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THEORY AND PHENOMENOLOGY OF THE QCD VACUUM

1. INTRODUCTION

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НОВОСИБИРСК

THEORY AND PHENOMENOLOGY OF THE QCD VACUUM

1. INTRODUCTION

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Abstract

This preprint is the first of the series of papers devoted to discussion of various properties of the QCD ground state, the QCD vacuum. After preface (section 1.1) we present elementary consideration of QCD in general and explain the notations (section 1.2). Section 1.3 is devoted to general discussion of energy density caused by nonperturbative fluctuations. The last two sections 1.4 and 1.5 contain some introductory discussion of two main qualitative properties of QCD vacuum: confinement and spontaneous breakdown of chiral symmetry.

1.1. Preface

The quantum chromodynamics (QCD) was discovered about a decade ago and now it is widely considered to be the true fundamental theory of strong interactions.

Its short history clearly demonstrates the radical changes taking place in character of modern science. Most of important results were obtained by groups of authors, often simultencously in different centers. Leaders appear from time to time, but are soon overtaken by their multiple followers. It is typical that discovery and development of QCD, being among the most fundamental achievements of modern science, is not so far marked by Nobel or other prizes, and it is even rather difficult to name the laureats.

Considering the question more quantitatively, I will take the present work as an example. The list of references contains mainly "current literature" of last 3-5 years, with earlier references only to "classical works". Obviously, only some particular topics are discussed and only some fraction of most important papers is mentioned. Still it contains about 600 works*) with about 10^3 authors, not counting authors of experimental works. Probably no other field have ever gathered so many high level theoreticians, and their collective work is impressive.

With so large ensemble it becomes meaningful to consider some distributions. What is most striking, they are extremely

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*) I feel uneasy when think that the number of good works missed by some reason is also counted in hundreds, but hope that their authors will prescribe this only to my limited ability to read and understand papers.

inhomogeneous. The obvious explanation to strong tendency for clustering observed is the interaction between the individuals of attractive type which, like gravity in Universe, gather them in stars and galaxies. The typical scenario looks as follows: some attractive idea captures more and more people and after some critical mass we find well formed cluster (or "club") of people, with very intense information exchange inside it, in the form of preprints, topical meetings etc.

The drawback of such process is the fact that it soon becomes very difficult for a distant observer to tell what is going on inside such group, so for him it looks like some black hole. Another defect of such irregular structure is seen in time perspective. As soon as the progress becomes slower than critical, the cluster suddenly explodes, with typically only small remnant left. As an example to be considered in this work I may mention the instanton theory, extremely popular at late seventies, but nearly stopped around 1980.

Obviously, scientific work can not be organized as that of factory workers, divided proportional to their productivity beforehand with nothing being forgotten. However, scientists are not also a set of gold-diggers. Their final aim is hoped to be some beatiful construction with its parts well fitted to each other, so the information exchange between separate groups is very important. Too much clustering is dangerous - remember canonical example of the Babilon tower.

The main goal of the present work is to provide some overview of various attempts to understand the nonperturbative QCD. In order to make it readable I have included elementary introduction with simplest examples, than comes brief discussion of main ideas, problems and results. Note, that no derivation of the results

is made in the present work. It is not a textbook, the presentation is very brief: it is a kind of status report, exposing to the reader general ideas and problems, as well as rather large list of references.

Another motivation is to emphasize multiple connections between QCD developments and more traditional physics. Twenty years ago all hopes were connected with some "crazy ideas", but in fact most of the methods used in QCD have their analogs, say, in theory of solids and liquids.

In particular, discussion of the correlation function, measured in various scattering processes, as well as their relations to spectroscopy of elementary excitations is widely used in this context - with obvious analogy to QCD sum rules, relating correlators in QCD vacuum to hadronic spectroscopy.

Lattice formulation of QCD have many common points with the theory of spin systems, while the numerical methods used for its studies have been previously developed for classical models of statistical mechanics.

An obvious method of investigation of ordinary matter is its heating up to certain temperature, with measurements of thermodynamical parameters, localisation of phase transitions etc. It is not easy to do so in QCD for necessary energy density is very large, but now such investigations are planned with collisions of heavy ions at ultrarelativistic energies.

Whatever particular method of investigation of some matter is used by some individual, he feels common goals with others studying the same object (say, nucleus, metal or liquid He^4) by different methods. However, situation is similar in QCD, where the unifying concept is <u>the problem of QCD vacuum structure</u>.

At first sight, vacuum is rather exotic object for investigation. However, the experience of macroscopic physics shows that good understanding of the ground state structure implies also natural explanation for many phenomenological facts concerning the excitations -- superconductivity theory being an example. In the absence of good theory of ground state the theory of excitations is essentially phenomenological -- with superfluid He^4 as a good example. And, after all, field theory is just the theory of coupled oscillators in different points, so the question about its ground state wave function can be posed in rather straigtforward manner in lattice formulation of QCD.

Although the power of modern computational methods is very impressive, I think that hopes connected with their straightforward applications to QCD are very naive. It is hardly possible to understand so complicated matter as the QCD vacuum without long process of trying some simplified models and their testing using experimental facts, as generations of physisists did before us. We are just at the beginning of the story, and should not be too proud to consider only theoretical results, following directly from the Lagrangian, but also take into account many (conflicting) experimental facts. In short, the way to theory of the QCD vacuum inevitably goes through its phenomenology - explaining the title of this work.

Finally, few remarks about the contents of other sections of the Introduction, which may well be skipped by more specialized reader. Elementary facts concerning QCD and some notations are given in section 1.2, while section 1.3 is devoted to preliminary discussion of energy density of nonperturbative fluctuations in QCD vacuum. Last two sections 1.4 and 1.5 contain some discussion of confinement and chiral symmetry breaking.

1.2. Quantum chromodynamics

The modern theory of strong interactions, QCD, is quantum field theory describing the interaction of a set of fermions, the quarks, with vector gauge fields which glue them together in hadrons, therefore being called the gluonic ones. Theory of such kind with nonabelian gauge groups were first suggested by Yang and Mills [1.1] . The main idea is the principle of local gauge invariance, which in electrodynamics means that phase of the wave function can be defined in arbitrary way in any space-time point. In nonabelian theories arbitrary phase is generalized to arbitrary rotation in the group of internal symmetry. It is believed now that all interactions in Nature are of such type.

Internal symmetry group of QCD is connected with the specific quantum number of quarks, considered first in Refs.[1.2] and later called "colour" by Gell-Mann. There are three colours of a quark, so the group is $SU(3)_c$.

The basic ideas of QCD are discussed in reviews [1.3] and there is no need to repeat them here. So, apart of some necessory notations, we only make some remarks about the main tendencies of QCD development during its short history.

In condenced notations the QCD Lagrangian looks similar to that of QED, namely $(F = u, d, s \ldots)$

$$\mathcal{L} = -\frac{1}{4}(G^a_{\mu\nu})^2 + \sum_{F,i,j} \bar{\psi}_{F,i} (i \mathcal{D}^{ij}_\mu \gamma_\mu - m_F) \psi_{F,j} \qquad (1.1)$$

but in nonabelian theory field strength is expressed in terms of the potentials as follows

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \qquad (1.2)$$

Here g is the coupling constant and index a counts 8 generators of $SU(3)_c$ rotations, f^{abc} is the so called structure constants of this group.

The covariant derivative in quark part of the Lagrangian contains colour indexes

$$i D_\mu^{i,j} = i \partial_\mu \delta^{ij} + \frac{g}{2} \cdot t^a_{i,j} A^a_\mu \qquad (1.3)$$

where t^a are the Gell-Mann matrixes. It is probably worth to mention that gauge invariance corresponds to transformation of the following form:

$$\psi' = \Omega \psi$$
$$t^a A^{a'}_\mu = \Omega^{-1} (A^a_\mu t^a) \Omega + \frac{2i}{g} \Omega^{-1} \partial_\mu \Omega \qquad (1.4)$$

where $\Omega(x)$ is the SU(3) transformation with arbitrary dependence on space-time point x .

In the next two chapters, devoted to theoretical considerations, we have to use the formalism with imaginary time, so that Minkowski space is substituted by four-dimensional Euclidean one. Unfortunately, only in such formalism we are now able to deal with this complicated problem. This defect makes the predictive power of the theory rather restricted, by its predictions still can be compared to data, see sections 4 and 5. So, we give now explicit relation between Minkowski and Euclidean notations, because this point often leads to misunderstanding. The coordinates and gauge fields are related as follows

$$X^E_m = X^M_m \quad m = 1,2,3 \quad ; \quad X^M_0 = -i X^E_4$$
$$A^E_m = -A^M_m \quad m = 1,2,3 \quad ; \quad A^M_0 = +i A^E_4 \qquad (1.5)$$

Note that if A_μ is changed as X_μ, than one should change sign in all covariant derivatives, which is less convenient. The field strength is changed as follows

$$G^E_{mn} = G^M_{mn} \,, \quad m,n = 1,2,3 \quad ; \quad G^M_{0n} = -i G^E_{4n} \qquad (1.6)$$

while the quark fields are related by

$$\psi^M = \psi^E \,, \quad \bar\psi^M = -i \bar\psi^E \qquad (1.7)$$

with Euclidean gamma matrixes defined as follows

$$\gamma^M_0 = \gamma^E_4 \,, \quad \gamma^E_m = -i \gamma^M_m \quad m = 1,2,3$$
$$\{ \gamma^E_\mu \gamma^E_\nu \}_+ = 2 \delta_{\mu\nu} \quad \mu,\nu = 1,2,3,4 \qquad (1.8)$$

As a result, one obtains the Euclidean action in the form

$$S^E = -i S^M = \int dx \left[\frac{1}{4} (G^E_{\mu\nu})^2 + \bar\psi^E (-i \gamma^E_\mu \mathcal{D}^E_\mu - im) \psi^E \right] \qquad (1.9)$$

We also remind the reader, that in the formalism with functional integrals the vacuum average of any operator $O(A,\bar\psi,\psi)$ is given by the following expression

$$\langle O \rangle = \frac{\int \mathcal{D}A \mathcal{D}\bar\psi \mathcal{D}\psi \exp(iS^M)\, O(A,\bar\psi,\psi)}{\int \mathcal{D}A \mathcal{D}\bar\psi \mathcal{D}\psi \exp(iS^M)} \qquad (1.10)$$

which, in fact, is just symbolic notation, and in order to prescribe to it some well defined meaning a lot of work is needed. Even the question how many degrees of freedom are left when one tries to use gauge invariance is very nontrivial. In perturbative context we have essentially two polarizations of gluons (as those of photons), although even in this case the general gauge leads to formulation with auxillary fields - Faddeev-Popov ghosts

[1.6]. Outside the perturbative context there apear questions concerning uniqueness of the choice of independent degrees of freedom, see e.g. work by Gribov [1.7] . However, there is no need to reduce the number of variables. For example, in lattice formulation to be discussed in chapter 3 the integration is made over all four A_μ , and extra factor - the volume of the gauge group in large power - is canceled in the ratio like (1.10).

Another comment, very important for what follows, is that transition to Euclidean formulation changes the oscillating weight $\exp(iS^M)$ to $\exp(-S^E)$, which is more convenient for calculations and leads to analogies to statistical mechanics, see sect.3.1. There are some questions concerning transition to Euclidean formulation, which we are not going to discuss here. One comment is that in theories with different number of right and left handed fermions (like the Weinberg-Salam model) we even do not know how to perform it, but fortunately in QCD the situation is more safe. Only the meaning of the integration over fermions is changed: $\bar\psi$ and ψ are independent and not related by (absent) complex conjugation.

QCD became the candidate for the role of strong interaction theory when the so called asymptotic freedom [1.9,1.10] was discovered. It is rather specific renormalization behaviour of the charge in nonabelian gauge theories. Up to two-loop effects, the charge at (momentum) scale K is equal to

$$\frac{8\pi^2}{g^2(K)} = b \ln\frac{K}{\Lambda} + \frac{b'}{b} \ln\ln\frac{K}{\Lambda} + O\left(\frac{1}{\ln(K/\Lambda)}\right)$$

$$b = \frac{11}{3}N - \frac{2}{3}N_f \ , \quad b' = \frac{17}{3}N^2 - \frac{13}{6}NN_f + \frac{1}{2}N_f/N \qquad (1.11)$$

where N, N_f are the number of colours (3) and flavours (u,d,...).

At large K or at small distances the charge vanishes, so in this region perturbative calculations make sense. We remind that in QED the trend is quite the opposite, the charge at small distances grows. This principal difference is related to nonabelian nature of gauge fields, due to which they are selfinteracting.

Formula (1.11) contains some parameter Λ , fixing the scale where the interaction becomes strong. It is the most fundamental parameter of QCD and it fixes the scale of all hadronic physics.

Several different definitions of Λ are used in literature. Λ_{MS}, defined by minimal subtraction scheme [1.12], was slightly modified to $\Lambda_{\overline{MS}}$ in [1.13]. See also [1.20], as well as for the definition of Λ_{MOM} . $\Lambda_{\overline{MS}}$ is practically equal to Λ_{PV} , corresponding to Pouli-Villars regularization method. The next popular constant Λ_L is used in lattice formulation, its more precise definition see in [1.16] . Finally, we have included here two nonstandard lambdas, Λ_c and $\Lambda_{e^+e^-}$, which are defined so that they absorb the two-loop corrections to Coulomb forces between heavy quarks [1.14] and to cross section of e^+e^- annihilation into light quarks [1.15]. They all differ just by some constants, approximately equal to $(N=3,\ N_f=4)$

$$\Lambda_{\overline{MS}} \simeq \Lambda_{PV} \simeq 2.67\,\Lambda_{MS} \simeq 0.47\,\Lambda_{MOM} \simeq$$
$$\simeq 39\,\Lambda_L \simeq .6\,\Lambda_c \simeq .62\cdot\Lambda_{e^+e^-} \qquad (1.12)$$

Unfortunately, in spite of much efforts of theorists and experimentalists, the precise value of lambda remains unknown. It is of the order of

$$\Lambda_{\overline{MS}} = (100\ \text{MeV})\cdot 2^{\pm 1} \qquad (1.13)$$

The general reason for this is that perturbative formulae are reliable only when coupling is sufficiently small, so its effect is difficult to measure. Reviews of various applications of perturbative QCD can be found in Refs.[1.18-1.22].

For the present work it is useful to outline from the start where the perturbative analysis becomes applicable. Different applications point toward virtuality region $p^2 > \mu^2$ with $\mu^2 \simeq 1\ \text{GeV}^2$, in some cases to be discussed below - with μ^2 about one order of magnitude larger. Quarks and gluons with such virtuality can reasonably be considered as free ones, with small and calculable corrections.

Field harmonics with smaller virtualities $p^2 \lesssim \mu^2$ make a problem, for (even in Euclidean formulation) we do not know how to treat them. Traditional approach to field theory starting from linear oscillators (Gaussian functional integrals) is not useful. We have to face here the problem with many degrees of freedom coupled in the nontrivial way.

Let us also note, that due to confinement phenomenon (see more in section 1.4) fields at large enough distances are uncorrelated as well. Using momentum representation one may say that there exist finite strip of virtualities

$$\mu^2 \gtrsim p^2 \gtrsim \Lambda^2 \qquad (1.14)$$

with nontrivial dynamics, so important degrees of freedom can safely be localized. Lattice theories are the explicit example, in which lattice spacing $a \lesssim \frac{1}{\mu}$ and lattice size $L \gtrsim \frac{1}{\Lambda}$.

However, straightforward lattice calculation is not the most simple way to understand nonperturbative effects, and historically the problem was first attacked by analytical methods from small distance side. The method used by such an approach is Wilson operator product expansion, which we discuss in greater details in section 4.2. Roughly speaking, this method consider propagation of quarks and gluons in some medium --the QCD vacuum -- affecting the propagation amplitude. If such propagation is evaluated for small enough distances, such effects produce only small corrections which can be computed perturbatively. We have called these effects nonperturbative above, which may lead to misunderstanding. By this we mean that local properties of the medium, say the dispersion of the gluonic field strength fluctuations, are not computed. Their values were extracted from comparison with data, to be considered in chapter 5, by means of the so called QCD sum rule method developed by Shifman, Vainshtein and Zakharov [5.13].

The upper boundary of the strip (1.14) μ is in fact determined as the place where such corrections become noticable, say 30%. It is very fortunate and nontrivial fact that μ is relatively large, so that perturbative series in $\alpha_s(\mu)$ still produce small effect, and discussion in terms of quarks and gluons still makes sense. It is not so at the lower boundary (1.14) $p^2 \sim \Lambda^2$ where we have to consider strong coupling regime explicitly.

Violation of the asymptotic freedom at small distances depends mainly on the <u>general intensity</u> of nonperturbative effects in QCD vacuum, but as soon as we become interested in effects at larger distances we come across the problem of the <u>space-time-colour</u> structure of vacuum fluctuations.

Related questions are much discussed below, and we only comment here that again the main question considers the size of the fluctuations. If they are small, with $\rho \sim \frac{1}{\mu}$, one may use smallness of $\alpha_s(\mu)$ and apply semiclassical theory, to be discussed in chapter 2. If the main fluctuations are of dimensions $\rho \sim \frac{1}{\Lambda}$, we need strong coupling methods like numerical simulations on the lattice, see chapter 3.

The idea that there are two scales of nonperturbative phenomena in QCD, say μ and Λ considered above, seems to appear in several different contexts, for example in consideration of relative importance of spontaneous breakdown of chiral symmetry and confinement [1.38] . As we will discuss in chapter 6, such two scales in vacuum seems to explain the substructure of hadrons in form of "constituent quarks".

In order to awoid misunderstanding it should be emphasized that, generally speaking, only Λ is the fundamental parameter of QCD and therefore all others (including μ) should be proportional to it. However, in some particular cases the numerical coefficients may turns out to be rather large: but we are far from the level of understanding at which such numericals will be properly understood.

1.3. The vacuum energy and vacuum structure

Starting discussion of some quantum system and its ground state it is reasonable to discuss its energy spectrum and, in particular, the energy level of the ground state. In field theory such quantity should be properly defined.

The first obvious point is that vacuum is Lorentz invariant, so its stress tensor $T_{\mu\nu}$ should be the same in all frames. As a result, its vacuum average value can only be of the type

of Einstein cosmological constant

$$\langle 0 | T_{\mu\nu} | 0 \rangle = \varepsilon_{vac}\, g_{\mu\nu}$$

Another well known problem is connected with the fact that field theory has infinitely many degrees of freedom, and all of them have some zero point oscillations. Even for free photon field we have strongly divergent vacuum energy

$$\int \frac{d^3 K\, V}{(2\pi)^3} \left(\frac{K}{2}\right) \sim K_{max}^4 \qquad (1.15)$$

For gluonic field, which is selfinteracting, one has in addition to (1.15) the series in $\alpha_s(K_{max})$.

In both cases we get rid of this infinite energy by "renormalization". Its physical basis is the following: we are not at all interested in the energy of very high frequency modes, for they are never excited in physical processes under consideration. As a result their zero point energy is unimportant additive constant which can be put equal to zero by some convention.

What is different in nonabelian theory as compared to QED, the zero point oscillations of coupled soft modes produce finite energy density of the order of Λ^4 . We call such effects the nonperturbative ones, for in terms of K_{max} they look as

$$K_{max}^4 \exp\left[-\frac{8\pi^2}{b}\frac{4}{g^2(K_{max})}\right]$$

Obviously, such energy density can also be subtracted by definition, so that physical ground state being considered as posessing zero energy. But it does not solve the problem.

First, soft modes are rearranged in excited states of QCD vacuum (say, at large temperatures) and variation of their energy should inevitably be considered. Second (more practical) difficulty is the fact that in this case we do not know what precizely to subtract.

However, vacuum energy density of soft modes (up to some boundary scale μ , called the normalization point) can be determined phenomenologically. In order to understand how it was made we have to consider some more theoretical questions.

It is convenient to start with the theory with massless quarks, so that our Lagrangian contains no dimensional parameters. As a result, at classical level theory becomes <u>scale invariant</u>. Discussion of its general consequences can be found e.g. in [1.27] and we are not going to discuss them in details. By general rules one may find the "dilatational" current using infinitisemal scale transformation

$$x'_\mu = (1+\delta\lambda)\, x_\mu$$
$$A'_\mu = (1-\delta\lambda)\, A_\mu \qquad (1.16)$$
$$\psi' = (1-\tfrac{3}{2}\delta\lambda)\, \psi$$

and calculating the variation of the action

$$\delta S \equiv \delta\lambda \int dx\, \partial_\mu j^{dil}_\mu \qquad (1.17)$$
$$j^{dil}_\mu = x_\nu T_{\mu\nu}$$

where $T_{\mu\nu}$ is the stress tensor. Classical invariance implies the conservation of this current, therefore

$$\partial_\mu j^{dil}_\mu = T_{\mu\mu} = 0 \qquad (1.18)$$

However, in quantum theory scale invariance is violated. The reason is that charge becomes renormalized and explicitly dependent on the scale considered, see e.g. (1.11). As a result, <u>the stress tensor obtains the nonzero trace</u>, which we are going to calculate now following Ref. [1.29] (in perturbative context

this relation was derived previously in the works [1.28] from evaluation of "triangular" diagrams).

It is convenient to change notations so that dependence on the renormalized charge becomes simpler. Therefore we define

$$\bar{A}^a_\mu = g A^a_\mu \; ; \qquad \bar{G}^a_{\mu\nu} = \partial_\mu \bar{A}^a_\nu - \partial_\nu \bar{A}^a_\mu + f^{abc} \bar{A}^b_\mu \bar{A}^c_\nu \qquad (1.19)$$

and rewrite the gauge field action

$$S = -\frac{1}{4}\int dx\, \frac{1}{g^2} (\bar{G}^a_{\mu\nu})^2 \qquad (1.20)$$

The standard account for quantum effects goes as follows. The vector potential $\bar{A}_\mu$ is split into two parts, quantum $\bar{A}^{(q)}_\mu$ and classical $\bar{A}^{(c)}_\mu$, and to calculate the <u>effective action</u> in terms of $\bar{A}^{(c)}_\mu$ by integration over $\bar{A}^{(q)}_\mu$:

$$S_{eff}(\bar{A}^{(c)}) = i \ln \left\{ \int \mathcal{D}\bar{A}^{(q)} \exp\left[i S(\bar{A}^{(c)} + \bar{A}^{(q)}) \right] \right\} \qquad (1.21)$$

If we have succeeded to calculate it in gauge invariant way, the resulting S_{eff} is expressed in terms of gauge invariant operators, $(\bar{G}^{a(c)}_{\mu\nu})^2$ etc. By definition, the coefficient of $(\bar{G}^{a(c)}_{\mu\nu})$ is related to <u>renormalized charge</u> $g^2(K)$(where K is some scale factor depending on G , say $K^2 \sim G$ for constant G etc.)

$$S_{eff} = -\frac{1}{4}\int dx\, \frac{1}{g^2(K)} (\bar{G}^{a(c)}_{\mu\nu})^2 .$$

Again by definition, its dependence on the scale is expressed in terms of the so called Gell-Mann-Low function

$$\beta(g) \equiv \frac{d g(K)}{d \ln K} \qquad (1.22)$$

Now everything is ready for derivation of the final result. Let us calculate the variation of the effective action with respect to infinitisemal scale transformation (1.16) and, comparing the result with general expression (1.17), find the divergence of dilatational current or the trace of the stress tensor

$$\partial_\mu J_\mu^{dil} = T_{\mu\mu} = \frac{\beta(g)}{2g^3}\left(G_{\mu\nu}^a\right)^2 \qquad (1.24)$$

We remind that Gell-Mann-Low function can be computed at small g perturbatively

$$\beta(g) = -\frac{b g^3}{16\pi^2} + O(g^5) \qquad (1.25)$$

and this result is the basis of the asymptotic freedom (1.11).

Simplicity of general derivation of (1.24) is a little bit misleading, for in our formal manipulation we have ignored the fact that both sides of this relation are in fact infinite. This point is more clearly seen if one explicitly calculates the radiative corrections to stress tensor by Feynman diagrams, but it is present in effective Lagrangian (1.21) as well due to existence of quantum fields $A_\mu^{(q)}$ with arbitrary large wave length K. As always, we need renormalization of S_{eff}, which means that modes with $K^2 > \mu^2$ should not be counted. As a result, all quantities are, generally speaking, μ dependent, say the coupling constant g in (1.24) should read $g(\mu)$, etc. (However, it can be proven that the particular combination of $g(\mu)$ and $(G_{\mu\nu}^a)_\mu$ entering (1.24) is in fact μ independent in perturbative context).

The renormalization of relation (1.24) may be considered as some subtraction of infinite terms from both sides, so one may reasonably ask whether we are sure that the remaining finite parts are in fact equal to each other. Considering radiative corrections perturbatively one may really see this order by order, but for the nonperturbative part (producing effect of the order of Λ^4) it is not at all clear.

This point was considered in exactly solvable models, in particular, 1+1 dimensional sigma model at large N was considered by Novikov et al. [4.14] with the result that nonperturbative terms are indeed the same in both sides of (1.24).

If the reader finds our discussion of anomaly relation (1.24) too sketchy, it may be helpful to look first at its perturbative derivation in Ref. [1.28]. There exist also simple way of its derivation in background field formalism, to be discussed in section 4.2. Finally, it may be just skiped by those interested by general line of reasoning rather than details.

What is important, the r.h.s. of relation (1.24) is the so called <u>nonperturbative gluon condensate</u> first introduced and <u>phenomenologically evaluated</u> by Shifman, Vainshtein and Zakharov [5.13], therefore we have numerical estimate of the energy density caused by nonperturbative fluctuations in the QCD vacuum:

$$\varepsilon_{vac} \simeq -\frac{b}{128\pi^2}\langle 0|(gG_{\mu\nu}^a)^2|0\rangle \simeq 0.5 \text{ GeV/fm}^3 \qquad (1.26)$$

Accuracy of this estimate will be discussed in section 8.1. As far as theoretical methods are concerned, we note that they in principle produce result in the units of Λ, very uncertain because of poor accuracy in Λ numerical value. Still it may be useful to comment from the start, that modern variational quasiclassical approach [2.22] seems to underestimate ε_{vac}, while lattice calculations (so far ignoring virtual quarks) overestimate it approximately by one order.

Note the negative sign of $\mathcal{E}_{vac}$ (1.26): it means that nonperturbative vacuum is lower than that given by account for only perturbative phenomena. The absolute value of $\mathcal{E}_{vac}$ set a scale for excitation energy density sufficient for rearrangement of the vacuum structure. More details on related questions can be found in sections 7.2, 7.3.

Obviously, the physical nature of phenomena leading to this energy density is of great interest. We discuss related questions throughout the whole work, with a kind of resumé in section 8.2. Now we are going to make brief discussion of different ideas discussed in current literature.

The most simple type of models was triggered by the observation made by Savvidi [1.50]. Let us consider again the effective action (1.21), including radiative corrections. For constant abelian magnetic field H it is completely analogous to well known Heisenberg-Euler effective lagrangian in QED, with the obvious difference: the charge is renormalized in different way. Simple calculation leads to the following expression for the energy density related to field H

$$\mathcal{E}(H) = \frac{H^2}{2} + \frac{8g^2}{32\pi^2} H^2 \left(\ln \frac{H}{\Lambda^2} - \frac{1}{2} \right) \qquad (1.27)$$

The observation mentioned above is that for small enough H the radiative correction may compensate the original term $H^2/2$ and lead to negative $\mathcal{E}(H)$. Savvidi has concluded that "empty" vacuum is unstable according to creation of some constant field giving the minimum of (1.27).

However, a lot of critical remarks can be made here. First of all, perturbative relation (1.27) is valid only if the second term is relatively small. Second, spatial and colour direction

of the field H^a_m is arbitrary, and in order to have isotropic vacuum some special models were proposed. Third, less trivial remark is the following: constant magnetic field is stable in QED, in contrast to electric field which produces e^+e^- pairs. However, magnetic moment of vector particle (the gluon) is such that lowest state in magnetic field is at negative energy, making the magnetic field to be unstable too.

Somethat similar models were discussed in Ref. [1.51], but the effective potential as a function of scalar combination $(G^a_{\mu\nu})^2$ (rather than $G^a_{\mu\nu}$) was considered. In somethat different context such potential was also discussed in Ref. [1.52].

The common feature of vacuum structure, suggested by these works, is rather simple local field configuration (say, constant quasiabelian field $G^a_{\mu\nu} = n^a F_{\mu\nu}$). As soon as field derivatives are not included, they are assumed to be rather small and inessential. Let us call such picture of the vacuum field <u>the homogeneous vacuum</u>.*)

Quite different picture of the vacuum fields is suggested by the semiclassical methods, to be discussed in chapter 2. Such methods are applicable if vacuum fluctuations have very strong fields, in some sense classical ones, with relatively small quantum fluctuations around them. Different studies of such models, using phenomenology [2.19, 2.20] or variational method [2.22] agree that only few percent of space-time is occupied by the field. Let us call such picture <u>the twinkling vacuum</u>.

*) It will be discussed in section 8.4 that such type of vacuum seems to be present in the limit of large number of colours $N \to \infty$. However, the constant vacuum "master field" is in this case the infinitely large coulour matrix.

These two alternatives will be repeatedly compared below in different contexts, and it can be commented here that both seem to have some truth in them, but at the moment it is difficult to say what type of the fields dominates, say, in vacuum energy density considered above.

Even from this introductory discussion it becomes clear that we are only at the beginning of the long way, leading to understanding of QCD vacuum. In addition, we have not so far mentioned the role of quark degrees of freedom, which are very important (see section 8.3), but are even worse understood.

1.4. Colour confinement

The coloured objects - quarks and gluons - are not observed as physical excited states, in contrast to colourless hadrons. This fact, known as colour confinement is the most striking and famous property of the QCD vacuum. Its mechanism is not so far understood.

There exist examples of field theories in smaller number of space-time dimensions where similar phenomena are found, see e.g. 3-dimensional model in Ref.[2.4] . However, as far as I understand, none of the mechanisms considered can be generalized to the case of QCD.

Completely different approach was suggested by Wilson [3.6] who has shown that confinement is natural in strong coupling $(g(a)\gg 1)$ regime of lattice gauge theories (see chapter 3). This conclusion is not however directly relevant to reality because continious limit of the lattice theory is connected with the opposit case $g(a)\to 0$. So the question was raised about continuity of confinement property when one goes from large to small g.

The next step in this problem was connected with numerical simulations on the lattice, pioneered by Creutz [3.48]. We also discuss these works in chapter 3, and here comment that up to smallest g which can be technically studied at the lattice at present the continuity mentioned above is really present. Moreover, there are strong evidence that with such g we are in fact in the domain of the perturbative renormalization group analysis. So, unless something completely unexpected happens, confinement in nonabelian gauge theories is demonstrated numerically. Evidently, analytic proof is also needed, as well as better understanding.

So far only some phenomenological models are developed, with the aim to include confinement effects in data analysis.

The first is the so called string model, which assumes that quarks are connected by some string with finite tension (energy per unite length), Recently related questions were much discussed in Ref.[1.46-1.49] (see also earlier references therein) in the course of efforts to derive string-like formulation of QCD. We do not discuss this program below for it is not so familiar to the author and, in any case, it is far from being completed.

Another popular model currently used in hadronic physics is the MIT bag model, suggested in Ref.[6.9] . It assumes that some bag (or bubble) is formed around the coloured objects, so that they can not escape from it. As soon as quarks are pulled apart to sufficiently large distance, the bag becomes of the string shape, so these two models are in some way related.

The central idea of the bag model is the existence of some positive energy of the bag, proportional to its volume:

$$E_{bag} = V_{bag} B_{bag} \qquad (1.28)$$

where B_{bag} is some positive constant, also called the vacuum pressure on the bag. At first sight V_{bag} is not well defined, but in such model quark and gluon fields have discontinuity at the boundary, at which the vacuum pressure is balanced.

Some qualitative idea about the origin of this energy was suggested in the works [6.11] . It can be explained using analogy with Meissner effect in superconductores. As it is well known, Cooper pairing of electrons with opposite spins makes the ground state energy lower, but external magnetic field works against it, for it tends to make spins parralel. As a result, magnetic field is expeled from the superconductors or, if strong enough, return the metal into normal state with finite conductivity. Suppose Dirac monopoles exist in nature. Than a pair of monopoles inside superconductors are connected by a string with finite tension, in which one unite of magnetic flux is transfered.

Now, we remind that nonperturbative energy of the QCD vacuum discussed above is also negative. So, the first problem is to demonstrate that applied colour field works against nonperturbative fluctuations. In Refs. [6.11] it was shown that for strong enough field and particular type of fluctuations - the instantons - it is really so. Of course, it is far from being a general case, but the qualitative idea may turn useful.

Our last remark concerning confinement effects is based on the phenomenology and will be repeatedly emphasized below. For some unknown reason confinement effects are weak in natural scale of nonperturbative phenomena. We return to this point in chapter 6, and now only mention one simple observation made in

my work [6.11] . The vacuum pressure B_{bag} needed for quark confinement inside bags (determined from fit to data) is at least one order of magnitude smaller than $\mathcal{E}_{vac}$ (discussed in the preceeding section).

1.5. Spontaneous breakdown of chiral symmetry

This phenomenon is not so famous as confinement, but in theoretical literature it is discussed for much longer period and, I believe, this discussion is more fruitful. The new wave of interest to this phenomenon, SBCS for brevity, was stimulated by QCD sum rules which clearly indicate its large contribution to nonperturbative corrections to correlation functions at small distances. In other terms, this phenomenon seems to be numerically much more important for masses of ordinary hadrons (made of light quarks) than confinement.

In this chapter we are not going to make a detailed discussion of the problem: its relation to various theoretical approaches and phenomenology will be discussed below. Here we only provide some elementary introduction, probably necessary for some of the readers.

Let us start from general discussion of chiral symmetry and its relation to QCD. Assume that some theory is considered, with N_f fermions with identical interaction and masses. Obviously, $U(N_f)$ symmetry is present in this case, for axis in flavour space can be chosen in arbitrary manner. Now, let all masses be equal to zero. In this case the symmetry is wider for theories with vector gauge fields: left and right handed fermions are in fact inconnected. As a result the flavour symmetry is doubbled which is also seen as appearance of chiral symmetry with γ_5

matrix in the transformation law, $\psi' = e^{i\alpha\gamma_5}\psi$.

Since the spatial parity transformation exchange the chiralities, the evident consequence of the chiral symmetry is the following: all states of the theory should be parity degenerate. At first sight even this simple point contradicts to what we observe in hadronic world, with natural suspicion that such symmetry has little to do with it.

However, a number of facts shows that u,d and s quarks are very light, so that probably nothing is seriously changed if one consideres the chiral limit in which their masses are put to zero. Their discussion and references to earlier works can be found in reviews [1.32,1.33], and here we just mention some numbers from the latter work [1.33]:

$$m_u = 5.1 \pm 1.5 \text{ MeV}, \quad \frac{m_d}{m_u} = 1.76 \pm .13, \quad \frac{m_s}{m_u} = 19.6 \pm 1.6$$

Note that ratios are known better than the absolute value. We also have to note that quark masses are renormalized by radiative corrections, so the given numbers correspond to the normalization point about 1 GeV.

The resolution of the conflict between light masses and the absence of apparent consequences of the chiral symmetry is made by the following conclusion: chiral symmetry is spontaneously broken in QCD vacuum. AS it is well known from Goldstone theorem and multiple examples, such asymmetric vacuum should have the massless excitations. In QCD they are the pseudoscalar mesons. If massless, they can be added to any state, with the change of the parity without any change of energy; solving the degeneracy problem.

Many consequences of chiral symmetry and SBCS were understood at sixties. This activity is known as discussion of partial

conservation of axial current (PCAC) . Indeed, from the equations of motion one finds

$$i\partial_\mu (\bar\psi \gamma_\mu \gamma_5 \psi) = m\bar\psi\psi \qquad (1.29)$$

and for small quark masses the axial current is nearly conserved.

Using the smallness of quark masses one may consider the so called chiral perturbation theory. One of the most important relations obtained in this way is that for the mass of pseudoscalar meson, being nonzero only due to nonzero quark masses.

$$m_\pi^2 f_\pi^2 = -2(m_u + m_d)\langle 0|\bar\psi\psi|0\rangle \qquad (1.30)$$

which follows from (1.29) and definition of the pion decay constant

$$\langle \pi^+ | \bar d \gamma_\mu \gamma_5 u | 0\rangle = -i p_\mu f_\pi$$
$$f_\pi \simeq 133 \text{ MeV} \qquad (1.31)$$

Important, that the so called quark condensate $\langle 0|\bar\psi\psi|0\rangle \neq 0$ comes into play. Note, that it connects left and right handed quarks, explicitly demonstrating that vacuum is chirally asymmetric. Its value can be evaluated from (1.30) and independent estimates for quark masses.

Not going into discussion of PCAC (see reviews [1.31-1.33]) we just make one comment. Generalization of its results for strange quark (say, of relation (1.30) for the kaon) is not so clear, for m_s is not so small. In chapter 5 we will show that its account can sometimes change the result by 100% (see some further discussion in section 8.3).

Rather serious difficulty for the theory was noticed by Weinberg [1.39] : octet of pseudoscalar mesons π, K, η well satisfies the mass relations following from $SU(3)_f$ breaking Lagrangian, while the nineth member of the multiplet η' is too heavy. This point is known as the U(1) problem, because η' meson is $SU(3)_f$ singlet.

Important step toward the understanding of this problem was made in Refs. [1.23], in which the so called Adler-Bell-Jackiw anomaly was found. It turns out that $SU(3)_f$ singlet axial current is not conserved, as it is suggested by the naive equations of motion (1.29), for in r.h.s. there appears extra term constructed of gluon fields. This point we consider in more details in section 4.2. However, it was not the whole story, for redefinition of singlet axial current was shown to lead to the conserved one.

The next step was made by t'Hooft in his classical work on instantons [2.8] . First, in the presence of instanton-type configurations the redefinition of the current does not help. Second, fermion zero modes (to be discussed in details in chapter 2) produce some instanton-induced interaction between light quarks (their role discussed in section 5.7), which explicitly violates the U(1) symmetry.

Thus, the U(1) problem is (in principle) solved by instantons. Of course, it still remains a great problem to make the instanton calculus quantitative and to calculate the η' mass from first principles.

Now we turn to the problem of SBCS for $SU(3)_f$ chiral symmetry. The classical work by Nambu and Jona-Lasinio [1.34] and subsequent works were driven by the analogy with the theory of superconductivity. At the chiral limit energy states with positive and negative energies do not have a gap in between, like those on the surface of Fermi sphere of a metal. So, it was reasonable to check whether some attractive interaction can qualitatively

change the ground state properties, producing some finite gap, as it takes place in superconductors. The condensate in vacuum can only has vacuum quantum numbers, so the interaction should be connected with scalar operators. The obvious candidate at that time was the interaction of the type

$$\mathcal{L}_{int} = \lambda (\bar{\psi}\psi)^2 \qquad (1.32)$$

By considering the Bethe-Salpeter equation for the bound states and looking for solutions with imaginary energy one may find the conditions for he vacuum stability. It turns out, that at some large enough λ such solutions are really present and SBCS takes place. Note the important difference with the superconductivity problem: here infrared divergence leads to "vacuum" rearrengement at arbitrary small λ .

Now we believe that no fundamental interaction of the type (1.32) exists among quarks, but it can well be some effective interaction generated in QCD vacuum. Opinions differ in current literature about its physical origin.

The first viewpoint is based on perturbative effect -- the one gluon exchange between quark and antiquark. At small enough distances it leads to well known Coulomb attraction (with relativistic corrections), which however is too small to produce the needed effect. Some extrapolations (see e.g. [1.35]) shows that the instability may take place due to one-gluon exchange, but only in the strong coupling domain, where its application is not justified.

The second possibility connects SBCS with instanton-induced interactions. Unlike the U(1) chiral symmetry, the $SU(3)_f$ one under consideration is not violated by t'Hooft interaction directly. However, as it was pointed out in Ref. [2.23,2.24], this interaction provides an attraction in the scalar channel. Some phenomenological estimates for its strength in QCD vacuum to be discussed below shows that they seem to be of necessary order of magnitude and are able to produce SBCS.

Finally, there is the third viewpoint, according to which confinement effects are also important for this phenomenon. We discuss this point in more details in section 7.4, and here only make some general comment on it. There are examples that SBCS can take place without confinement, while confinement seems to imply SBCS(in theories with light quarks).

So, we have a number of candidates for the interaction between quark and antiquark in scalar channel, leading to instability of chirally symmetric vacuum and SBCS. Now it is difficult to get information on their quantitative importance in real world. Lattice numerical calculations show, that account for virtual quarks is badly needed in order to make reliable information, and masses of quarks are not at all some irrelevant parameters, as it was previously believed. As a result, our main source of information is now the QCD sum rules, see chapter 5.

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