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ANALYSIS OF HYPERON NON-LEPTONIC AMPLITUDES WITH THE HELP OF QCD SUM RULES
II.FOUR-POINT CORRELATOR

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НОВОСИБИРСК

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II. FOUR-POINT CORRELATOR

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ABSTRACT

The correlator of two barion, axial-vector and four-quark currents is studied in quantum chromodinamics. The non-zero vacuum expectation values (VEV's) of the field operator products are taking into account. Saturating the correlator by the lower barion states and by pion allows one to find the pole contribution from these states into P-waves. Results are consistent with those obtained in the first part of the work using three-point correlator. This is also important for the possible use of the four-point correlator in the analysis of resonance processes. The advantages of the sum rules considered as compared to the three-point ones are: 1) long-distance contribution decreased, so OPE is effective in calculation of power corrections of all orders; 2) scale of power correction decreased; 3) continuum contribution diminished.

1. Introduction

In the first part of the work (hereafter I) the matrix elements of weak hamiltonian H_W between octet baryons were calculated with QCD sum rules (SR) method, first suggested in ref. [1] . For that the correlator of three currents, two baryonic and one four-fermionic ones, H_W , was considered. The double Borel transformation in the baryon channels was used, external light-like momentum being introduced into H_W . Then the possibility to calculate the power corrections with the help of operator product expansion (OPE) appears for operators of dimension $d \leqslant 8$. Using the values of $\langle B_2 | H_W | B_1 \rangle$ obtained, the ground-state pole contribution into P -waves can be found.

The same pole contribution can be found with the help of fourfold Borel transformation applied to the correlator of the barionic, axial and four-fermion currents. In doing so we obtain directly the products of matrix elements of axial current and of H_W from QCD. The SR obtained possess the advantage as compared to the three-point SR in that the main contribution into the power corrections of all orders is calculated within OPE. This contribution is due to operators including four quark fields Ψ, $\overline{\Psi}$ in the leading order in α_S. The scale of power corrections thus calculated is smaller than that in the three-point correlator, that allows one to restrict oneself to a few first terms in OPE with greater reliability. Besides, continuum contribution decreases, because the spectral density grows more slowly. The approach used is consistent with the three-point one.

The paper is organized as follows. The method is described in the next section. In sect. 3 SR in the lower order are formulated. In sect. 4 models of continuum are considered and the duality estimate is made. In sect. 5 the nearest power correction is calculated. In sect. 6 the pole contribution of operators I_3, I_6 into P -waves is calculated and comparative analysis of the SR based on three- and four-point correlators is given. In sect. 7 we conclude.

2. The method

The object of interest is the four-point correlator

$$K_\mu = \int \langle 0|T\{\eta_f(z) A_\mu(y) H_W(0) \bar{\eta}_i(x)\}|0\rangle \cdot \exp(-iq_i x + iky + iq_f z)\, d^4x\, d^4y\, d^4z \qquad (1)$$

Three currents, η_i, η_f, H_W were considered in I. In addition, the axial current A_μ is introduced, into which pion gives the residue $\langle 0|A_\mu|\pi\rangle = i f_\pi k_\mu$, $f_\pi = 133\,MeV$. As far as we are interested in the three-particle amplitude $\langle B_f \pi|H_W|B_i\rangle$ physical states B_i, B_f, π must be singled out in three channels which we shall indicate as i, f, π (see figs. 1a,b). There also two intermediate channels p (fig. 1a) and p' (fig. 1b) are indicated. $q_i, q_f, q_p, q_{p'}$ denote momenta in the corresponding channels; k denote pion momentum. Imaginary part of the correlator in q_i^2 is contributed not only by real states in channel i

(figs. 1a,b) but also by those in channel p (fig. 1a), provided condition that momentum is conserved in the weak vertex, i.e. $q_p = q_i$. Analogous situation holds for the imaginary part in q_i^2. Consequently, to single out real states in channels i, f, one must violate the momentum conservation in H_W introducing the momentum $-\ell = q_f + k - q_i \neq 0$ into H_W, as in I. As a result, seven kinematic variables q_i^2, q_f^2, q_p^2, $q_{p'}^2$, k^2, $(k+\ell)^2$, ℓ^2 appear, six of which are independent:

$$(k+\ell)^2 + q_p^2 + q_{p'}^2 = q_i^2 + q_f^2 + k^2 + \ell^2 \qquad (2)$$

The correlator becomes a nontrivial function of six variables.

OPE determines K_μ at large Euclidean momenta. Applying the Borel transformation in $S_i = -q_i^2$, $S_f = -q_f^2$ and $S_\pi = -k^2$ one can single out the lower states in the channel i, f, π. The amplitude obtained in this way is of interest at $\ell^2 = 0$, $(k+\ell)^2 = m_\pi^2 \approx 0$, $q_p^2 = m_i^2$ $(q_{p'}^2 = m_f^2)$. Thus q_p^2 $(q_{p'}^2)$ is in the vicinity of real hadron singularities m_p^2 $(m_{p'}^2)$ which can not be obtained by any diagrammatic calculation. Studying the amplitude at $-q_p^2 \gg 0$ $(-q_{p'}^2 \gg 0)$ we can single out (by the Borel transformation in $S_p = -q_p^2$ or in $S_{p'} = -q_{p'}^2$) the lower-state pole contribution (in case of P -wave) into such the amplitude. After that one needs to extrapolate to $\ell^2 = 0$, $(k+\ell)^2 = 0$. Situation is the following. We study quite definite the γ -matrix structure in K_μ which is not contributed by asymptotic (free-quark) loop. The main contribution into the correlator occurs in this case at the level of two quark loops broken due to fermion condensate. As a result, putting $\ell^2 = 0$, $(\ell+k)^2 = 0$ does not lead for kinematical reasons to appearance of the long-distance contribution in these diagrams in the leading order in α_S.

The amplitude is parametrized as

$$\langle B_f \pi|iH_W|B_i\rangle = \bar{u}_f (A + B\gamma_5) u_i \qquad (3)$$

where P -wave part B is of interest. Consider more general case $\ell \neq 0$. In the pole approximation we have

$$
T = \langle B_f \pi | i H_W^{PC} | B_i \rangle = \bar{u}_f \left(\gamma_5 \frac{b_p}{\not{q}_p - m_p} + \right.
$$

$$
\left. + \frac{-b_{p'}}{\not{q}_{p'} - m_{p'}} \gamma_5 \right) u_i \tag{4}
$$

then

$$
B \approx B_{pole} = \frac{b_p}{m_i - m_p} + \frac{-b_{p'}}{m_f - m_{p'}} \tag{5}
$$

The contribution of the lower states in the channels i, f, π, p, p' to the correlator takes the form:

$$
K_\mu(S_i, S_p(S_{p'}), S_f, S_\pi) = -i \frac{\tilde{\beta}_i \tilde{\beta}_f}{(2\pi)^4} \frac{f_\pi k_\mu}{S_\pi} \cdot \left\{ b_p \gamma_5 \frac{\not{q}_f + m_f}{S_f + m_f^2} \gamma_5 \cdot \right.
$$

$$
\cdot \frac{\not{q}_p + m_p}{S_p + m_p^2} \frac{\not{q}_i + m_i}{S_i + m_i^2} \gamma_5 - b_{p'} \gamma_5 \frac{\not{q}_f + m_f}{S_f + m_f^2} \frac{\not{q}_{p'} + m_{p'}}{S_{p'} + m_{p'}^2} \gamma_5 \cdot \tag{6}
$$

$$
\left. \cdot \frac{\not{q}_i + m_i}{S_i + m_i^2} \gamma_5 \right\}
$$

Here $\tilde{\beta}_i, \tilde{\beta}_f$ are residues of B_i, B_f into currents η_i, η_f

Applying the Borel transformation in $S_i, S_f, S_\pi, S_p(S_{p'})$ to (6) suppresses the higher-state contribution in the corresponding channels. Consider the Borel transformation in S_i, S_f, S_π, S_p. In the second term in (6) substitute $S_{p'} = S_i + S_f + S_\pi - S_p$ according to (2) at $(k+\ell)^2 = \ell^2 = 0$. Then the pole of this term in S_p is $S_p^{(0)} = m_{p'}^2 + S_i + S_f + S_\pi > 0$. At large S_i, S_f, S_π it lies far to the right in the plane of complex S_p and the Borel transformation in S_p yields zero. The Borel transformation of $f(S)$ in S is determined as [1]

$$
f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} f(s) \exp\left(\frac{s}{t}\right) \frac{ds}{t} \tag{7}
$$

where c is positive real and can be small. The Borel transformation is trivial in the first term. We separate the structure

$$
-i k_\mu \frac{1}{2} (\not{q}_i \not{q}_p \not{q}_f - \not{q}_f \not{q}_p \not{q}_i) \gamma_5 = k_\mu \varepsilon_{\rho\nu\lambda\omega} q_f^\rho q_p^\nu q_i^\lambda \gamma^\omega \tag{8}
$$

and find for the coefficient $C(S_i, S_p, S_f, S_\pi)$ of it in K_μ

$$
\int_{c-i\infty}^{c+i\infty} \exp\left(\frac{S_i}{t_i} + \frac{S_p}{t_p} + \frac{S_f}{t_f} + \frac{S_\pi}{t_\pi}\right) \frac{dS_i \, dS_p \, dS_f \, dS_\pi}{(2\pi i)^4 t_i t_p t_f t_\pi} \cdot
$$

$$
\cdot C(S_i, S_p, S_f, S_\pi) \equiv C(t_i, t_p, t_f, t_\pi) = \tag{9}
$$

$$
= \frac{\tilde{\beta}_1 \tilde{\beta}_2}{(2\pi)^4 t_i t_p t_f t_\pi} b_p \exp\left(-\frac{m_i^2}{t_i} - \frac{m_p^2}{t_p} - \frac{m_f^2}{t_f}\right)
$$

Equating (9) to the result of calculation of $C(t_i, t_p, t_f, t_\pi)$ within OPE, one finds b_p. Analogously, the Borel transformation in $S_i, S_f, S_\pi, S_{p'}$ leads to the SR for $b_{p'}$.

3. Lower order calculation

Asymptotic (free-quark) diagrams don't contribute to $C(t_i, t_p(t_{p'}), t_f, t_\pi)$ and the largest contribution comes from the VEV $\langle \psi\bar{\psi}\psi\bar{\psi} \rangle$ (figs. 2a,b,c). Consider first the diagram c). The Borel transformations in $S_{p'}$ and S_p look quite symmetrically. For corresponding coefficients $b_p^{c)}$, $-b_{p'}^{c)}$ we get equality in SU (3) limit. Corresponding contribution into $B(B_i \to B_f \pi)$ vanishes according to (5) at $m_{p'} = m_i$, $m_p = m_f$. Thus, the diagram of fig.2c can be ommitted in SU (3) limit.

Further, consider contribution of fig. 2b into b_p. Reduce it to the integral over Feynman parameters x, y, z, α shown in fig. 2b. The nontrivial dependence upon the momenta is absorbed in the denominator $(x\alpha S_f + y\alpha S_{p'} + \alpha z S_i + y z S_\pi)^N$. Accounting for the kinematic restriction (2) we see that zero of the denominator in S_p lies far to the right in the plane of complex S_p. Consequently, applying the Borel transformation in S_p gives zero, and $b_p^{b)} = 0$. Analogously $b_{p'}^{a)} = 0$, and we are left with $b_p^{a)} \neq 0, b_{p'}^{b)} \neq 0$ in agreement with intuitive approach.

Result of Borel transformation of the diagram a) in S_i, S_f, S_π

S_p or b) in $S_i, S_f, S_\pi, S_{p'}$ is representable as the rational function of the parameters t_i, t_f, t_π, t_p ($t_{p'}$).

All the above said in this section is equally applicable to the graphs with soft gluons studied in sect. 5.

The VEV $\langle \psi\bar{\psi}\psi\bar{\psi} \rangle$ is considered within factorization. In the absence of the gluon exchange only operator $I_3 = -\mathcal{O}_1$ in H_W contributes to the correlator. Anomalous dimensions are taken into account (see ref. [2]) by the factor

$$L_M^{4/9} a_3(M) = a_3(\mu) \qquad (10)$$

where $L_M = \ln(M/\Lambda)/\ln(\mu/\Lambda)$, $a_3(M) = (\ln(m_W/\Lambda)/\ln\frac{M}{\Lambda})^{4/9}$ is the coefficient of I_3 in $H_W(M)$ (see (8) I), M^2 is the typical scale of the Borel parameters $t_i, t_f, t_\pi, t_p (t_{p'})$ μ = 0.5 GeV is the normalization point of OPE, Λ = 0.15GeV.

Then one arrives at the following SR in which the S -wave part $a_p, a_{p'}$ is also presented for generality

$$a_p + b_p\gamma_5 = a_3(\mu)\mathcal{D}\exp\left(\frac{m_i^2}{t_i} + \frac{m_p^2}{t_p} + \frac{m_f^2}{t_f}\right)\cdot$$
$$\cdot(d_{3pL}\cdot J_{pL}\cdot L + d_{3pR}\cdot J_{pR}\cdot R) \qquad (11)$$
$$a_{p'} + b_{p'}\gamma_5 = a_3(\mu)\mathcal{D}\exp\left(\frac{m_i^2}{t_i} + \frac{m_{p'}^2}{t_{p'}} + \frac{m_f^2}{t_f}\right)\cdot$$
$$\cdot(d_{3p'L}\cdot J_{p'L}\cdot L + d_{3p'R}\cdot J_{p'R}\cdot R)$$
$$\mathcal{D} = \frac{\sqrt{2}}{6\pi^2} cs \frac{a^2}{m_\pi^2 f_\pi} \frac{1}{\beta_i \beta_f}$$

Here $c = \cos\theta_c$, $s = \sin\theta_c$; $R = \frac{1-\gamma_5}{2}$, $L = \frac{1+\gamma_5}{2}$ are the right (left) projection operators, $J_{(p,p')(L,R)}$ are the following functions of the Borel parameters

$$J_{pL} \equiv J_{pL}(t_i, t_p, t_f, t_\pi) = t_p t_f y(1+z-y)$$
$$J_{pR} \equiv J_{pR}(t_i, t_p, t_f, t_\pi) = J_{pL}(t_i, t_f, t_p, t_\pi) \qquad (12)$$
$$J_{p'(L,R)}(t_i, t_{p'}, t_f, t_\pi) = J_{p(L,R)}(t_f, t_{p'}, t_i, t_\pi)$$
$$y = t_i t_f/\Delta,\ z = t_i t_p/\Delta,\ \Delta = t_i(t_p + t_f + t_\pi) + t_p t_f$$

$d_{3(p,p')(L,R)}$ are the numerical coefficients presented in table 1. Pole contribution into P -wave is then defined by (5). The formulas (5), (11), (12) are expressed in units of $G m_\pi^2$.

4. Accounting for continuum and duality estimate

Reduce SR (11) to the form typical for the two-point SR. Put $t_i = t_p = t_{p'} = t_f = 3t$ and denote $3\overline{m^2}_{p(p')} = m_i^2 + m_{p(p')}^2 + m_f^2$. Then we get, for example, in case of Σ_+^+ :

$$-B(\Sigma_+^+) = \frac{a_3(\mu)\mathcal{D}}{m_\Sigma - m_N}\exp(\overline{m^2}/t)\cdot J$$
$$\overline{m^2} = \frac{1}{3}m_\Sigma^2 + \frac{2}{3}m_N^2,\ J = J_{(p,p')(L,R)} = 3t^2\frac{9t}{9t + t_\pi} \qquad (13)$$

It follows from general properties of the multiple Borel transformation that reduced to such the form multipoint SR resemble the two-point ones not only by the exponential factor but also by the scale of power corrections.

Due to the absence of the exponential dependence upon t_π^{-1} in (13) (in chiral limit, $m_\pi = 0$) there is no platou in t_π. This drawback is overcome by introducing the duality interval S_0 for pion with subsequent putting $t_\pi = \infty$. The SR obtained weekly depend upon S_0. Really, we have (to introduce the duality interval we first find the spectral density $\rho(s)$):

$$\frac{9t}{9t + t_\pi} = \int_0^\infty \rho(s)\exp\left(-\frac{s}{t_\pi}\right)ds,$$
$$\rho(s) = \delta(s) - \frac{1}{9t}\theta(s)\exp\left(-\frac{s}{9t}\right) \qquad (14)$$
$$\lim_{t_\pi\to\infty}\int_0^{S_0}\rho(s)\exp\left(-\frac{s}{t_\pi}\right)ds = \exp\left(-\frac{S_0}{9t}\right) \approx 1$$

at $t \gtrsim 1\mathrm{GeV}^2$, $S_0 \sim 1\mathrm{GeV}^2$. Formally one must substitute 0 for t_π in (13). Hereafter all the SR will be given in the limit $t_\pi = \infty$.

Further, take into account continuum in the baryon channels. For that we introduce the triple spectral functions

$$J_{p(L,R)}(t_i, t_p, t_f) \equiv J_{p(L,R)}(t_i, t_p, t_f, 0) =$$
$$= \int_0^\infty\int_0^\infty\int_0^\infty \rho_{p(L,R)}(s_i, s_p, s_f)\exp\left(-\frac{s_i}{t_i} - \frac{s_p}{t_p} - \frac{s_f}{t_f}\right)ds_i\,ds_p\cdot \qquad (15)$$
$$\cdot ds_f$$

and analogous with the replacement $p \to p'$. These are

$$\rho_{pR}(s_i, s_p, s_f) = 2s_f\delta(s_i - s_f)\delta(s_p - s_f) + s_f\cdot\delta'(s_i - s_f)\cdot$$
$$\cdot\theta(s_p - s_f) \qquad (16)$$

$$\rho_{pL}(s_i, s_p, s_f) = \rho_{pR}(s_i, s_f, s_p)$$

$$\rho_{p'(L,R)}(s_i, s_{p'}, s_f) = \rho_{p(L,R)}(s_f, s_p, s_i)$$

Then we restrict the integration in (15) to some region Ω in the octant $s_i, s_p, s_f \geq 0$. We consider two in some sense opposite variants for Ω : tetrahedron

$$s_i + s_p + s_f < 3 s_o^t \tag{17a}$$

and cube

$$s_i < s_o^c, \; s_p < s_o^c, \; s_f < s_o^c \tag{17b}$$

As a result, we get at $t_i = t_p = t_{p'} = t_f = 3t$

$$J \equiv J_{(p,p')(L,R)} = 3t^2 E_2(s_o/t) \tag{18a}$$

$$J \equiv J_{(p,p')(L,R)} = 3t^2\left[\frac{1}{2}E_2\left(\frac{s_o}{t}\right) + \frac{3}{4}E_1\left(\frac{s_o}{3t}\right) - \frac{1}{4}E_1\left(\frac{s_o}{t}\right)\right], \quad E_n(x) = \frac{1}{(n-1)!}\int_0^x \exp(-y)\, y^{n-1} dy \tag{18b}$$

and putting $t = \infty$

$$J = \frac{3}{2} s_o^2 \tag{19a}$$

$$J = s_o^2 \tag{19b}$$

Return ourselves to Σ_+^+. Evaluate $B(\Sigma_+^+)$ on condition on that all with the exception of the pole denominator is considered as being in $SU(3)$ limit. Use the cube variant for Ω, then $J = s_o^2$. The same s_o is used in the SR for $\tilde{\beta}^2$. For $\tilde{\beta}^2$ we choose SR contributed by $\langle\bar{\psi}\psi\rangle$ [2]:

$$m_{\tilde{\beta}}^2 = a \exp(m^2/t)\, t^2 E_2(s_o/t) \xrightarrow{t\to\infty} \frac{1}{2} a\, s_o^2 \tag{20}$$

Substituting (19b), (20) in (13) we get

$$-B(\Sigma_+^+) = \frac{\sqrt{2}}{3}\, cs \frac{f_\pi}{(m_u + m_d)_\mu}\, a_3(\mu)\, \frac{2m}{\Delta m} = 20 \tag{21}$$

Here the known result of PCAC is used [6] ,

$$\langle\bar{\psi}\psi\rangle = -\frac{1}{2}\,\frac{m_\pi^2 f_\pi^2}{m_u + m_d} \tag{22}$$

and the numerical estimate is given for m = 1 GeV, $\Delta m = 0.25$ GeV, $cs = 0.215$, f_π = 133 MeV, $(m_u + m_d)_{0.5 GeV}$ = 11 MeV, Λ = 0.15 GeV. If Ω is the tetrahedron of the same volume then one gets $\sqrt[3]{6}$ times smaller value for $B(\Sigma_+^+)$.

Thus,

$$-B(\Sigma_+^+) = 11 \div 20 \tag{23}$$

Confront this result with the value found using the duality estimates for S -waves (see (30) I) and the axial constants f = 0.428, d = 0.823 known from leptonic hyperon decays [3] :

$$-B(\Sigma_+^+) = \frac{2d}{9}\sqrt{2}\, cs \frac{f_\pi}{(m_u + m_d)_\mu}\, a_3(\mu)\, \frac{2m}{\Delta m} = 11 \tag{24}$$

It is consistent with (23). Fitting procedure (see below) gives the result close to (24).

5. Correction due to quark-gluon condensate

The nearest considerable correction is due to the VEV $\langle\psi\bar{\psi}\psi G\bar{\psi}\rangle$ considered here within factorization:

$$\langle u_i^a \bar{u}_k^b d_l^c \bar{d}_m^d\, g G_{\mu\nu}^n \rangle = -\frac{m_o^2 \langle\bar{u}u\rangle^2}{48^2}\Big[\delta^{ab}\delta_{ik} \cdot (t^n)^{cd} \cdot (\sigma_{\mu\nu})_{lm} + (t^n)^{ab}(\sigma_{\mu\nu})_{ik}\, \delta^{cd}\delta_{lm}\Big],$$

$$i\langle g(\bar{\psi}\sigma_{\mu\nu} t^n G_{\mu\nu}^n \psi)(\bar{\psi}\psi)\rangle \equiv m_o^2 \langle\bar{\psi}\psi\rangle^2 \tag{25}$$

$$\sigma_{\mu\nu} = \frac{1}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$$

Then the diagrams like those of figs. 3a,b appear. The diagram of fig. 2b leads to arising the contribution of $I_6 = (\bar{d}\gamma_\mu u_L)(\bar{u}\gamma^\mu s_L) + (\bar{u}\gamma_\mu u_L)(\bar{d}\gamma^\mu s_L)$ to the correlator. Calculation of graphs including soft gluons is performed in the fixed-point gauge [4] . The coordinate origin is taken in H_W or, to check the answer, in η_1, η_2 or A_μ . Instead of (11) we have now four formulas, where indices 3 or 6 correspond to I_3 or I_6 pieces:

$$a_{(3,6)(p,p')} + b_{(3,6)(p,p')} \cdot \gamma_5 = a_{(3,6)}(M) \cdot L_M^{4/9} \cdot \mathcal{D} \cdot$$
$$\cdot \exp\left(\frac{m_i^2}{t_i} + \frac{m^2_{p(p')}}{t_{p(p')}} + \frac{m_f^2}{t_f}\right) \cdot$$
$$\cdot \left(d_{(3,6)(p,p')L} \cdot J_{(3,6)(p,p')L} \cdot L + d_{(3,6)(p,p')R} \cdot \right. \tag{26}$$
$$\left. \cdot J_{(3,6)(p,p')R} \cdot R\right)$$

Functions $J_{(3,6)(p,p')(L,R)}(t_i, t_{p(p')}, t_f)$ are as follows

$$J_{3pL} \equiv J_{3pL}(t_i,t_p,t_f) = t_p t_f\, y_o(1+z_o-y_o) -$$
$$- \frac{m_o^2}{48}\left[11 t_f - 2(y_o - z_o)t_p\right] L_M^{-4/9}$$
$$J_{3pR} \equiv J_{3pR}(t_i,t_p,t_f) = t_p t_f\, z_o(1+y_o-z_o) -$$
$$- \frac{m_o^2}{48}\left[3 t_p - 10(y_o - z_o)t_p\right] L_M^{-4/9}$$
$$J_{6pL} = \frac{m_o^2}{12}\left[t_f - 2(y_o - z_o)t_p\right] L_M^{-4/9}$$
$$J_{6pR} = \frac{m_o^2}{12}\left[t_p + 2(y_o - z_o)t_p\right] L_M^{-4/9} \tag{27}$$
$$J_{(3,6)p'(L,R)}(t_i, t_{p'}, t_f) = J_{(3,6)p(L,R)}(t_f, t_{p'}, t_i)$$
$$y_o = \frac{t_i t_f}{\Delta_o}, \quad z_o = \frac{t_i t_p}{\Delta_o}, \quad \Delta_o = t_i(t_p + t_f) + t_p t_f$$

Coefficients $d_{(3,6)(p,p')(L,R)}$ at them are given in table 1.
Then the pole contribution into ρ -wave is

$$B = \frac{b_{3p} + b_{6p}}{m_i - m_p} + \frac{b_{3p'} + b_{6p'}}{m_{p'} - m_f} \tag{28}$$

6. Pole contribution into ρ -waves

6.1. Stability of the results with respect to the choice of the model of continuum.

Accounting for continuum is made with two choices of Ω (17a,b). For the tetrahedral variant of Ω we get at $t_i = t_p = t_{p'} = t_f = 3t$ $(x = s_o^t/t)$:

$$J_{3L} \equiv J_{3(p,p')L}(3t,3t,3t) = 3t^2 E_2(x) - \frac{11}{16} m_o^2 t E_1(x)$$
$$J_{3R} = 3t^2 E_2(x) - \frac{3}{16} m_o^2 t E_1(x) \tag{29}$$
$$J_{6L} = J_{6R} = \frac{1}{4} m_o^2 t E_1(x)$$

and for the cubic one ($x = s_o^c/t$)

$$J_{3L} = 3t^2\left[\frac{1}{2}E_2(x) + \frac{3}{4}E_1\left(\frac{x}{3}\right) - \frac{1}{4}E_1(x)\right] -$$
$$- \frac{m_o^2}{16} t\left[12 E_1\left(\frac{x}{3}\right) - E_1(x)\right]$$
$$J_{3R} = 3t^2\left[\frac{1}{2}E_2(x) + \frac{3}{4}E_1\left(\frac{x}{3}\right) - \frac{1}{4}E_1(x)\right] -$$
$$- \frac{m_o^2}{16} t\left[8 E_1\left(\frac{x}{3}\right) - 5E_1(x)\right] \tag{30}$$
$$J_{6L} = \frac{m_o^2}{4} t\left[2E_1\left(\frac{x}{3}\right) - E_1(x)\right], \quad J_{6R} = \frac{m_o^2}{4} t E_1(x)$$

Then the thresholds S_o^t or S_o^c are selected for which the platou in the dependence $b_3(t)$ exists (b_6 is estimated below). In fig. 4. example of the dependence $-B(\Sigma_t^+) = B(t) \equiv \frac{-b_{3N}(t)}{m_\Sigma - m_N}$ is given in the tetrahedral version of continuum for three choices of S_o^t. At S_o^t = 1.6 GeV^2 we obtain good fit $b_{3N}(t) \approx$ const. Power correction at $t \geqslant 1$GeV^2 doesn't exceed 15%, so the condition of applicability of OPE is fulfilled [1]. The relation between the optimal values of the threshold is $S_o^t = (0.87 \div 0.90) S_o^c$ and the typical values are $S_o^c = 2 \div 2.5$ GeV^2 for Σ, Λ decays and $S_o^c = 2.5 \div 3$ GeV^2 for Ξ decays. The results obtained for the cubic Ω at $m_o^2 = 0, 0.6$ and 1 GeV^2 are average 6, 11 and 13 percents correspondingly smaller than those for the tetrahedral Ω . For definiteness, results are presented for the tetrahedral Ω.

6.2. I_3 contribution

Write out SR for the concrete ρ -wave amplitude in the ground state pole approximation obtained from the three-point correlator (31a) (considered in I) and from the four-point one (31b)(considered in this paper):

$$-B(\Sigma_{+}^{+}) = \frac{1}{\Delta m}\frac{cs}{\sqrt{2}\,\pi^2 m_\pi^2 f_\pi}\frac{a_3(M)}{\tilde{\beta}^2}\exp\left(\frac{m^2}{t}\right)\cdot$$

$$\cdot\begin{cases} \frac{10}{9}d\left[a^2t^2\left(1-\frac{16}{48}\frac{m_o^2}{t}\right)+\frac{18}{25}t^5\right] & (31a)\\ a^2t^2\left(1-\frac{7}{48}\frac{m_o^2}{t}\right) & (31b)\end{cases}$$

Leading order in $SU(3)$ violation, $O(\Delta m^{-1})$ is used here. Proportional to t^5 asymptotic term in (31a) is several times larger than the most essential correction due to $\langle\psi\bar{\psi}\psi\bar{\psi}\rangle$. However, as it was obtained in [in the definite models for continuum this term is strongly compensated by continuum leading to the contribution of the order of 10-20% into the whole answer. Both the SR are consistent at $d \approx 0.9$ which is quite reasonable.

Four-point SR (31b) differs from (31a) by 1) diminished scale of power corrections and by 2) considerably diminished continuum contribution due to the absence of the asymptotic term. The second circumstance is essential because the accuracy of compensation of large asymptotic term must be sufficiently great to ensure the smallness of it's final contribution into the answer. One can ask if this compensation can be violated by modification of the model of continuum. SR (31b) implies directly that the asymptotic term is indeed almost entirely due to the higher states and thus not very essential in (31a) too.

In calculation of the pole contribution into the p -wave the masses of Σ and Λ propagating in one and the same channel p or p' are replaced by the same mass m_p or $m_{p'}$. Therefore, when comparing the results of the four-point approach with those of the three-point one calculations within the latter must be done in the same assumption, i.e. $m_\Lambda = m_\Sigma$ in the denominator. This circumstance is accounted for in table 2 where in the first column of the numerical data the results of the first part of the work are given for f =0.428, d =0.823 [3], and in the second column presented are the results of analogous calculation in assumption $m_\Lambda = m_\Sigma$=1.19 Gev in the denominator. Further, possible is the large error due to partial can-

cellation of different graphs for the four-point correlator. One can circumvent this difficulty ommiting the diagrams, the sum of which vanishes in $SU(3)$ limit. Then it turns out that only one graph contributes to each process considered. The results of this approach are presented in the third column of the data of table 2. Secondly, one can average intermediate scale introducing $\frac{1}{2}(m_p^2+m_{p'}^2)$ into the exponent in the SR instead of m_p^2 and $m_{p'}^2$. Then the diagrams mentioned cancel authomatically, as if they were $SU(3)$ symmetrical. Appropriate results are given in the fourth column. Approximations made are the more natural, as accounting for the $SU(3)$ violation only due to the difference between the mass scales m_p and $m_{p'}$ and neglecting $SU(3)$ violating VEV's would be exceeding the accuracy.

The maximal discrepance between the second and the third or fourth columns of the data appears in Σ_-^- case. Large error appears here due to the strong cancellation between the different poles: $B(\Sigma_-^-) \sim (f-0.5d)$ for $SU(3)$ symmetrical strong and weak coupling constants. As a whole, we observe rather good agreement between the two approaches, three- and four-point ones. In particular, consistent are the dependences upon m_o^2.

6.3. I_6 contribution

The contribution of I_6 is determined by VEV $\langle\psi\bar{\psi}\psi G\bar{\psi}\rangle$ from the SR (26). Dependence upon t in (26) is due to the factor

$$f(t) = t\,E_1(S_o/t)\exp(m^2/t) \qquad (32)$$

We use $f'(t)=0$, $f''(t)=0$ as the conditions of maximal constancy of $f(t)$. These lead to $t=\infty$, $S_o = 2\overline{m^2}$. Note that the value $S_o \sim 2$ GeV^2 is quite reasonable and at $t=\infty$ higher power corrections vanish. As a result, $f(t) = 2\overline{m^2}$. Put $\overline{m^2} = m_N^2$, $\tilde{\beta}_i = \tilde{\beta}_f = \tilde{\beta}_N$. Calculated with the help of (26), (27), (28) and table 1 pole contribution of I_6 into P -waves is given in table 3, where the results of the three-point approach are also presented. Estimates are made for m_o^2 = 0.6 GeV^2, d = 0.823,

$\xi = 0.428$.

In general, we observe the discrepancy between the two approaches. As for the agreement with experiment, the $\Delta T = 3/2$ pole ρ -wave amplitudes are calculated in table 4 in both the approaches. Presented is also the separable contribution to these amplitudes [5] . One can see that accounting for the pole contribution in addition to the separable one improves the agreement with the experiment in the three-point approach and leads to disagreement in the four-point one. The reason for this is probably the roughness of the factorization formula (25) for $\langle u\bar{u}dG\bar{d}\rangle$. Last circumstance influences the result to a considerably smaller extent in the framework of the three-point approach. Really, irrespectively of the structure of VEV's three-point correlator leads to parametrization of matrix elements of I_6 by only one parameter ε (see (24) [ ]). This is not so for the four-point correlator, in which relations between the contributions of I_6 to different processes depend upon the structure of VEV's used. Therefore, aiming at the agreement of the four-point approach with the three-point one, one can, in principle, study the structure of complex VEV's.

7. Conclusion

In the paper the pole contribution of operators I_3, I_6 in H_W to P -waves was calculated in the framework of four-point correlator. The results for I_3 are consistent with those obtained from the three-point correlator. They confirm some principial aspects of the calculations made in I , in particular, 1) the possibility to use for calculation the OPE with limited interval of applicability $(d \leq 8)$ and 2) that in the multiloop diagrams the asymptotic term though originally dominant is quite accurately compensated by the continuum and thus finally not very essential.

At the same time we have shown the consistency of the whole approach in analysis of resonance processes. We have found the most singular (ground-state pole) piece of the amplitude with the help of QCD SR. Next step can be finding the contribution of continuum and thus determining the whole amplitude from QCD SR.

Multipoint correlators allow one to separate the structures with a large number of momenta which are contributed mainly by VEV's of high dimensional operators. Imposed phenomenologically, for example, by pole model, relations between the correlators of different number of currents allow one to study the structure of these VEV's. We have found that the consistency requirements with respect to I_6 contribution impose some restrictions on the structure of the VEV $\langle u_i^a \bar{u}_k^b d_\ell^c G_{\mu\nu}^n \bar{d}_m^d \rangle$ for which the factorization hypothesis turns out to be only rather rough approximation.

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Figure captions

Fig. 1. Pole graphs for the four-point correlator.
Fig. 2. Diagrams proportional to the contribution $\langle \psi\bar{\psi}\psi\bar{\psi} \rangle$ to the correlator.
Fig. 3. Diagrams proportional to the contribution $\langle \psi\bar{\psi}\psi G\bar{\psi} \rangle$ to the correlator; circles outlined by dots stand for the factorized doublet $\langle \psi\bar{\psi} \rangle$ or triplet $\langle \psi G\bar{\psi} \rangle$ of vacuum fields.
Fig. 4. Function $-B(\Sigma_+^+) = B(t)$ for the three-values of the continuum threshold S_0.

Table captions

Table 1. Coefficients entering the sum rules (11) and (26).
Table 2 Pole contribution of I_3 to P -waves in the three- and four-point approaches.
Table 3 Pole contribution of I_6 to P -waves.
Rable 4 Deviations from the $\Delta T = 1/2$ rule for P -waves.

Table 1.

Mode	$a_{(\cdot)L} \cdot L + a_{(\cdot)R} \cdot R$			
	$(\cdot)=3p$	$(\cdot)=3p'$	$(\cdot)=6p$	$(\cdot)=6p'$
$-\Sigma_+^+$	$L-R$	0	$L+R$	0
$-\Sigma_0^+\sqrt{2}$	$L-R$	R	$-L+R$	$-R$
$+\Sigma_-^-$	0	R	0	R
$+\Lambda_-^0\sqrt{6}$	$-L+R$	$2L-R$	$L+R$	$2L+R$
$-\Lambda_0^0\sqrt{12}$	$-L+R$	$2L-R$	$3(L-R)$	$-2L-R$
$-\Xi_-^-\sqrt{3/2}$	0	L	0	0
$+\Xi_0^0\sqrt{3}$	0	L	$L-R$	0

Table 2.

Mode	m_0^2, GeV^2	Amplitude B			
		Three-point correlator		Four-point correlator	
		I^*	II^*	III^*	IV^*
$-\Sigma_+^+$	0	12.6	15.2	15.2	18.5
	0.6	11.0	13.0	14.1	16.6
	1	9.5	10.9	12.5	15.2
$-\Sigma_0^+$	0	7.20	7.20	5.40	6.53
	0.6	6.00	6.00	4.54	5.00
	1	4.88	4.88	3.42	4.24
$-\Sigma_-^-$	0	2.48	5.05	10.3	9.25
	0.6	2.58	4.54	10.9	9.22
	1	2.63	4.08	11.3	9.25
$+\Lambda_-^0$	0	1.10	5.98	4.30	3.46
	0.6	1.96	5.62	3.70	2.88
	1	2.64	5.26	2.97	2.30
$-\Xi_-^-$	0	12.4		12.6	11.2
	0.6	10.7		11.2	10.2
	1	9.8		9.8	8.6

* For explanations see subsect. 6.2

Table 3.

Mode	Three-point correlator	Four-point correlator
Σ_+^+	-0.97	0
Σ_0^+	-0.10	$+0.26$
Σ_-^-	-0.25	-0.38
Λ_-^0	$+0.93$	$+0.16$
Λ_0^0	-0.35	-0.55
Ξ_-^-	0	0
Ξ_0^0	0	-0.59

Table 4.

Mode	Pole contribution		Separable contribution [5]	Experiment
	Three-point correlator	Four-point correlator		
$\Lambda^0_- + \Lambda^0_0\sqrt{2}$	+0.44	− 0.44	−0.95	0.03±1.03
$\Xi^-_- + \Xi^0_0\sqrt{2}$	0	+ 0.64	+0.23	0.39±0.75
$\Sigma^+_0\sqrt{2} - \Sigma^+_+ + \Sigma^-_-$	+0.58	0	+ 0.49	2.76±0.96

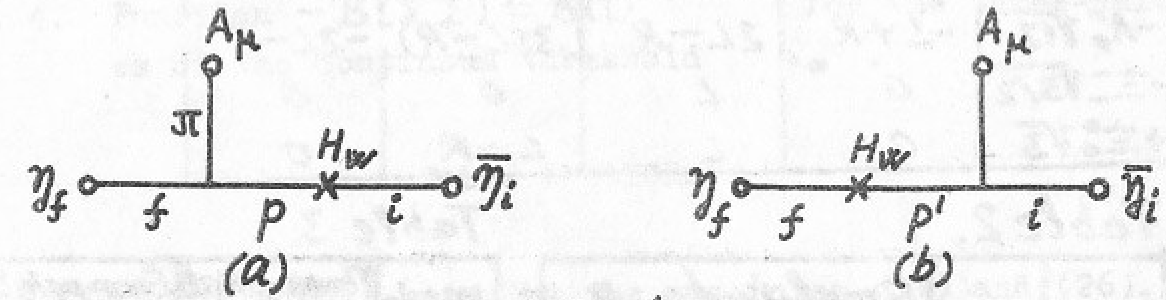

Fig.1

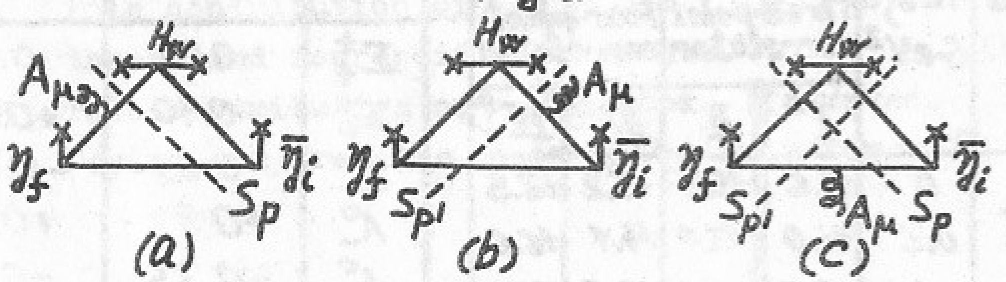

Fig.2

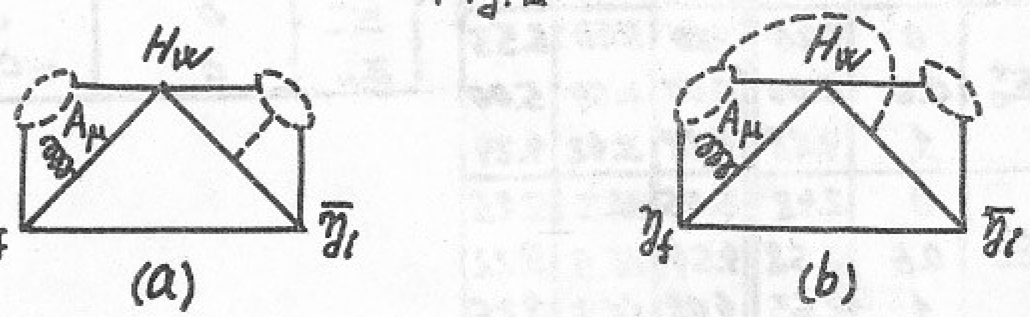

Fig.3

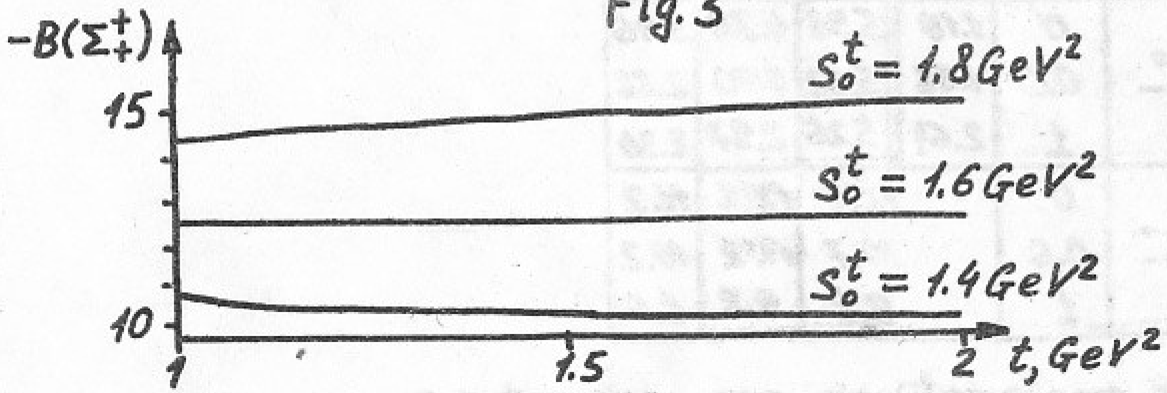

Fig.4

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