ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

E.V.Shuryak

TOWARD QUANTITATIVE THEORY OF TOPOLOGICAL EFFECTS IN GAUGE THEORIES

PREPRINT 84-157

НОВОСИБИРСК

TOWARD QUANTITATIVE THEORY OF
TOPOLOGICAL EFFECTS IN GAUGE THEORIES

E.V.Shuryak

Institute of Nuclear Physics,
630090, Novosibirsk, U S S R

Abstract

New variational calculation of vacuum properties of $SU(2)$ gauge theory is reported, based on topological "tunneling" phenomena. Eliminating some defect of the trial function used previously by Dyakonov and Petrov we have found much deeper ground state.

The understanding of vacuum structure of gauge theories is the main problem of nowdays quantum field theory. Impressive theoretical work was made using lattice formulation and numerical methods, while important phenomenological information was obtained by means of QCD sum rules. Recent review, containing multiple references to original works, is given in ref. [1]. Facing huge amount of experimental and computer data, we now have to understand their physical meaning.

Unfortunately, only one type of nonperturbative effects is so far understood, at least qualitatively: is the "tunneling" between topologically different classical vacua. This phenomenon was discovered by Polyakov and coworkers [2] in semiclassical framework 9 years ago.

Dividing this time into three periods we may say, that in the first one, marked by primary enthusiasm, we have learned that some long-standing problems (like the $U(1)$ one can be solved and that new ones (like the CP conservation) can be posed. The second period (1976-1981) was rather pessimistic: it was realized that practically no observable effect can really be described by semiclassical theory, relevant only for very rare fluctuations. The third period was marked by phenomenological investigations of topological effects. In ref. [3] strong evidences were given suggesting that these effects are responsible for the strongest nunperturbative corrections to correlators at small distances, and in refs [4] some "instanton liquid" model was shown to reproduce these observations well enough. Multiple attempts to detect topological effects on the lattice have not so far produced comprehensive results. Depending on the method used, results on "tunneling" space-time density differ by about two orders of magnitude, see refs. [5].

Recently, Dyakonov and Petrov [6] have started new attack on this problem using the so called Feynman variational principle. In short, their ideas are as follows. Suppose one starts with some arbitrary "ansatz" field $A^{(cl)}_{a,\mu}$ and then try to evaluate effect of quantum fluctuations around it. If such field is not a solution of classical equations $j_{a,\mu} \equiv (\mathcal{D}^{ab}_{\mu} G_{b\mu\nu}) \neq 0$, one finds the nonzero linear term $\delta S = \int dx\, j_{a,\mu} a_{a,\mu}$ (where $a_{a,\mu}$

is quantum field) and therefore there is no reason for $a_{a,\mu}$ to be small as compared to $A^{(cl)}_{a,\mu}$. The radical step suggested in [6] is <u>to add an external current compensating the linear term</u>, so that one may enjoy small quantum oscillations. The price is that the vacuum problem is substituted by an other one, with $j^{(ext)}_{a,\mu} \neq 0$. Nevertheless the following nonequality exists

$$\mathcal{E}_{vac} < \mathcal{E}(j^{ext}) \qquad (1)$$

which can be used as the basis for variational approach.

I am now working on more straightforward approach to "instanton liquid" problem. The first step is the introduction of <u>collective variables,</u> being a "tunneling" centers $z^{(i)}_\mu$ and $\rho_{(i)}$, $O^{ab}_{(i)}$ determined by conditions

$$A_{a,\mu}(x) \xrightarrow[y=x-z_i\to 0]{} \left(\frac{2}{g}\right) O^{ab}_{(i)} \bar{\eta}^{b}_{\mu\nu} y_\nu / y^2 \qquad (2)$$

$$G_{a,\mu,\nu}(x) \xrightarrow[y\to 0]{} \left(-\frac{8}{g\rho_i^2}\right) O^{ab}_{(i)} \bar{\eta}^{b}_{\mu\rho} (y_\mu y_\rho / y^2 - \delta_{\mu\rho}/4) + (\mu \leftrightarrow \rho) \qquad (3)$$

where the r.h.s. corresponds to instantons in singular gauge. The second step is evaluation of <u>"effective action"</u> in space of collective variables, made by evaluation of <u>quantum fluctuations</u> in background <u>"interpolating field"</u>, the conditional extremum of the action. The third step is the <u>integration over collective variables.</u> So far this program is not carried out completely, therefore in this letter I only make some comments demonstrating how one may essentially improve the variational calculations presented in [6].

Dyakonov and Petrov have used the simplest possible "ansatz DP"

$$A^{(DP)}_{a,\mu}(x) = \sum_i A^{(+)}_{a,\mu}(x-z_i) + \sum_i A^{(-)}_{a,\mu}(x-\bar{z}_i) \qquad (4)$$

where $A^{(\pm)}_{a,\mu}$ is the instanton (antiinstanton) solution in singular gauge. Obviously, condition (2) is satisfied, but (3) is violated. It is easy to see that $(G^a_{\mu\nu})^2$ is infinite at the centers because delicate cancellation of terms typical for one-instanton solution is spoiled!

It is quite simple to avoid this defect and to write down ansatz consistent with (3). Considering for simplicity only one instanton-antiinstanton pair we may mimic the 't Hooft solution for two instantons and write down the following "ansatz A":

$$A^{(A)}_{a,\mu}(x) = \left(-\frac{2}{g}\right)\left(\bar{\eta}^{a}_{\mu\nu} y_\nu / y^4 + O^{ab} \eta^{b}_{\mu\nu} \bar{y}_\nu / \bar{y}^4\right) / F(x) \qquad (5)$$

$$y = x - z, \quad \bar{y} = x - \bar{z}, \quad F(x) = 1 + \rho^2/y^2 + \bar{\rho}^2/\bar{y}^2$$

Putting it into action, one may numerically evaluate its dependence on $R^2 = (z-\bar{z})^2$ the distance between centers, and on other parameters. At fig. 1 results of such calculation is compared to DP results [6]. For comparison, we have plotted also results for more elaborate "ansatz B"

$$A^{(B)}_{a,\mu} = A^{(A)}_{a,\mu}(x) + C_{a,\mu} \exp(-C_L x_L^2 - C_T x_T^2)/F(x) \qquad (6)$$

$$x_T = x - x_L \; ; \quad x_L = (z-\bar{z})(xz - x\bar{z})/(z-\bar{z})^2$$

in which 14 parameters $C_{a,\mu}$, C_L, C_T were minimized numerically for each R. It is seen, that although repulsion between instanton and antiinstanton is essentially reduced, it is still present at small enough R. Evaluating the current we have found that $\langle j_{a,\mu}^2 \rangle$ is typically reduced by a factor 20, if one compares ansatz A to DP one.

So, qualitative improvement of the ansats leads to strong qualitative changes in the instanton interaction! Assuming (together with DP) that quantum determinant is a product of individual ones, we now come to integration over the instanton parameters. DP have solved this problem analytically, with one more variational principle. I have chosen to make straightforward numerical model for the "instanton liquid" using standard Metropolis algorithm. It allows to check accuracy of various approximations explicitly.

But before we come to results, given in Table 1, let me explain the notations used. The nonperturbative vacuum energy density $\mathcal{E}_{vac}$ is determined from the statistical sum Z as follows

$$Z \sim \exp\left(-\mathcal{E}_{vac} \cdot VT\right) \tag{7}$$

where VT is space-time volume. Its value following from DP work [6] is 1)

$$\mathcal{E}^{(DP)}_{vac} = -0.45\ \Lambda^4_{PV} \tag{8}$$

Important, that general "anomaly" relation between $\mathcal{E}_{vac}$ and gluonic condensate

$$\mathcal{E}_{vac} = -\left(\frac{11\ N_{colours}}{384\pi^2}\right)\langle (gG^a_{\mu\nu})^2\rangle \tag{9}$$

is automatically fulfilled.

Coming to results given in Table 1 one may see, that making more accurate calculations we gradually obtain deeper vacuum state, so that the total increase in one order of magnitude is reported. However, lattice results [7] for the gluon condensate suggest (via relation (9)) $|\mathcal{E}_{vac}|$ still few times larger (see the last line in the Table).

Now, do we really need the crude model reported, if we have lattice data? First of all, we need "understanding", which is more or less sinonim to "reduction of degrees of freedom". The model considered above makes such reduction by the large factor 10^4 (per unite volume), while the results for $\mathcal{E}_{vac}$ are similar, (inside order-of-magnitude accuracy of both calculations). Second, one may now claim that topological tunneling occurs at least few times per $V_0 = \Lambda^{-4}_{PV}$, while lattice results have much larger uncertainty, more than two orders of magnitude, on this issue. And finally, let us hope that the art of theoretical physics will not be completely substituted by "brute force" methods!

1) We use units Λ_{PV}, where PV stands for Pauli-Villars regularization method.

TABLE 1

VACUUM PARAMETERS FOR SU(2) GAUGE THEORY

REF.	ANSATZ	COMMENTS	$-\mathcal{E}_{vac}$ 1)	n 2)	$\bar{R}/\bar{\rho}$ 3)
[6]	DP	MEAN FIELD AVERAGED OVER POSITION AND ORIENTATION	0.45	0.24	4.3
[6]	DP	THE SAME PLUS INSTANTON SHAPE VARIATION	0.7-0.9		
THIS WORK	DP	POSITION AND ORIENTATION AVERAGED INTERACTION	0.4	0.2	4.7
THIS WORK	DP	ORIENTATION AVERAGED INTERACTION	0.7	0.4	3.1
THIS WORK	DP	FULL INTERACTION	1.5	0.8	2.0
THIS WORK	A	FULL INTERACTION	3.6	2.3	1.25
[7]		LATTICE CALCULATION	21+/- 4) 1	?	?

1) $\mathcal{E}_{vac}$ - NONPERTURBATIVE VACUUM ENERGY DENSITY, IN Λ^4_{PV}.
2) n - DENSITY OF TOPOLOGICAL TUNNELING EVENTS PER $V_0 = \Lambda^{-4}_{PV}$
3) RATIO OF THE AVERAGE SPACING R TO AVERAGE RADIUS ρ.
4) ERRORS GIVEN ARE ONLY STATISTICAL, SYSTEMATICS IS ESSENTIALLY LARGER

Figure Captions

(a) Action of interaction of the instanton and antiinstanton separated by the distance R. S_{int} is measured in units of S_0, the instanton action, while R in unites of the instanton radius, taken equal for instanton and antiinstanton. Points correspond to field configurations as indicated, the dashed lines just shown to guide the eye, "max" ("min") are different orientations.

(b) Averaged value of the current squared $\langle (j_{a,\mu})^2\rangle$ as a function of R. The averaging prescription corresponds to ensemble of points in space-time with density proportional to the action.

References

1. E.V.Shuryak, Theory and phenomenology of the QCD vacuum. preprints INP, Novosibirsk 1984 and Phys.Reports, in press.

2. A.M.Polyakov, Phys.Lett. 59B(1975)82, A.A.Belavin et al. Phys.Lett.59B(1975)85.

3. V.A.Novikov et al. Nucl.Phys. B191(1981)301.

4. E.V.Shuryak, Nucl.Phys. B203(1982)93,116,160,214B(1983)237.

5. P. di Vecchia et al. Nucl.Phys.B192(1981)392.

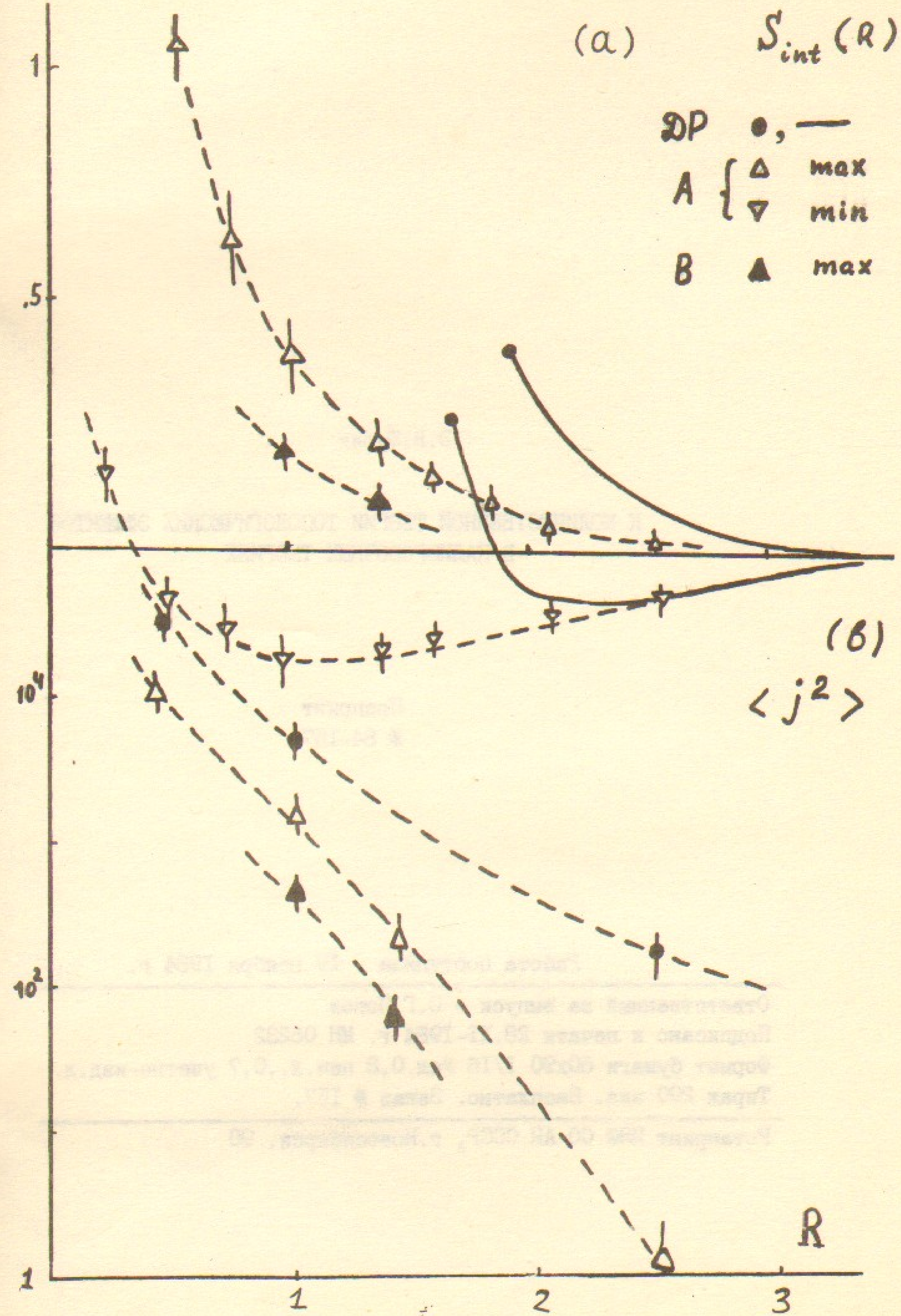
K.Ishikawa et al. Phys.Lett. 128(1983)309.

P.Woit, Phys.Rev.Lett.51(1983)638, G.Bhanot et al, Nucl. Phys. B230 FS10 (1984) 291.

Y.Iwasaki and T.Yoshie, Phys.Lett. 143(1984)449.

6. D.I.Dyakonov and V.Yu.Petrov, Instanton-based vacuum from Feynman variational principle, preprint LINP-900, Leningrad 1983, Nucl. Phys. B245 (1984) 259.

7. Ishikawa K. et al. Nucl.Phys. B227(1983)221.



Э.В.Шуряк

К КОЛИЧЕСТВЕННОЙ ТЕОРИИ ТОПОЛОГИЧЕСКИХ ЭФФЕКТОВ В КАЛИБРОВОЧНЫХ ТЕОРИЯХ

Препринт
№ 84-157

Работа поступила - 19 ноября 1984 г.

Ответственный за выпуск - С.Г.Попов
Подписано к печати 28.XI-1984 г. МН 06232
Формат бумаги 60x90 1/16 Усл.0,8 печ.л.,0,7 учетно-изд.л.
Тираж 290 экз. Бесплатно. Заказ № 157.

Ротапринт ИЯФ СО АН СССР, г.Новосибирск, 90