ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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THE $g_{\omega\rho\pi}$ COUPLING CONSTANT FROM QCD SUM RULES BY TRIPLE BORELIZATION

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НОВОСИБИРСК

The QCD sum rules method originally suggested in the pioneering paper [1] proved to be very effective for determination of masses and couplings of low lying mesonic [1,2] and barionic [3] states. Two-point functions of various currents were used for that. In refs. [4,5] three-point functions were considered, the sum rules based on a double dispersion relation for these functions were obtained and the pion, ρ - and A_1-meson formfactors were determined. In ref. [6] the analogous approach was undertaken to determine the $g_{\omega\rho\pi}$ coupling constant from QCD sum rules. The Borel transformation was used to handle these sum rules. It was originally proposed in ref. [1] in the case of one variable and generalized in ref. [4] to the case of two variables. The Borel transformation in two variables is convenient when formfactors are calculated. It allows one to suppress high order power corrections in the operator expansion and the contributions of higher resonances in the channels of incoming and outgoing mass-shell particles. It doesn't distort the resonance contributions in the third channel.

when all three particles are considered as lying on mass shell the contributions of higher resonances must be suppressed in all three channels, i.e. it is convenient to use the Borel transformation in three variables. An additional advantage of this procedure, from our viewpoint, is its technical simplicity. In the present paper the triple Borel transformations is used for determination of $g_{\omega\rho\pi}$ coupling constant from QCD sum rules. Earlier this constant was found in ref. [6] with the help of the double Borel transformation. Our result differs from that of ref. [6].

Consider the procedure of the triple borelization of an arbitrary triangular one-loop Feynman diagram (Fig. 1). When calculating this diagram one obtaines integrals of the form

$$I(s_1,s_2,s_3) = \int \Gamma(N)\,\mathcal{D}^{-N} \prod_{i=1}^{3} s_i^{k_i} x_i^{l_i}\, dx_1 dx_2 \qquad (1)$$

Here $\mathcal{D} = x_1 x_2 s_3 + x_1 s_2 x_3 + s_1 x_2 x_3$, $s_i = -q_i^2$ are the kinematic variables, x_i are the Feynman parameters, $x_1 + x_2 + x_3 = 1$. The Borel transform of some function $f(s)$ in variable s is determined as

$$(Bf)(t) = \frac{1}{2\pi i}\int\limits_{c-i\infty}^{c+i\infty} f(s)\exp\frac{s}{t}\,\frac{ds}{t}, \qquad (2)$$

where t is the Borel parameter and the integration contour runs to the right of all the singularities of the function $f(s)$ [1]. We want to calculate $(B_1B_2B_3I)(t_1,t_2,t_3)$. We have

$$(B_1B_2B_3I)(t_1,t_2,t_3) = \prod_{i=1}^{3}\left(-t_i\frac{\partial}{\partial t_i}t_i\right)^{k_i}(B_1B_2B_3I')(t_1,t_2,t_3), \qquad (3)$$

where $I' = \int \Gamma(N)\mathcal{D}^{-N}\prod_{i=1}^{3}x_i^{l_i}dx_1dx_2$. Borelizing I' in s_3 and substituting $x_3x_1^{-1}=u$, $x_3x_2^{-1}=v$ we obtain expression of the form $\int f(u,v)\exp(-us_1t_3^{-1}-vs_2t_3^{-1})du\,dv$. It can be easily borelized in two remaining variables:

$$(B_1B_2B_3I')(t_1,t_2,t_3) = \prod_{i=1}^{3}\lambda_i^{l_i+1}t^{-N}, \qquad (4)$$

where $\lambda_i = t_i(t_1+t_2+t_3)^{-1}$, $t = t_1t_2t_3(t_1+t_2+t_3)^{-2}$. The formulas (3), (4) allow one to perform the borelization in three variables by simply writing out the expressions for diagrams in terms of Feynman parameters and differentiating in t_i, if it is necessary.

Now we turn to calculation of the vertex function of two vector and one axial vector currents

$$A_{\mu\nu\lambda}(q_1,q_2) = -\int d^4x\,d^4y\,\exp(iq_2y-iq_1x)\cdot \langle 0|T\{j_\mu^{(\omega)}(x)j_\nu^{(\rho)}(y)j_\lambda^{5}(0)\}|0\rangle \qquad (5)$$

where $j_\mu^{(\omega)}=\frac{1}{6}(\bar u\gamma_\mu u+\bar d\gamma_\mu d)$, $j_\nu^{(\rho)}=\frac{1}{2}(\bar u\gamma_\nu u-\bar d\gamma_\nu d)$, $j_\lambda^5=\frac{1}{2}(\bar u\gamma_\lambda\gamma_5u-\bar d\gamma_\lambda\gamma_5 d)$.

$A_{\mu\nu\lambda}$ can be represented as the sum of six independent structures [6], which can be chosen as $S_{1\mu\nu\lambda}=\varepsilon_{\mu\nu\lambda\omega}q_{1\omega}$, $S_{2\mu\nu\lambda}=\varepsilon_{\mu\nu\lambda\omega}q_{2\omega}$, $q_{3\lambda}T_{\mu\nu}$, $q_\lambda T_{\mu\nu}$, $q_{3\mu}T_{\nu\lambda}-q_{3\nu}T_{\lambda\mu}$, $q_\mu T_{\nu\lambda}-q_\nu T_{\lambda\mu}$. Here $q_3=q_1-q_2$, $q=q_1+q_2$. So we can write

$$A_{\mu\nu\lambda} = f_{1A}S_{1\mu\nu\lambda}+f_{2A}S_{2\mu\nu\lambda}+f_\rho q_{3\lambda}T_{\mu\nu}\ldots, \qquad (6)$$

where f's are some functions of kinematical variables s_1,s_2,s_3. f_ρ is contributed by 0^{-+} and 1^{++} states in the axial vector channel, whereas f_{1A}, f_{2A} - by 1^{++} states only. Saturating the vector channels by ω and ρ mesons and the axial vector channel - by pion we come to the following model structure function:

$$f_\rho = if_\pi\frac{m_\omega^2}{g_\omega}\frac{m_\rho^2}{g_\rho}g_{\omega\rho\pi}\frac{1}{s_1+m_\omega^2}\,\frac{1}{s_2+m_\rho^2}\,\frac{1}{s_3},$$

$$-i(B_1B_2B_3f_\rho)(t_1,t_2,t_3) = f_\pi m_\omega^2g_\omega^{-1}m_\rho^2g_\rho^{-1}g_{\omega\rho\pi}\cdot t_1^{-1}t_2^{-1}t_3^{-1}\exp(-m_\omega^2t_1^{-1}-m_\rho^2t_2^{-1}) \qquad (7)$$

The constant $g_{\omega\rho\pi}$ is defined by the amplitude of $\omega\to\rho\pi$ transition, $-g_{\omega\rho\pi}\varepsilon_{\mu\nu\alpha\beta}q_{1\mu}q_{2\nu}e_{1\alpha}e_{2\beta}$, where e_1, e_2 are the polarization vectors of ω and ρ, respectively. The constants f_π and g_V, where $V=\omega$ or ρ, are defined by the relations $\langle 0|j_\lambda^5|\pi^0\rangle = if_\pi q_{3\lambda}$, $f_\pi = 93\,\mathrm{MeV}$, $\langle 0|j_\mu|V\rangle = m_V^2g_V^{-1}e_\mu$, where e_μ is the polarization vector of meson V.

Alternatively, f_ρ can be computed with the help of the operator expansion. The ground term is given by the quark loop diagrams of fig. 1. Calculation with the help of formulas (3), (4) leads to

$$-i(B_1B_2B_3f_\rho^{(0)})(t_1,t_2,t_3) = \frac{1}{16\pi^2}\,\frac{t_3(t_1+t_2)+4t_1t_2}{(t_1+t_2+t_3)^3} \qquad (8)$$

In the particular case $t_1=t_2$ this expression can be derived from (8) of ref. [6] by means of borelization in $Q^2=s_3$.

Gluonic corrections are computed in the fix point gauge,

$x_\mu A_\mu(x)=0$. It was first introduced by Shwinger [7] for the motion of particles in the arbitrary external field and used in ref. [4] for the calculation of QCD power corrections in the case of three-point functions. Choosing the coordinate origin in the axial vector vertex we obtain the following expressions for each of four diagrams presented in figs. 2a-2d:

$$
\begin{aligned}
-iB_1B_2B_3 f_\rho^{(2a)} &= \langle \frac{\alpha_s}{\pi}G^2\rangle \frac{1}{288}\Big[\frac{t_1+t_2}{t_1t_2t_3^2}-\frac{(t_1-t_2)^2}{(t_1+t_2+t_3)t_1t_2t_3^2}\Big],\\
-iB_1B_2B_3 f_\rho^{(2b)} &= \langle \frac{\alpha_s}{\pi}G^2\rangle \frac{1}{288}\Big[-\frac{1}{t_1t_2t_3}+\frac{(t_2-t_1)(t_2-t_3)}{(t_1+t_2+t_3)t_1t_2^2t_3}\Big],\\
-iB_1B_2B_3 f_\rho^{(2c)} &= \langle \frac{\alpha_s}{\pi}G^2\rangle \frac{1}{288}\Big[-\frac{1}{t_1t_2t_3}+\frac{(t_1-t_2)(t_1-t_3)}{(t_1+t_2+t_3)t_1^2t_2t_3}\Big],\\
-iB_1B_2B_3 f_\rho^{(2d)} &= \langle \frac{\alpha_s}{\pi}G^2\rangle \frac{1}{288}\Big[-\frac{4}{t_1t_2t_3}-\frac{t_1+t_2}{t_1t_2t_3^2}+\\
&\quad+\frac{(t_1t_2-t_3^2)(t_1-t_2)^2}{(t_1+t_2+t_3)t_1^2t_2^2t_3^2}\Big].
\end{aligned}
\tag{9}
$$

Adding these contributions we arrive at the simple answer

$$-iB_1B_2B_3 f_\rho^{(g)} = -\frac{1}{48}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{1}{t_1t_2t_3}. \tag{10}$$

This result doesn't agree with that of ref. [6]. So the calculations were repeated with the coordinate origin taken in the vector vertex. Calculations, although more complicated in this nonsymmetrical approach, led to the same answer.

Quark condensate corrections are given by the diagrams of fig. 3 (only those diagrams are shown which aren't nullified by the triple borelization). The result is

$$-iB_1B_2B_3 f_\rho^{(q)} = \frac{2\pi}{27}\alpha_s\langle\bar u u\rangle^2\frac{1}{t_1t_2t_3}\Big(\frac{5}{t_1}+\frac{5}{t_2}-\frac{2}{t_3}\Big) \tag{11}$$

Equating the model and calculated structure functions we come to the following sum rule:

$$
\begin{aligned}
&f_\pi \frac{m_\omega^2}{g_\omega}\frac{m_\rho^2}{g_\rho} g_{\omega\rho\pi}\, exp(-m_\omega^2 t_1^{-1}-m_\rho^2 t_2^{-1}) =\\
&= \frac{1}{16\pi^2}\frac{t_3(t_1+t_2)+4t_1t_2}{(t_1+t_2+t_3)^3}t_1t_2t_3-\frac{1}{48}\langle\frac{\alpha_s}{\pi}G^2\rangle+\\
&+\frac{2\pi}{27}\alpha_s\langle\bar u u\rangle^2\Big(\frac{5}{t_1}+\frac{5}{t_2}-\frac{2}{t_3}\Big).
\end{aligned}
\tag{12}
$$

In principle, one may fit this sum rule varying all three parameters t_1, t_2, t_3 independently. More simple way is to put $t_1=t_2=t_3=t$. Masses, constants [1] and vacuum averages [4] are as follows: $4\pi g_\rho^{-2}=0.41$, $4\pi g_\omega^{-2}=0.046$, $\alpha_s\pi^{-1}\langle G^2\rangle = 0.012\,GeV^4$, $\alpha_s\langle\bar u u\rangle^2 = 8\cdot 10^{-5}GeV^6$, $m_\omega^2 \approx m_\rho^2 \approx 0.6\,GeV^2$.

Then we come to the following sum rule (fig. 4, curve 1):

$$g_{\omega\rho\pi} = 3.74\Big(t^2-0.18+\frac{0.11}{t}\Big)exp\frac{1.2}{t} \tag{13}$$

It is seen from fig. 4 that there is some region at $t \simeq$ 0.80÷0.85 GeV^2 corresponding to $g_{\omega\rho\pi}^{(0)} \simeq$ 10 GeV^{-1} where the curve is close to horizontal one. Gluonic correction reaches 25÷30% of the total result in this region.

Now we shall try to estimate the correction to the obtained value of $g_{\omega\rho\pi}$ due to the A_1 meson contribution in the axial vector channel. To do this, write down the sum rules for f_{1A}, f_{2A} and f_ρ entering (6). The amplitude of $\omega\to\rho A_1$ transition has the form $M_{\omega\rho A_1} = -g_1 e_{1\alpha}e_{2\beta}\cdot e_{3\gamma}\, i\varepsilon_{\alpha\beta\gamma\delta}q_{1\delta} - g_2 e_{1\alpha}e_{2\beta}e_{3\gamma}\, i\varepsilon_{\alpha\beta\gamma\delta}q_{2\delta}$ with two constants g_1, g_2; $e_{1\alpha}, e_{2\beta}, e_{3\gamma}$ are ω, ρ, A_1 polarizations. Saturating the axial vector channel by π and A_1 contributions one obtaines

$$
\begin{aligned}
-iB_1B_2B_3 A_{\mu\nu\lambda}^{(res)} &= t_1^{-1}t_2^{-1}t_3^{-1} m_\omega^2 g_\omega^{-1} m_\rho^2 g_\rho^{-1} m_A^2 g_A^{-1}\cdot\\
&\cdot\Big[\Big(\frac{1}{2}g_1+\frac{m_\rho^2+m_\omega^2-m_A^2}{4m_\omega^2}g_2\Big)S_{1\mu\nu\lambda}+\\
&+\Big(\frac{1}{2}g_2+\frac{m_\rho^2+m_\omega^2-m_A^2}{4m_\rho^2}g_1\Big)S_{2\mu\nu\lambda}+
\end{aligned}
\tag{14}
$$

$$+\left(\frac{g_1}{4m_\rho^2}+\frac{g_2}{4m_\omega^2}-\frac{g_1+g_2}{m_A^2}\right)q_{3\lambda}T_{\mu\nu}\Big]\cdot$$
$$\cdot\exp(-m_\omega^2 t_1^{-1}-m_\rho^2 t_2^{-1}-m_A^2 t_3^{-1})+$$
$$+t_1^{-1}t_2^{-1}t_3^{-1}m_\omega^2 g_\omega^{-1}m_\rho^2 g_\rho^{-1}f_\pi g_{\omega\rho\pi}q_{3\lambda}T_{\mu\nu}\cdot \qquad (14)$$
$$\cdot\exp(-m_\omega^2 t_1^{-1}-m_\rho^2 t_2^{-1})$$

Further, one obtaines in addition to (8) from the quark loop diagram of fig. 1

$$-iB_1B_2B_3 f_{1A}^{(a)}=\frac{1}{16\pi^2}\,\frac{t_2t_3(t_3-t_2)}{(t_1+t_2+t_3)^3}, \qquad (15)$$
$$-iB_1B_2B_3 f_{2A}^{(a)}=\frac{1}{16\pi^2}\,\frac{t_1t_3(t_3-t_1)}{(t_1+t_2+t_3)^3}.$$

Putting $t_1=t_2=t_V$, $t_3=t_A$ or t_π, $g_1=g_2=g$, $m_\omega^2=m_\rho^2=m_V^2$ and equating $A_{\mu\nu\lambda}^{(res)}$ and theoretical $A_{\mu\nu\lambda}$ one arrives at the following sum rules:

$$m_\omega^2 g_\omega^{-1}m_\rho^2 g_\rho^{-1}m_A^2 g_A^{-1}(1-\tfrac{1}{4}m_A^2 m_V^{-2})g\,t_A^{-1}t_V^{-2}\cdot$$
$$\cdot\exp(-2m_V^2 t_V^{-1}-m_A^2 t_A^{-1})\simeq$$
$$\simeq(16\pi^2)^{-1}t_A t_V\cdot(t_A-t_V)(2t_V+t_A)^{-3}$$
$$m_\omega^2 g_\omega^{-1}m_\rho^2 g_\rho^{-1}m_A^2 g_A^{-1}(-2m_A^{-2})(1-\tfrac{1}{4}m_A^2 m_V^{-2})\cdot \qquad (16)$$
$$\cdot g\,t_\pi^{-1}t_V^{-2}\exp(-2m_V^2 t_V^{-1}-m_A^2 t_\pi^{-1})+$$
$$+m_\omega^2 g_\omega^{-1}m_\rho^2 g_\rho^{-1}f_\pi g_{\omega\rho\pi}t_\pi^{-1}t_V^{-2}\exp(-2m_V^2 t_V^{-1})\simeq$$
$$\simeq(16\pi^2)^{-1}2t_V(2t_V+t_A)^{-2}$$

Choosing the following typical scales $t_\pi=t_V=0.6\div 0.8\,GeV^2$, $t_A=m_A^2=1.44\,GeV^2$ we obtain the (positive) correction 9÷10% to $g_{\omega\rho\pi}$ on account of A_1 meson contribution. Roughly speaking, the residue of A_1

pole in $f_\rho^{(res)}$ is twice as small as compared to that of π pole.

As for the $\rho'(1,6)$ and ω' contributions to $f_\rho^{(res)}$, they are opposite in sign to those of ρ and ω [6], as it follows from asymptotic behaviour of the corresponding formfactors (according to the quark counting rule there are no terms $\sim$(momentum)$^{-2}$ in their asymptotics). To find the upper limit for the effect of ρ', ω' on $g_{\omega\rho\pi}$ put their residues in $f_\rho^{(res)}$ equal to the residue of $\omega\to\rho\pi$ in absolute value. Then, saturating $f_\rho^{(res)}$ by $\omega\rho\pi$, $\omega'\rho\pi$, $\omega\rho'\pi$ and $\omega\rho A_1$ contributions one obtaines the following sum rule (fig. 4, curve 2)

$$g_{\omega\rho\pi}=[1-2\exp(-1.97\cdot t^{-1})-\tfrac{1}{2}\exp(-1.44\cdot t^{-1})]^{-1}g(t) \qquad (17)$$

where $g(t)$ stands for the right hand side of (13). Here A_1 residue is put to be equal to 0.5 of π residue, as it was established before; the values $m_{\rho'}^2\approx m_{\omega'}^2\approx 2.56\,GeV^2$, $m_A^2\approx 1.44\,GeV^2$ were used. Arrow A depicted in fig. 4 indicates t for which both the leading (gluonic) power correction and continuum contribution reach 30% of the total. We may choose the best value of $g_{\omega\rho\pi}$ as the middle point between the curves 1 (no continuum) and 2 (maximal effect of continuum) at this t and write down $g_{\omega\rho\pi}\simeq 12\,GeV^{-1}$ with the accuracy 30%.

It is worth noting that the experimental restriction on $g_{\omega\rho'\pi}$ can be extracted from the $\rho'\to\pi^+\pi^-\pi^0\pi^0$ decay width $\Gamma(\rho'\to\pi^+\pi^-2\pi^0)<\Gamma(\rho'\to 4\pi)\simeq$ 200÷400 MeV (the author is indebted to I.B.Khriplovich who pointed out this circumstance). In turn, $\Gamma(\rho'\to\omega\pi)<\Gamma(\rho'\to\pi^+\pi^-2\pi^0)$ (in fact, $\rho'\to A_1\pi$ is probably the leading mechanism [8]). So we get 5 GeV^{-1} as upper boundary for $g_{\omega\rho'\pi}$. Corresponding correction to $g_{\omega\rho\pi}$ on account of ρ' will be $\lesssim$ 5% in our calculations. However, there possibly exists some structure ($\rho(1.25)?$) at $\simeq$ 1.25 GeV in $\omega\pi$ channel with $\Gamma(\rho(1.25)?\to\omega\pi)\simeq 200\,MeV$ [8], so $g_{\rho(1.25)?\omega\pi}\simeq$ 10 GeV^{-1}. It can well contribute to the ρ-channel giving rise to the correction $\sim$ 30% to $g_{\omega\rho\pi}^{(0)}$ in the framework of our approach.

We present here the experimental values of $g_{\omega\rho\pi}$ used in ref. [6] (the superscript indicates the decay from which the value of $g_{\omega\rho\pi}$ was extracted): $g^{(\omega\to 3\pi)}_{\omega\rho\pi} = 15\,GeV^{-1}$, $g^{(\omega\to\pi^0\gamma)}_{\omega\rho\pi} = 14\,GeV^{-1}$, $g^{(\rho^0\to\pi^0\gamma)}_{\omega\rho\pi} = 12\,GeV^{-1}$, $g^{(\pi^0\to 2\gamma)}_{\omega\rho\pi} = 12\,GeV^{-1}$ where $g^{(\omega\to\pi\gamma)}_{\omega\rho\pi}$ and $g^{(\rho^0\to\pi^0\gamma)}_{\omega\rho\pi}$ are about 20% underestimated due to neglect of higher resonances in VDM. The value obtained $g_{\omega\rho\pi} \simeq$ $\simeq 12$ GeV^{-1} agrees well with these data. It doesn't agree with the result of ref. [6] (17 GeV^{-1}). This disagreement is connected mainly with the difference in the formulas for the gluonic correction, in particular, with the difference in their signs.

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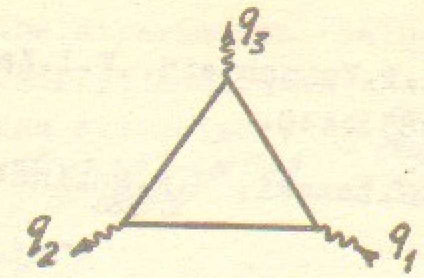
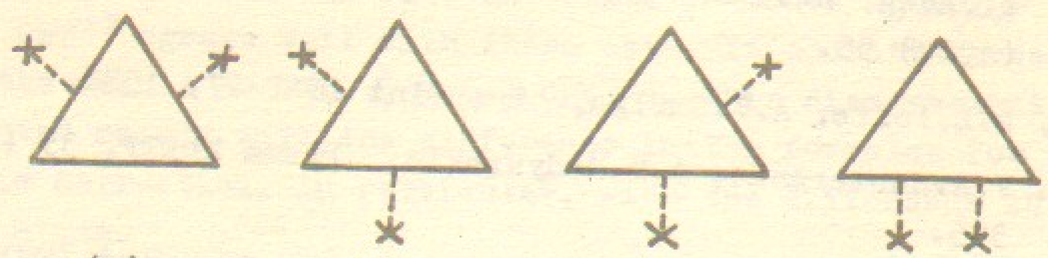
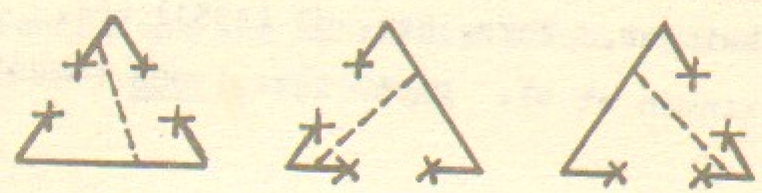


Fig. 1. Quark loop diagram



(a) (b) (c) (d)

Fig. 2. Gluon condensate corrections



(a) (b) (c)

Fig. 3. Quark condensate corrections

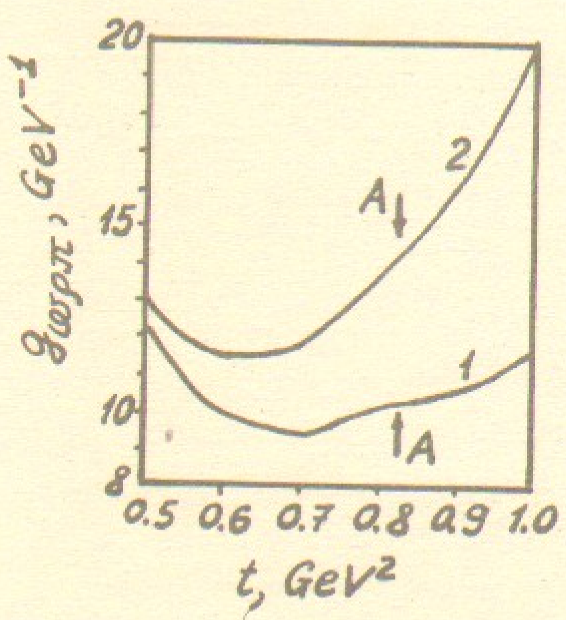


Fig. 4. The t dependence of the r.h.s. of the sum rules (13) (curve 1) and (17) (curve 2). Arrow A indicates t for which the leading power correction as well as continuum contribution reaches 30% of the total result.

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ВЫЧИСЛЕНИЕ КОНСТАНТЫ $g_{\omega\rho\pi}$ ИЗ ПРАВИЛ СУММ КХД ПОСРЕДСТВОМ ТРОЙНОЙ БОРЕЛИЗАЦИИ

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