those shown at figs.9.2a and 9.2b has the form:

$$\langle\rho^-_\perp(p)\pi^+(p')|S|\Psi(Q)\rangle = i\,\delta(Q-p-p')\, I_0 M_3, \quad I_0 = e_{\mu\nu\lambda\sigma}\, p'_\mu \Psi^\perp_\nu p_\lambda \rho^\perp_\sigma,$$

$$M_3=-\frac{10}{27}\left(4\pi\alpha_s\right)^2\left\{\frac{f_\psi f_\pi f_{3A}}{M_\psi^5}I_{3\rho}+\frac{f_\psi f_\rho^T f_{3\pi}}{M_\psi^5}I_{3\pi}\right\},$$

$$I_{3\rho}=\int_{-1}^{1}d\zeta\int_0^1 d_3x\,\frac{\varphi_\pi^A(\zeta)\,\varphi_{3\rho}(x)}{x_3x_ux_d(1-\zeta)\left[1-\zeta(2x_d-1)\right]},\qquad(9.5)$$

$$I_{3\pi}=2\int_{-1}^{1}d\zeta'\int_0^1 d_3y\,\frac{\varphi_\rho^T(\zeta')\,\varphi_{3\pi}(y)}{y_1y_2y_3(1-\zeta'^2)}\,\frac{(1-\zeta')y_2+(1+\zeta')y_1}{\left[1-\zeta'(2y_2-1)\right]\left[1+\zeta'(2y_1-1)\right]},$$

$$\varphi_{3\rho}(x)=\varphi_{3A}(x_u,x_d,x_3)-\frac{f_{3V}}{f_{3A}}\varphi_{3V}(x_u,x_d,x_3).$$

In (9.5): $\Psi_\nu^\perp$ is the Ψ meson polarization vector, $\zeta=2y_1-1$, $\zeta'=2x_1-1$. The total decay width of Ψ into lihgt hadrons is:

$$\Gamma(\Psi\to3g)=\frac{40(\pi^2-9)}{243}\alpha_s^3\frac{|f_\psi^2|}{M_\psi}\simeq0.8\,\Gamma(\Psi\to all).\qquad(9.6)$$

The leading twist pion and $\rho_\perp$ meson wave functions $f_\pi\varphi_\pi^A(\zeta)$ and $f_\rho^T\cdot\varphi_\rho^T(\zeta')$ have been found above in ch.4. The properties of π and $\rho_\perp$ three-particle wave functions are described below in the sect.9.5. We want to emphasize that the relative signs of all these wave functions can be determined unambiguously from the QCD sum rules for suitable non-diagonal correlators. The results are: $(f_{3\pi}/f_\pi)>0,\ (f_{3A}/f_\rho)>0,\ (f_{3V}/f_\rho)>0,\ (f_\rho^T/f_\rho)>0.$ Therefore, the amplitudes $M_3^{(\rho)}$ and $M_3^{(\pi)}$ add to each other and the total amplitude $M_3\equiv M_3^{(\rho)}+M_3^{(\pi)}$ has a definite sign. (This property was used above in the sect.5.4 for the calculation of the $\Psi\to\pi^0\gamma$ decay).

Using the explicit form of all these wave functions (see

(4.10),(4.27) and the sect,9.5),one obtains from (9.5),(9.6) (for $\bar\alpha_s\simeq0.3$)*: $Br(\Psi\to\rho\pi)_{3\,part}\simeq0.8\,\Gamma(\Psi\to\rho\pi)/\Gamma(\Psi\to3g)=$

$$0.8\frac{20\pi^3}{3(\pi^2-9)}\bar\alpha_s\left(\frac{f_\pi f_{3A}}{M_\psi^3}\Omega\right)^2\simeq1.4\%,\qquad(9.7)$$

$$\Omega=I_{3\rho}+\frac{f_\rho^T f_{3\pi}}{f_\pi f_{3A}}I_{3\pi}\simeq I_{3\rho}+I_{3\pi}\simeq5.9\cdot10^2,$$

$$I_{3\rho}\simeq3.0\cdot10^2,\quad I_{3\pi}\simeq2.9\cdot10^2,\quad f_\pi f_{3A}/M_\psi^3\simeq2.7\cdot10^{-5}.$$

The experimental value is /2.6/:

$$Br(\Psi\to\rho\pi)=(1.22\pm0.12)\%.$$

The calculation of the contributions due to the loop diagrams like those shown at fig.9.1 is lacking at present. The estimate is (the loop integration in the fig.9.1 diagram gives no logarithmic enhancement $\sim\ln M_\psi^2/\mu^2$,because the corresponding Born diagram is zero):

$$M_2\sim\left(4\pi\alpha_s\right)^2\frac{\alpha_s}{\pi}\frac{f_\psi f_\pi f_\rho m_\rho}{M_\psi^5}I_\pi I_\rho,\qquad(9.8)$$

$$I_\pi=\int_{-1}^{1}\frac{d\zeta\,\varphi_\pi^A(\zeta)}{1-\zeta^2},\quad I_\rho=\int_{-1}^{1}\frac{d\zeta'}{1-\zeta'^2}\varphi_{2\rho}(\zeta'),\quad\int_{-1}^{1}d\zeta'\,\varphi_{2\rho}(\zeta')=1,$$

where the function $\varphi_{2\rho}(\zeta')$ represents conditionally the $\rho_\perp$

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* For the non-relativistic wave functions: $\varphi_\pi^A(\zeta)=\delta(\zeta)$, $\varphi_\rho^T(\zeta')=\delta(\zeta')$, $\varphi_{3A}(x)=\delta(x_1-1/3)\delta(x_2-1/3)=\varphi_{3\pi}(x)$, $\varphi_{3V}(x)=0$, one obtains: $I_{3\rho}=27$. $I_{3\pi}=36$.

meson two-particle wave functions of the twist 3 (see below the sects.9.2 and 9.5). Using for the estimate of (9.9): $I_\pi \simeq 2.5,\ I_\rho \simeq 2$,one has from (9.9),(9.5):

$$\left|\frac{M_2}{M_3}\right| \sim \frac{\alpha_s}{\pi}\left|\frac{f_\rho m_\rho}{f_{3A}}\,\frac{27}{2\Omega}\right| \lesssim \frac{\alpha_s}{\pi} = 0.1 . \qquad (9.9)$$

Therefore,according to the estimate (9.9),the two-particle contributions,fig.9.1,are small (mainly due to the additional loop at the fig.9.1),and the $\Psi\to\rho\pi$ decay is dominated by the three-particle contributions,fig.9.2. As a result,the experimental measurment of the $\Psi\to\rho\pi$ decay width is the direct observation of the π and ρ meson three-particle wave functions.

9.2 CALCULATION OF THE $\gamma\pi\rho$ FORM FACTOR

Let us rewrite for a convenience the definition of the form factor (the Breit frame is used, $\vec p + \vec p\,' = 0$):

$$\gamma^\perp_\mu \langle \pi^+(p')\,|\,J_\mu(0)\,|\,\rho^+_\perp(p)\rangle = I_0\, F_{\pi\rho}(Q^2), \qquad (9.10)$$

$$I_0 = e_{\mu\nu\lambda\sigma}\, p'_\mu\, p_\nu\, \rho^\perp_\lambda\, \gamma_\sigma\,, \quad q = p' - p, \quad Q^2 = -q^2 .$$

The photon and ρ meson are transversally polarized, $|\lambda_\gamma| = |\lambda_\rho| = 1$, $\gamma^\perp_\sigma$ and $\rho^\perp_\lambda$ are their polarization vectors.

As was pointed above in the sect.2.3.4, $F_{\pi\rho}(Q^2) \sim 1/Q^4$, because there is an additional suppression $\sim 1/Q$. Hence,analogously to $\Psi\to\rho\pi$ decay,two types of contributions "come into a game": the quark transverse momentum $(\sim \bar k_\perp/Q)$ or additional transversally polarized gluon $(\sim \bar B_\perp/Q)$. And in analogy with $\Psi\to\rho\pi$ decay,the answer includes the product of two-particle leading twist wave functions with non-leading twist wave functions, both two- and three-particle. But unlike to $\Psi\to\rho\pi$ decay,

these two types of contributions both appear now at the Born level and enter the answer on equal footing. For this reason, the obtaining of the operator expansion for the form factor $F_{\pi\rho}(Q^2)$ is a much more complicated problem than all previously considered above. We describe below in detail the technical methods needed for a solution of this problem. These methods are universal ones and can be applied in other cases as well.

In the case under consideration the "rules of a game" are as follows ($p_+ \sim p'_- \sim Q \to \infty$).

1. The unphysical longitudinal components of the gluon field, $B_\pm$,can temporarily be put equal zero. As was explained above in the sect.3.3,this can be done in the planar gauge. It is clear from the gauge-invariance,however,that in any gauge the unphysical longitudinally polarized gluons $B_\pm$ will enter the final answer only through $\partial_\pm \to D_\pm$ or inside of $G_{\mu\nu}$,and this replacement can be done at the end of calculations. Therefore,it is sufficient for our purposes to retain only the transverse components $B_\perp$ of the gluon field operator,and the Heisenberg equations of the motion take a form $(B = B^a\lambda^a/2)$, fig.9.3

$$(x_1\hat p + \hat k_\perp + g\hat B_\perp)u = 0, \quad \bar d(-x_2\hat p + \hat k_\perp + g\hat B_\perp) = 0,$$
$$\bar u(y_1\hat p' + \hat l_\perp + g\hat B_\perp) = 0, \quad (-y_2\hat p' + \hat l_\perp + g\hat B_\perp)d = 0. \qquad (9.11)$$

2. Write now the operator expression for the fig.9.3 diagram (and for three analogous diagrams). The external quark lines at fig.9.3 are understooded as the operators satisfying the equations (9.11). The quark and gluon propagators at fig.9.3 are the propagators in the external operator field $B_\perp$. For instance,the quark propagator at fig.9.3 is:

$$1/\left(\hat{P}-y_2\hat{P}'+\hat{\ell}_\perp+g\hat{B}_\perp\right). \qquad (9.12)$$

3. Expand the quark and gluon propagators into a power series of transverse components $K_\perp$, $\ell_\perp$ and $gB_\perp$. (It is sufficient in our case to keep only the first order terms). It should be remembered that the gluon operators $B_\perp$ act, at the same time, as the displacement operators in the longitudinal momentum space, because a gluon carries a finite fraction of the meson momentum (x_3p or y_3p', depending on what meson the gluon belongs to). The decomposition of propagators over $gB_\perp$ corresponds to the pert.th. diagrams like those shown at figs.9.4,9.5.

4. Even having performed an expansion of propagators, one still can't do directly the factorization like $\langle\pi|O_2|0\rangle\langle 0|O_1|\rho\rangle$. The reason is that the small components $K_\perp$, $\ell_\perp$ and $B_\perp$ are contained implicitly in the Heisenberg quark operators Ψ_i. For instance, the operator u gives contributions of two types into the operator expansion. To see this, let us use (9.11) and rewrite u in the form: $u=-\left(1/x_1\hat{P}_+\right)\left(\hat{K}_\perp+g\hat{B}_\perp\right)u$. The contributions of the first type are obtained by factorizing the operator $\left(\hat{K}_\perp+g\hat{B}_\perp\right)u$ as a whole into the $\rho_\perp$ meson state (in the pert.th. this corresponds to the diagrams like those shown at fig.9.6). The contributions of the second type are obtained by factorizing the operator u into the $\rho_\perp$ meson state and $B_\perp$ into the pion state (in the pert.th. this corresponds to the giagrams like those shown at fig.9.7). Hence, one should be carefull and account for all types of contributions when performing the final factorization.

The above described scheme for obtaining operator expansions is a general one. In the case under consideration, however, the answer can be obtained by a more conventional method, using partly the pert.th. diagrams. The receipe is as follows.

a) The diagrams like those at fig.9.7. These contributions can be calculated by the usual methods. The initial (final) meson state is substituted by a set of free quarks plus a free transversally polarized gluon carring the logitudinal momentum fractions x_i (y_i), and neglecting all transverse momenta. A factorization of operators is performed in a standard way without any complications. The three-particle pion and $\rho_\perp$ meson wave functions are introduced with a help of the formulae (9.1)-(9.4). For instance, the contribution of the fig.9.8 diagram has the form*

$$T=(ig)^2\left[\frac{-i}{x_2y_2q^2}\right]\left[\frac{i}{(1-x_3)y_2q^2}\right]\left[\frac{i}{y_2q^2}\right]\left(e_u=\frac{2}{3}\right)\cdot$$

$$\left[\bar{d}\frac{\lambda^a}{2}\gamma_\mu d\right]\left[\bar{u}\gamma_\perp\hat{P}ig\hat{B}_\perp(-y_2\hat{P}')\gamma_\mu\frac{\lambda^a}{2}u\right]= \qquad (9.13)$$

$$-4\pi\alpha_s e_u\frac{\left[\bar{d}\frac{\lambda^a}{2}\gamma_\mu d\right]\left[\bar{u}\gamma_\perp g\hat{B}_\perp\gamma_\mu\frac{\lambda^a}{2}u\right]}{x_2(1-x_3)y_2^2q^4}.$$

Introducing the leading twist pion wave function: $\left(d_\alpha^i\bar{u}_\beta^j\right)\to -i\frac{1}{12}f_\pi\delta^{ij}(\hat{P}'\gamma_5)_{\alpha\beta}\varphi_\pi^A(\zeta')$, $\zeta'=2y_1-1$, one has**:

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* We give the expressions for separate diagrams in the Feynman gauge. The total answer (9.22) is, of course, gauge-invariant.

** Because the additional suppression $\sim 1/Q$ is ensured by a presence of the three-particle $\rho_\perp$ meson wave function, it is sufficient to retain only the leading twist pion wave function φ_π^A.

$$T=-(4\pi\alpha_s)e_u\,\frac{i f_\pi\,\varphi_\pi^A(\zeta')}{9x_2(1-x_3)(1-\zeta')^2q^4}\Big[\bar d\,g\hat B_\perp\hat\gamma_\perp\hat p'\gamma_5 u\Big] \qquad (9.14)$$

Finally, introducing the three-particle $\rho_\perp$- meson wave functions (9.3), (9.4), one obtains the contribution of the fig.9.8 diagram into the form factor $F_{\pi\rho}(Q^2)$:

$$F_{\pi\rho}(Q^2)\Big|_{fig.9.8}=(4\pi\alpha_s)e_u\,\frac{f_\pi f_{3A}}{9Q^4}\int_{-1}^{1}\frac{d\zeta'\,\varphi_\pi^A(\zeta')}{(1-\zeta')^2}\int_0^1 d_3x\,\frac{\varphi_{3\rho}(x)}{x_1x_3(1-x_3)}. \qquad (9.15)$$

All other contributions of this type (fig.9.5 and other) can be calculated analogously.

b) The rest types of contributions which are not accounted yet, correspond in the pert.th. to the diagrams like those shown at figs.9.4 and 9.6 and the two-particle Born diagram, fig.9.9. It is convenient to write their summary contribution in the form:

$$T=(ig)^2\Big[\frac{-i}{x_2y_2q^2}\Big]\Big[\frac{i}{(1-x_1)q^2}\Big]e_u\cdot \qquad (9.16)$$

$$\Big[\bar d\frac{\lambda^a}{2}\gamma_\mu d\Big]\Big[\bar u\gamma_\mu(\hat p'-x_2\hat p+\hat k_1+g\hat B_\perp)\hat\gamma_\perp\frac{\lambda^a}{2}u\Big]+\text{"perm"},$$

where "perm" denotes three analogous contributions. Explicitly shown in (9.16) are the contributions which have the same topology as the fig.9.9 diagram. (For instance, the contributions like fig.9.13 are included therein, because the quark operators in (9.16) are the Heisenberg ones, while those like the figs.9.4, 9.6 enter the "perm").

Consider at first the term $\hat p'$ in the numerator in (9.16).

The leading twist pion wave function φ_π^A gives zero contribution for the term under consideration and hence, the suppression $\sim 1/Q$ will bu due to the pion. It is sufficient, therefore, to keep only the leading twist $\rho_\perp$- meson wave function φ_ρ^T (see (4.13)): $(u^i_\alpha\bar d^j_\beta)\to\frac{1}{12}\delta^{ij}(\hat\rho_\perp\hat p)_{\alpha\beta}f_\rho^T\varphi_\rho^T(\zeta)$, $\zeta=2x_1-1$.

After this, this contribution becomes:

$$-\frac{4\pi\alpha_s e_u}{x_2y_2q^4}\Big[\bar u\frac{\lambda^a}{2}\gamma_\mu\hat p'\hat\gamma_\perp\Big(\frac{1}{12}f_\rho^T\varphi_\rho^T(\zeta)\hat\rho_\perp\hat p\Big)\gamma_\mu\frac{\lambda^a}{2}d\Big]=$$

$$-\frac{4\pi\alpha_s e_u}{(1-\zeta)^2y_2q^4}I_0\,\frac{16}{9}f_\rho^T\varphi_\rho^T(\zeta)\Big[\bar u i\gamma_5 d\Big].$$

Introducing the two-particle pion wave function of the nonleading twist 3 $(\zeta'=2y_1-1)$:

$$\langle\pi^+(p')|\bar u i\gamma_5 d|0\rangle\to f_\pi\frac{m_\pi^2}{m_u+m_d}\varphi_\pi^P(\zeta'),\quad\int_{-1}^{1}d\zeta'\varphi_\pi^P(\zeta')=1, \qquad (9.17)$$

one has finally for this contribution into $F_{\pi\rho}(Q^2)$:

$$-\frac{128}{9Q^4}e_uf_\rho^Tf_\pi\frac{m_\pi^2}{m_u+m_d}\int_{-1}^{1}\frac{d\zeta\,\varphi_\rho^T(\zeta)}{(1-\zeta)^2}\int_{-1}^{1}\frac{d\zeta'\,\varphi_\pi^P(\zeta')}{1-\zeta'}. \qquad (9.18)$$

The terms ΔT in (9.16) which are unaccounted yet, have the suppression $\sim 1/Q$ due to $\rho_\perp$- meson and so, it is sufficient to keep only the leading twist pion wave function $\varphi_\pi^A(\zeta')$. One obtains:

$$\Delta T=i\frac{8\pi\alpha_s e_u}{9q^4}\frac{f_\pi\varphi_\pi^A(\zeta')}{y_2}\Big[\frac{\bar d\hat p'(-x_2\hat p+\hat k_1+g\hat B_\perp)\hat\gamma_\perp\gamma_5u}{(1-x_1)x_2}\Big]= \qquad (9.19)$$

$$i\frac{8\pi\alpha_s e_u}{9q^4}\frac{f_\pi\varphi_\pi^A(\zeta')}{y_2}\Big[\frac{q^2}{x_2}\bar d\hat\gamma_\perp\gamma_5u+\frac{1}{1-x_1}\bar d\Big(\frac{1}{x_2+x_3}-\frac{1}{x_2}\Big)g\hat B_\perp\hat p'\hat\gamma_\perp\gamma_5u\Big].$$

The equations of the motion in the form: $\bar{d} f(x_2)(-x_2\hat{p}+\hat{k}_1)= -\bar{d} g \hat{B}_\perp f(x_2+x_3)$ were used in obtaining (9.19), where f is an arbitrary function and x_1, x_2 and x_3 are understooded as the operators $x_{1,2,3}=\left(\frac{2p'_\mu}{Q^2} i D_\mu^{(1,2,3)}\right)$ acting on the fields $u, \bar{d}$ and $B_\perp$ respectively.

Introduce now the $\rho_\perp$- meson two-particle wave functions of the twist 3:

$$\langle 0|\bar{d}\gamma_\mu^\perp u|\rho_\perp(p)\rangle \to f_\rho^V m_\rho \varphi_\rho^{V,\perp}(\zeta), \quad \zeta = x_1 - x_2 = x_u - x_d,$$

$$\langle 0|\bar{d}\gamma_\mu^\perp \gamma_5 u|\rho_\perp(p)\rangle \to \frac{i}{q^2} f_\rho^V m_\rho e_{\mu\nu\lambda\sigma} p_\nu p'_\lambda \rho_\sigma^\perp \frac{1}{\zeta}\varphi_\rho^A(\zeta), \tag{9.20}$$

$$\int_{-1}^{1} d\zeta\, \varphi_\rho^{V,\perp}(\zeta) = \int_{-1}^{1} d\zeta\, \varphi_\rho^A(\zeta) = 1,$$

$$\varphi_\rho^{V,\perp}(\zeta) = \varphi_\rho^{V,\perp}(-\zeta), \quad \varphi_\rho^A(\zeta) = \varphi_\rho^A(-\zeta).$$

The normalization condition for $\varphi_\rho^{V,\perp}$ is the definition of the constant $f_\rho^V \equiv f_\rho \simeq 200\,\text{MeV}$ (see ch.4), while the normalization condition for φ_ρ^A follows from the equations of the motion. Using the definitions (9.20), (9.3) and (9.4), one has:

$$\Delta T = - I_0 \frac{16\pi\alpha_s}{9 q^4} e_u f_\pi \int_{-1}^{1} \frac{d\zeta'\, \varphi_\pi^A(\zeta')}{1-\zeta'} f_{3A} \times$$

$$\left\{ f_\rho^V m_\rho \int_{-1}^{1} \frac{d\zeta\, \varphi_\rho^A(\zeta)}{\zeta(1-\zeta)} + \int_0^1 d_3x \frac{\varphi_{3\rho}(x)}{x_u(1-x_d)^2} \right\}.$$

Proceeding in an analogous way and adding up all contributions, one obtains finally ($e_u+e_d = 1/3$):

$$F_{\pi\rho}(Q^2) \to (e_u+e_d)\frac{64\pi\alpha_s}{9}\frac{f_\pi f_\rho^V m_\rho}{Q^4}\left[\left(\Sigma_2^\rho - \Sigma_3^\rho\right) + \left(\Sigma_2^\pi - \Sigma_3^\pi\right)\right],$$

$$\Sigma_2^\rho = \int_{-1}^{1} \frac{d\zeta\, \varphi_\rho^{V,\perp}(\zeta)}{1-\zeta^2} \int_{-1}^{1} \frac{d\zeta'\, \varphi_\pi^A(\zeta')}{(1-\zeta')^2} + \int_{-1}^{1} \frac{d\zeta\, \varphi_\rho^A(\zeta)}{1-\zeta^2} \int_{-1}^{1} \frac{d\zeta'\, \varphi_\pi^A(\zeta')}{(1-\zeta'^2)^2},$$

$$\Sigma_2^\pi = 2\frac{f_\rho^T}{f_\rho^V}\frac{m_\pi^2}{(m_u+m_d)m_\rho} \int_{-1}^{1} \frac{d\zeta\, \varphi_\rho^T(\zeta)}{(1-\zeta)^2} \int_{-1}^{1} \frac{d\zeta'\, \varphi_\pi^P(\zeta')}{1-\zeta'},$$

$$\Sigma_3^\rho = \frac{9 f_{3A}}{16 f_\rho^V m_\rho}\left\{ \int_{-1}^{1} \frac{d\zeta'\, \varphi_\pi^A(\zeta')}{(1-\zeta')^2} \int_0^1 d_3x\, \varphi_{3\rho}(x)\left[\frac{1}{x_u(1-x_d)^2} + \frac{1}{9}\frac{1}{x_3(1-x_3)x_u} + \right.\right.$$

$$\left.\frac{1}{9}\frac{1}{x_3(1-x_d)^2}\right] - \frac{1}{9}\int_{-1}^{1} \frac{d\zeta'\, \varphi_\pi^A(\zeta')}{1-\zeta'} \int_0^1 d_3x\, \varphi_{3\rho}(x) \left.\left[\frac{1}{x_3 x_d(1-x_3)} + \frac{4}{x_u(1-x_d)^2}\right]\right\},$$

$$\Sigma_3^\pi = \frac{9 f_\rho^T f_{3\pi}}{8 f_\pi f_\rho^V m_\rho}\left\{ \int_{-1}^{1} \frac{d\zeta\, \varphi_\rho^T(\zeta)}{(1-\zeta)^2} \int_0^1 d_3y\, \varphi_{3\pi}(y)\left[\frac{1}{y_d(1-y_u)^2} + \right.\right.$$

$$\left.\frac{1}{9}\frac{1}{y_3(1-y_u)^2}\right] + \frac{1}{9}\int_{-1}^{1} \frac{d\zeta\, \varphi_\rho^T(\zeta)}{1-\zeta} \int_0^1 d_3y\, \varphi_{3\pi}(y) \left.\frac{1}{y_3 y_d(1-y_u)}\right\},$$

$$\varphi_{3\rho}(x_u,x_d,x_3) = \varphi_{3\rho}^A(x_u,x_d,x_3) - \frac{f_{3V}}{f_{3A}}\varphi_{3\rho}^V(x_u,x_d,x_3). \tag{9.22}$$

It is evident that the asymptotic behaviour of any form factor $\langle \lambda_2=0|J_\mu|\,|\lambda_1|=1\rangle$ can now easily be found by analogous calculations.

Let us now estimate the expression (9.22) numerically (at $Q^2 \simeq 10\ \text{GeV}^2$). The leading twist wave functions have been found above in ch.4: $\varphi_\pi^A(\zeta) \simeq \frac{15}{4}(1-\zeta^2)\zeta^2$, $\varphi_\rho^T(\zeta) \simeq \frac{15}{16}(1-\zeta^2)^2$. The properties of the three-particle wave functions are described below in the sect.9.5. We use for the estimate the asymptotic form of two-particle wave functions of the twist 3 (see sects.9.3 and 9.5): $\varphi^{p}_{\pi}(\zeta)=\frac{1}{2}$, $\varphi^{V,\perp}_{\rho}(\zeta)=\frac{3}{8}(1+\zeta^2)$, $\varphi^{A}_{\rho}(\zeta)=\frac{3}{2}\zeta^2$. Unfortunately, the integrals over the two-particle wave functions entering (9.22) include in this case logarithmic divergences and for this reason their estimates can have uncertainties $\sim 50\%$. We estimate*:

$$\int_{-1}^{1}\frac{d\zeta\,\varphi^{A}_{\pi}(\zeta)}{(1-\zeta)^2}=\int_{-1}^{1}\frac{d\zeta\,(1+\zeta^2)\,\varphi^{A}_{\pi}(\zeta)}{(1-\zeta^2)^2}\simeq 9,\quad \int_{-1}^{1}\frac{d\zeta\,\varphi^{A}_{\pi}(\zeta)}{(1-\zeta^2)^2}\simeq 5,$$
$$\int_{-1}^{1}\frac{d\zeta\,\varphi^{V,\perp}_{\rho}(\zeta)}{1-\zeta^2}\simeq 1.75,\quad \int_{-1}^{1}\frac{d\zeta\,\varphi^{A}_{\rho}(\zeta)}{1-\zeta^2}\simeq 2.0,\quad \int_{-1}^{1}\frac{d\zeta\,\varphi^{p}_{\pi}(\zeta)}{1-\zeta}\simeq 1.3. \qquad (9.23)$$

The dimensional constants entering (9.22) are equal:

$$f_\pi\simeq 133\,\text{MeV},\quad f^{V}_{\rho}\simeq 200\,\text{MeV},\quad f^{T}_{\rho}\simeq 200\,\text{MeV},\quad \frac{m_\pi^2}{(m_u+m_d)\,m_\rho}\simeq 2.3,$$
$$f_{3A}\simeq 0.6\cdot 10^{-2}\,\text{GeV}^2,\quad f_{3\pi}\simeq 0.4\cdot 10^{-2}\,\text{GeV}^2,\quad f_{3V}\simeq 0.25\cdot 10^{-2}\,\text{GeV}^2. \qquad (9.24)$$

As a result, one obtains (at $\bar{\alpha}_s\simeq 0.35$)**:

$$\Sigma^{\rho}_{2}\simeq 25.75,\quad \Sigma^{\rho}_{3}\simeq 22.4,\quad \Sigma^{\pi}_{2}\simeq 14.2,\quad \Sigma^{\pi}_{3}\simeq 10.6,$$
$$F_{\pi\rho}(Q^2)\simeq (e_u+e_d)\,1.2\,\text{GeV}^3/Q^4 = 0.4\,\text{GeV}^3/Q^4, \qquad (9.25)$$
$$\frac{\sigma(e^+e^-\to\rho^\pm\pi^\mp)}{\sigma(e^+e^-\to\pi^+\pi^-)}=\left[\frac{Q F_{\pi\rho}(Q^2)}{F_\pi(Q^2)}\right]^2\simeq\left(\frac{0.85\,\text{GeV}}{Q}\right)^2\simeq 0.075;\quad Q^2\simeq 10\,\text{GeV}^2,$$

(out of resonances).

---

* The first of the integrals in (9.23) appeared above in (5.30) at the description of the D,F mesons weak decays.

** Let us note that the contribution of the wave function $\varphi^{V}_{3\rho}(x)$ into Σ^{ρ}_{3} is very essential, unlike the case of $\Psi\to\rho\pi$ decay.

It is seen from (9.22) and (9.25) that two-particle and three-particle contributions are nearly equál and tend to cansel each other. (Therefore, the estimate of the cross sections ratio in (9.25) sould be considered as an order of magnitude estimate). Hence, the three-particle wave functions give contributions of the same order as the two-particle ones, in spite of the smallness of the scale constants $f_{3A,3V}$ and $f_{3\pi}$ in comparison with $f^{V}_{\rho} m_\rho \simeq 26 f_{3A}$.

Using (9.25) and the relation $F_{\pi\omega}(Q^2)=3F_{\pi\rho}(Q^2)$ one has the estimate:

$$Br\left(\frac{\Psi\to\pi^0\omega}{\Psi\to e^+e^-}\right)=\frac{9}{32}\left(M_\Psi F_{\pi\omega}(M^2_\Psi)\right)^2\simeq\frac{9}{32}\left(\frac{1.2\,\text{GeV}^3}{M^3_\Psi}\right)^2\simeq 0.45\cdot 10^{-3}. \qquad (9.26)$$

The form factor $\gamma\pi^\pm A_2^\mp$ will look like (9.22) with a replacment of the $\rho_\perp$-meson wave functions by the $A_2^\perp$-meson ones and $(e_u+e_d)\to(e_u-e_d)$ (due to an opposite G-parity). To obtain a rough estimate, one can put therefore:

$$Br\left(\frac{\Psi\to\pi^\pm A_2^\mp}{\Psi\to e^+e^-}\right)=\frac{9}{16}\left(M_\Psi F_{\pi A_2}(M^2_\Psi)\right)^2\simeq 0.9\cdot 10^{-3},$$
$$Br\left(\frac{\Psi\to\pi^\pm A_2^\mp}{\Psi\to all}\right)\simeq (0.6-0.7)\cdot 10^{-2}\,\%. \qquad (9.27)$$

Logarithmic divergences of the integrals over two-particle wave functions in (9.22) signal that there will appear additional "non-standard" logs in loop corrections, because the Born hard kernel is too singular at $|\zeta,\zeta'|\to 1$. From the formal viewpoint this means that the standard renormalization group or the LLA approximation are both break down. However, the "improved approximation" described in the sect.3.5 in which "non-leading" internal Sudakov effects are taken into account, remains applicable, and these Sudakov effects suppress the contributions from the regions $|\xi,\xi'|\to 1$.

In practice, however, the power corrections $\sim\left(K_\perp^2+\mu^2/Q^2\right)$ in the denominators of the quark and gluon propagators are more important at not too large Q^2, as compared with the Sudakov effects. Just this correction terms serve as regulators and suppress the contributions from the regions $|\xi,\xi'|\to 1$ at not too large Q^2, while the Sudakov effects are the most important ones at very large Q^2.

Besides, on account for loop logarithmic corrections, the two-particle and three-particle wave functions mix with each other. The experience shows, however, that these mixing effects are negligible. Since the accuracy of calculations is not high at present, we neglect logarithmic effects altogether.

9.3 POWER CORRECTIONS TO THE PION FORM FACTOR

There are two types of the power corrections $\sim 1/Q^4$ to $F_\pi(Q^2)$. First type corrections are due to the pion wave functions $\varphi_\pi^P(\xi)$ and $\varphi_{3\pi}(x)$ of the twist 3 (γ_5 - sector). In this case the total lose $\sim 1/Q^2$ is a result of the loses $\sim 1/Q$ in the initial and the final states sumiltaneously, so that the answer includes a product of two wave functions of the twist 3.

For second type corrections ($\gamma_\mu\gamma_5$ - sector) the total lose $\sim 1/Q^2$ is due to either initial or final meson states, so that the answer includes a product of two wave functions of the twists 2 and 4.

It is not difficult to calculate all corrections in the γ_5-sector, using the methods described in the previous section.*

* It is convenient to use the following relation for the pion wave functions (f is an arbitrary function):

$$\int_0^1 d_2x\langle 0|\bar d_2[\hat P',\hat P]\gamma_5 f(x_1)u_1|\pi(p)\rangle=\int_0^1 d_2x\langle 0|q^2\bar d_2(2x_1-1)f(x_1)u_1|\pi(p)\rangle+$$
$$2\int_0^1 d_3x\langle 0|\bar d_2\hat P' g\hat B_\perp\gamma_5\big(f(x_1)-f(1-x_2)\big)u_1|\pi(p)\rangle. \qquad (9.28)$$

Therefore, we give here the results only. The contribution of the wave function $\varphi_\pi^P(\xi)$ (9.17) into $F_\pi(Q^2)$ has the form in the LLA /9.1/:

$$\Delta F_\pi(Q^2)\to\frac{128}{9q^4}\left(-\frac{2\langle 0|\sqrt{\alpha_s}\,\bar u u|0\rangle}{f_\pi}\right)^2\left|\int_{-1}^{1}\frac{d\xi}{1-\xi}\varphi_\pi^P(\xi,Q^2)\right|^2 \qquad (9.29)$$

(the relation $f_\pi^2 m_\pi^2/(m_u+m_d)\simeq-\langle 0|\bar u u+\bar d d|0\rangle$ was used). The asymptotic form of φ_π^P is /9.1/: $\varphi_\pi^P(\xi,\mu\to\infty)=1/2$. Hence, the integral in (9.29) is sensitive to the region $|\xi|\to 1$, and "non-leading" logs are of importance here at large Q^2 (see the sect. 3.5). Based on the results described in the sect. 3.5, one should expect that on account of "non-leading" log effects the integral in (9.29) is replaced by:

$$I_P=\int_{-1}^{1}\frac{d\xi_1}{1-\xi_1}\int_{-1}^{1}\frac{d\xi_2}{1-\xi_2}\varphi_\pi^P(\xi_1,\sigma^2)\,S(Q^2,\sigma^2)\,\varphi_\pi^P(\xi_2,\sigma^2), \qquad (9.30)$$

$$\sigma^2=\left(\frac{1-\xi_1}{2}\right)\left(\frac{1-\xi_2}{2}\right)Q^2+\mu^2,\quad S(Q^2,\sigma^2)\simeq\left(\frac{\sigma^2}{Q^2}\right)^{C_F\tau},\quad \tau=\frac{1}{b_0}\ln\frac{\alpha_s(\sigma^2)}{\alpha_s(Q^2)}.$$

(see the sect. 3.5). It has been shown in /9.2/ by calculating explicitly the Feynman giagrams, that this is indeed the case.

The properties of the wave function $\varphi_\pi^P(\xi,\mu)$ are not investigated in detail up to now. Using the equations of the motion and the relation (9.28), one can obtain the following relation ($\mu\sim 1\,GeV$, $(m_u+m_d)_{1\,GeV}\simeq 10\,MeV$):

$$f_\pi\frac{m_\pi^2}{m_u+m_d}\int_{-1}^{1}d\xi\left(\xi^2-\frac{1}{3}\right)\varphi_\pi^P(\xi,\mu)=4f_{3\pi}(\mu)\simeq 1.6\cdot 10^{-2}\,GeV^2,$$

$$\langle\xi^2\rangle_\pi^P=\int_{-1}^{1}d\xi\,\xi^2\varphi_\pi^P(\xi,\mu)=\frac{1}{3}+\frac{4f_{3\pi}(m_u+m_d)}{f_\pi m_\pi^2}\simeq 0.33+0.06$$

(the value $f_{3\pi} \simeq 0.4 \cdot 10^{-2}\,\mathrm{GeV}^2$ was used, see the sect. 9.5). It is seen that the value $\langle \xi^2 \rangle_\pi^\rho$ at $\mu \sim 1\,\mathrm{GeV}$ differs only slightly from the asymptotic value $\langle \xi^2 \rangle_{as} = 1/3$. Therefore, we use in (9.30): $\varphi_\pi^\rho(\xi) \simeq \varphi_{as}(\xi) = 1/2$ and estimate (here and below in this section all estimates are for $Q^2 \simeq 10\ \mathrm{GeV}^2$, see the discussion after the formula (9.27): $I_\rho \simeq 1.7$. Taking into account the leading twist contribution (5.16c) and using $\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2 \simeq 1.8 \cdot 10^{-4}\,\mathrm{GeV}^6$, one has from (9.29), (9.30):

$$F_\pi(q^2) \to \left\{ -\frac{0.53\ \mathrm{GeV}^2}{q^2} + \frac{3\ \mathrm{GeV}^4}{q^4} \right\} \quad \text{at } |q^2| \simeq 10\ \mathrm{GeV}^2 \tag{9.31}$$

Hence, this correction is $\simeq 60\%$ of the leading twist contribution, if the wave function (4.10) is used for the leading term.*
In comparison, for instance, with the VDM :

$$F_\pi(q^2)\Big|_{VDM} \simeq \frac{m_\rho^2}{m_\rho^2 - q^2} \simeq \left\{ -\frac{0.6\ \mathrm{GeV}^2}{q^2} - \frac{0.36\ \mathrm{GeV}^4}{q^4} \right\}$$

the correction in (9.31) is surprisingly large.

Using the technique described in the previous section, we have calculated all rest corrections $\sim 1/q^4$ in the γ_5-sector (they all include the three-particle pion wave function $\varphi_{3\pi}(x)$). The qualitative picture here is the same as in the case of $\gamma\rho\pi$ form factor (sect. 9.2): all contributions are of the same order and have different signs. For instance, the contribution of the diagrams like that shown at fig. 9.10, is**:

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* It was supposed in /9.1, 9.2/ that $\varphi_\pi^A(\xi, \mu \sim 1\,\mathrm{GeV}) \simeq \varphi_{as}(\xi) = \frac{3}{4}(1-\xi^2)$ and as a result, the leading twist contribution was estimated as $\simeq -0.15\,\mathrm{GeV}^2/q^2$, instead of $\simeq -0.53\,\mathrm{GeV}^2/q^2$ for the wave function (4.10).

** Of course, this contribution is not gauge-invariant by itself and is given for an illustration only (we use the Feynman gauge).

$$\Delta F_\pi(q^2) = -2\pi\bar{\alpha}_s \frac{|f_{3\pi}|^2}{q^4} \int_0^1 d_3x \frac{\varphi_{3\pi}(x)}{x_2 x_3 (1-x_1)} \int_0^1 d_3y \frac{\varphi_{3\pi}(y)}{y_3} \left[\frac{1}{y_2^2} + \frac{2}{(1-y_1)^2} \right] \simeq$$

$$- (\pi\bar{\alpha}_s)\, \frac{1.2\ \mathrm{GeV}^4}{q^4} \simeq -\frac{1.5\ \mathrm{GeV}^4}{q^4}. \tag{9.32}$$

Let us present also the rough estimate of the power correction from the $\gamma_\mu\gamma_5$ - sector, which is connected with the two-particle wave function of the twist 4. For this purpose, let us return to the leading twist contribution and account partly for the power correction due to quark transverse momenta. The gluon propagator has the form:

$$\frac{1}{\left(\frac{1-\xi_1}{2}\right)\left(\frac{1-\xi_2}{2}\right)q^2 - \Delta} \simeq \frac{1}{\left(\frac{1-\xi_1}{2}\right)\left(\frac{1-\xi_2}{2}\right)q^2} + \frac{\Delta}{\left[\left(\frac{1-\xi_1}{2}\right)\left(\frac{1-\xi_2}{2}\right)q^2\right]^2},$$

$$\Delta = \frac{1-\xi_2}{1-\xi_1}\vec{k}_\perp^{\,2} + \frac{1-\xi_1}{1-\xi_2}\vec{\ell}_\perp^{\,2} - 2\vec{k}_\perp\vec{\ell}_\perp, \tag{9.33}$$

where $\vec{k}_\perp$ and $\vec{\ell}_\perp$ are the transverse momenta of the initial and final quarks. The first term in (9.33) is the leading twist contribution (see (2.12)), the second one is the power correction we are interested in. On account for the rotation symmetry: $\Delta \to 2\left(\frac{1-\xi_2}{1-\xi_1}\right)\vec{k}_\perp^2$ in (9.33). It is shown in the Appendix B that the asymptotic wave function for $\bar{d}\gamma_\mu\gamma_5 D_\perp^2 u$ is: $\varphi_4(\xi_1) = \frac{15}{16}(1-\xi_1^2)^2$, and we use it for the estimate below. The final wave function is the leading twist wave function $\varphi_\pi^A(\xi_1)$, (4.10). Then the correction (9.33) can be written as (the leading term is shown for comparison):

$$\Delta F_{\pi}(q^2) \to -\frac{32\pi\bar{\alpha}_s}{9q^2}|f_\pi|^2\left\{\int\limits_{-1}^{1}\frac{d\xi_1\,\varphi_\pi^A(\xi_1)}{1-\xi_1^2}\int\limits_{-1}^{1}\frac{d\xi_2\,\varphi_\pi^A(\xi_2)}{1-\xi_2^2}+\right.$$
$$\left.\frac{8\langle\vec{K}_\perp^2\rangle_\pi^A}{q^2}\int\limits_{-1}^{1}\frac{d\xi_1(1+3\xi_1^2)\varphi_4(\xi_1)}{(1-\xi_1^2)^3}\int\limits_{-1}^{1}\frac{d\xi_2\,\varphi_\pi^A(\xi_2)}{1-\xi_2^2}\right\}.$$

Using the above wave functions $\varphi_\pi^A(\xi) \simeq \frac{15}{4}(1-\xi^2)\xi^2$ and $\varphi_4(\xi) \simeq \frac{15}{16}(1-\xi^2)^2$ one obtains (see (5.16c), (9.23) and (8.15),

$$F_\pi(q^2) \to \left\{-\frac{0.53\,\mathrm{GeV}^2}{q^2} - \frac{0.8\,\mathrm{GeV}^4}{q^4}\right\}. \qquad (9.34)$$

Therefore (see (9.31), (9.32) and (9.34), all correction terms are of the same order and have different signs. Hence, considerable cancellations are possible, analogously to the case of $\gamma\pi\rho$ form factor (see the previous section). The accuracy of estimates is, however, low and it seems impossible to give more reliable estimates at present.

We have investigated also the corrections $\sim 1/q^4$ to the ρ_L-meson form factor $F_\rho(q^2)$. Analogous estimates show that these corrections are typically $\lesssim$ 20-30% of the leading term (for the wave function (4.19)). It seems, that there are every reasons to expect that the pion and ρ_L- meson form factors can not differ greatly.

Summarizing the results presented in this section, we would say that the question about the magnitude of the power correction $\sim 1/q^4$ to the pion form factor remains unanswered at present. Because considerable cancellations between different terms can not be excluded at present, further investigations are required to elucidate this question.

9.4 CORRECTIONS TO THE $\gamma\gamma\pi^0$ FORM FACTOR

Let us consider the properties of the operator expansion for the form factor $F_{\gamma\pi}(q_1^2, q_2^2)$ in more detail (see (2.22) and (3.46)), /9.3 /. Each derivative $\overleftrightarrow{D}_\mu$ enters the expansion over the local operators in the form: $(2Q_\mu\overleftrightarrow{D}_\mu/Q^2)$, $Q=q_1-q_2$, $P=q_1+q_2$. After the matrix element is taken, $\langle 0|\ldots\overleftrightarrow{D}_\mu\ldots|\pi^0(P)\rangle \sim P_\mu$ and hence, the expansion parameter is: $\omega=(2QP/Q^2)=(q_1^2-q_2^2/q_1^2+q_2^2)$. Therefore, the operators which include derivatives $\overleftrightarrow{D}_\mu$ give no contribution at $q_1^2=q_2^2$, and it is a dimensionality, not a twist, which determines the role of each given operator in this case. This means that not only $x^2 \sim (1/Q^2) \to 0$, but also $xP \to 0$ in the co-ordinate representation, so that we approach to the local limit. As a result, the answer will not include the integrals over the longitudinal momentum fractions x_i of various pion wave functions, but only their values at the origin. This simplifies the problem of a calculation of power corrections to the leading term. The power correction $\sim 1/q^2$ at $q_1^2=q_2^2=q^2$ has been calculated in the paper /8.1/, and the radiative correction $\sim \alpha_s/\pi$ has been obtained in / 9.4 /. The result has the form:

$$F_{\gamma\pi}(q^2,q^2) \to -\frac{\sqrt{2}f_\pi}{3q^2}\left[1-\frac{5}{6}\frac{\alpha_s(q^2)}{\pi}+\frac{8}{5}\frac{\langle\vec{K}_\perp^2\rangle_\pi^A}{q^2}\right] \qquad (9.35)$$

where $\langle\vec{K}_\perp^2\rangle_\pi^A \simeq (323\,\mathrm{MeV}^2) \simeq 0.1\,\mathrm{GeV}$ is the mean square of the quark transverse momentum in the pion, see the sect. 8.1.

At present, the formula (9.35) is the only example of a complete calculation of the power correction $\sim 1/q^2$ to the leading term, and it is seen from (9.35) that the correction is not large.

9.5 NONLEADING TWIST WAVE FUNCTIONS*

9.5.1 A qualitative discussion

The three particle pion and $\rho_\perp$ meson wave functions of the twist 3 have been defined above by the formulae (9.1),(9.2) and (9.3),(9.4). There arise the additional difficulties when one tries to apply the method of the QCD sum rules for the determination of the moment of these wave functions. Let us take, as an example, the pion wave function $\varphi_{3\pi}(x)$ and consider the correlator:

$$I_\rho^{n_1n_2n_3}(q,z) = i\int dx\, e^{iqx}\langle 0|T\, J_\rho^{n_1n_2n_3}(x,z)\, J_\rho^{+\,000}(0,z)|0\rangle = (zq)^{n_1+n_2+n_3+4}\, I_\rho(q^2),\quad z^2=0, \tag{9.36}$$

$$J_\rho^{n_1n_2n_3}(x,z) = \bar u(x)(iz\overleftarrow{D})^{n_2}\left[(iz\overrightarrow{D})^{n_3} g\, G^a_{\mu\nu}(x)\frac{\lambda^a}{2} z_\mu\right]\sigma_{\nu\lambda} z_\lambda \gamma_5 (iz\overrightarrow{D})^{n_1} d(x). \tag{9.37}$$

1. The Born diagram, fig.9.11, has two loops and is very small numerically. For this reason, the scale at which non-perturbative power corrections are $\simeq$ 20% of the Born contribution is, as a rule, large: $\bar M^2 \simeq 3\text{-}5\ GeV^2$. This leads, in its turn, to the low sensitivity of the sum rules to the lowest resonance contribution.

2. When calculating the contributions like those shown at fig.9.12, there appear the vacuum averages of the type: $\langle 0|\bar u \sigma_{\mu\nu}\lambda^a u\, \bar d \sigma_{\mu\nu}\lambda^a d|0\rangle$, for which the factorization hypothesis, seems, doesn't works**.

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* This section is written in collaboration with I.R.Zhitnitsky

** All the earlier considered in this paper four-fermionic vacuum matrix elements were of the form: $\langle 0|\bar u\gamma_\mu(1\pm\gamma_5)\lambda^a u\, \bar d\gamma_\mu(1\pm\gamma_5)\lambda^a d|0\rangle$, and only for such operators the factorization hypothesis was checked, see /1.42/.

3. The spectral density $\mathrm{Im}\, I_\rho^{n_1n_2n_3}(s)$ in (9.36) has the dimension $[M^2]$ and the behaviour $\sim s^1$ at large s, unlike the dimensionless spectral density for the twist 2 operators. Hence, the role of the "background" increases at the treatment of the sum rules, if the scale $\bar M$ is large enough, and this also lowers the sum rule sensitivity to the lowest resonance contribution.

Due to these (and other analogous) difficulties, it becomes very important to make a good choice of the correlator which has a good sensitivity to the lowest resonance. The following trick was used.

Return to the correlator (9.36),(9.37). The large scale of power corrections in this correlator ($\bar M^2 \simeq 3\text{-}5\ GeV^2$, which is mainly due to the fig.9.12 contribution), means that the true spectral density coincides with the smooth asymptotic curve at $s > 5\ GeV^2$ only, while there are appresiable resonance maxima and dips between them at $s < 5\ GeV^2$.

Let us consider now the superposition of the currents with the opposite P-parities: $J_\rho \to J_{\rho+s}$, instead of the current J_ρ in (9.37), replacing in (9.37)

$$\sigma_{\mu\nu}\gamma_5 \to \sigma_{\mu\nu}(\gamma_5 + i). \tag{9.38}$$

It seems at the first sight that we have made matter worse, when adding the opposite parity contributions. In fact, the situation improved essentially. The main power correction, fig. 9.12, is zero for the correlator of the mixed-parity currents (9.38). As a result, the difficulty with the possible non-factorization of these contributions disappeared and, moreover, the characteristic scale $\bar M$ decreased essentially. Rest power corrections are now $\simeq$ 20% of the Born contribution at $\bar M^2 \simeq 0.6\text{-}1\ GeV^2$.

From the physical viewpoint, the following had happened. The admixture of opposite-parity contributions filled dips in the spectral density. Hence, the spectral density became much more smooth in the region $1.5 \lesssim s \lesssim 5\ \mathrm{GeV}^2$, and looks now like the asymptotic spectral density beginning from $s \gtrsim 1.5\,\mathrm{GeV}^2$. As a result, we can now parametrize with a better accuracy the spectral density by the standard form: the contribution of one (or two) lowest resonance plus the asymptotic continuum, which starts now at $s_0 \simeq 1.5\,\mathrm{GeV}^2$. Such sum rules have much better sensitivity to the lowest resonance contribution.

9.5.2 QUANTITATIVE RESULTS AND THE MODEL WAVE FUNCTIONS

The π meson matrix element is:

$$\langle 0 | J^{n_1 n_2 n_3}_{P+S}(0,z) | \pi(q) \rangle = (zq)^{n_1+n_2+n_3+2}\, 2 f_{3\pi} \langle x_1^{n_1} x_2^{n_2} x_3^{n_3} \rangle, \quad z^2 = 0,$$

$$\langle x_1^{n_1} x_2^{n_2} x_3^{n_3} \rangle = \int_0^1 d_3x\, x_1^{n_1} x_2^{n_2} x_3^{n_3} \varphi_{3\pi}(x), \quad \langle x_1^0 x_2^0 x_3^0 \rangle = 1,$$

$$\varphi_{3\pi}(x_1, x_2, x_3) = \varphi_{3\pi}(x_2, x_1, x_3). \tag{9.39}$$

The spectral density is taken in the standard form:

$$\frac{1}{\pi} \mathrm{Im}\, I^{n_1 n_2 n_3}_{P+S}(s) = |2 f_{3\pi}|^2 \langle x_1^{n_1} x_2^{n_2} x_3^{n_3} \rangle \delta(s) + \theta(s - s_{n_1 n_2 n_3}) \left[\frac{1}{\pi} \mathrm{Im}\, I^{n_1 n_2 n_3}_{P+S} \right]_{\text{pert.th.}}, \tag{9.40}$$

where the first term is the pion contribution and the second one is the asymptotic contribution determined by the Born diagram, fig.9.11, the free parameters $s_{n_1 n_2 n_3}$ determine the duality intervals.

The sum rules have the form $\left(\bar{\alpha}_s(\bar{M} \simeq 1\,\mathrm{GeV}) \simeq 0.3,\ \ln \bar{M}^2/\mu^2 \simeq 3\right)$:

$$(\bar{\alpha}_s)^{-1} |f_{3\pi}|^2 \langle x_1^{n_1} x_2^{n_2} x_3^{n_3} \rangle = \frac{M^4}{720\pi^3} \left\{ 1 - e^{-H}(1+H) \right\} A_{n_1 n_2 n_3} + \frac{(\ln M^2/\mu^2)}{810\pi} \langle \frac{\alpha_s}{\pi} G^2 \rangle B_{n_1 n_2 n_3} + \frac{\langle \sqrt{\alpha_s}\, \bar{u}u \rangle^2}{27 M^2} C_{n_1 n_2 n_3}, \quad H = \frac{s_{n_1 n_2 n_3}}{M^2}, \tag{9.41}$$

and the values of the coefficients $A_{n_1 n_2 n_3}$, $B_{n_1 n_2 n_3}$ and $C_{n_1 n_2 n_3}$ are given in Table 9.1.

Don't going into further details, we give in Table 9.2 the results which follow from these sum rules (the mean normalization point of the moment values and $f_{3\pi}$ is $\bar{\mu} \simeq 1\,\mathrm{GeV}$). For a comparison, the moments of the asymptotic wave function, $\varphi_{as}(x) = 360 x_1 x_2 x_3^2$ are also presented (x_1 and x_2 are the momentum fractions carried by the quarks and x_3 carried by the gluon). It is seen from Table 9.2 that the mean momentum fractions carried by quark are larger and that by the gluon is considerably smaller than the corresponding values for $\varphi_{as}(x)$.

As usually, the model wave function is chosen as the linear combination of few lowest polinomials

$$\varphi_{3\pi}(x, \mu \simeq 1\,\mathrm{GeV}) = \varphi_{as}(x) \left[a(x_1^2 + x_2^2) + b x_3^2 + c x_3 + d \right].$$

The wave function $\varphi_{3\pi}(x)$ we have used above in this chapter, has the form ($\varphi_{as}(x) = 360 x_1 x_2 x_3^2$):

$$\varphi_{3\pi}(x, \mu \simeq 1\,\mathrm{GeV}) = \varphi_{as}(x)\, 14 \left[1.5(x_1^2 + x_2^2) + 4.5 x_3^2 - 3.72 x_3 + 0.38 \right], \tag{9.42}$$

and the values of its moments are also presented in Table 9.2.

The method of the QCD sum rules gives a possibility to determine not only the absolute values, but the relative signs of various dimensional constants as well, using suitable non-diagonal correlators. All the signs were determined in this way:

$$(f_{3\pi}/f_\pi) > 0, \quad (f_{3A}/f_\rho) > 0, \quad (f_{3V}/f_\rho) > 0, \quad (f_\rho^T/f_\rho) > 0.$$

The properties of the ρ_L- meson three-particle wave functions $\varphi^A_{3\rho}(x)$, $\varphi^V_{3\rho}(x)$ (see (9.3), (9.4)) have been investigated by analogous methods, and the results are presented in Tables 9.3 and 9.4. We have used above in this chapter the following model wave functions:

$$\varphi^{A}_{3\rho}(x,\mu\simeq 1\,\mathrm{GeV})=\varphi_{as}(x)\,14\left[-0.75(x_1^2+x_2^2)+5.25x_3^2-5.7x_3+1.55\right], \quad (9.43)$$

$$\varphi^{V}_{3\rho}(x,\mu\simeq 1\,\mathrm{GeV})=\varphi_{as}(x)\,7\left[(x_1-x_2)(7-15x_3)\right], \quad (9.44)$$

and the values of their moments are also given in Tables 9.3 and 9.4.

The two-particle pion and $\rho_{\perp}$- meson wave functions of the twist 3 are determined by the formulae (9.17) and (9.20). The asymptotic form of these wave functions (and the three-particle ones above) can easily be found using the method described in the Appendix B:

$$\varphi^{P}_{\pi}(\zeta,\mu\to\infty)=\frac{1}{2}\,;\quad \varphi^{V,\perp}_{\rho}(\zeta,\mu\to\infty)=\frac{3}{8}(1+\zeta^2);\quad \varphi^{A}_{\rho}(\zeta,\mu\to\infty)=\frac{3}{2}\zeta^2. \quad (9.45)$$

It is shown above in (9.31) that $\varphi^{P}_{\pi}(\zeta,\mu\simeq 1\,\mathrm{GeV})$ is somewhat wider than $\varphi^{P}_{\pi}(\zeta,\mu\to\infty)$,but the difference is not large. The preliminary investigation of the properties of $\varphi^{V,\perp}_{\rho}(\zeta,\mu\simeq 1\,\mathrm{GeV})$ and $\varphi^{A}_{\rho}(\zeta,\mu\simeq 1\,\mathrm{GeV})$ shows that these wave functions are also somewhat wider then their asymptotic forms. The difference is mild,however,and so we have used their asymptotic forms for the estimates above in this chapter.

9.7 CONCLUSIONS

The problem of the calculation of non-leading twist proceses and corresponding wave functions is much more complicated in comparison with those of a leading twist. The methods for obtaining the corresponding operator expansions were described above in this chapter and a number of applications was considered. As a result,the following was elucidated.

1. The contributions of three-particle wave functions play,in general,an essential (and sometimes dominant) role in non-leading twist processes. For instance,the $\Psi\to\rho\pi$ decay amplitude

is dominated by the contributions of just three-particle wave functions.

The question arises:how can this property be combined with a success of the "valence quark model" in a description of the hadronic spectroscopy? The answer is as follows. Three-particle wave functions have a much smaller scale (i.e. the values at the origin) than two-particle ones,the additional smallness is $\lesssim (f_\pi/m_\rho)$. Hence,if there are no enhancement factors, the role of three-particle (and,all the more,of many-particle) wave functions is small. This is the case,for instance,for the low energy hadronic spectroscopy,for the deep inelastic structure function $F_2(x)$ at $(1-x)\ll 1$,etc. The three-particle wave functions enter,however, the asymptotic expressions for exclusive processes through the integrals of the form: $\int_0^1 d_3x\,\varphi_3(x)/\prod_{i=1}^{3} x_i^{k_i}$, $\int_0^1 d_3x\,\varphi_3(x)=1$. The mean longitudinal momentum fractions,$\bar{x}_i$,are $\bar{x}_i\simeq 1/3$ here,while $\bar{x}_i \simeq 1/2$ for two-particle wave functions and,moreover,the number of x_i in the denominator (i.e. the number of propagators) is larger here in comparison with the corresponding integrals over two-particle wave functions. These are the enhancement factors which can compensate for the smallness $\sim f_\pi/m_\rho$. Such a situation we observe when considering the $\gamma\pi\rho$ form factor. The three-particle contributions can even become dominant,if the two-particle ones are suppressed for some reason. This is just the case for the $\Psi\to\rho\pi$ decay.

2. Unfortunately,the precise calculation of the power corrections $\sim 1/q^2$ to the leading twist contributions is lacking at present. The difficulties are connected with an appearance of the wave functions of the twist 4,with the logarithmic divergences of integrals over the non-leading twist wave functions,

with possible cancellations between separate contributions, etc. All this complicates the accurate estimates of power corrections. Nevertheless, the arguments and the estimates presented in this chapter and in ch.8 indicate, from our point of view, that power corrections constitute $\lesssim 30\%$ of the leading twist contributions at $Q^2 \simeq 10\ \mathrm{GeV}^2$ (if the latter are not suppressed for some reason).

3. The method of QCD sum rules allows one to find not only the values of wave functions at the origin and the realistic model wave functions, but all relative signs between various wave functions of the same hadron as well. The knowledge of relative signs is of principal importance, if there are various interfering contributions into a amplitude. For instance, the change of the relative signs of the three- and two-particle π and $\rho_\perp$ wave functions leads to the increase of $\sigma(e^+e^- \to \pi\rho)$ and $\Gamma(\Psi \to \pi\omega)$ (see (9.25),(9.26)) by a factor ~ 100 , and this looks unrealistic. Moreover, changing for instance the relative sign of $f_{3\pi}$ and f_{3A} , see (9.2),(9.4), one obtains the value of $\Gamma(\Psi \to \rho\pi)$ ~ 100 times smaller than the experimental value.

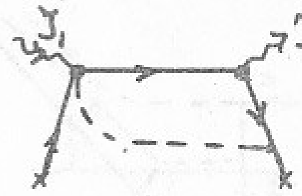
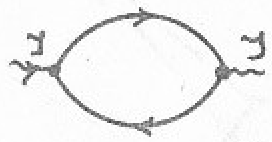


fig. 8.1



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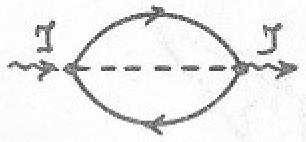

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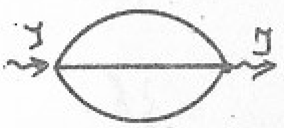

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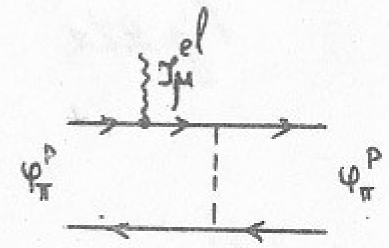

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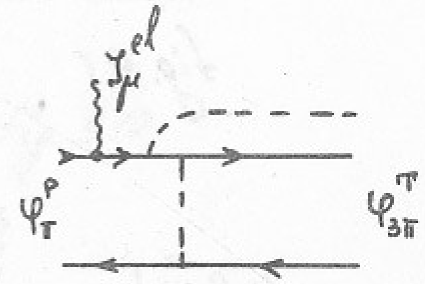
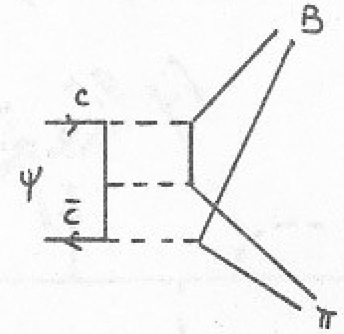


fig 8.6



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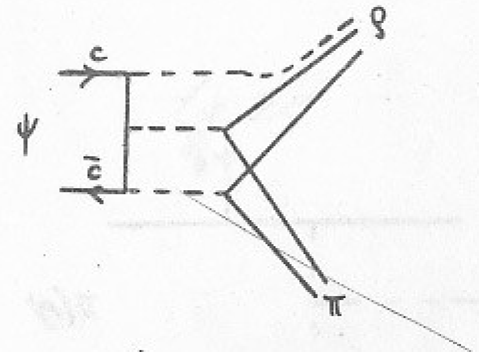

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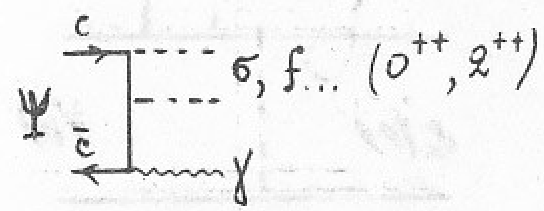

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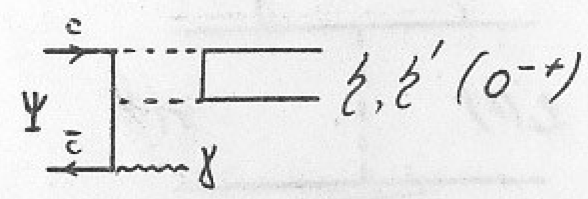
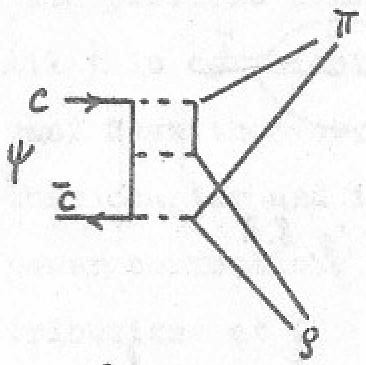


fig. 8.10



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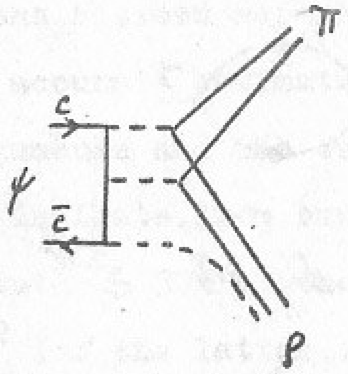

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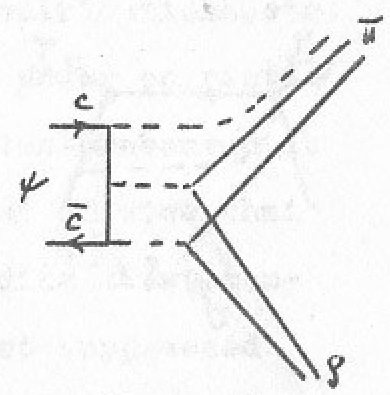
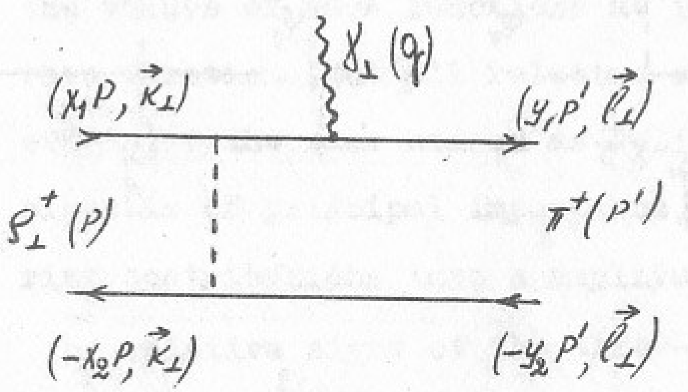


fig. 9.2b



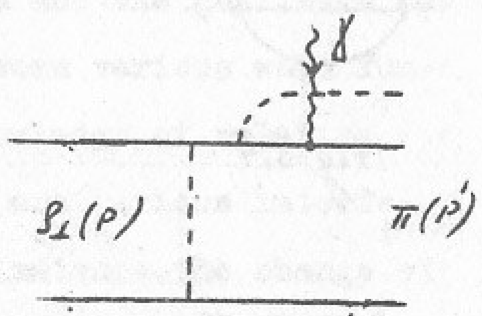
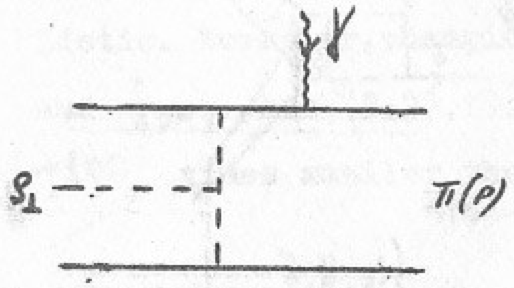


fig. 9.4



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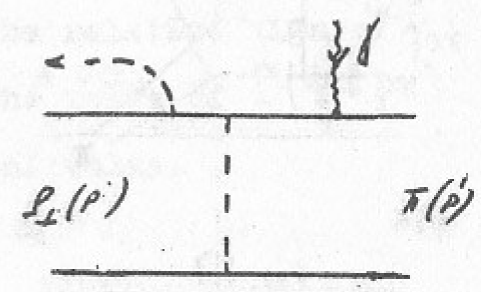

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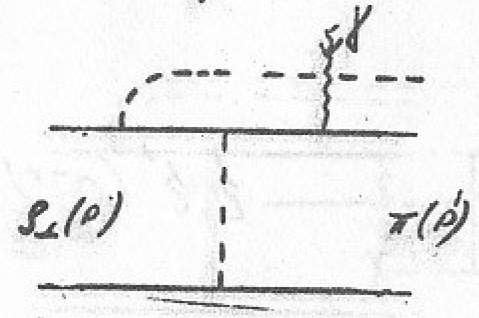

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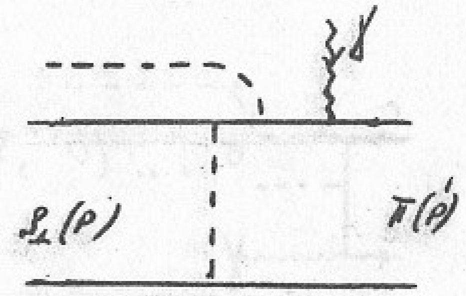

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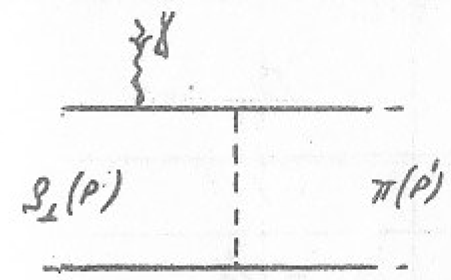
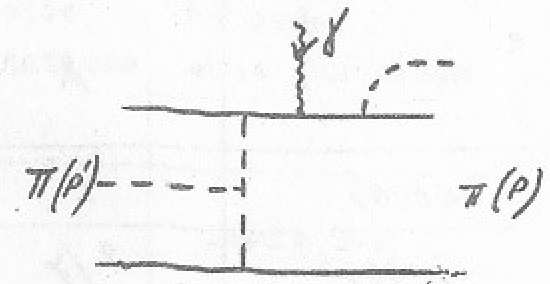


fig. 9.9



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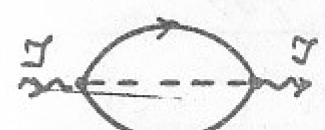

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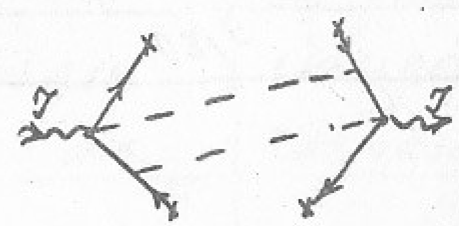

fig. 9.12

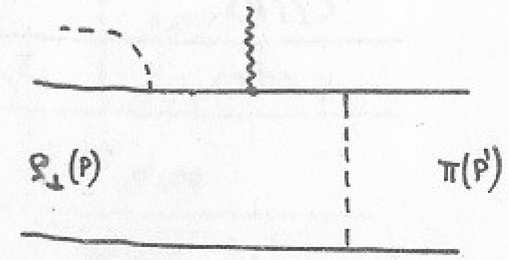

fig. 9.13

Table 9.1

	A	B	C
⟨000⟩	1	5/2	−1
⟨100⟩	2/7	17/16	−1/3
⟨001⟩	3/7	3/8	−1/3
⟨200⟩	3/28	23/40	−17/108
⟨002⟩	3/14	3/20	−1/10
⟨110⟩	1/14	3/8	−8/135
⟨101⟩	3/28	9/80	−7/60

	Result of sum rules	Moments of the asymptotic wave functions	Moments of the model wave functions
			Table 9.2
$f_{3\pi}$	$+(0.3\div0.4)10^{-2}\,GeV^2$		
⟨100⟩	0.34÷0.44	2/7 ≃ 0.29	0.44
⟨001⟩	0.06÷0.22	3/7 ≃ 0.43	0.12
⟨200⟩	0.16 ÷ 0.26	3/28 ≃ 0.11	0.26
⟨002⟩	0.04 ÷ 0.15	3/14 ≃ 0.21	0.04
⟨110⟩	0.12 ÷ 0.19	1/14 ≃ 0.07	0.14
⟨101⟩	0.02 ÷ 0.06	3/28 ≃ 0.11	0.04
			Table 9.3
f_{3A}	$+(0.5\div0.6)10^{-2}\,GeV^2$		0.4
⟨100⟩	0.34 ÷ 0.4	2/7 ≃ 0.29	0.4
⟨001⟩	0.11 ÷ 0.22	3/7 ≃ 0.43	0.2
⟨200⟩	0.16 ÷ 0.22	3/28 ≃ 0.11	0.2
⟨002⟩	0.08 ÷ 0.12	3/14 ≃ 0.21	0.1
⟨110⟩	0.12 ÷ 0.16	1/14 ≃ 0.07	0.15
⟨101⟩	0.02 ÷ 0.05	3/28 ≃ 0.11	0.05
			Table 9.4
f_{3V}	$+(0.2\div0.3)10^{-2}\,GeV^2$		
⟨101⟩	0.02 ÷ 0.08	1/6 ≃ 0.17	0.08
⟨200⟩	0.35 ÷ 0.45	1/3 ≃ 0.33	0.42

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АСИМПТОТИЧЕСКОЕ ПОВЕДЕНИЕ ЭКСКЛЮЗИВНЫХ ПРОЦЕССОВ В КХД

8. Величина поперечного импульса кварков в адронах. Роль степенных поправок (качественное описание и численные оценки).
9. Процессы неведущего твиста и вычисление степенных поправок.

Препринт
№ 83-107

Работа поступила - 28 июня 1983 г.

Ответственный за выпуск - С.Г.Попов
Подписано к печати 15.08-1983 г. МН 03304
Формат бумаги 60x90 1/16 Усл.2,7 печ.л., 2,2 учетно-изд.л.
Тираж 290 экз. Бесплатно. Заказ № 107.

Ротапринт ИЯФ СО АН СССР, г.Новосибирск, 90