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Abstract

Parity violating $\pi$NN vertex is calculated in the framework of the standard model using the QCD sum rules method. The result obtained is consistent with the experimental data.

## 1. Introduction

Recently the QCD sum rules method suggested in the paper [1] is widely used to describe properties of the lowest hadron states. The masses of mesons [1-2] and of baryons [3-5], formfactors and meson coupling constants [6-8] have been calculated with this method. In refs. [9,10] the QCD sum rules for polarization operator of nucleon current in an external electromagnetic field first suggested in ref. [9] were used for calculation of the magnetic moments of octed baryons. In ref. [11] using analogous sum rules in an external axial field the axial constants of octed baryons were calculated.

In this paper the QCD sum rules method for correlator of two nucleon currents in the presence of weak interaction is applied to calculation of the parity violating $\pi$NN vertex (in the soft pion limit). The given vertex is interesting as that acquiring the dominant contribution from neutral currents [20,21]. Parity violating meson-nucleon interactions provide with the unique possibility of studying all the components of the standard model at low energies (in nuclear reactions).

We stick to the usual (at our definition of the pion field*)) parametrization of this vertex:

$$\langle \pi^- p | i H_W^{PV} | n \rangle = G m_\pi^2 A_\pi \phi_\pi \bar{u}_p u_n \tag{1}$$

Experimental bounds on $A_\pi$ taking into account theoretical uncertainties while extracting this value from experiment are [18] (see also ref. [19])

$$1.5 \leqslant A \leqslant 2.5 \tag{2}$$

As for the theoretical estimates of $A_\pi$, up to now they have been done in various quark models and are scattered in a rather

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*) The pion residue $\langle \pi^- | \bar{d}(x) \gamma_5 u(x) | 0 \rangle = -i f_\pi m_\pi^2 \cdot (m_u + m_d)^{-1} \phi_\pi(x)$ where $\phi_\pi(x)$ is the pion wavefunction. Then if $\langle \pi^- | \pi^+(x) | 0 \rangle = \phi_\pi(x)$ then $\pi^+(x) = +i(m_u+m_d) f_\pi^{-1} m_\pi^{-2} \bar{d}(x) \gamma_5 u(x)$ is the interpolating pion field.

large interval, $A_\pi \simeq 0.5 \div 5$ (see refs. [13,14,18-22]). The origins of this discrepancy are 1) dependence of the result on a choice of normalization point of the effective weak Hamiltonian $H_W^{PV}$, 2) dependence on a model used and 3) dependence on the constants characterizing this model, say on quark masses in the MIT bag.

The following calculation allows one to overcome these difficulties to a considerable extent and, within the accuracy expected, results in agreement with the upper limit for $A_\pi$ in (2).

## 2. The method

Using PCAC in the soft pion limit leads to

$$\langle \pi^- p | H_W^{PV} | n \rangle = \frac{i}{f_\pi} \langle p | H_W' | n \rangle \qquad (3)$$

where

$$H_W' = \left[ H_W^{PV}, \int u^+(x) \gamma_5 d(x) d^3 \bar{x} \right] \qquad (4)$$

Then

$$A_\pi = - \frac{\alpha}{G_F m_\pi^2 f_\pi} \qquad (5)$$

where $\langle p | H_W' | n \rangle = \alpha \bar{u}_p u_n$. The value of $\alpha$ is obtainable from QCD sum rules. Acting in the same way as at the derivation of QCD sum rules in the constant external fields [9-11] we would consider the T-product of proton $\eta_p$ and neutron $\bar{\eta}_n$ currents and Wilson operator expansion for it in the presence of weak interaction; the latter being effectively described here by the Hamiltonian $H_W'$ (see (4)):

$$K(q) = i \int dx\, e^{iqx} \langle 0 | T\{ \eta_p(x) \bar{\eta}_n(0) \} | 0 \rangle_{H_W'} = \sum_n (C_n \not{q} + D_n) \langle 0 | O_n | 0 \rangle \qquad (6)$$

Here $O_n$ are local operators, and coefficients $C_n$, $D_n$ at the two independent $\gamma$-matrix structures $\not{q}$, 1 are power



functions of $s = -q^2$. One can speak about the correlator placed in an external charged scalar field coupled to $H_W'$.

Complication arising in our case is connected with occurence of the single pole subtraction terms needed to regularize UV divergences in the correlator (6) (see, e.g., Fig. 1). They correspond to the case when only one of the currents creates from vacuum or annihilates real hadronic states. So they are not relevant to the problem under consideration. To get rid of these terms one must place the correlator into the variable external field allowing one to make distinction between momentum variables of initial and final baryons. In the first order in the weak interaction we arrive at the correlator

$$K(q_1, q_2) = \int dx\, dy \exp(i q_2 y - i q_1 x) \cdot \langle 0 | T\{ \eta_p(y) H_W'(0) \bar{\eta}_n(x) \} | 0 \rangle \qquad (7)$$

The matrix element calculated becomes a function of $k^2$, $k = q_1 - q_2$. This circumstance is not essential for us as soon as we put $k^2 = 0$.

Let us expand $K(q_1, q_2)$ in the four independent $\gamma$-matrix structures

$$T_0 = \not{q}_1 - \not{q}_2, \quad T_1 = \frac{\not{q}_1 + \not{q}_2}{2}, \quad T_2 = \frac{\not{q}_1 \not{q}_2 - \not{q}_2 \not{q}_1}{2}, \quad T_3 = -1 \qquad (8)$$

in the following way:

$$K(q_1, q_2) = G_F \sqrt{2} \frac{\pi^4}{(2\pi)^{10}} \sum_{i=0}^{3} k_i(s_1, s_2) T_i \qquad (9)$$

Then the Laplace transformation $L_1 L_2$ of $k_i(s_1, s_2)$ in the two variables $s_1 = -q_1^2$, $s_2 = -q_2^2$ (first used in ref. [6])

$$k_i(t_1, t_2) \equiv L_1 L_2 k_i(s_1, s_2) = \int_{\varepsilon - i\infty}^{\varepsilon + i\infty} \frac{ds_1}{2\pi i t_1} \int_{\varepsilon - i\infty}^{\varepsilon + i\infty} \frac{ds_2}{2\pi i t_2} \cdot \exp\left( \frac{s_1}{t_1} + \frac{s_2}{t_2} \right) \cdot k_i(s_1, s_2), \quad \varepsilon > 0, \qquad (10)$$

turns out to be adequate to the problem, since it cancels single pole terms. Contour integration (10) can be performed with the help of the double dispersion relation for $k_i(s_1, s_2)$:

$$k_i(s_1,s_2)=\int_0^\infty\int_0^\infty \frac{\rho_i(u_1,u_2)\,du_1du_2}{(u_1+s_1)(u_2+s_2)} \qquad (11)$$

Then

$$k_i(t_1,t_2)=\int_0^\infty\int_0^\infty \rho_i(u_1,u_2)\exp\left(-\frac{u_1}{t_1}-\frac{u_2}{t_2}\right)\frac{du_1}{t_1}\frac{du_2}{t_2} \qquad (12)$$

Phenomenological expressions for $k_i$ follow from saturation of $K(q_1,q_2)$ by physical hadronic states:

$$K(q_1,q_2)=-\frac{\tilde{\beta}^2}{(2\pi)^4}(-\gamma_5)\frac{i}{\not{q}_2-m}\,\alpha\,\frac{i}{\not{q}_1-m}\gamma_5+\text{higher states} \qquad (13)$$

where $m$, $\tilde{\beta}$ are the nucleon mass and residue, respectively, $\langle 0|\eta_N|N\rangle=(2\pi)^{-2}\tilde{\beta}\gamma_5 u_N$, baryon spinors $u_N$ being normalized in a usual way, $\bar{u}_N u_N=2m$. Transformation $L_1L_2$ (10),(12) improves convergence of OPE series and cancels single pole subtraction terms in (13). Besides, it suppresses the higher states contribution in (13). Therefore let us omit this contribution at the first step. Consider, for example, the structure $T_1=(\not{q}_1+\not{q}_2)/2$. Equating both the phenomenological and theoretical expressions for $k_1$ yields with taking into account (5),(9),(13) the amplitude $A_\pi$ of interest

$$A_\pi=\frac{1}{64\sqrt{2}\,m_\pi^2 f_\pi m\tilde{\beta}^2\pi^2}\,t_1t_2k_1(t_1,t_2)\exp\frac{m^2}{t},$$
$$t=\frac{t_1t_2}{t_1+t_2} \qquad (14)$$

The higher states contribution is accounted for in a model way as continuum. Transferring the sum over higher states from the LHS (phenomenological part) to the RHS (theoretical part) of the sum rules is suggested to be equivalent to limitation of integration in spectral representation (11),(12) in the RHS by some region $\Omega$. We adopt the following two models for $\Omega$: the square

$$\Omega=\{(u_1,u_2)\,|\,0<u_{1,2}<s_0\} \qquad (15)$$

or the triangle



$$\Omega=\{(u_1,u_2)\,|\,u_1+u_2<2s_0,\ u_{1,2}>0\} \qquad (16)$$

Varying $s_0$ we aim at the existence of a platou in the dependence of $A_\pi$ on $t=t_1/2=t_2/2$. The difference in the results of such the fitting procedure for the two choices of $\Omega$, (15), (16), turns out to be small, $\simeq 1 \div 2\%$. For definiteness, we shall work further with the variant (15) for $\Omega$.

We can unambiguously determine the contribution to $k_i(t_1,t_2)$ of the diagrams like those depicted in Figs. 1a,2 responsible for the weak interaction at short distances. These diagrams are given by explicit formulas as functions of $s_1$, $s_2$. At the same time effects connected with the weak interaction at large distances (see Figs. 3,4) are included into a definition of the vacuum expectation values (VEV's) of the operators in a weak field ("weak condensates").

We are able to calculate these weak condensates only when $k=0$ and $s_1=s_2$. We can also compute a few terms in the expansion of the corresponding part of correlator in the $(s_1-s_2)$ series around the point $s_1=s_2$. It is not sufficient to perform $L_1L_2$ since the double spectral density $\rho(s_1,s_2)$ must be known for that. However, it is sufficient to perform the ordinary Laplace transformation $L_t$ (in the variable $s=s_1=s_2$ and with a parameter $t$). This transformation had been successfully applied to calculation of the baryonic couplings to the two-quark external fields [9-11] characterized by the absence of strong UV divergences like those considered above.

Thus, weak interactions at different distances, namely, at short and at large ones require different techniques, namely the double and the ordinary Laplace transformations, respectively, for analysis of the corresponding contributions into the physical amplitude of interest. It is desirable, from the formal viewpoint, to reproduce the corresponding results in the framework of a single methodic, namely, using the double Laplace transformation. With this end in view consider the general form of the double spectral density $\rho(s_1,s_2)$ relevant to the effects of weak condensate.

First, a connection between $\rho(s_1,s_2)$ and a model used for the continuum should be noted. Confining ourselves by the region $s_1 = s_2$ we lose a considerable part of information on the continuum contribution. The latter was earlier attributed to the complement of $\Omega$ in the quadrant $s_1 \geqslant 0$, $s_2 \geqslant 0$. In case of the two-quark external fields [9,11] an acceptable model for continuum is that resulting in the following sum rules:

$$\tilde{\beta}^2(A+Bt) = \left(\exp\frac{m^2}{t}\right)\sum_n C_n t^n E_n\left(\frac{S_0}{t}\right) \qquad (17)$$

where

$$E_n(x) = \begin{cases} \frac{1}{(n-1)!}\int_0^x y^{n-1} e^{-y} dy, & n \geqslant 1 \\ 1, & n \leqslant 0 \end{cases} \qquad (18)$$

Here A is the constant to be determined, B and $S_0$ parametrize single poles in the LHS and continuum contribution, respectively. In particular case of vector field the coefficients $C_n$ in (17) were shown in ref. [11] to reproduce those in the expansion for $\tilde{\beta}^2$; therefore the specific form of dependence on $S_0$ exhibited by (17) is the only one required to ensure nonrenormalizability of vector constant $A = g_V = 1$. The same form of dependence on $S_0$ leads to consistency with experiment in cases of external axial [11] and electromagnetic [9] fields. In particular, calculation with the power corrections suppressed results in the neutron and proton magnetic moments being equal to $(-\frac{4}{3})$ and $(+\frac{8}{3})$, respectively; taking into account these corrections yields an agreement within 10% with experiment [9]. Further, a connection between (17) and $\rho(u_1, u_2)$ is as follows

$$\sum_n C_n t^n E_n\left(\frac{S_0}{t}\right) =$$

$$= \int_\Omega \rho(u_1, u_2)\, \frac{\exp\left(-\frac{u_2}{t}\right) - \exp\left(-\frac{u_1}{t}\right)}{u_1 - u_2}\, \frac{du_1 du_2}{t} \qquad (19)$$

where the region $\Omega$ depends on $S_0$ as on a parameter. The



RHS of (19) does not contain any subtraction terms. Really, the only necessary subtraction in the diagrams of interest like those of Fig. 3 is that which must be done in the loop I, and corresponding subtraction term is cancelled by $L_t$; the shaded blob does not add any new divergences since it survives, by definition, only at small $p_1^2$, $p_2^2$ and at $p_1 = p_2 = 0$ it comes to the finite matrix element of a definite local operator in external field. It is possible to show that (19) implies the following general form of $\rho(S_1, S_2)$:

$$\rho(S_1, S_2) = f(S_1)\,\delta(S_1 - S_2) \qquad (20)$$

With this condition the double Laplace transformation is reduced to the ordinary one in $S = S_1 = S_2$ with the parameter $t = t_1 t_2 / (t_1 + t_2)$.

Our choice (20) for general form of $\rho(S_1, S_2)$ relevant to weak condensates is supported, firstly, by an analogy with the two-quark external fields. Secondly, such the form of $\rho$ appears at explicit calculation of graphs of Fig. 2 containing the weak vertex at short distances. While lowering virtualities of the weakly interacting quarks the graphs of such the kind turn into those of interest of Fig. 3 relevant to the weak condensates. Finally, thirdly, ansatz (20) seems quite reasonable from physical point of view: once interaction is soft, it can not appreciably change the invariant mass squared $s_1$ of incoming three-quark state. This circumstance is just reflected by $\delta$-function in (20).

### 3. Derivation of the sum rules

The sum rules are derived for the structure $T_1 = (\not{A}_1 + \not{A}_2)/2$, since it is nonsingular at $q_1 = q_2$ (which is important for the analysis of the weak condensate effects) and, in addition, resulting sum rules are comparatively low sensitive to the continuum contribution.

The nucleon currents take the form

$$\eta_p = (u^a C \gamma_\mu u^b)\gamma^\mu d^c \varepsilon_{abc}$$

$$\eta_n = -(d^a C\gamma_\mu d^b)\gamma^\mu u^c \varepsilon_{abc},\quad C=\gamma_0\gamma_2 \qquad (21)$$

The structure of the effective Hamiltonian $H_W^{PV}(Q_q)$ is determined by a typical quark momentum $Q_q$. The latter squared, $Q_q^2$, is substituted by $M^2$, some scale of order of $t_1, t_2$ once Laplace transformation is applied to the correlator. An expression for the interesting for us $\Delta S = 0$ Hamiltonian $H_W^{PV}(M)$ at $M \gtrsim m_c$ is found in ref. [12] (the general case $M < m_c$ is considered in refs. [13-15]). As it is easy to check, transition $n \to p\pi^-$ is contributed only by $\Delta T = 1$, flavour octet part of $H_W^{PV}$:

$$\frac{H_W^{PV}(m_c)}{G_F\sqrt{2}} = s_c^2\left(\frac{1}{20}K_W^{d_{84}}O_1^S + \frac{1}{4}K_W^{d_{20}}O_1^A\right) + (1-2s_W^2)\left(-\frac{1}{20}K_Z^{d_{84}}O_1^S + \frac{1}{4}K_Z^{d_{20}}O_1^A\right) + \left(-\frac{1}{3}s_W^2\right)[1,0,0,0]\,K_Z^{d_{15}}O_1^8 \qquad (22)$$

where
$$s_c = \sin\theta_c,\quad s_W = \sin\theta_W$$

$$K_{W,Z} = \alpha_S(m_c)/\alpha_S(m_{W,Z})$$

The matrices $d_R$ (proportional to the anomalous dimension matrices of operators $O^R$) take the form

$$d_{84} = -\frac{6}{25},\quad d_{20} = +\frac{12}{25},$$

$$d_{15} = -\frac{1}{25}\begin{bmatrix} -\frac{2}{3} & 2 & -3 & 9 \\ \frac{11}{6} & -\frac{11}{2} & \frac{9}{2} & \frac{21}{2} \\ -\frac{11}{3} & 11 & 0 & 0 \\ \frac{9}{2} & \frac{21}{2} & \frac{9}{2} & -\frac{27}{2} \end{bmatrix} \qquad (23)$$

The operators ($O_1^S$, $O_1^A$ and a column $O_1^8$) are defined in the following way:

$$\begin{bmatrix} O_1^S \\ O_1^A \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix} O_1^8$$



$$O_1^8 = \frac{1}{2}\begin{bmatrix} O(\lambda_3, 1) \\ O^c(\lambda_3, 1) \\ O(1, \lambda_3) \\ O^c(1, \lambda_3) \end{bmatrix} \qquad (24)$$

where

$$O(M,N) = (\overline{\Psi}\gamma_\mu\gamma_5 M\Psi)(\overline{\Psi}\gamma^\mu N\Psi)$$
$$O^c(M,N) = (\overline{\Psi}^a\gamma_\mu\gamma_5 M\Psi^b)(\overline{\Psi}^b\gamma^\mu N\Psi^a) \qquad (25)$$

$\lambda^n (2t^n)$ are the Gell-Mann matrices in $SU(3)_{fl}$ $(SU(3)_c)$. The quark field $\Psi$ carries the flavour ($\alpha, \beta, \gamma, \ldots = 1,2,3$), colour ($a,b,c,\ldots = 1,2,3$) and bispinor ($i,k,l,\ldots = 1,2,3,4$) indices. $q$ means a definite (light) flavour, $u_i^a, d_i^a$ or $s_i^a$. Expressions like $\overline{\Psi}\Psi$ imply the summation over all the indices, while those like $\Psi\overline{\Psi}$ do not, i.e.

$$\overline{\Psi}\Psi \equiv \overline{\Psi}^a_{\alpha i}\Psi^a_{\alpha i},\quad \Psi\overline{\Psi} \equiv \Psi^a_{\alpha i}\overline{\Psi}^b_{\beta j} \qquad (26)$$

We have analysed contribution of operators with dimensions $d \leq 8$ in the operator expansion. Consider first the graphs like those of Figs. 1a,d containing the weak vertex at short distances. It turns out that these graphs do not vanish only due to operators from $H_W^{PV}$ representable as the products of right- and left-handed currents. Let us note just here that their contribution into the whole amplitude is small, of order of 1%. Therefore we have taken into account only contributions of operators I (asymptotic loop of Fig. 1a) and $\Psi\overline{\Psi}\Psi\overline{\Psi}$ (Figs. 1d, 2b). The latter can be, generally speaking, anomalously large due to the loss of loop smallness, but in case of the transition $n \to p$ it equals to zero identically.

The dominant contribution comes from the weak interaction at large distances. Corresponding graphs are shown in Fig. 3 and have only two valence quarks weakly interacting at large distances. Contribution of the configurations with four valence quarks weakly interacting at large distance $x$ (see Fig. 4) is suppressed by an additional fermion propagators. As a result, the graphs of such the topology are essential in higher orders of operator expansion, namely, in orders corresponding to 6 units larger operator dimensions than in the two-quark case. Therefore we disregard these graphs.

We have analysed the following VEV's in the charged weak field (in notations of (26)):

$$\langle \nabla_\mu u\bar{d}\rangle,\ \langle \nabla_\mu \nabla_\nu \nabla_\lambda u\bar{d}\rangle\ (\langle \nabla_\mu u\bar{d}G\rangle),$$
$$\langle u\bar{d}q\bar{q}\rangle \approx \langle u\bar{d}\rangle\langle q\bar{q}\rangle \tag{27}$$
$$\langle u\bar{d}q\bar{q}G\rangle \approx \langle u\bar{d}G\rangle\langle q\bar{q}\rangle + \langle u\bar{d}\rangle\langle q\bar{q}G\rangle$$

the two latter being estimated using the factorization hypothesis. Let us put for estimate

$$\langle \bar{d}\sigma_{\mu\nu}t^n ig_s G^n_{\mu\nu}u\rangle = m_0^2\langle \bar{d}u\rangle \tag{28}$$

where $m_0^2$ is the same as in the definition

$$\langle \bar{q}\sigma_{\mu\nu}t^n ig_s G^n_{\mu\nu}q\rangle = m_0^2\langle \bar{q}q\rangle \tag{29}$$

Of course, this estimate is rather rough. It turns, however, that the corresponding contribution into the sum rules (due to the first term in $\langle u\bar{d}q\bar{q}G\rangle$ (27)) is about 10÷15% to the main term. Consequently, the roughness of the estimate (28) can not essentially tell on the accuracy of the final result. By means of the equations of motion

$$i\not{\nabla} q = m_q q + \frac{\partial H_W}{\partial \bar{q}} \tag{30}$$

the considered weak condensates can be differentiated in $G_F$ and reduced to the pure QCD VEV's, for example:

$$\langle u_i^a \bar{d}_k^b\rangle = -\frac{\delta^{ab}\delta_{ik}}{12}\langle 0|\frac{\bar{d}\frac{\partial H'_W}{\partial \bar{u}} - \frac{\partial H'_W}{\partial d}u}{m_d - m_u}|0\rangle \tag{31}$$

The RHS of (31) is the VEV of some four-quark operator normalized at $M \simeq m_c$. Expressing this VEV through those normalized at $\mu < m_c$ with the help of renormalization group (RG) we explicitly take into account perturbative contribution of the fields of virtuality $p$ from the region $\mu < p < m_c$. Virtualities below $\mu$ are included into a definition of the matrix element itself. The latter is calculated by vacuum insertion at low $\mu$. Putting $\alpha_s(\mu) = 1$ yields $\mu$ = 0.2 GeV at $\Lambda$ =



= 0.1 GeV. We have checked stability of the results with respect to the choice of $\mu$. Enhancing $\mu$ from 0.2 to 0.5 GeV leads to lowering $A_\pi$ by no more than 5%. Such the stability is explained by mutual compensation of the following two effects caused by enhancing $\mu$. The first effect consists in diminishing factors accounting for the renormalization from $m_c$ to $\mu$ of the considered pure QCD VEV's. The second one is enhancing the normalized at $\mu$ quark condensates $\langle \bar{q}q\rangle$ appearing while these pure QCD VEV's are factorized. To make use of the RG in the region $\mu < p < m_c$ with more confidence the larger $\mu$, $\mu$ = 0.5 GeV, is chosen.

For the sake of simplicity we present here the results of calculation of $k_1(t_1,t_2)$ without taking into account dressing with gluons in the whole region $\mu < p < m_{W,Z}$:

$$t_1t_2k_1 = \frac{2}{3}s_W^2\left[\frac{3}{5}\frac{t_1^3t_2^3}{t_1+t_2} + \frac{16}{3}a^2t^2\left(1-\frac{1}{3}\frac{m_0^2}{t}\right) + \frac{64}{9}\frac{a^3}{\Delta}t\left(1-\frac{1}{4}\cdot\frac{m_0^2}{t}\right)\right],\quad t = \frac{t_1t_2}{t_1+t_2} \tag{32}$$

where $a = -(2\pi)^2\langle \bar{q}q\rangle$,

$$\frac{1}{\Delta} = \frac{\langle(\bar{u}u - \bar{d}d)\rangle}{\langle \bar{u}u\rangle(m_d - m_u)} \tag{33}$$

The first term in square brackets in (32) is given by asymptotic loop of Fig. 1a. The second one is the contribution of the weak condensates $\langle \nabla_\mu u\bar{d}\rangle$ and $\langle \nabla_\mu u\bar{d}G\rangle$, $\langle \nabla_\mu\nabla_\nu\nabla_\lambda u\bar{d}\rangle$ (see Figs. 3a-c) and the third one is that of $\langle u\bar{d}q\bar{q}\rangle$ and $\langle u\bar{d}q\bar{q}G\rangle$, $\langle \nabla_\mu\nabla_\nu u\bar{d}q\bar{q}\rangle$, $\langle u\bar{d}\nabla_\mu\nabla_\nu q\bar{q}\rangle$ (Figs. 3d-h). The complete form of the sum rules taking into account the gluonic corrections is given in Appendix.

## 4. Numerical analysis of the sum rules

In what follows the knowledge of $\Delta$ (see (33)) is important. In refs. [16,17] some estimates of $\langle(\bar{u}u - \bar{d}d)\rangle$ consistent with each other had been obtained by different methods. In particular, the result of ref. [16] provides an elegant formula

$$\Delta \approx \frac{m_u+m_d}{m_\pi^2} m_\rho^2 = 330\ \mathrm{MeV} \tag{34}$$

at $m_u+m_d$ = 11 MeV. Calculation using this estimate results in that the main contribution into the sum rules comes from the operator $u\bar{d}q\bar{q}$ . The largeness of this correction does not violate, however, an applicability of the operator expansion. Really, the given correction is singled out both for geometrical reasons (it is determined by Born graph of Fig. 2d, not by loop one) and for accidental reasons for it is sensitive to $\Delta$ . This anomaly is not iterated in the next orders of operator expansion: as it is seen from (32), the subsequent (quark-gluon) corrections are comparatively small, as usually.

We adopt the following values for the rest of parameters*:

$$s_c=0.219\,,\ s_W^2=0.23\,,\ m_W=80\ \mathrm{GeV}$$
$$a=0.546\ \mathrm{GeV}^3,\ \Lambda=0.1\ \mathrm{GeV},\ m_0^2=1\ \mathrm{GeV}^2 \tag{35}$$

The nucleon residue is determined from QCD sum rules obtained in refs. [3,4]**:

$$\tilde{\beta}^2 = 1.0\ \mathrm{GeV}^6 \tag{36}$$

The dependence of $A_\pi$ on $t = t_1/2 = t_2/2$ is shown in Fig. 5 for the three values of the continuum threshold $S_0$ .

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* The normalization point of the presented VEV's corresponds here to $\alpha_s$ = 0.7.

** The quantity 0.25 GeV$^6$ for $\frac{1}{4}\tilde{\beta}^2$ we use in this paper is slightly smaller than the number 0.30 GeV$^6$ obtained in refs. [3,4] (with taking into account some corrections, see ref. [11]). This difference is connected with that we issue everywhere from the experimental nucleon mass whereas in refs. [3,4] it is extracted from QCD sum rules parallel with $\tilde{\beta}^2$ and turns out to exceed it's experimental value.



Analysis of the sum rules is performed in the region $0.5 \leqslant t \leqslant 1.5$ GeV$^2$: on the one hand, sum rules are the most sensitive to the lowest resonance contribution just at $t \simeq m^2 \approx 1$ GeV$^2$, on the other hand, the nearest power correction to the main term is sufficiently small and is 25% at $t$ = 1 GeV$^2$ and 15% at $t$ = 1.5 GeV$^2$. We have found that $A_{\pi_2}$ is practically constant at $S_0$ = 1.5 GeV$^2$, $1.0 \leqslant t \leqslant 1.5$ GeV$^2$ and equals to

$$A_\pi = 2.7 \tag{37}$$

(at $\mu$ = 0.5 GeV; at $\mu$ = 0.2 GeV $A_\pi$ = 2.8). This resut is consistent with the experimental bounds (2) for $A_\pi$ within the accuracy expected. The latter seems to be about 25% if one takes his cue from the ratio of the nearest power correction to the main term or from the continuum contribution to this term (being equal to 20÷40% at $t$ = 1÷1.5 GeV$^2$, respectively).

Finally, the sum rules demonstrate the dominant role of neutral currents and importance of perturbative gluonic corrections in $A_\pi$ (see refs. [13,14,20,21]). For example, the contribution of charged currents in $A_\pi$ is only about 1%. Disregarding perturbative dressing with gluons in the whole range $\mu < p < m_{W,Z}$ yields

$$A_\pi(\alpha_s=0) = 0.8 \tag{38}$$

instead of (37). Accounting for the renormalizations of only $H_W^{PV}$ or only in matrix elements of the type of (31) leads to

$$A_\pi(m_c < p < m_{W,Z}) = 1.7 \tag{39}$$

$$A_\pi(\mu < p < m_c) = 1.2 \tag{40}$$

respectively.

## 5. Conclusion

Thus, the QCD sum rules method allows one to overcome to a considerable extent the difficulties listed in introduction inherent in the previous methods of calculation of parity nonconserving $\pi$ NN vertex and results in an agreement with experiment in the framework of the standard model.

In conclusion, the author is indebted to I.B.Khriplovich who has drawn my attention to the problem involved, for consideration of the work; to V.L.Chernyak, A.I.Vainshtein and A.R.Zhitnitsky for helpful discussions and reading the manuscript.



## Appendix

The general form of the function $t_1 t_2 k_1$ entering the sum rule (14) for $A_\pi$ with taking into account dressing with gluons is as follows:

$$t_1t_2k_1 = s_c^2\left(K_w^{0.48}A - \frac{1}{5}K_w^{-0.24}B\right) + (1-2s_w^2)\left(K_Z^{0.48}A + \frac{1}{5}K_Z^{-0.24}B\right) + \frac{2}{3}s_w^2[1,0,0,0]K_Z^{d_{15}} \cdot$$

$$\cdot\left[\frac{3}{5}L_M^{-4/9}\frac{t_1^3t_2^3}{t_1+t_2}\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix} + \frac{16}{3}L_M^{-4/9}a^2t^2X - \frac{16}{9}L_M^{-4/9}a^2m_0^2t\begin{bmatrix}1\\3\\-1\\-3\end{bmatrix} + \frac{32}{27}a^3\frac{1}{\Delta}tD_1Y\left(L_M^{+4/9} - \frac{m_0^2}{4t}\right)\right] \quad (1A)$$

where the scalars A,B and vector columns $X$ , $Y$ are

$$A = \frac{8}{3}L_M^{-4/9}a^2t^2[1,-1,-1,1]X + \frac{64}{27}L_M^{+4/9}\frac{a^3}{\Delta}t[1,0,0,0]Y$$

$$B = \frac{8}{3}L_M^{-4/9}a^2t^2[1,1,1,1]X + \frac{64}{27}L_M^{+4/9}\frac{a^3}{\Delta}\cdot t[0,1,0,0]Y \quad (2A)$$

$$X = \frac{1}{4}D_2K^{d_S}\begin{bmatrix}3\\16\\0\\0\end{bmatrix} + \frac{1}{2}K^{8/9}\begin{bmatrix}1\\3\\-3\\-1\end{bmatrix}, \quad Y = K^{d_1}\begin{bmatrix}0\\0\\16\\3\end{bmatrix}$$

The anomalous dimension matrices $bd_R$ are introduced in accordance with (23) and with

$$d_S = \frac{1}{9}\begin{bmatrix}0 & \frac{3}{2} & 0 & 0\\ \frac{16}{3} & 5 & 0 & -1\\ 0 & -\frac{2}{3} & 0 & -\frac{11}{6}\\ 0 & -\frac{32}{9} & -\frac{16}{3} & \frac{2}{9}\end{bmatrix}, \quad d_1 = \frac{1}{9}\begin{bmatrix}\frac{34}{9} & \frac{1}{9} & \frac{1}{6} & 0\\ \frac{10}{9} & -\frac{23}{9} & -\frac{5}{6} & 0\\ \frac{4}{3} & -\frac{2}{3} & 6 & \frac{16}{3}\\ 0 & 0 & -\frac{3}{2} & 0\end{bmatrix} \quad (3A)$$

The matrices $D_1$ , $D_2$ are

$$D_1=\begin{bmatrix} 1 & 1 & 0 & 2 \\ -1 & 1 & 1 & +\frac{2}{3} \\ 1 & 1 & 0 & -2 \\ -1 & 1 & -1 & -\frac{2}{3} \end{bmatrix}, \quad D_2=\begin{bmatrix} \frac{2}{3} & 0 & \frac{1}{3} & \frac{1}{16} \\ \frac{2}{9} & \frac{1}{3} & \frac{2}{9} & \frac{7}{48} \\ -\frac{2}{3} & 0 & \frac{1}{3} & \frac{1}{16} \\ -\frac{2}{9} & -\frac{1}{3} & \frac{2}{9} & \frac{7}{48} \end{bmatrix} \tag{4A}$$

Finally,

$$K_{w,z}=\frac{\alpha_s(m_c)}{\alpha_s(m_{w,z})}, \quad K=\frac{\alpha_s(\mu)}{\alpha_s(m_c)}, \quad L_M=\frac{\alpha_s(\mu)}{\alpha_s(M)} \tag{5A}$$



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## Figure captions

Fig. 1. The diagrams (a) and (d) are responsible for weak interaction of the four valence quarks at short distances; the graphs (b,c) and (e), respectively, correspond to the subtraction terms to these diagrams.

Fig. 2. The diagrams relevant to weak interaction of the two valence quarks at short distances.

Fig. 3. The graphs accounting for weak interaction of the two valence quarks at large distances. The shaded blob stands for interaction of the soft fields with an external (herewith a weak) field.

Fig. 4. The diagram relevant to weak interaction of the four valence quarks at large distances.

Fig. 5. $A_{\pi}$ versus $t$ for three choices of the continuum threshold $S_0$ .

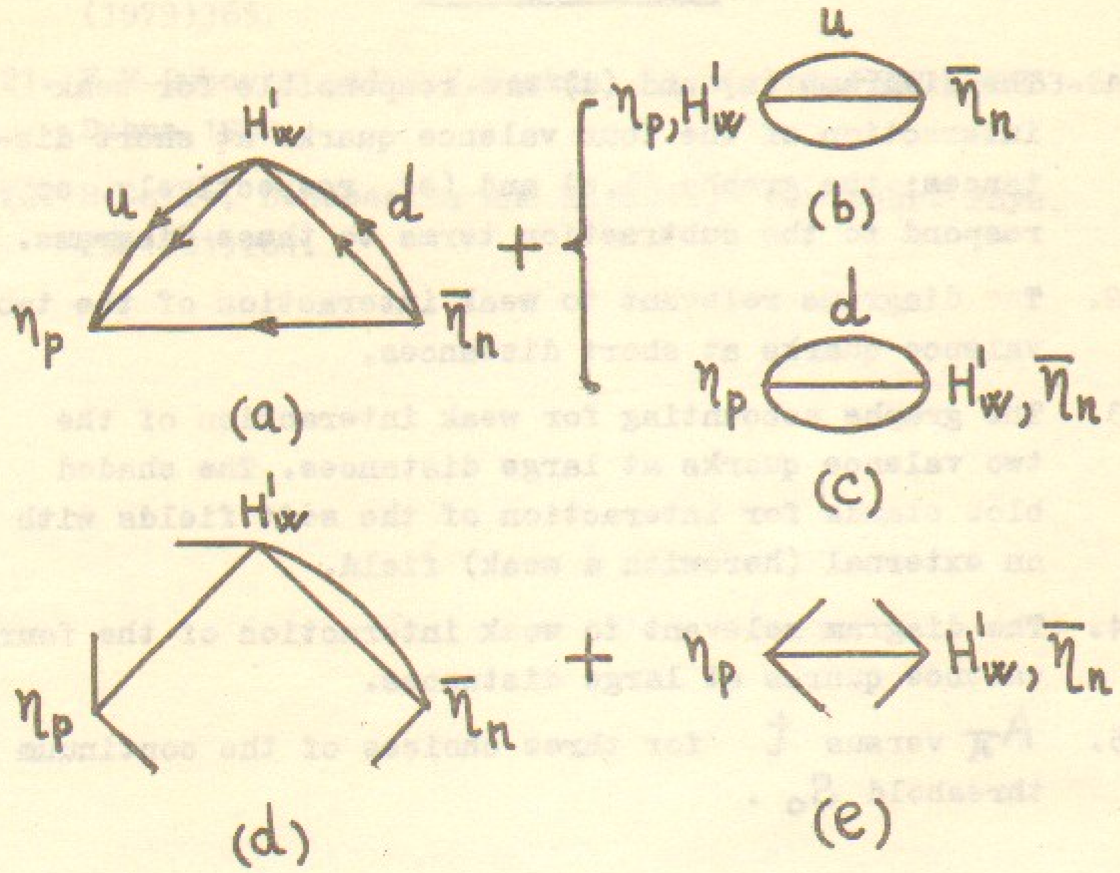


Fig.1

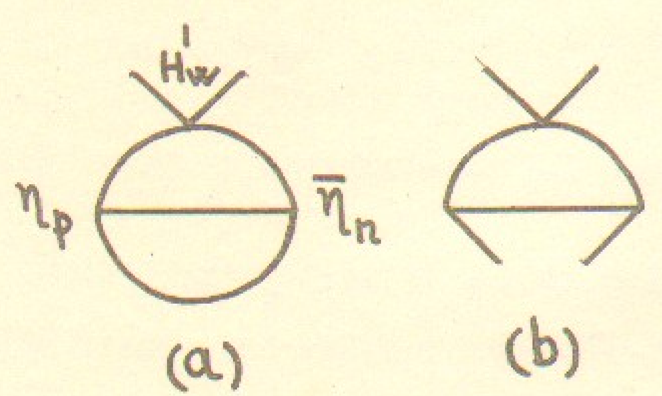


Fig.2

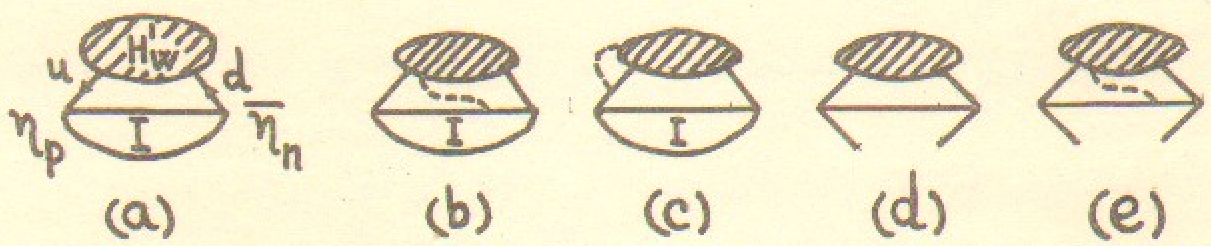


Fig.3




Fig.3

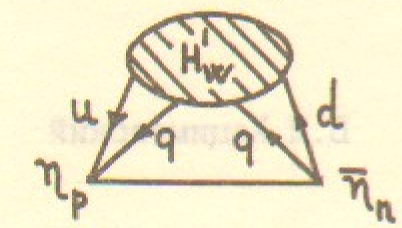


Fig.4

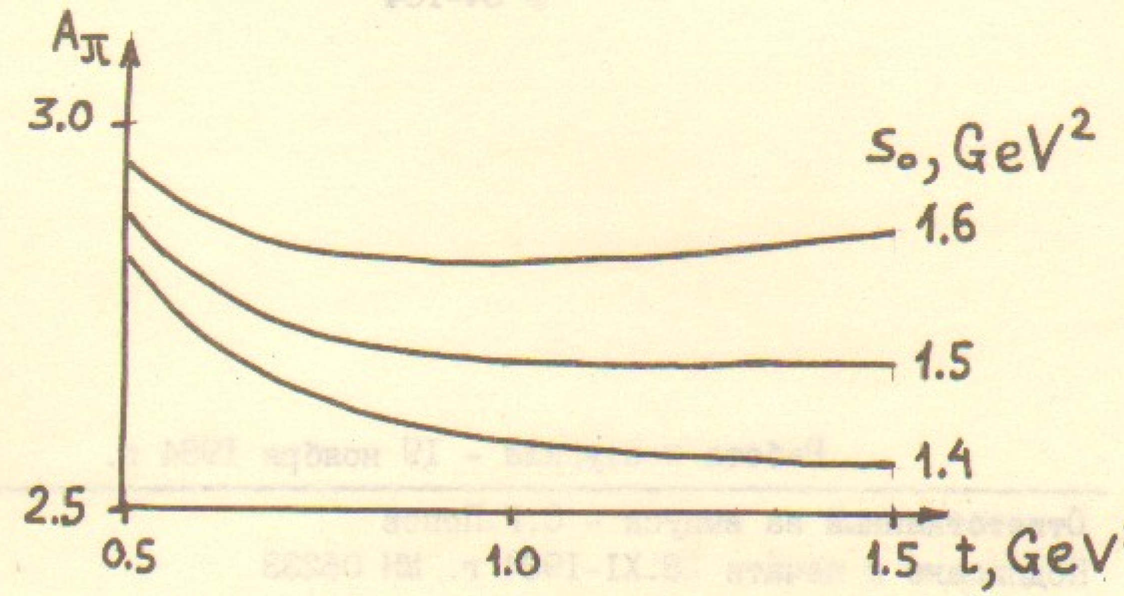


Fig.5

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НАРУШАЮЩАЯ ЧЕТНОСТЬ ВЕРШИНА $\pi NN$ В КХД

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