ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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NUCLEAR ANAPOLE MOMENTS

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ABSTRACT

In a simple model an analytical expression is obtained for the parity-violating electromagnetic, so-called anapole, moment of a spherical nucleus. The result is confirmed by numerical computation in the Saxon-Woods potential for the nuclei ^{133}Cs, $^{203,205}Tl$, ^{207}Pb, ^{209}Bi. The effective coupling constant of the P-odd electron-nucleus interaction constitutes 0.3÷0.4 of the Fermi constant. Measurement of the nuclear anapole moments is possible in atomic and molecular experiments.

The weak interaction of electrons with nucleus due to neutral currents was discovered at Novosibirsk in an atomic experiment [1-3] and then observed in many experiments both high-energy [4,5] and atomic [6-11]. In all these experiments only the effects of parity violation independent of nuclear spin were studied in fact. As to the P-odd phenomena in heavy atoms, dependent on the nuclear spin, the difficulty of their investigation stems from the fact that they are suppressed, roughly speaking, by Z times in comparison with the effects already detected and measured. Moreover, the parameters of the standard electroweak model are of such values that the dimensionless constant $\varkappa_2$ that determines the nuclear spin-dependent P-odd correlations (its definition is given below) is numerically small, $\varkappa_2 \approx 0.05$.

However, some years ago it was noted [12] that the P-odd nuclear spin dependent effects in atom can arise due to the electromagnetic interaction of electron with P-odd multipole moments of nuclei. The latter arise due to parity violating nuclear forces. The dimensionless effective constant $\varkappa_a$ caused by such an interaction with the lowest of these multipoles - nuclear anapole moment was estimated in Ref. [12] for the case of heavy nuclei as

$$\varkappa_a \sim 0.1 - 1 \qquad (1)$$

And it is considerably larger than the above mentioned contribution to the effect of neutral weak currents.

Now when the measurement of the nuclear anapole moments has become not only important, but sufficiently realistic experimental problem a more detailed calculation of this nuclear characteristic is quite relevant, so much the more that the estimate (1) was many times reproached to be over-optimistic.

The nuclear anapole moment $\vec{a}$ is expressed through the electromagnetic current density $\vec{j}(\vec{r})$ as follows [12]:

$$\vec{a} = -\pi \int d^3r \; r^2 \vec{j}(\vec{r}) \qquad (2)$$

We shall use in our calculations the simple shell model of a nucleus. The parity-violating weak interaction of the external nucleon with the core in a heavy nucleus we shall describe by the Hamiltonian

$$\hat{W} = \frac{Gg}{\sqrt{2}} \frac{1}{2m} \left\{ \vec{\sigma}\vec{p}\, \rho(r) + \rho(r)\, \vec{\sigma}\vec{p} \right\} \tag{3}$$

Here $G = 10^{-5} m^{-2}$ is the Fermi constant of weak interaction, m, $\vec{\sigma}$ and $\vec{p}$ are the mass, spin and momentum of the external nucleon, $\rho(r)$ is the core density normalized by the condition $\int d^3r\, \rho(r) = A$ (we assume that the atomic number $A \gg 1$). The dimensionless constant g is expressed in the following way through the constants of weak meson-nucleon interaction (we use the notations of Ref.[13]):

$$g_p = 2.0 \cdot 10^5 W_\rho \left[176 \frac{W_\pi}{W_\rho} f_\pi - 19.5 h_\rho^0 - 4.7 h_\rho^1 + 1.3 h_\rho^2 - 11.3 (h_\omega^0 + h_\omega^1) - 1.7 h_\rho^{1'} \right] \tag{4}$$

$$g_n = 2.0 \cdot 10^5 W_\rho \left[-118 \frac{W_\pi}{W_\rho} f_\pi - 18.9 h_\rho^0 + 8.4_\rho^1 - 1.3 h_\rho^2 - 12.8 (h_\omega^0 - h_\omega^1) + 1.1 h_\rho^{1'} \right]$$

The parameters W_ρ and W_π are introduced to take into account the nucleon-nucleon repulsion at short distances and non--locality of the true interaction potential. The latter point is especially important for the π-meson exchange since m_π^{-1} is comparable with the characteristic scale of the nucleon wave-function variation in a nucleus. Basing on the calculation of the weak interaction between the neutron and α-particle performed in Ref.[14], we take $W_\rho = 0.4$ and $W_\pi = 0.16$. These values are in reasonable agreement with those previously obtained: $W_\rho = 0.4$, $W_\pi = 0.14$ [15] and $W_\rho = 0.4$, $W_\pi = 0.25$ [16]. The possible interval of variation of the constants f_π and h_i was estimated in Ref.[13]. Using the values of these constants considered as the best values in Ref.[13], we find that for the proton $g_p = 4.6$, for the neutron $g_n = 0.2$. The smallness of g_n is caused by the cancellation between π- and ρ-contributions. In the situation when all the constants are determined very roughly, this compensation should not be taken too seriously. E.g., at the values of f_π and h_i

from Ref.[17] $g_p = 2.5$, $g_n = 1.0$.

We start from a simple model leading to an analytical expression for the anapole moment of a heavy nucleus. We shall take in the formula (3) the density $\rho(r)$ as a constant all over the space coinciding with the average nuclear density ρ_0. This approximation is quite reasonable in the situation when the wave function of external nucleon is localized mainly inside the core. In this approximation the solution of the Schroedinger equation

$$\left[-\frac{1}{2m}\Delta + U(r) + \hat{W}(\vec{r}) \right] \Psi = E\Psi \tag{5}$$

to the first order in the perturbation $\hat{W}$ is found in an elementary way:

$$\Psi(\vec{r}) = \left\{ 1 - i \frac{Gg\rho_0}{\sqrt{2}} \vec{\sigma}\vec{r} \right\} \Psi_0(\vec{r}) \tag{6}$$

Here $\Psi_0(\vec{r})$ is the unperturbed wave function of the external nucleon. One could suspect that the interaction (3) at $\rho(r) = \rho_0 =$ const, being equivalent to the electromagnetic interaction with the constant vector-potential $\vec{A} = -\frac{Gg\rho_0}{\sqrt{2}} \vec{\sigma}$, should not lead to any physical consequences at all. And indeed, the orbital contribution to $\vec{a}$ vanishes in this approximation, as one can check easily also by direct calculation using the corresponding formulae of Ref.[12]. However, the spin contribution to the current density

$$\vec{j}^{\,s}(\vec{r}) = \frac{e\mu}{2m} \vec{\nabla} \times \Psi^+ \vec{\sigma} \Psi \tag{7}$$

(μ is the nucleon magnetic moment) is quite operative even in this approximation, due to non-commutativity of σ-matrices. Simple calculation with the formulae (2), (6) and (7) lead to the following expression:

$$\vec{a} = \frac{Gg\rho_0}{\sqrt{2}} \frac{2\pi e\mu}{m} \frac{K\vec{I}}{I(I+1)} \langle r^2 \rangle \tag{8}$$

where I is the spin of the nucleus, $K = (I+1/2)\,(-1)^{I+1/2-l}$, l is the orbital angular momentum of the external nucleon. The mean square radius of the external nucleon $\langle r^2\rangle$ is known to coincide to good accuracy with the square of the nuclear charge radius $\frac{3}{5} r_0^2 A^{2/3}$. Then with $\rho_c = \left(\frac{4\pi}{3} r_0^3\right)^{-1}$ we get the following result:

$$\vec{a} = \frac{G}{\sqrt{2}}\, g\, \frac{9}{10}\, \frac{e\mu}{m r_0}\, A^{2/3}\, \frac{K\vec{I}}{I(I+1)} \tag{9}$$

The dependence of the anapole moment on the atomic number is quite natural. In the very first paper[18] where the notion of an anapole moment was introduced, it was mentioned to correspond to the magnetic field configuration created by a toroidal winding. Clearly the magnitude of an anapole moment should be proportional to the magnetic flux, i.e. to the torus cross-section. Hence the dependence on $\langle r^2\rangle$ in the formula (8) and on $A^{2/3}$ in the formula (9) follows. Note that the approximation (6) for the wave-function allows also to find explicit expressions for higher P-odd multipoles.

The vector - potential of a nucleus $\vec{A}(\vec{r})$ created by an anapole moment $\vec{a}$ equals to [12]

$$\vec{A}(\vec{r}) = \vec{a}\,\delta(\vec{r}) \tag{10}$$

The Hamiltonian of the electron interaction with this field we present as

$$e\vec{\alpha}\vec{a}\,\delta(\vec{r}) = \frac{G}{\sqrt{2}}\,\frac{K\vec{I}\vec{\alpha}}{I(I+1)}\,\varkappa_a\,\delta(\vec{r}) \tag{11}$$

(electron charge is $-e$, $\vec{\alpha}$ are the Dirac matrices). The dimensionless constant $\varkappa_a$ equals to

$$\varkappa_a = \frac{9}{10}\, g\, \frac{\alpha\mu}{m r_0}\, A^{2/3}, \tag{12}$$

$\alpha = e^2 = 1/137$. The values of this constant for various

nuclei at $r_0 = 1.2\cdot10^{-13}$ cm, $g_p = 4$ and $g_n = 1$ are presented in the third column of the table.

In the second column of the table we present the values of $\varkappa_a$ obtained with a more realistic description of the core density $\rho(r)$ and with the use of the Saxon-Woods potential, spin-orbit coupling including, for the calculation of the wave function and Green function of the external nucleon. The agreement between the two calculations is quite satisfactory. At any rate, the difference between the results is comparable with the expected accuracy of the nuclear shell model used in both approaches.

If one omits spin-orbit interaction in numerical calculations, their result becomes very close to the analytical one (12). It would be natural therefore to try to take into account the spin-orbit coupling

$$U_{ls} = -\lambda\,\frac{1}{r}\,\frac{dU}{dr}\,\vec{l}\vec{\sigma} \tag{13}$$

in the analytical calculations as well. In the same approximation of the constant density we find to the first order in λ, instead of (6),

$$\Psi = \left[1 - \frac{Gg}{\sqrt{2}}\left(i\rho_0\,\vec{\sigma}\vec{r} + 2\rho_0 K\lambda\,\vec{\sigma}\vec{p}\right)\right]\Psi_0 \tag{14}$$

where spin-orbit coupling is taken into account already in Ψ_0. The correction to the analytical values of anapole moments, found in this way, is indeed of the necessary sign, but its magnitude is about twice larger than necessary for agreement with the numerical calculations. The disagreement can be caused, probably, by the fact that the density at the boundary of a nucleus where U_{ls} works is smaller than ρ_c. The computed numerically values of $\vec{a}$ and $\varkappa_a$ are well reproduced by the formulae

$$\vec{a} = \frac{Gg\rho_0}{\sqrt{2}}\,\frac{2\pi e\mu}{m}\,\frac{K\vec{I}}{I(I+1)}\left\{\langle r^2\rangle - \frac{\lambda}{2}\left[2I(I+1) + \frac{1}{2} - K\right]\right\}$$
$$\varkappa_a = \frac{9}{10}\, g\,\frac{\alpha\mu}{m r_0}\, A^{2/3}\left\{1 - \frac{5\lambda\left[2I(I+1) + 1/2 - K\right]}{6A^{2/3} r_0^2}\right\} \tag{15}$$

Here $\lambda = 0.37$ fm^2 and according to the above remark we have introduced the factor $1/2$ into the spin-orbit correction obtained from equations (7), (14). Orbital contribution to $\vec{a}$ and $\varkappa_a$ again can be neglected since it contains two small factors: λ and $1/3\,\mu$.

Note that the P-odd interaction of an electron with the spin of a nucleus due to neutral currents is described by the expression

$$\frac{G}{\sqrt{2}}\,\frac{(1/2-K)}{I\,(I+1)}\,\varkappa_2\,\delta(\vec{r}) \tag{13}$$

akin to (11) (see, e.g., Ref. [19]). For external proton and neutron the signs of the constants $\varkappa_2$ are opposite:

$$\varkappa_{2p} = -\varkappa_{2n} \approx -0.05$$

It can be easily seen that the contribution (13) of neutral currents, at any rate in the case of an external proton, leads to small numerical increase of the P-odd interaction of an electron with the spin of a nucleus, caused mainly by an anapole moment.

In Ref. [20] it was noted that T-odd multipoles in some non-spherical nuclei can be enhanced because closely to the ground state there is a level of opposite parity and the same angular momentum. Due to the same reason anapole moments could be enhanced in the nuclei ^{161}Dy, ^{237}Np, ^{155}Gd, ^{233}U where the interval between the levels of opposite parity does not exceed 100 KeV. We have performed the corresponding calculations using the wave functions of an external nucleon in the deformed nucleus found in the Nilsson model. Although the mixing of the levels of opposite parity in these nuclei is indeed by one or two orders of magnitude larger than in spherical nuclei, the contribution of the nearest opposite parity level to the anapole moment does not exceed the value given by (9). The reason of the suppression can be explained in the following way. The spin contribution of the nearest level $|1\rangle$ to the nuclear anapole moment can be written as

$$\vec{a} = \frac{2\pi e\mu}{m}\,\frac{\vec{I}}{I+1}\,\frac{\langle 0|H_w|1\rangle\langle 1|(\vec{r}\times\vec{\sigma})_z|0\rangle}{E_0-E_1}$$

where the matrix elements $\langle 0|\ \ |1\rangle$ are calculated in the frame rotating with the nucleus and the factor $I/(I+1)$ corresponds to the transition to laboratory frame. One of the reasons of the suppression of $\vec{a}$ is that the dominating components in the Nilsson functions $|0\rangle$ and $|1\rangle$ have angular momenta differing by two units and a vector operator cannot transform them into each other. Another reason is specific for the oscillator potential $U = \sum_k \frac{m\omega_k^2 r_k^2}{2}$ used in the Nilsson model. In this case $r_k = \frac{1}{m\omega_k^2}\frac{\partial U}{\partial r_k} = \frac{i}{m\omega_k^2}[p_k, H]$ and the matrix element

$$\langle 0|r_k|1\rangle = \frac{i}{m\omega_k^2}\langle 0|p_k|1\rangle(E_1-E_0)$$

is itself proportional to the small energy interval between the levels $|0\rangle$ and $|1\rangle$. If we neglect the spin-orbit coupling the matrix element of $(\vec{r}\times\vec{\sigma})_z$ is suppressed as well. However, the real nuclear potential differs from the oscillator one. It is a possible reason why the calculated E1 amplitude of transition between the lowest levels in ^{161}Dy is five times smaller than the experimental one. If, being led by this argument, we increase by a factor of five the matrix element $\langle 1|(\vec{r}\times\vec{\sigma})_z|0\rangle$, the anapole moment of ^{161}Dy will exceed the value given by (9) almost by an order of magnitude.

In conclusion we stress once more that the detection of nuclear anapole moments is an extremely interesting problem lying on the border of the modern experimental facilities. The measurement of anapole moments would give a unique information on the P-odd interaction potential between a nucleon and heavy nucleus.

Various directions of investigation are possible here. We mean the difference in the magnitude of P -odd effects at different hyperfine components of optical transitions in heavy atoms[21], optical activity in the radio band[22-24], the effects of parity violation in molecules[25-26] both in optics and radio band. Especially we wish to mention the study of P -odd effects in rare earth atoms. Here one can come across anomalously close levels of opposite parity with the total angular momentum of electrons differing by unity that can be mixed only by the interaction of interest to us.

And at last, nuclear anapole moments can be observed by studying P -odd correlations in μ -mesoatoms[27]. Even higher P -odd multipoles may happen to be of interest here.

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References

1. L.M.Barkov and M.S.Zolotorev. Pis'ma Zh. Eksp. Teor. Fiz. 27 (1978) 379 (JETP Lett. 27 (1978) 357).
2. L.M.Barkov and M.S.Zolotorev. Pis'ma Zh. Eksp. Teor. Fiz. 28 (1978) 544 (JETP Lett. 28 (1978) 524).
3. L.M.Barkov and M.S.Zolotorev. Phys. Lett. 85B (1979) 308.
4. C.Y.Prescott et al., Phys. Lett. 77B (1978) 347.
5. C.Y.Prescott et al., Phys. Lett. 84B (1979) 524.
6. R.Conti, P.Bucksbaum, S.Chu, E.D.Commins and L.Hunter, Phys. Rev. Lett. 42, (1979), 343.
7. P.Bucksbaum, E.Commins and L.Hunter, Phys. Rev. Lett. 46(1981) 640.
8. J.H.Hollister, G.R.Apperson, L.L.Lewis, T.P.Emmons, T.G.Vold and E.N.Fortson. Phys. Rev. Lett. 46 (1981) 643.
9. T.P.Emmons, J.M.Reeves and E.N.Fortson. Phys. Rev. Lett. 51 (1983) 2089.
10. M.-A.Bouchiat, J.Guena, L.Hunter and L.Pottier. Phys. Rev. Lett. 117B (1982) 358.
11. M.-A.Bouchiat, J.Guena, L.Hunter and L.Pottier, Phys. Rev. Lett. 134B (1984) 463.
12. V.V.Flambaum and I.B.Khriplovich. Zh. Eksp. Teor. Fiz. 79 (1980) 1656 (JETP 52 (1980) 835).
13. B.Desplanques, J.F.Donoghue and B.R.Holstein, Ann. Phys. 124 (1980) 449.
14. V.F.Dmitriev, V.V.Flambaum, O.P.Sushkov and V.B.Telitsin, Phys. Lett. 125B (1983) 1.
15. B.H.J.McKellar. Phys. Rev. Lett. 20 (1968) 1542.
16. G.A.Lobov. Izv. AN SSSR (ser. fiz.) 44 (1980) 2364.
17. V.M.Dubovik, S.V.Zenkin. Preprint JINR E2-83-922 (Dubna, 1983).
18. Ya.B.Zel'dovich. Zh. Eksp. Teor. Fiz. 33 (1957) 1531 (JETP 6(1957) 1184).

19. I.B.Khriplovich. Parity non-conservation in atomic phenomena (in Russian) (Nauka, Moscow, 1981).

20. G.Feinberg, Trans. N.Y. Ac. Sc., ser. II 38(1977)26.

21. V.N.Novikov, O.P.Sushkov, V.V.Flambaum and I.B.Khriplovich. Zh. Eksp. Teor. Fiz. 73 (1977) 802 (JETP 46 (1977) 420).

22. V.N.Novikov and I.B.Khriplovich. Pis'ma Zh. Eksp. Teor. Fiz. 22 (1975) 162 (JETP Lett. 22 (1975) 74).

23. V.E.Balakin and S.I.Kozhemyachenko. Pis'ma Zh. Eksp. Teor. Fiz. 31 (1980) 326.

24. E.L.Al'tshuler et al. In XIX Winter School of Leningrad Institute of Nuclear Physics, (Leningrad, 1984) p. 91.

25. L.N.Labzovsky. Zh. Eksp. Teor. Fiz. 75 (1978) 856 (JETP 48 (1978) 434).

26. O.P.Sushkov and V.V.Flambaum. Zh. Eksp. Teor. Fiz. 75 (1978) 1208 (JETP 48 (1978) 608).

27. J.Missimer, L.M.Simons. SIN preprint PR-84-03, January 1984.

Table

Nucleus	Saxon-Woods potential	Simple model	
		without $\mathcal{U}_{es}$	$\mathcal{U}_{es}$ included
^{133}Cs	0.25	0.33	0.25
$^{203,205}Tl$	0.38	0.44	0.43
^{209}Bi	0.31	0.45	0.33
^{207}Pb	-0.09	-0.08	-0.08

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