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FOUR PARTICLES QED PROCESSES IN HIGH ENERGY ELECTRON-POSITRON COLLISIONS

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Abstract

Differential cross sections of the processes $e^+e^- \to \mu^+\mu^-\gamma\gamma$, $e^+e^-\gamma\gamma$, $\gamma\gamma\gamma\gamma$, $\mu^+\mu^- q^+ q^-$, $\pi^+\pi^-\gamma\gamma$ are calculated in a kinematical region in which angles between any pair of particle momenta in c.m. are large compared to their mass-to energy ratio. All cross sections are expressed through explicit universal combinations of four-momenta components. Two check-up tests are suggested: soft photon limit and collinear kinematics of photon emission. Both are fulfilled. The cross sections of the processes $e^+e^- \to \gamma\gamma\gamma$, $\mu^+\mu^-\gamma$, $e^+e^-\gamma$, $\pi^+\pi^-\gamma$ are presented for completeness.

The motivation for an investigation of inelastic QED processes is two-fold. The first one is an examination of validity of QED predictions in high orders of perturbation theory. For this purpose storage rings with a total energy $E \sim 1$ GeV are preferable as compared with $E \gtrsim 20$ GeV ones, since the cross sections of large-angle QED processes decrease as E^{-2}.

The second reason is the necessity to know in detail the QED background in studying of strong and weak interactions in e^+e^- experiments. The traditional method in which the modulus of the matrix element squared is calculated becomes ineffective, since it requires trace calculations by "brute force".

Recently the elegant method for calculation of helicity amplitudes was suggested [1-2] and applied to cross section calculation of QED processes $2 \to 3$ and some of $2 \to 4$: $e^+e^- \to 4\gamma$ [3], $\mu^+\mu^-\gamma\gamma$ [4]; numerical calculation of $e^+e^- \to e^+e^-e^+e^-$, $e^+e^-\mu^+\mu^-$ cross sections was performed in [6]. In our work [5] the cross sections of $e^+e^- \to \mu^+\mu^-\gamma\gamma$ and $e^+e^- \to \gamma\gamma e^+e^-$ were computed. The quantum chromodynamics (QCD) three and four jet production processes were considered in [7].

In this work we obtain the cross sections of $e^+e^- \to 4\gamma$, sect. 2, $\mu^+\mu^-\gamma\gamma$ (sect. 3), $e^+e^-\gamma\gamma$ (sect. 4) $\pi^+\pi^-\gamma\gamma$ (sect. 5). In sect 6 we obtain an analytical expression for the cross section of two different fermion pairs production. For completeness in this section we present cross sections of $2 \to 3$ processes $e^+e^- \to 3\gamma$, $\mu^+\mu^-\gamma$, $e^+e^-\gamma$, $\pi^+\pi^-\gamma$. All our results are given in terms of bilinear combinations of spinors of initial particles, which are calculated in Appendices A,B. The description of photon and fermion helicity states is given in sect. 3,4. The results obtained agree with those of straight forward calculation with applying of soft photon approximation as well as the quasireal electron approximation for the kinematical region in which one of the photons is soft or emitted close to one of the charged particles. These tests are performed in sect. 3.

The helicity amplitudes of double bremsstrahlung in the production of a charged pair have a factorized form in the case when the chirality states of photons are the same. Remaining amplitudes have rather cumbersome form.

We consider the case then the energies of final particles

$\mathcal{E}_i$ are of the same order, whereas their relative angles as well as the angles of their directions to a beam axis $\vec{P}_-$ are large compared to $m_r/\mathcal{E}_i$. In this case all the particles can be considered as massless:

$$\mathcal{E}_i \sim \mathcal{E},\quad \theta_i \sim 1,\quad m_i = 0. \qquad (1.1)$$

The cross section of three quantum annihilation has the form

$$d\sigma^{e^+e^-\to 3\gamma} = \frac{1}{3!}\frac{\alpha^3}{8\pi^2 s}\frac{[(\chi_1^2+\chi_1'^2)\chi_1\chi_1'+(\chi_2^2+\chi_2'^2)\chi_2\chi_2'+(\chi_3^2+\chi_3'^2)\chi_3\chi_3']}{\chi_1\chi_1'\,\chi_2\chi_2'\,\chi_3\chi_3'}\times \qquad (1.2)$$

$$\times\frac{d^3k_1d^3k_2d^3k_3}{\omega_1\,\omega_2\,\omega_3}\,\delta^{(4)}(p_++p_--k_1-k_2-k_3),$$

where

$$\chi_i = p_-k_i,\quad \chi_i' = p_+k_i,\quad s=(p_++p_-)^2.$$

The cross section of single bremsstrahlung in e^+e^- annihilation into a pair of muons has the form

$$d\sigma^{e^+e^-\to\mu^+\mu^-\gamma} = \frac{\alpha^3}{8\pi^2 s}\cdot W\cdot\frac{(t^2+t_1^2+u^2+u_1^2)}{ss_1}\,\frac{d^3q_+d^3q_-dk}{q_{+0}\,q_{-0}\,k_0}\times \qquad (1.3)$$

$$\times\,\delta^{(4)}(p_++p_--q_+-q_--k),$$

where

$$W=-(v_p-v_q)^2=-\left(-\frac{p_-}{p_-k}+\frac{p_+}{p_+k}+\frac{q_-}{q_-k}-\frac{q_+}{q_+k}\right)^2=[2ss_1(t+t_1)+2tt_1(s+s_1)+$$

$$+u_1(st_1+s_1t)+u(st+s_1t_1)]\,[4kp_+\cdot kp_-\cdot kq_+\cdot kq_-]^{-1}, \qquad (1.4)$$

$$s=(p_++p_-)^2,\ t=(p_--q_-)^2,\ u=(p_--q_+)^2, \qquad (1.5)$$

$$s_1=(q_++q_-)^2,\ t_1=(p_+-q_+)^2,\ u_1=(p_+-q_-)^2.$$

The cross section of single bremsstrahlung in e^+e^- annihilation into a $\pi^+\pi^-$ pair has the form:

$$d\sigma^{e^+e^-\to\pi^+\pi^-\gamma} = \frac{\alpha^3}{8\pi^2 s}\times W\times\frac{tu+t_1u_1}{ss_1}\times$$

$$\times\frac{d^3q_+d^3q_-d^3k}{q_{+0}\,q_{-0}\,k_0}\,\delta^{(4)}(p_++p_--q_+-q_--k). \qquad (1.6)$$

The cross section of single bremsstrahlung in e^+e^- scattering has the form

$$d\sigma^{e^+e^-\to e^+e^-\gamma} = \frac{\alpha^3}{8\pi^2 s}\times W\times\frac{[ss_1(s^2+s_1^2)+tt_1(t^2+t_1^2)+uu_1(u^2+u_1^2)]}{ss_1tt_1}\times \qquad (1.7)$$

$$\times\frac{d^3q_+d^3q_-d^3k}{q_{+0}\,q_{-0}\,k_0}\,\delta^{(4)}(p_++p_--q_+-q_--k).$$

The expressions (1.2-1.7) are valid in region (1.1). The characteristic feature of the processes with two charged particles in a final state is the factorized form of cross sections. One of the factor, W, is known factor of accompanying emission. It usually arises in processes of soft photon emission. In this case the photon is not soft.

2. Process $e^+e^-\to 4\gamma$

The four quantum annihilation process $e^+(p_+)+e^-(p_-)\to\gamma(k_1)+\gamma(k_2)+\gamma(k_3)+\gamma(k_4)$ [3] is described by 24 Feynman diagrams, which we will numerate by an order of their emission along an electron line. Chiral states of electron and positron have the form (A.5)

$$u_\lambda(p_-)\equiv u_\lambda=\omega_\lambda u(p_-),\quad v_\lambda(p_+)\equiv v_\lambda=\omega_\lambda v(p_+), \qquad (2.1)$$

$$\omega_\lambda=\tfrac{1}{2}(1+\lambda\gamma_5),\quad \lambda=\pm1.$$

Chiral states of photons:

$$\hat{e}^*_\lambda(k)=2N(k)[\hat{p}_+\hat{p}_-\hat{k}\,\omega_{-\lambda}-\hat{k}\hat{p}_+\hat{p}_-\omega_\lambda], \qquad (2.2)$$

$$N_k=2^{-\frac{3}{2}}[s\cdot kp_+\cdot kp_-]^{-\frac{1}{2}}.$$

Chiral amplitudes are defined in such a way

$$\mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4}_{\lambda_+\lambda_-} = (4\pi\alpha)^{-2} M^{\lambda\gamma_1\lambda\gamma_2\lambda\gamma_3\lambda\gamma_4}_{\lambda e^+\lambda e^-} = \overline{V}_{\lambda e+} O_{\mu_1\mu_2\mu_3\mu_4} U_{\lambda e-} e^{*\mu_1}_{\lambda_1}(k_1)\cdot$$

$$\cdots e^{\mu_4}_{\lambda_4}(k_4), \quad O_{\mu_1\cdots\mu_4} = \gamma_{\mu_4}\frac{-p_+ + k_4}{-2p_+k_4}\gamma_{\mu_2}\frac{p_- - k_1 - k_3}{(p_- - k_1 - k_3)^2}\gamma_{\mu_1}\frac{p_- - k_3}{-2p_-k_3}\gamma_{\mu_3} + \cdots \equiv \tag{2.3}$$

$$\equiv O_{3124} + \cdots$$

where $M^{\{\lambda\}}_{\{\lambda\}}$ is the matrix element of the process. Due to parity conservation one has for $\sum\limits_{cn}|M|^2$:

$$\sum_{\lambda=\pm}\left|M^{\lambda_1\lambda_2\lambda_3\lambda_4}_{\lambda e+\lambda e-}\right|^2 = 2(4\pi\alpha)^4\sum_{\lambda=\pm}\left|\mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4}_{+-}\right|^2 \equiv$$

$$\equiv 2(4\pi\alpha)^4\sum_{\lambda=\pm}\left|\mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4}\right|^2 = 8(4\pi\alpha)^4\cdot M, \tag{2.4}$$

and for cross section:

$$d\sigma^{e^+e^-\to 4\gamma} = \frac{\alpha^4\cdot 2^{-6}}{4!\,\pi^4 E^2} M \prod_1^4 \frac{d^3k_i}{\omega_i}\,\delta^{(4)}(p_+ + p_- -$$

$$-k_1 - k_2 - k_3 - k_4), \tag{2.5}$$

where $2E$ - is the total c.m. energy, $(4!)^{-1}$ arises due to the identity,

$$M = \frac{1}{4}\sum_{\lambda_i}\left|\mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4}\right|^2. \tag{2.6}$$

Chiral amplitudes vanish in the case when the chirality states of all photons are the same (it follows directly from 2.1-3):

$$\mathcal{M}^{++++} = \mathcal{M}^{----} = 0. \tag{2.7}$$

It's convenient to cast the remaining 14 amplitudes into a tree class:

$$(\mathcal{M}^{+++-} + \ldots), (\mathcal{M}^{---+} + \ldots), (\mathcal{M}^{++--} + \ldots). \tag{2.8}$$

It's sufficient to compute one amplitude per class, while the others can be obtained by permutations of the photon momenta. The Feynman diagrams 3124, 1324, 2134, 2314, 3214 contribute to $\mathcal{M}^{+++-}$. The first pair contains terms proportional to $(p_- - k_1 - k_3)^{-2}$

$$2^4\prod_{i=1}^4 N(k_i)\,\overline{V}(p_+)k_4 p_+ p_- \frac{-p_+ + k_1}{-2p_+k_1} p_+ p_- k_2 \frac{p_- - k_1 - k_3}{(p_- - k_1 - k_3)^2}\left[p_+ p_- k\frac{p_- - k_3}{-2p_-k_3}\times\right.$$

$$\left. p_+ p_- k_3 + p_+ p_- k_3 \frac{p_- - k_1}{-2p_-k_1} p_+ p_- k_1\right]\omega_- U(p_-).$$

Rearranging the Dirac matrices and using the Dirac equation $p_- U = \overline{V}p_+ = 0$, one can reduce this expression to the form

$$\Delta_{13}^{-2}\,\overline{V}(p_+)(-p_-)(-p_+ + k_4)p_+ p_- k_2 (p_- - k_1 - k_3)p_+ p_-\left[k_1(p_- - k_3) + \right.$$

$$\left. + k_3(p_- - k_1)\right]p_+ \omega_- U(p_-) = s\overline{V}p_- k_4 p_+ p_- k_2 k_4 p_+ \omega_- U, \quad \Delta_{ij} = p_- - k_i - k_j.$$

So in the sum of 3124 and 1324 diagram's contributions the pole term $(p_- - k_1 - k_3)^{-2}$ disappears. The same fact has place for two remaining pairs of diagrams. As a result (see App. (B4)):

$$\mathcal{M}^{+++-} = 2^4\left(\prod_1^4 N(k_i)\right)s\,\overline{V}\cdot\left[p_- k_4 p_+ p_- k_2 k_4 p_+ + p_- k_4 p_+ p_- k_1 k_4 p_+ \right.$$

$$\left. + p_- k_4 p_+ p_- k_3 k_4 p_+\right]\omega_- U = 2^4\left(\prod_1^4 N(k_i)\right)(-2s^3)(p_+k_4)\cdot \tag{2.9}$$

$$\cdot\overline{V}(p_+)k_4\omega_- U(p_-) = 2^4\left(\prod_{i=1}^4 N(k_i)\right)s^4 k_{4+}k_{4\perp}.$$

A similar result can be obtained for the second class amplitudes:

$$\mathcal{M}^{---+} = \left(\prod_1^4 N(k_i)\right)\cdot 2s^3\cdot k_4 p_-\cdot\overline{V}k_4\omega_- U =$$

$$= 2^4\left(\prod_1^4 N(k_i)\right)\cdot s^4(-k_{4-})k_{4\perp}. \tag{2.10}$$

Consider now the third class amplitude $\mathcal{M}^{++--}$. Feynman diagrams contribute 1234, 1324, 1243, 1423, 2134, 2314, 2143, 2413. Four of them contain the pole Δ_{12}^{-2} terms, which disappear in their sum:

$$\sim + S^3 (P_- - k_1 - k_2)^2 \bar{V} (P_- - k_1 - k_2) \omega_- U =$$
$$= 2\varepsilon \cdot S^3 \Delta_{12}^2 \cdot (k_1 + k_2)_\perp .$$

The contributions of another diagrams contain pole terms $\sim \Delta_{13}^{-2}$, Δ_{23}^{-2}, Δ_{14}^{-2}, Δ_{24}^{-2}, $\Delta_{ij} = P_- - k_i - k_j$. So the 1324 diagram's contribution:

$$2^4 \Big(\prod_1^4 N(k_i) \Big) \Delta_{13}^{-2} \cdot \bar{V} \cdot P_- (-P_+ + k_4) P_+ P_- k_2 (P_- - k_1 - k_3) k_3 P_+ P_- (P_- - k_1) P_+ \omega_- U .$$

can be transformed, using (1.3), to the form:

$$-2^4 \Big(\prod_1^4 N(k_i) \Big) \Delta_{13}^{-2} \bar{V} P_- P_+ k_4 P_- k_2 \Delta_{13} k_3 P_+ P_- P_+ k_1 \omega_- U = -S^2 \bar{V} k_4 \omega_- P_- k_2 \Delta_{13} k_3 \omega_- P_+ k_1 \omega_- U \sim - S^2 \bar{V} k_4 \omega_- U \cdot \bar{U} k_2 \Delta_{13} k_3 \omega_- V \cdot \bar{V} k_1 \omega_- U =$$
$$= -2\varepsilon S^3 k_{1\perp} k_{4\perp} Z^*(k_2, \Delta_{13}, k_3).$$

The expression for M (2.6) takes the form (all the notations are given in App. B (B.9)):

$$M = 4 \sum_{i=1}^4 k_{i+} k_{i-} (k_{i+}^2 + k_{i-}^2) + \varepsilon^{-2} \Big[|F_{1234}|^2 + |F_{1324}|^2 + |F_{1432}|^2 + \qquad (2.11)$$
$$+ |F_{3214}|^2 + |F_{3412}|^2 + |F_{4231}|^2 \Big],$$

$$F_{1234} = \Delta_{12}^2 \cdot (k_1 + k_2)_\perp - \Delta_{13}^{-2} \cdot k_{1\perp} \cdot k_{4\perp} \cdot Z^*(k_2, \Delta_{13}, k_3) - \Delta_{23}^{-2} \cdot k_{2\perp} \cdot \qquad (2.12)$$
$$\cdot k_{4\perp} \cdot Z^*(k_1, \Delta_{23}, k_3) - \Delta_{14}^{-2} \cdot k_{1\perp} \cdot k_{3\perp} \cdot Z^*(k_2, \Delta_{14}, k_4) - \Delta_{24}^{-2} k_{2\perp} k_{3\perp} Z^*(k_1, \Delta_{24}, k_4).$$

The remaining quantities F can be obtained from (2.12) by means of permutations. The expression (2.11) coincides with those obtained in [3]. This is evident if one takes into account the identity:

$$-k_{1\perp} Z^*(k_2, \Delta_{13}, k_3) = 2\varepsilon k_{2-} k_{1\perp} k_{3\perp}^* + Z(k_2, k_1) Z^*(k_1, k_3). \qquad (2.13)$$

In the case, when one of the photons is soft (say $k_4 \to 0$), the

second term in (2.11) coincides with the first one:

$$(M)_{soft} = 8 \sum_{i=1}^3 k_{i+} k_{i-} (k_{i+}^2 + k_{i-}^2). \qquad (2.14)$$

The photon-identity factor $^1/4!$ must be replaced by $^1/3!$ This result can be obtained using the classical current approximation as well.

3. The process $e^+(P_+) + e^-(P_-) \to \mu^+(q_+) + \mu^-(q_-) + \gamma(k_1) + \gamma(k_2)$.

We shall use everywhere the photon polarization vectors:

$$\hat{e}_{q\lambda}^* = 2N_q \big[\hat{q}_- \hat{q}_+ \hat{k} \omega_{-\lambda} - \hat{k} \hat{q}_- \hat{q}_+ \omega_\lambda \big], \quad N_q = \tfrac{1}{4} \big[q_+ q_- \cdot q_+ k \cdot q_- k \big]^{-1/2}. \qquad (3.1)$$

The expression (3.1) is well suited for the case when the photon is emitted by muons. In the case when photon is emitted by electron or positron it's convenient to put (3.1) in the form:

$$\hat{e}_{q\lambda}^* = 2B_\lambda N_P \big[\hat{P}_+ \hat{P}_- \hat{k} \omega_{-\lambda} - \hat{k} \hat{P}_+ \hat{P}_- \omega_\lambda \big] = e_{P\lambda}^* \cdot B_\lambda , \quad |B_\lambda| = 1. \qquad (3.2)$$
$$N_P = \tfrac{1}{4} \big[P_+ P_- \cdot P_+ k \cdot P_- k \big]^{-1/2}, \quad B_+ = B_-^* = -(e_{q+}^* e_{P+}) = -2S N_P N_q Z(k, q_-, q_+, k).$$

Let us define the chiral amplitudes in such a way

$$\mathfrak{M}_{a\, \lambda_{e^+} \lambda_{e^-}}^{\lambda_{\mu^+} \lambda_{\mu^-}, \lambda_1 \lambda_2} = (4\pi\alpha)^{-2} M_{\lambda, -\lambda}^{\nu, -\nu, \lambda_1 \lambda_2} = e_\mu^{*\lambda_1}(k_1) e_\nu^{*\lambda_2}(k_2) \bar{V}_{\lambda e^+}^{\alpha} \cdot \qquad (3.3)$$
$$U_{\lambda e^-}^{\beta} \bar{U}_{\lambda_{\mu^-}}(q_-)^{\gamma} V_{\lambda_{\mu^+}}^{\delta}(q_+) O_a{}^{\mu\nu}{}_{\alpha\beta\gamma\delta}.$$

When calculating the square of matrix element modulus summed over spin states:

$$\sum_{\{\lambda, \nu\}} |\mathfrak{M}_a|^2 = 2 (4\pi\alpha)^4 \sum_{\nu, \lambda_1, \lambda_2} |\mathfrak{M}_{a\, +-}^{\nu\, -\nu\, \lambda_1 \lambda_2}|^2 = \qquad (3.4)$$
$$= 2 (4\pi\alpha)^4 \sum_{\nu, \lambda} |\mathfrak{M}_a^{\nu -\nu \lambda_1 \lambda_2}|^2.$$

Only the amplitudes $\mathfrak{M}_a^{\pm\mp++}$, $\mathfrak{M}_a^{\pm\mp--}$, $\mathfrak{M}_a^{\pm\mp+-}$ must be considered. Remarkable simplification takes place in the case, when the chiral states of photons are the same. Consider, for example, M^{+-++}. Six Feynman diagrams contribute to it:

$$\frac{1}{4}M^{+-++}=\bar V_+\gamma_\mu\frac{\Delta_-}{\Delta_-^2}p_+p_-\left(-k_2(p_--k_1)-k_1(p_--k_2)\right)p_+u_-\cdot\bar u_-(q_-)\gamma_\mu V_+(q_+)\cdot A_1$$

$$-B_1\bar V\gamma_\mu(p_--k_1)p_+\omega_-u\cdot\bar u(q_-)\gamma_\mu(-q_++k_2)q_-\omega_-v(q_+)-C_1\bar V\gamma_\mu(p_--k_2)p_+\omega_-u\cdot \quad (3.5)$$

$$\cdot\bar u(q_-)\gamma_\mu(-q_+-k_1)q_-\omega_-V(q_+)+D_1\bar V\gamma_\mu\omega_-u\cdot\bar u(q_-)\gamma_\mu\frac{\overbrace{\Delta_+'}^{q_--q_+}}{\Delta_+'^2}\left(k_1(-q_++k_2)+k_2(-q_+-k_1)\right)q_-\omega_-V(q_+)$$

notations introduced are: $\Delta_-=p_--k_1-k_2$,

$$A_1=\frac{N_{1p}N_{2p}B_{1+}B_{2+}}{s_1},\quad B_1=\frac{N_{1p}N_{2q}B_{1+}}{\tilde q_2^2},\quad C_1=\frac{N_{1q}N_{2p}B_{2+}}{\tilde q_1^2},\quad D_1=\frac{N_{1q}N_{2q}}{s},$$

$$s=(p_++p_-)^2,\ s_1=(q_++q_-)^2,\ \tilde q_1=q_++q_--k_1,\ \tilde q_2=q_++q_--k_2,\ \Delta_+'=-q_+-k_1-k_2. \quad (3.6)$$

To perform the summation over a virtual photon vector index in the first term (3.5) we insert the quantity (equal to unit)

$$\left(8p\,p_-p_+q_-p_+\omega_-\right)^{-1}\cdot\bar u_-p_+u_-(q_-)\cdot\bar u_-(q_-)p_+u_-=1. \quad (3.7)$$

Using the completeness properties of spinors $u_-(q)\bar u_-(q)=\omega_-\hat q$, $V_+(q)\bar V_+(q)=\omega_-\hat q$ one can transform it to the expression

$$-\frac{2s}{s_1}N_{1p}N_{2p}B_{1+}B_{2+}H,\quad H=\bar V q_-\omega_-V(q_+)\cdot\bar u(q_-)p_+\omega_-u. \quad (3.8)$$

The same calculation for the remaining three terms in (3.5) leads to a result

$$\frac{1}{4}M_a^{+-++}=-\frac{2}{ss_1}H\left[sN_{1p}B_{1+}+s_1N_{1q}\right]\left[sN_{2p}B_{2+}+s_1N_{2q}\right]. \quad (3.9)$$

Taking into account the relation found in [2]

$$|sN_pB_++s_1N_q|^2=-\frac{1}{8}(v_p-v_q)^2=-\frac{1}{8}\left(\frac{p_+}{p_+k}-\frac{p_-}{p_-k}-\frac{q_+}{q_+k}+\frac{q_-}{q_-k}\right)^2 \quad (3.10)$$

The square of modulus of M^{+-++} can be written as

$$|M_a^{+-++}|^2=\frac{(2p_+q_-)^2}{ss_1}(v_{p_1}-v_{q_1})^2(v_{p_2}-v_{q_2})^2.$$

Three another amplitudes with the equal chiral states of photons have a factorized form. Their contribution is

$$|M_a^{+-++}|^2+|M_a^{+---}|^2+|M_a^{-+++}|^2+|M_a^{-+--}|^2=$$

$$=\frac{1}{ss_1}\left(t^2+t_1^2+u^2+u_1^2\right)(v_{p_1}-v_{q_1})^2(v_{p_2}-v_{q_2})^2, \quad (3.11)$$

where

$$t=(p_--q_-)^2,\quad u=(p_--q_+)^2,\quad s=(p_++p_-)^2,$$

$$t_1=(p_+-q_+)^2,\quad u_1=(p_+-q_-)^2,\quad s_1=(q_++q_-)^2.$$

Remind that the quantity $-(v_p-v_q)^2$ is the known accompanying emission multiplier which emerges when a soft photon is radiated in charged particles interaction. Here both photons are hard.

The compensation of pole terms Δ^{-2}, $\tilde q^{-2}$ doesn't have place in the case of unequal chiralities of photons, so corresponding amplitudes are more complicated. Consider the amplitude M^{+-+-}. Using the photon polarization vector in (3.1-2) one has:

$$\frac{1}{4}M_a^{+-+-}=A\bar V\Big\{\gamma_\mu\frac{\Delta_{12}}{\Delta_{12}^2}(-k_2p_+p_-)\frac{p_--k_1}{-2p_-k_1}p_+p_-k_1+(-k_2p_+p_-)\frac{-p_++k_2}{-2p_+k_2}\gamma_\mu\frac{p_--k_1}{-2p_-k_1}p_+p_-k_1$$

$$+(-k_2p_+p_-)\frac{-p_++k_2}{-2p_+k_2}p_+p_-k_1\frac{-p_++k_1+k_2}{\tilde\Delta_{12}^2}\gamma_\mu\Big\}\omega_-u\cdot\bar u(q_-)\gamma_\mu\omega_-V(q_+)+B\bar V\gamma_\mu\frac{p_--k_1}{-2p_-k_1}p_+\cdot$$

$$p_-k_1\omega_-u\cdot\bar u(q_-)(-k_2q_-q_+)\frac{q_-+k_2}{2q_-k_2}\gamma_\mu\omega_-V(q_+)+C\bar V(-k_2p_+p_-)\frac{-p_++k_2}{-2p_+k_2}\cdot \quad (3.12)$$

$$\gamma_\mu\omega_-u\cdot\bar u(q_-)\gamma_\mu\frac{-q_+-k_1}{2q_+k_1}q_-q_+k_1\omega_-V(q_+)+D\bar V\gamma_\mu\omega_-u\cdot\bar u(q_-)\Big\{-k_2q_-q_+\cdot$$

$$\frac{q_-+k_2}{2q_-k_2}q_-q_+k_1\frac{\Delta_{12}'}{\Delta_{12}'^2}\gamma_\mu+(-k_2q_-q_+)\frac{q_-+k_2}{2q_-k_2}\gamma_\mu\frac{-q_+-k_1}{2q_+k_1}q_-q_+k_1+\gamma_\mu\frac{\tilde\Delta_{12}'}{\tilde\Delta_{12}'^2}(-k_2q_-q_+)\frac{-q_+-k_1}{2q_+k_1}q_-q_+k_1\Big\}\omega_-V(q_+),$$

where

$$A=\frac{1}{s_1}N_{1p}N_{2p}B_{1+}B_{2-},\ B=\frac{N_{1p}N_{2q}B_{1+}}{(p_++p_--k_1)^2},\ C=\frac{N_{1q}N_{2p}B_{2-}}{(p_++p_--k_2)^2},\ \mathcal{D}=\frac{N_{1q}N_{2q}}{s}. \tag{3.13}$$

Performing the vector index μ summation with the help of relation

$$\bar{V}A\omega_-U\cdot\bar{U}(q_-)B\omega_-V(q_+)=\frac{L}{s^2uu_1}\,Sp\,p_+Ap_-p_+q_-Bq_+p_-\omega_-,$$

$$L=\bar{V}p_-\omega_-V(q_+)\cdot\bar{U}(q_-)p_+\omega_-U,\ |L|=s\sqrt{uu_1}, \tag{3.14}$$

one can express (3.12) in terms of bilinear combinations of spinors of initial particles Z, Z_U, Z_V (App. B, (B9),(B10)). For example, let us transform the trace (3.14) appearing for the term in (3.12) which is proportional to $A\,\Delta_{12}^{-2}$:

$$Sp\,p_+\gamma_\mu\Delta_{12}k_2p_+p_-(p_--k_1)p_+p_-p_+q_-\gamma_\mu q_+p_-\omega_-=4s\cdot(p_+q_-)\cdot Sp\,\omega_-p_+k_1\cdot$$

$$\cdot\omega_-p_-\cdot\omega_+p_+k_2\Delta_{12}q_+\omega_+p_-=-2su_1\cdot\bar{U}_+V_+\cdot\bar{V}_+k_1U_-\cdot\bar{U}_-V_-\cdot\bar{V}_-k_2\cdot\Delta_{12}$$

$$\cdot q_+U_+=-2s^3u_1k_{1\perp}\cdot Z^*(q_+,\Delta_{12},k_2).$$

In the same way (3.12) takes the form:

$$M_a^{+-+-}=\frac{4L}{s^2uu_1}\Big\{\Big[-\frac{2s^3u_1}{\Delta_{12}^2}k_{1\perp}Z^*(q_+,\Delta_{12},k_2)+\frac{2s^3u}{\tilde{\Delta}_{12}^2}k_{2\perp}Z^*(k_1,\tilde{\Delta}_{12},q_-)+2s^3Z^*(-p_++k_2,q_-)\cdot$$

$$\cdot Z(q_+,p_--k_1)\Big]A+B(-4s^2\varepsilon)Z(q_+,p_--k_1)Z_V(q_-+k_2,q_+,q_-)+ \tag{3.15}$$

$$+C(-4s^2\varepsilon)Z^*(-p_++k_2,q_-)Z_U(-q_+-k_1,q_-,q_+)+\mathcal{D}\Big[\frac{4ss_1u\varepsilon}{\Delta_{12}'^2}Z_V(\Delta_{12}',k_1,q_+,k_2,q_-)$$

$$+\frac{4ss_1\varepsilon u_1}{\tilde{\Delta}_{12}'^2}Z_U(\tilde{\Delta}_{12}',k_2,q_-,k_1,q_+)+2s^2Z_V(q_-+k_2,q_+,q_-)\cdot Z_U(-q_+-k_1,q_-,q_+)\Big]\Big\},$$

where

$$L=\bar{V}p_-\omega_-V(q_+)\cdot\bar{U}(q_-)p_+\omega_-U,\ \ \Delta_{12}=p_--k_1-k_2,$$

$$\tilde{\Delta}_{12}=-p_++k_1+k_2,\ \Delta_{12}'=q_-+k_1+k_2,\ \tilde{\Delta}_{12}'=q_++k_1+k_2. \tag{3.16}$$

Consider now the amplitude M^{-++-}. Using the same quantities A-D which were accepted in (3.13), one can write

$$\frac{1}{4}M_a^{-++-}=A\bar{V}\Big\{\gamma_\mu\frac{\Delta_{12}}{\Delta_{12}^2}k_2p_+p_-(p_--k_1)p_+-p_-(-p_++k_2)\gamma_\mu(p_--k_1)p_++p_-(-p_++k_2)p_+\cdot$$

$$p_-k_1\frac{\tilde{\Delta}_{12}}{\tilde{\Delta}_{12}^2}\gamma_\mu\Big\}\omega_-U\cdot\bar{U}(q_-)\gamma_\mu\omega_+V(q_+)+B\bar{V}\gamma_\mu(p_--k_1)p_+\omega_-U\cdot\bar{U}(q_-)\gamma_\mu(q_++k_2)\cdot \tag{3.16}$$

$$q_-\omega_+V(q_+)+C\bar{V}p_-(p_+-k_2)\gamma_\mu\omega_-U\cdot\bar{U}(q_-)q_+(q_-+k_1)\gamma_\mu\omega_+V(q_+)+$$

$$+\mathcal{D}\bar{V}\gamma_\mu\omega_-U\cdot\bar{U}(q_-)\Big\{-\Delta_{12}'^{-2}q_+(q_-+k_1)q_-q_+k_2\Delta_{12}'\gamma_\mu+q_+(q_-+k_1)\gamma_\mu(q_++k_2)q_--$$

$$-\tilde{\Delta}_{12}'^{-2}\gamma_\mu\tilde{\Delta}_{12}'k_1q_-q_+(q_++k_2)q_-\Big\}\omega_+V(q_+).$$

Summation over a vector index in full analogy with M^{+-+-} can be performed by means of identity

$$\bar{V}A\omega_-U\cdot\bar{U}(q_-)B\omega_+V(q_+)=\frac{\bar{V}\omega_+V(q_+)\cdot\bar{U}(q_-)\omega_-U}{tt_1}\cdot Sp\,p_+Ap_-q_-Bq_+\omega_-. \tag{3.17}$$

The resulting expression can be obtained by means of "line turning" operation applied to muon line also. Using explicit expressions for spinors (A.10),(A.12) one can be convinced in the validity of the relation

$$\bar{U}(q_-)\gamma_{\mu_1}\cdots\gamma_{\mu_{2n+1}}\omega_+V(q_+)=-\bar{V}(q_+)\gamma_{\mu_{2n+1}}\cdots\gamma_{\mu_1}\omega_-U(q_-). \tag{3.18}$$

Applying (3.18) to (3.16) one can rewritten it in the form (3.12) with a replacement $V(q_+)\leftrightarrow U(q_-)$ and change the general sign. As a result:

$$M_a^{-++-}=\frac{-4\tilde{L}}{s^2tt_1}\Big\{A\Big[-\frac{2s^3t_1}{\Delta_{12}^2}k_{1\perp}Z^*(q_-,\Delta_{12},k_2)-\frac{2s^3t}{\tilde{\Delta}_{12}^2}k_{2\perp}Z^*(k_1,\tilde{\Delta}_{12},q_+)+$$

$$+2s^3Z^*(-p_++k_2,q_+)Z(q_-,p_--k_1)\Big]+B(-4s^2\varepsilon)Z(q_-,p_--k_1)Z_V(q_++k_2, \tag{3.19}$$

$$q_-,q_+)+C(-4s^2\varepsilon)Z^*(-p_++k_2,q_+)Z_U(-q_--k_1,q_+,q_-)+\mathcal{D}\Big[\frac{4ss_1t\varepsilon}{\tilde{\Delta}_{12}'^2}Z_V(\tilde{\Delta}_{12}',$$

$$k_1,q_-,k_2,q_+)+\frac{4ss_1t_1\varepsilon}{\Delta_{12}'^2}Z_U(\Delta_{12}',k_2,q_+,k_1,q_-)+2s^2Z_V(q_++k_2,q_-,q_+)Z_U(-q_+-k_1,$$

$$q_+,q_-);$$

$$\tilde{L}=\bar{V}p_-\omega_-U(q_-)\cdot\bar{V}(q_+)p_+\omega_-U,\ \ |\tilde{L}|=s\sqrt{tt_1} \tag{3.20}$$

Expressions for amplitudes $\mathcal{M}^{+--+}$ and $\mathcal{M}^{-+-+}$ can be obtained from (3.15) and (3.19) by relabelling photon momenta $k_1 \leftrightarrow k_2$:

$$\mathcal{M}_a^{+--+}(k_1,k_2) = \mathcal{M}_a^{+-+-}(k_2,k_1); \quad \mathcal{M}_a^{-+-+}(k_1,k_2) = \mathcal{M}_a^{-++-}(k_2,k_1). \tag{3.21}$$

Cross section of the process $e^+e^- \to \mu^+\mu^-\gamma\gamma$ has the form

$$d\sigma^{e^+e^-\to\mu^+\mu^-\gamma\gamma} = \frac{1}{2!}\,\frac{(4\pi\alpha)^4}{64 s (2\pi)^8}\, M \frac{d^3k_1}{\omega_1}\,\frac{d^3k_2}{\omega_2}\,\frac{d^3q_-}{q_{-0}}\,\frac{d^3q_+}{q_{+0}}\cdot \tag{3.22}$$

$$\cdot\,\delta^{(4)}(p_+ + p_- - q_+ - q_- - k_1 - k_2),$$

where

$$M = \frac{1}{ss_1}(t^2+t_1^2+u^2+u_1^2)(\mathcal{V}_{p_1}-\mathcal{V}_{q_1})^2(\mathcal{V}_{p_2}-\mathcal{V}_{q_2})^2 + \Big[|\mathcal{M}_a^{+-+-}|^2 +$$

$$+ |\mathcal{M}_a^{-++-}|^2 + (k_1 \leftrightarrow k_2)\Big]. \tag{3.23}$$

A factor ($^1/2!$) accepted in (3.22) is due to identity of photons, since in (3.22) implied the integration over total phase space of both photons. The expressions (3.15),(3.19) have rather cumbersome form. To check-up their validity we suggest to consider two different specific kinematics of final particles. First, consider the "soft limit": let the energy of one of photons be small, $k_2 \to 0$. In this case, the second term in r.h.s. (3.23) coincides with the first one. The multiplier $^1/2!$ must be omitted since the photons are nonequivalent. As a result one has

$$d\sigma_{soft}^{e^+e^-\to\mu^+\mu^-\gamma(\gamma)} = \frac{(4\pi\alpha)^4}{32 s (2\pi)^8}\cdot\frac{t^2+t_1^2+u^2+u_1^2}{ss_1}\cdot(\mathcal{V}_{p_1}-\mathcal{V}_{q_1})^2(\mathcal{V}_{p_2}-\mathcal{V}_{q_2})^2\cdot \tag{3.24}$$

$$\cdot\,\frac{d^3q_+\,d^3q_-\,d^3k_1\,d^3k_2}{q_{+0}\,q_{-0}\,\omega_1\,\omega_2}\,\delta^{(4)}(p_+ + p_- - q_+ - q_- - k_1).$$

This expression can be present in the form of the product of the cross-section of the process $e^+e^- \to \mu^+\mu^-\gamma$ (1.3) and the probability of soft photon emission:

$$d\sigma_{soft}^{e^+e^-\to\mu^+\mu^-\gamma(\gamma)} = d\sigma^{e^+e^-\to\mu^+\mu^-\gamma(k_1)}\,dW(k_2) \tag{3.25}$$

where

$$dW(k_2) = \frac{4\pi\alpha}{2(2\pi)^3}(-1)(\mathcal{V}_{p_2}-\mathcal{V}_{q_2})^2\,\frac{d^3k_2}{\omega_2}. \tag{3.26}$$

To check the expression (3.15) we note first that in a limit $k_2 \to 0$:

$$\mathcal{M}_a^{+-+-}\Big|_{k_2\to 0} = \frac{8L}{uu_1}\Big\{\tilde{A}\, s\, z^*(-p_+,q_-)\, z(q_+,p_- - k_1) - 2\varepsilon\tilde{B}\, z(q_+,p_- - k_1)\, z_v(q_-,$$

$$q_+,q_-) - 2\varepsilon\tilde{C}\, z^*(p_+,q_-)\, z_u(q_+ + k_1, q_-, q_+) - \tilde{D}\, z_v(q_-,q_+,q_-)\, z_u(q_+ + k_1, q_-, q_+)\Big\},$$

where

$$\tilde{A} = \frac{N_{1p}N_{2p}B_{1+}B_{2-}}{s_1},\quad \tilde{B} = \frac{N_{1p}N_{2q}B_{1+}}{s_1},\quad \tilde{C} = \frac{N_{1q}N_{2p}B_{2-}}{s},\quad \tilde{D} = \frac{N_{1q}N_{2q}}{s}.$$

Using the conservation low $p_+ + p_- = q_+ + q_- + k$ and the properties of z quantities (App. B). Expression for can be written cast in the form

$$\mathcal{M}_a^{+-+-}\Big|_{k_2\to 0} = \frac{8L}{uu_1}\, z(q_+,p_- - k_1)\,\frac{z(-p_+ + k_2, q_-)}{ss_1}\Big[sN_{1p}B_{1+} + \tag{3.27a}$$

$$+ s_1N_{1q}\Big]\Big[sN_{2p}B_{2-} + s_1N_{2q}\Big].$$

The similar calculation gives for the amplitude $\mathcal{M}_a^{-++-}$:

$$\mathcal{M}_a^{-++-}\Big|_{k_2\to 0} = \frac{-8\tilde{L}}{tt_1}\,\frac{z(q_-,p_- - k_1)\, z(-p_+ + k_2, q_+)}{ss_1}\Big[sN_{1p}B_{1+} + \tag{3.27b}$$

$$+ s_1N_{1q}\Big]\Big[sN_{2p}B_{2-} + s_1N_{2q}\Big].$$

Taking into the account the relations $\left|\frac{L}{uu_1}z(q_+q_-)\right| = \left|\frac{\tilde{L}}{tt_1}z(q_-q_+)\right| = \sqrt{ss_1}$

One easily obtains:

$$|m_a^{+-+-}|^2+|m_a^{-++-}|^2+|m_a^{+--+}|^2+|m_a^{-+-+}|^2\Big|_{k_2\to 0}=$$

$$=\frac{1}{ss_1}\left[t^2+t_1^2+u^2+u_1^2\right](\mathcal{V}_{p_1}-\mathcal{V}_{q_1})^2(\mathcal{V}_{p_2}-\mathcal{V}_{q_2})^2. \tag{3.28}$$

The independent check-up we suggest is the situation when one of the photons, is hard, and is emitted close to one of the charged particle direction. In this case the cross section of the process $e^+e^-\to a\bar{a}\gamma\gamma$ can be expressed through the cross-section of $e^+e^-\to a\bar{a}\gamma$ using the quasireal electron's method [8]. Let the photon with momentum k_2 move close to the direction of μ^+ meson. Kinematical invariant k_2q_+ which is small compared to ε^2, must be large compared to muon mass square μ^2 (see (1.1)). Photons are nonequivalent now. The cross section have the form:

$$d\sigma^{e^+e^-\to\mu^+\mu^-\gamma\gamma}\Big|_{k_2\parallel q_+}=d\sigma^{e^+e^-\to\mu^+\mu^-\gamma}(q_++k_2,q_-,k_1)\,dW_{q_++k_2}(k_2), \tag{3.29}$$

where the cross-section $d\sigma^{e^+e^-\to\mu^+\mu^-\gamma}$ can be obtained from (1.3) by substitution $q_+\to q_++k_2$;

$$dW_{q_++k_2}(k_2)=\frac{\alpha}{4\pi^2}\cdot\frac{\left[(\varepsilon_++\omega_2)^2+\varepsilon_+^2\right]}{\omega_2^2(\varepsilon_++\omega_2)(q_+k_2)}d^3k_2. \tag{3.30}$$

4. Process $e^+(p_+)+e^-(p_-)\to e^+(q_+)+e^-(q_-)+\gamma(k_1)+\gamma(k_2)$.

Comparing this process with $e^+e^-\to\mu^+\mu^-\gamma\gamma$ one has 20 additional Feynman diagrams of a scattering type [5]. They give an additional contribution m_s to chiral amplitudes $m^{+-\lambda_1\lambda_2}$ as well as a class of new amplitudes for which the chiral states of initial particles are the same $m^{\pm\pm\lambda_1\lambda_2}$. The scattering diagrams contribution to the amplitude $m_{+-}^{-+\lambda_1\lambda_2}=m^{-+\lambda_1\lambda_2}$ equals zero due chirality conservation:

$$m^{-+\lambda_1\lambda_2}=m_a^{-+\lambda_1\lambda_2}\ . \tag{4.1}$$

To compute the scattering diagrams contribution to chiral amplitude $m^{+-\lambda_1\lambda_2}$ the photon polarization vector is written in the form

$$\hat{e}_{q\lambda}^{*}=C_\lambda^{\text{э}}\,\hat{e}_{\text{э}\lambda}^{*},\quad C_\lambda^{\text{э}}=-(e_{q\lambda}^{*}e_{\text{э}\lambda}),\quad C_+^{\text{э}}=(C_-^{\text{э}})^*=-8\varepsilon\cdot(q_-k)\cdot N_qN_{\text{э}}z_u^{*}(q_-,q_+,k), \tag{4.2}$$

$$\hat{e}_{\text{э}\lambda}^{*}=2N_{\text{э}}\left[\hat{q}_-\hat{p}_-\hat{k}\,\omega_{-\lambda}-\hat{k}\hat{q}_-\hat{p}_-\omega_\lambda\right],\quad N_{\text{э}}=\frac{1}{4}\left[p_-q_-\cdot p_-k\cdot q_-k\right]^{-1/2};$$

in the case when photon is emitted by electron and in the form

$$\hat{e}_{q\lambda}^{*}=\mathcal{D}_\lambda^{\Pi}\,\hat{e}_{\Pi\lambda}^{*},\quad \mathcal{D}_\lambda^{\Pi}=-(e_{q\lambda}^{*}e_{\Pi\lambda}),\quad \mathcal{D}_+^{\Pi}=-8\varepsilon(q_+k)\cdot N_qN_{\Pi}z_v(q_+,q_-,k), \tag{4.3}$$

$$e_{\Pi\lambda}^{*}=2N_{\Pi}\left[\hat{p}_+\hat{q}_+\hat{k}\,\omega_{-\lambda}-\hat{k}\hat{p}_+\hat{q}_+\omega_\lambda\right],\quad N_{\Pi}=\frac{1}{4}\left[p_+q_+\cdot p_+k\cdot q_+k\right]^{-1/2},$$

when it is emitted by positron.

Using (3.11) for definition of kinematical invariants the scattering diagrams contribution is present in the form:

$$m_{s\lambda_{e_+}\lambda_{e_-}}^{\lambda_{e_+}'\lambda_{e_-}'\lambda_1\lambda_2}=\bar{u}_{\lambda_{e_-}'}O_\mu u_{\lambda_{e_-}}\cdot\bar{v}_{\lambda_{e_+}}\tilde{O}_\mu v_{\lambda_{e_+}'}\;e_{\lambda_1}^{*\sigma}(k_1)\,e_{\lambda_2}^{*}(k_2)^{\rho},$$

$$\begin{aligned}O_\mu*\tilde{O}_\mu={}&-\frac{1}{t_1}\Big\{\gamma_\mu\frac{\Delta_-}{\Delta_-^2}\Big[\gamma_\rho\frac{\hat{p}_--\hat{k}_1}{-2p_-k_1}\gamma_\sigma+\gamma_\sigma\frac{\hat{p}_--\hat{k}_2}{-2p_-k_2}\gamma_\rho\Big]+\gamma_\rho\frac{\hat{q}_-+\hat{k}_2}{2q_-k_2}\gamma_\mu\frac{\hat{p}_--\hat{k}_1}{-2p_-k_1}\gamma_\sigma+\gamma_\sigma\frac{\hat{q}_-+\hat{k}_1}{2q_-k_1}\gamma_\mu\cdot\\&\cdot\frac{\hat{p}_--\hat{k}_2}{-2p_-k_2}\gamma_\rho+\Big[\gamma_\rho\frac{\hat{q}_-+\hat{k}_2}{2q_-k_2}\gamma_\sigma+\gamma_\sigma\frac{\hat{q}_-+\hat{k}_1}{2q_-k_1}\gamma_\rho\Big]\frac{\Delta_-'}{\Delta_-'^2}\gamma_\mu\Big\}*\gamma_\mu-\frac{\gamma_\mu*}{t}\Big\{\Big[\gamma_\sigma\frac{-\hat{p}_++\hat{k}_1}{-2p_+k_1}\gamma_\rho+\\&+\gamma_\rho\frac{-\hat{p}_++\hat{k}_2}{-2p_+k_2}\gamma_\sigma\Big]\frac{\Delta_+}{\Delta_+^2}\gamma_\mu+\gamma_\sigma\frac{-\hat{p}_++\hat{k}_1}{-2p_+k_1}\gamma_\mu\frac{-\hat{q}_+-\hat{k}_2}{2q_+k_2}\gamma_\rho+\gamma_\rho\frac{-\hat{p}_++\hat{k}_2}{-2p_+k_2}\gamma_\mu\frac{-\hat{q}_+-\hat{k}_1}{2q_+k_1}\gamma_\sigma+\gamma_\mu\frac{\Delta_+'}{\Delta_+'^2}\Big[\gamma_\sigma\frac{-\hat{q}_+-\hat{k}_2}{2q_+k_2}\gamma_\rho+\\&+\gamma_\rho\frac{-\hat{q}_+-\hat{k}_1}{2q_+k_1}\gamma_\sigma\Big]\Big\}-(p_--q_--k_1)^{-2}\Big\{\gamma_\mu\frac{\hat{p}_--\hat{k}_1}{-2p_-k_1}\gamma_\sigma*\gamma_\rho\frac{-\hat{p}_++\hat{k}_2}{-2p_+k_2}\gamma_\mu+\gamma_\sigma\frac{\hat{q}_-+\hat{k}_1}{2q_-k_1}\gamma_\mu*\gamma_\rho\frac{-\hat{p}_++\hat{k}_2}{-2p_+k_2}\gamma_\mu+\gamma_\mu\frac{\hat{p}_--\hat{k}_1}{-2p_-k_1}\cdot\\&\cdot\gamma_\sigma*\gamma_\mu\frac{-\hat{q}_+-\hat{k}_2}{2q_+k_2}\gamma_\rho+\gamma_\sigma\frac{\hat{q}_-+\hat{k}_1}{2q_-k_1}\gamma_\mu*\gamma_\mu\frac{-\hat{q}_+-\hat{k}_2}{2q_+k_2}\gamma_\rho\Big\}-(p_--q_--k_2)^{-2}\Big\{\gamma_\mu\frac{\hat{p}_--\hat{k}_2}{-2p_-k_2}\gamma_\rho*\gamma_\sigma\frac{-\hat{p}_++\hat{k}_1}{-2p_+k_1}\gamma_\mu\\&+\gamma_\rho\frac{\hat{q}_-+\hat{k}_2}{2q_-k_2}\gamma_\mu*\gamma_\sigma\frac{-\hat{p}_++\hat{k}_1}{-2p_+k_1}\gamma_\mu+\gamma_\mu\frac{\hat{p}_--\hat{k}_2}{-2p_-k_2}\gamma_\rho*\gamma_\mu\frac{-\hat{q}_+-\hat{k}_1}{2q_+k_1}\gamma_\sigma+\gamma_\rho\frac{\hat{q}_-+\hat{k}_2}{2q_-k_2}\gamma_\mu*\gamma_\mu\frac{-\hat{q}_+-\hat{k}_1}{2q_+k_1}\gamma_\sigma\Big\},\end{aligned} \tag{4.4}$$

$$\Delta_\pm=\mp\left[p_\pm-k_1-k_2\right],\quad \Delta_\pm'=\mp(q_\pm+k_1+k_2),\quad q_{1,2}=p_--q_--k_{1,2}\,.$$

Consider first the amplitudes with the same chiralities of photons. The virtual photon's vector index summation may be performed using the identity (see (3.8)):

$$\bar{U}(q_-)A\omega_- U(p_-)\cdot\bar{V}(p_+)B\omega_- V(q_+) = -H\cdot\frac{Z^*(q_+,q_-)}{s_1 t u u_1^2}\,\mathrm{Sp}\,\hat{q}_- A \hat{p}_- \hat{q}_- \hat{p}_+ \cdot \tag{4.5}$$

$$\cdot B \hat{q}_+ \hat{p}_- \omega_- .$$

The scattering diagram's contribution to m^{+-++} comes from terms which are underlined in (4.4). Using (4.5) for traces calculation and the explicit expressions for photon polarization vectors (4.2,3) one can convince that the pole terms $\Delta_\pm^{-2}$, $\Delta_\pm'^{-2}$ in the total sum disappear. The result is

$$m_s^{+-++} = \frac{-8H\,\mathrm{Sp}\,\hat{q}_-\hat{p}_+\hat{q}_+\hat{p}_-\omega_-\cdot\mathrm{Sp}\,\hat{p}_+\hat{p}_-\hat{q}_+\hat{q}_-\omega_-}{s s_1 t t_1 u u_1}\left[t N_1^{\partial} C_{1+}^{\partial} + t_1 N_1^{\Pi} D_{1+}^{\Pi}\right]\cdot \tag{4.6}$$

$$\cdot\left[t N_2^{\partial} C_{2+}^{\partial} + t_1 N_2^{\Pi} D_{2+}^{\Pi}\right].$$

Using the relation [2]

$$t N_{\partial} C_\pm + t_1 N_\Pi D_\pm = -s N_p B_\pm - s_1 N_q\,, \tag{4.7}$$

and taking into account the annihilation diagrams contribution (3.9) the total amplitude will be:

$$m^{+-++} = \frac{-8H}{s s_1}\left[s N_{1p} B_{1+} + s_1 N_{1q}\right]\left[s N_{2p} B_{2+} + s_1 N_{2q}\right]\Big(1 + \tag{4.8}$$

$$+ s^2 (t t_1 u u_1)^{-1} q_{+\perp}^* q_{-\perp} Z^*(q_+ q_-)\Big).$$

One of multipliers in r.h.s. (4.8) can be written as:

$$1 + \frac{s^2 q_{+\perp}^* q_{-\perp} Z^*(q_+,q_-)}{t t_1 u u_1} = \frac{q_{+\perp}^* q_{-\perp}}{q_{--}\,q_{++}}.$$

The square of module of this amplitude can be written using (3.10) in the form

$$|m^{+-++}|^2 = \frac{u_1^3 u}{s s_1 t t_1}(v_{p_1} - v_{q_1})^2 (v_{p_2} - v_{q_2})^2. \tag{4.9}$$

Similar expression can be deduced for m^{+---}, so:

$$|m^{+---}|^2 + |m^{+-++}|^2 = \frac{u u_1 (u^2 + u_1^2)}{s s_1 t t_1}(v_{p_1} - v_{q_1})^2 (v_{p_2} - v_{q_2})^2. \tag{4.10}$$

From (3.11) and (4.1) we obtain

$$|m^{-+++}|^2 + |m^{-+--}|^2 = \frac{t t_1 (t^2 + t_1^2)}{s s_1 t t_1}(v_{p_1} - v_{q_1})^2 (v_{p_2} - v_{q_2})^2. \tag{4.11}$$

Consider now the scattering diagrams contribution to m^{+-+-}. To perform the contraction over index μ the equation (4.5) rewritten in the form

$$\bar{U}(q_-)A\omega_- U(p_-)\cdot\bar{V}(p_+)B\omega_- V(q_+) = \frac{L}{s t u u_1}\,\mathrm{Sp}\,\hat{q}_- A \hat{p}_- \hat{q}_- \hat{p}_+ B \hat{q}_+ \hat{p}_- \omega_- . \tag{4.12}$$

can be applied. Further trace calculations and writting them in terms of bilinear spinor quantities (App. B) can be done directly. The total amplitude m^{+-+-} has the form:

$$m^{+-+-} = m_a^{+-+-} + m_s^{+-+-}, \tag{4.13}$$

where m_a^{+-+-} has been written earlier (3.15) and m_s^{+-+-} has the form: $m_s^{+-+-} = (8L/s u u_1)\,A_s^{+-+-}$,

$$A_s^{+-+-} = \frac{1}{t_1} N_1^{\partial} N_2^{\partial} C_{1+}^{\partial} C_{2-}^{\partial}\Big[-ts(q_- + k_2)_\perp Z^*(q_+, p_- - k_1, q_-) - \frac{2\varepsilon t u_1}{(p_- - k_1 - k_2)^2} Z_u(k_1, q_-, k_2,$$

$$\Delta_{12}, q_+) - \frac{stu}{(q_- + k_1 + k_2)^2} Z(k_1, q_- + k_2) Z^*(k_2, q_-)\Big] + \frac{1}{t} N_1^{\Pi} N_2^{\Pi} D_{1+}^{\Pi} D_{2+}^{\Pi}\Big[\frac{s^2}{t} q_{-\perp}^* q_{+\perp}^* \cdot \tag{4.14}$$

$$(q_+ + k_1)_\perp Z(q_-, -p_+ + k_2, q_+) + \frac{2\varepsilon u t_1}{(-p_+ + k_1 + k_2)^2} Z_v(k_2, q_+, k_1, -p_+ + k_1 + k_2, q_-) -$$

$$\cdot \frac{s t_1 u_1}{(q_+ + k_1 + k_2)^2} Z(q_+, k_1) Z^*(q_+ + k_1, k_2)] + \frac{s N_1^\ni N_2^\Pi C_{1+}^\ni D_{2-}^\Pi}{(p_- - q_- - k_1)^2} Z^*(q_+, p_- - k_1, q_-) \cdot$$

$$\cdot Z(q_-, -p_+ + k_2, q_+) - \frac{s^2 N_{1\Pi} N_{2\ni} C_{2-}^\ni D_{1+}^\Pi}{(p_- - q_- - k_2)^2} q_{-\perp}^* q_{+\perp}^* (q_- + k_1)_\perp (q_- + k_2)_\perp .$$

The quantities $N_\ni, N_\Pi$ are defined in (4.2) and (4.3) and the remaining ones in Appendices A,B. Chiral amplitude M^{+--+} can be obtained from (4.13) by a replacement of photon momenta.

$$M^{+--+}(k_1, k_2) = M^{+-+-}(k_2, k_1). \tag{4.15}$$

Consider now the chiral amplitudes proper to scattering channel. It's sufficient to consider $M^{++\lambda_1\lambda_2}$. Since we are interested in squares of modulus of amplitudes $M^{++\lambda_1\lambda_2}$ the phase factor of emitted photon by a positron can be chosen as unit:

$$\hat{e}^*_{\Pi\lambda} = E_\lambda \hat{e}^*_{\ni\lambda}, \quad E^*_\lambda = -(e^*_{\ni\lambda} e_{\Pi\lambda}); \quad 2 s N_\Pi (N_\ni)^2 k_\perp \hat{Z}^*(q_-, k, q_+) = \tag{4.16}$$
$$= E^*_+ = E_- ,$$

where the quantities $\hat{e}^*_{\Pi\lambda}$, $\hat{e}^+_{\ni\lambda}$ are given in (4.2),(4.3). Calculate first the amplitudes with the equal chiralities of photons. For amplitude M^{++++} one has from (4.4)

$$M^{++++} = 4 \bar{U}_\alpha(q_-) U_\beta(p_-) \bar{V}_\gamma(p_+) V_\delta(q_+) O^{\alpha\beta\gamma\delta}, \quad O = -\frac{1}{t_1} N_1^\ni N_2^\ni E_{1+} E_{2+} \cdot$$
$$[p_-(q_- + k_2) k_1 + p_-(q_- + k_1) k_2] q_- p_- \frac{\Delta'_-}{\Delta'^2_-} \gamma_\mu \omega_+ * \gamma_\mu \omega_- - \frac{1}{t} N_1^\Pi N_2^\Pi \gamma_\mu \omega_+ *$$
$$* \gamma_\mu \frac{\Delta'_+}{\Delta'^2_+} p_+ q_+ [k_1(-q_+ - k_2) + k_2(-q_+ - k_1)] p_+ \omega_- - \frac{1}{q_1^2} N_1^\ni N_2^\Pi E_{1+} p_-(q_- + k_1) \gamma_\mu \omega_+ * \gamma_\mu (q_+ + k_2) \cdot \tag{4.17}$$
$$\cdot p_+ \omega_- - \frac{1}{q_2^2} N_1^\Pi N_2^\ni E_{2+} p_-(q_- + k_2) \gamma_\mu \omega_+ * \gamma_\mu (q_+ + k_1) p_+ \omega_-$$

This expression can be performed, using the identity

$$\bar{U}(q_-) A \omega_+ U(p_-) \cdot \bar{V}(p_+) B \omega_- V(q_+) = \frac{\tilde{H}}{s s_1} \mathrm{Sp}\, q_- A p_- p_+ B q_+ \omega_+ ,$$
$$\tilde{H} = \bar{V}(p_+) \omega_+ U(p_-) \cdot \bar{U}(q_-) \omega_- V(q_+), \tag{4.18}$$

can be rewritten in the form

$$\frac{1}{4} M^{++++} = \frac{8 \tilde{H} \cdot s^2}{s s_1 t t_1} Z^*(q_-, q_+) [t N_1^\ni E_{1+} + t_1 N_1^\Pi][t N_2^\ni E_{2+} + t_1 N_2^\Pi]; \tag{4.19}$$

M^{++--} has similar form. Their contribution is:

$$|M^{++++}|^2 + |M^{++--}|^2 = \frac{s s_1 (s^2 + s_1^2)}{s s_1 t t_1} (\mathcal{V}_{p_1} - \mathcal{V}_{q_1})^2 (\mathcal{V}_{p_2} - \mathcal{V}_{q_2})^2 . \tag{4.20}$$

To perform the summation over vector index μ in amplitude M^{+++-} it's convenient to apply the "turning line" operation to electron. Using the formula A.13,A.8,A.3 one has

$$\bar{U}(q_-) O_\mu \omega + U(p_-) = \bar{U}(p_-) \tilde{O}_\mu \omega_- U(q_-) \tag{4.21}$$

where $\tilde{O}_\mu$ differs from O_μ by opposite order of Dirac matrices. Applying "turning line" operation (4.21) with the identity

$$\bar{U}(p_-) A \omega_- U(q_-) * \bar{V}(p_+) B \omega_- V(q_+) = \frac{1}{s^2 t t_1} \bar{V}(p_+) p_- \omega_- U(q_-) \cdot \tag{4.22}$$
$$\cdot \bar{U}(p_-) p_+ \omega_- V(q_+) \cdot \mathrm{Sp}\, p_- A q_- p_- p_+ B q_+ p_+ \omega_-$$

to (4.4) the expression for M^{+++-} can be written in the form

$$M^{+++-} = (s^2 t t_1)^{-1} \bar{V}(p_+) p_- \omega_- U(q_-) \cdot \bar{U}(p_-) p_+ \omega_- V(q_+) \cdot \mathrm{Sp}\, Q ,$$
$$Q = -\frac{1}{t_1} N_1^\ni N_2^\ni E_{1+} E_{2-} \left\{ \frac{1}{\Delta_-^2} p_- q_- (p_- - k_2) p_- q_- k_1 \Delta_- \gamma_\mu q_- p_- p_+ \gamma_\mu q_+ p_+ \omega_- - \frac{1}{\Delta'^2_-} p_- \gamma_\mu \Delta'_- \cdot \right.$$
$$\cdot k_2 p_- q_- (q_- + k_1) p_- q_- p_+ \gamma_\mu q_+ p_+ \omega_- + p_- q_- (p_- - k_2) \gamma_\mu (q_- + k_1) p_- q_- p_- p_+ \cdot \tag{4.23}$$

$$\gamma_\mu \hat{q}_+ P_+ \omega_- \Big\} - \frac{1}{t} N_1^\pi N_2^\pi \Big\{ \frac{1}{\Delta_+^2} P_- \gamma_\mu \hat{q}_- P_- P_+ \hat{q}_+ (-\hat{p}_+ + \hat{k}_2) P_+ \hat{q}_+ \hat{k}_1 \hat{\Delta}_+ \gamma_\mu \hat{q}_+ P_+ \omega_-$$

$$- \frac{1}{\Delta_+'^2} P_- \gamma_\mu \hat{q}_- P_- P_+ \gamma_\mu \hat{\Delta}'_+ \hat{k}_2 P_+ \hat{q}_+ (-\hat{p}_+ - \hat{k}_1) P_+ \hat{q}_+ P_+ \omega_- + P_- \gamma_\mu \hat{q}_- P_- P_+ \hat{q}_+ (-\hat{p}_+ + \hat{k}_2) \cdot$$

$$\cdot \gamma_\mu (-\hat{q}_+ - \hat{k}_1) P_+ \hat{q}_+ P_+ \omega_- \Big\} + \frac{1}{q_1^2} N_1^\gamma N_2^\pi E_{1+} P_- \gamma_\mu \hat{q}_1 P_- \hat{q}_- P_- P_+ \hat{q}_+ (-\hat{p}_+ + \hat{k}_2) \gamma_\mu \hat{q}_+ P_+ \omega_- -$$

$$- \frac{1}{q_2^2} N_1^\pi N_2^\gamma E_{2-} P_- \hat{q}_- \hat{q}_2 \gamma_\mu \hat{q}_- P_- P_+ \gamma_\mu (-\hat{q}_+ - \hat{k}_1) P_+ \hat{q}_+ P_+ \omega_- .$$

In spite of complexity the traces in (4.23) can be easily expressed in terms of bilinear combinations Z (see App.B). The result is $|m^{+++-}| = (8/s\sqrt{tt_1})\,|A^{+++-}|$,

$$A^{+++-} = \frac{1}{t_1} A_1 \Big[-\frac{ts^2}{\Delta_-^2} Z(q_-, q_+)\, Z^*(k_2, q_-, k_1, \Delta_-) - \frac{ts^2}{\Delta_-'^2} 2\varepsilon Z(k_2, \Delta'_-, q_+) \cdot$$

$$Y^*(k_1, q_-) + ts^2 Z(q_-, p_- - k_2) Z(q_- + k_1, q_+) \Big] + \frac{1}{t} \mathcal{D}_1 \Big[\frac{ts^2}{\Delta_+^2} Z(q_-, q_+) \cdot \quad (4.24)$$

$$Z^*(-p_+ + k_2, k_1, q_+, k_2) + \frac{t_1 s^2}{\Delta_+'^2} X(k_1, q_+) \cdot 2\varepsilon Z^*(q_-, \Delta'_+, k_2) - t_1 s^2 Z(-p_+ + k_2, q_+) \cdot$$

$$\cdot Z(q_-, -q_+ - k_2) \Big] - \frac{ts^2}{q_1^2} B_1 Z(-p_+ + k_2, q_+) Z(q_- + k_1, q_+) - \frac{t_1 s^2}{q_2^2} C_1 Z(q_-, p_- - k_2) \cdot$$

$$\cdot Z(q_-, -q_+ - k_1),$$

where

$$A_1 = N_1^\gamma N_2^\gamma E_{1+} E_{2-}, \quad B_1 = N_1^\gamma N_2^\pi E_{1+}, \quad C_1 = N_1^\pi N_2^\gamma E_{2-},$$

$$\mathcal{D}_1 = N_1^\pi N_2^\pi .$$

Amplitude m^{++-+} can be obtained from (4.24) by means of replacement of photon momenta:

$$|m^{++-+}(k_1, k_2)| = |m^{+++-}(k_2, k_1)|. \quad (4.25)$$

Differential cross section of the process $e^+e^- \to e^+e^-\gamma\gamma$ has the form:

$$d\sigma^{e^+e^- \to e^+e^-\gamma\gamma} = \frac{(4\pi\alpha)^4}{2!\,(2\pi)^8 \cdot 64 s}\, M\, \frac{d^3q_+ d^3q_- d^3k_1 d^3k_2}{q_{+0}\, q_{-0}\, k_{10}\, k_{20}}\, \delta^{(4)}(p_+ + p_- - q_+ - q_- - k_1 - k_2),$$

$$M = \frac{1}{ss_1tt_1} \big[ss_1(s^2 + s_1^2) + tt_1(t^2 + t_1^2) + uu_1(u^2 + u_1^2) \big] (\mathcal{V}_{p_1} - \mathcal{V}_{q_1})^2 (\mathcal{V}_{p_2} - \mathcal{V}_{q_2})^2 +$$

$$+ \Big[|m^{+-+-}|^2 + |m^{-++-}|^2 + |m^{+++-}|^2 + (k_1 \leftrightarrow k_2) \Big]. \quad (4.27)$$

where

$$\mathcal{V}_p - \mathcal{V}_q = -\frac{p_-}{p_- k} + \frac{p_+}{p_+ k} + \frac{q_-}{q_- k} - \frac{q_+}{q_+ k},$$

and the quantities m^{+-+-}, m^{-++-}, m^{+++-} are given in (4.13),(4.14),(4.1) and (4.24) correspondingly.

In the limiting case when one of photons (k_2) is soft, the cross section (4.26) has the form (the second term in r.h.s. (4.30) coincides with the first one):

$$d\sigma^{e^+e^- \to e^+e^-\gamma(k_1)\gamma(k_2)}\Big|_{k_2 \to 0} = d\sigma^{e^+e^- \to e^+e^-\gamma(k_1)}\, dW(k_2) \quad (4.28)$$

where the cross section of process $e^+e^- \to e^+e^-\gamma$ is given in (1.7) and $dW(k_2)$ in (3.26). In the kinematical region, in which one of the photons, k_2, is emitted close to $\vec{q}_+$ direction the cross section is given by (3.29), where $d\sigma^{e^+e^- \to \mu^+\mu^-\gamma}$ must be replaced by $d\sigma^{e^+e^- \to e^+e^-\gamma}$.

5. Processes $e^+e^- \to \pi^+\pi^-\gamma,\ \pi^+\pi^-\gamma\gamma$.

Consider first

$$e^+(p_+) + e^-(p_-) \to \pi^+(q_+) + \pi^-(q_-) + \gamma(k). \quad (5.1)$$

Here we will assume pions to be point like

$$F_\pi(q^2) = 1 \quad (5.2)$$

The chiral amplitudes are defined in so of a way:

$$M^{\lambda\gamma}_{\lambda e_+\lambda e_-}=\bar V(p_+)^{\lambda e_+}_{\alpha}\,U^{\lambda e_-}_{\beta}(p_-)\,O^{\rho}_{\alpha\beta}\,e^{*\lambda\gamma}_{\rho}(k), \tag{5.3}$$

$$O=\frac{1}{s_1}\Big[\gamma_\mu\frac{\hat p_- -\hat k}{-2p_-k}\gamma_\rho+\gamma_\rho\frac{-\hat p_+ +\hat k}{-2p_+k}\gamma_\mu\Big](q_--q_+)_\mu+\frac{1}{s}\gamma_\mu\Big[(q_--q_++k)_\mu\cdot \tag{5.4}$$
$$\cdot\frac{(2q_-+k)_\rho}{2q_-k}+(q_--q_+-k)_\mu\frac{(-2q_+-k)_\rho}{2q_+k}-2g_{\rho\mu}\Big]\equiv M_e+M_\pi .$$

It's easy to verify that the quantity O (5.4) satisfies the gauge condition $\bar V(p_+)O^\rho U(p_-)k_\rho=0$. The polarization vector of photon is taken in the form:

$$e^{*\lambda}_{\pi\mu}=2N_\pi\big[q_-k\cdot q_{+\mu}-q_+k\cdot q_{-\mu}-i\lambda\varepsilon_{\mu\alpha\beta\gamma}q_-^{\alpha}q_+^{\beta}k^{\gamma}\big],$$
$$N_\pi=\frac{1}{4}\big[q_+q_-\cdot q_+k\cdot q_-k\big]^{-1/2}. \tag{5.5}$$

In the case when it is emitted by leptons we rewrite (5.5) in the form

$$\hat e^{*\lambda}_{e}=2N_e\big[\hat k\hat p_+\hat p_-\omega_{-\lambda}-\hat p_+\hat p_-\hat k\,\omega_\lambda\big],\quad N_e=\frac{1}{4}\big[p_+p_-\cdot p_+k\cdot p_-k\big]^{-1/2},$$
$$e^{*\lambda}_{\pi}=\phi_\lambda e^{*\lambda}_{e},\quad \phi_+=\phi_-^{*}=-(e^{*+}_{\pi}e^{+}_{e})=2N_eN_\pi S_p\,\hat k\hat q_+\hat q_-\hat k\hat p_+\hat p_-\omega_+=2sN_\pi N_e Z^{*}(k,q_+,q_-,k). \tag{5.6}$$

Only two chiral amplitudes $M^{+}_{+-}=M^{+},\ M^{-}_{+-}=M^{-}$ are to be computed. Starting from (5.4) and using (5.6), (5.5) and the results of Appendix B one has

$$M^{+}=\frac{4}{s_1}\big(sN_e\phi_++s_1N_\pi\big)X(q_-,q_+)$$
$$M^{-}=\frac{4}{s_1}\big(sN_e\phi_-+s_1N_\pi\big)Y(q_-,q_+). \tag{5.7}$$

Taking into account that $|X(q_-,q_+)|^2=\frac{s_1}{s}t_1u_1,\ |Y(q_-,q_+)|^2=\frac{s_1}{s}tu$ one can obtain for a square of modul of matrix element:

$$\sum_{ch}\big|M^{e^+e^-\to\pi^+\pi^-\gamma}\big|^2=2(4\pi\alpha)^3\big(|M^+|^2+|M^-|^2\big)=$$
$$=-\frac{4}{ss_1}(4\pi\alpha)^3\big(tu+t_1u_1\big)\big(\mathcal V_p-\mathcal V_q\big)^2. \tag{5.8}$$

The cross section of (5.1) has the form (see (1.5)):

$$d\sigma^{e^+e^-\to\pi^+\pi^-\gamma}=\frac{\alpha^3}{32\pi^2s}\cdot\frac{4(tu+t_1u_1)}{ss_1}(-1)(\mathcal V_p-\mathcal V_q)^2\cdot \tag{5.9}$$
$$\cdot\frac{d^3q_+d^3q_-d^3k}{q_{+0}\,q_{-0}\,k_0}\,\delta^{(4)}(p_++p_--q_+-q_--k),$$

$$-(\mathcal V_p-\mathcal V_q)^2=\big[4\,kp_+\cdot kp_-\cdot kq_+\cdot kq_-\big]^{-1}\big[2ss_1(t+t_1)+2tt_1(s+s_1)+uu_1(st+s_1t_1)+u_1(s_1t+t_1s)\big]. \tag{5.10}$$

Consider now the process $e^+e^-\to\pi^+\pi^-\gamma\gamma$

$$e^+(p_+)+e^-(p_-)\to\pi^+(q_+)+\pi^-(q_-)+\gamma(k_1)+\gamma(k_2). \tag{5.11}$$

We define the chiral amplitudes in such a way:

$$M^{\lambda_1\lambda_2}_{\lambda e_+\lambda e_-}=\bar V_{\lambda e_+}(p_+)\,O^{\rho\sigma}\,U_{\lambda e_-}(p_-)\,e^{*\lambda_1}_{\rho}(k_1)\,e^{*\lambda_2}_{\sigma}(k_2), \tag{5.12}$$

where

$$O^{\rho\sigma}=\frac{1}{s_1}\Big[\gamma_\mu\frac{\Delta_-}{\Delta_-^2}\Big(\gamma_\sigma\frac{\hat p_--\hat k_1}{-2p_-k_1}\gamma_\rho+\gamma_\rho\frac{\hat p_--\hat k_2}{-2p_-k_2}\gamma_\sigma\Big)+\gamma_\sigma\frac{-\hat p_++\hat k_2}{-2p_+k_2}\gamma_\mu\frac{\hat p_--\hat k_1}{-2p_-k_1}\gamma_\rho+\gamma_\rho\frac{-\hat p_++\hat k_1}{-2p_+k_1}\gamma_\mu\cdot$$
$$\frac{\hat p_--\hat k_2}{-2p_-k_2}\gamma_\sigma+\Big(\gamma_\sigma\frac{-\hat p_++\hat k_2}{-2p_+k_2}\gamma_\rho+\gamma_\rho\frac{-\hat p_++\hat k_1}{-2p_+k_1}\gamma_\sigma\Big)\frac{\Delta_+}{\Delta_+^2}\gamma_\mu\Big](q_--q_+)_\mu+\frac{1}{(q_++q_-+k_2)^2}\Big(\gamma_\mu\frac{\hat p_--\hat k_1}{-2p_-k_1}\gamma_\rho+$$
$$+\gamma_\rho\frac{-\hat p_++\hat k_1}{-2p_+k_1}\gamma_\mu\Big)\Big((q_--q_++k_2)_\mu\frac{q_{-\sigma}}{q_-k_2}+(q_--q_+-k_2)_\mu\frac{(-q_{+\sigma})}{q_+k_2}-2g_{\mu\sigma}\Big)+ \tag{5.13}$$
$$+\frac{1}{(q_++q_-+k_1)^2}\Big(\gamma_\mu\frac{\hat p_--\hat k_2}{-2p_-k_2}\gamma_\sigma+\gamma_\sigma\frac{-\hat p_++\hat k_2}{-2p_+k_2}\gamma_\mu\Big)\Big((q_--q_++k_1)_\mu\frac{q_{-\rho}}{q_-k_1}+(q_--q_+-k_1)_\mu\frac{(-q_{+\rho})}{q_+k_1}-2g_{\mu\rho}\Big)+$$
$$\frac{\gamma_\mu}{s}\Big\{\frac{(q_--q_++k_1+k_2)_\mu}{(q_-+k_1+k_2)^2}\Big[\frac{2(q_-+k_2)_\rho}{q_-k_2}q_{-\sigma}+2\frac{(q_-+k_1)_\sigma}{q_-k_1}q_{-\rho}-2g_{\rho\sigma}\Big]+\frac{(q_--q_+-k_1-k_2)_\mu}{(q_++k_1+k_2)^2}\Big[\frac{2(q_++k_2)_\rho q_{+\sigma}}{q_+k_2}+$$
$$+\frac{2(q_++k_1)_\sigma q_{+\rho}}{q_+k_1}-2g_{\rho\sigma}\Big]-\frac{(q_--q_++k_1-k_2)_\mu}{q_-k_1\cdot q_-k_2}q_{-\rho}q_{+\sigma}-\frac{(q_--q_++k_2-k_1)_\mu}{q_-k_2\cdot q_+k_1}q_{-\sigma}q_{+\rho}-$$
$$-2q_{-\sigma}g_{\rho\mu}/(q_-k_2)-2q_{-\rho}g_{\mu\sigma}/(q_-k_1)+2q_{+\sigma}g_{\mu\rho}/(q_+k_2)+2q_{+\rho}g_{\sigma\mu}/(q_+k_1)\Big\}.$$

One may verify that the gauge condition is satisfied: the r.h.s. (5.12) vanishes at substitution $e_1\to k_1$ (or $e_2\to k_2$). It's sufficient to calculate only four chiral amplitudes $M^{++}_{+-}\equiv M^{++}$, M^{--}, M^{+-}, M^{-+} . It turns out that chiral amplitudes M^{++}, M^{--} have the factorized form.

Consider first $\mathfrak{M}^{++}$. The corresponding contributions are of four types. In the case when both photons are emitted by leptons (the first term in r.h.s. (5.13) is relevant) one has

$$4N_{1e}N_{2e}\frac{\Phi_{1+}\Phi_{2+}}{\Delta_-^2 s_1}\bar{V}(p_+)\gamma_\mu(\hat{p}_- - \hat{k}_1 - \hat{k}_2)\Big[\hat{p}_+\hat{p}_-\hat{k}_2\frac{\hat{p}_- - \hat{k}_1}{-2p_-k_1}\hat{p}_+\hat{p}_-\hat{k}_1 + \hat{p}_+\hat{p}_-\hat{k}_1\frac{\hat{p}_- - \hat{k}_2}{-2p_-k_2}\hat{p}_+\hat{p}_-\hat{k}_2\Big] \tag{5.14a}$$

$$\omega_- U(p_-)\cdot(q_- - q_+)_\mu = \frac{4s}{s_1}\bar{V}(p_+)(\hat{q}_- - \hat{q}_+)(\hat{p}_- - \hat{k}_1 - \hat{k}_2)\hat{p}_+\omega_- U(p_-)\Phi_{1+}\Phi_{2+}N_{1e}N_{2e}$$

$$= \frac{8s}{s_1}\Phi_{1+}\Phi_{2+}N_{1e}N_{2e}\,\bar{V}(p_+)\hat{q}_-\hat{q}_+\hat{p}_+\omega_- U(p_-).$$

In the case when photon k_1 is emitted by fermions and the other by pions one deduces from (5.13)

$$\frac{4N_1^e N_2^\pi \Phi_{1+}}{(q_+ + q_- + k_2)^2}\bar{V}\gamma_\mu\frac{\hat{p}_- - \hat{k}_1}{-2p_-k_1}(-\hat{p}_+\hat{p}_-\hat{k}_1)\omega_- U\Big\{2q_{-\mu}\,q_+(q_- + k_2) - 2q_{+\mu}\,q_-(q_+ + k_2)$$

$$+2i\varepsilon_{\mu\alpha\beta\gamma}q_-^\alpha q_+^\beta k_2^\gamma\Big\}$$

This expression, using the identity

$$2\gamma^\mu\big[(q_+k)q_{-\mu} - (q_-k)\cdot q_{+\mu} + i\lambda\varepsilon_{\mu\alpha\beta\gamma}q_+^\alpha q_-^\beta k^\gamma\big] = \hat{k}\hat{q}_+\hat{q}_-(1+\lambda\gamma_5) -$$
$$-\hat{q}_+\hat{q}_-\hat{k}(1-\lambda\gamma_5) - 2\lambda\hat{k}\gamma_5(q_+q_-),$$

can be presented in the form

$$8N_{1e}N_{2\pi}\Phi_{1+}\,\bar{V}\,\hat{q}_-\hat{q}_+\hat{p}_+\omega_- U. \tag{5.14b}$$

The similar expression

$$8N_{1\pi}N_{2e}\Phi_{2+}\,\bar{V}\hat{q}_-\hat{q}_+\hat{p}_+\omega_- U, \tag{5.14c}$$

arises for the case when the second photon (with momentum k_2) is emitted by fermions and the first by pions. When both photons are emitted by pions, one finds after some algebra:

$$\frac{4s}{s_1}\bar{V}\gamma_\mu\omega_- U\cdot N_{1\pi}N_{2\pi}\Big\{-2q_{+\mu}\cdot q_-(q_+ + k_1 + k_2) + 2q_{-\mu}\cdot q_+(q_- + k_1 + k_2) \tag{5.14d}$$
$$-i\varepsilon_{\mu\alpha\beta\gamma}q_-^\alpha q_+^\beta(q_+ + q_- + k_1 + k_2)^\gamma = \frac{8s_1}{s}N_{1\pi}N_{2\pi}\bar{V}\big[(\hat{p}_+ + \hat{p}_-)\hat{q}_+\hat{q}_-\omega_+$$

$$-\hat{q}_+\hat{q}_-(\hat{p}_+ + \hat{p}_-)\omega_-\big]\omega_- U = \frac{8s_1}{s}N_{1\pi}N_{2\pi}\,\bar{V}\hat{q}_-\hat{q}_+\hat{p}_+\omega_- U.$$

Taking into account $\bar{V}\hat{q}_-\hat{q}_+\hat{p}_+\omega_- U = s\,\bar{X}(q_-, q_+)$, one has for sum (5.14a-d)

$$\mathfrak{M}^{++} = \frac{8}{ss_1}\cdot s\,\bar{X}(q_-, q_+)\,(sN_{1e}\Phi_{1+} + N_{1\pi}s_1)(N_{2e}\Phi_{2+}\cdot s + N_{2\pi}s_1). \tag{5.14}$$

The analogous calculation gives for $\mathfrak{M}^{--}$:

$$\mathfrak{M}^{--} = \frac{8}{ss_1}\,s\,Y(q_-, q_+)\,(N_{1e}s\Phi_{1-} + N_{1\pi}s_1)(N_{2e}s\Phi_{2-} + N_{2\pi}s_1). \tag{5.15}$$

Consider now amplitude $\mathfrak{M}^{+-}$. One may write it in the form

$$\mathfrak{M}^{+-} = \frac{4N_{1e}N_{2e}\Phi_{1+}\Phi_{2-}}{s_1}A_{ee} + \frac{4N_{1e}N_{2\pi}\Phi_{1+}}{(q_+ + q_- + k_2)^2}A_{e_1\pi_2} + \frac{4N_{1\pi}N_{2e}\Phi_{2-}}{(q_+ + q_- + k_1)^2}A_{e_2\pi_1} \tag{5.16}$$
$$+ \frac{4N_{1\pi}N_{2\pi}}{s}A_{\pi\pi},$$

where the quantities A_i can be found from (5.13) using (5.5), (5.6), and (5.16), the Appendix B. The result is

$$A_{ee} = -\frac{s^2 k_{2\perp}}{\Delta_-^2}Z_V(q, \Delta_-, k_1) + \frac{s^2 k_{1\perp}}{\Delta_+^2}Z_U(q, \Delta_+, k_2) + 2\varepsilon s\,Z(\Delta_+, q, \Delta_-);$$

$$A_{e_1\pi_2} = 2s\,Z_U(-p_+ + k_1, q_+, q_-, q_+ + q_- + k_2), \quad A_{e_2\pi_1} = -2s\,Z_V(q_+ + q_- + k_1, \tag{5.17}$$
$$q_+, q_-, p_- - k_2); \quad A_{\pi\pi} = 4s_1\varepsilon\Big\{-\frac{\Delta'_{-\perp}}{\Delta_-'^2}\big[Z^*(k_2, q_-, k_1, q_+) + Z^*(q_+, k_1, q_-, k_2)\big]$$
$$+\frac{\Delta'_{+\perp}}{\Delta_+'^2}\big(Z^*(k_2, q_+, k_1, q_-) + Z^*(q_-, k_1, q_+, k_2)\big) + Z(k_2, q_+, q_-) - Z(q_+, q_-, k_1) + \frac{s_1}{2}(q_+ + k_2 - q_- - k_1)_\perp\Big\},$$

where

$$\Delta'_\pm = q_\pm + k_1 + k_2, \quad \Delta_\pm = \mp(p_\pm - k_1 - k_2), \quad q = q_- - q_+, \tag{5.18}$$
$$\Delta_{1,2} = p_+ + p_- - k_{1,2}.$$

Amplitude $\mathfrak{M}^{-+}$ can be obtained from (5.16) by relabeling the photons momenta

$$\mathcal{M}^{-+}(k_1,k_2) = \mathcal{M}^{+-}(k_2,k_1). \qquad (5.19)$$

The cross section of process has the form

$$d\sigma^{e^+e^-\to\pi^+\pi^-\gamma\gamma} = \frac{1}{2!}\,\frac{\alpha^4}{64\pi^4 s}\, M\, \frac{d^3q_+ d^3q_- d^3k_1 d^3k_2}{q_{+0}\, q_{-0}\, k_{10}\, k_{20}}\, \delta^{(4)}(p_+ + p_- - q_+ - q_- - k_1 - k_2),$$

$$M = \frac{(tu + t_1u_1)}{ss_1}(\mathcal{V}_{p_1} - \mathcal{V}_{q_1})^2(\mathcal{V}_{p_2} - \mathcal{V}_{q_2})^2 + \left(|\mathcal{M}^{+-}|^2 + \quad (k_1 \leftrightarrow k_2)\right), \qquad (5.20)$$

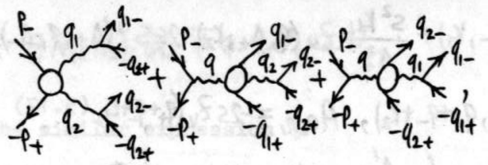
where ($^1/2!$) takes into account the identity of photons, the quantity $\mathcal{M}^{+-}$ is drawn in (5.16–5.18). In the soft k_2 photon limit $|\mathcal{M}^{+-}|$ coincides with the modul of expression (5.15) and $|\mathcal{M}^{-+}|$ – (5.14). So the second term in r.h.s. (5.21) coincides with the first one.

6. Process $e^+e^-\to\mu^+\mu^-\bar{q}q$.

Consider the process of creation of two different fermion pairs in e^+e^- annihilation

$$e_+(p_+) + e_-(p_-) \to \mu_+(q_{1+}) + \mu_-(q_{1-}) + q_+(q_{2+}) + q_-(q_{2-}). \qquad (6.1)$$

It is described by six Feynman diagrams:



$$q_1 = q_{1-} + q_{1+}, \quad q_2 = q_{2-} + q_{2+}, \quad q = p_+ + p_-. \qquad (6.2)$$

It's necessary to calculate four chiral amplitudes:

$$\sum_{Cn} |M|^2 = 2(4\pi\alpha)^4 Q^2\left[|\mathcal{M}^{++}|^2 + |\mathcal{M}^{+-}|^2 + |\mathcal{M}^{-+}|^2 + |\mathcal{M}^{--}|^2\right], \qquad (6.3)$$

where we believe the charge of q particle eQ, $e^2 = 4\pi\alpha$, and such a definition of chiral amplitudes (see (6.2)):

$$\mathcal{M}^{\lambda_1\lambda_2} = (\bar{V}(p_+))_\alpha\, (\omega_- U(p_-))_\beta * (\bar{U}(q_{1-}))_\gamma\, (\omega_{-\lambda_1} V(q_{1+}))_\delta * (\bar{U}(q_{2-}))_\varepsilon\, (\omega_{-\lambda_2} V(q_{2+}))_\sigma \cdot$$

$$O^{\alpha\beta\gamma\delta\varepsilon\sigma}, \quad O = \frac{1}{q_1^2 q_2^2}\left[\gamma_\sigma \frac{\hat{p}_- - \hat{q}_1}{(p_- - q_1)^2}\gamma_\rho + \gamma_\rho \frac{\hat{p}_- - \hat{q}_2}{(p_- - q_2)^2}\gamma_\sigma\right] * \gamma_\rho * \gamma_\sigma + \qquad (6.4)$$

$$+ \frac{1}{sq_2^2}\gamma_\rho * \left[\gamma_\sigma \frac{(\hat{q}_{1-} + \hat{q}_2)}{(q_{1-} + q_2)^2}\gamma_\rho + \gamma_\rho \frac{(-\hat{q}_{1+} - \hat{q}_2)}{(q_{1+} + q_2)^2}\gamma_\sigma\right] * \gamma_\sigma + \frac{1}{sq_1^2}\gamma_\rho * \gamma_\sigma * \left[\gamma_\sigma \frac{\hat{q}_{2-} + \hat{q}_1}{(q_{2-} + q_1)^2} \qquad (6.5)\right.$$

$$\left.\cdot\gamma_\rho + \gamma_\rho \frac{(-\hat{q}_{2+} - \hat{q}_1)}{(q_{2+} + q_1)^2}\gamma_\sigma\right] \equiv \frac{O^a_{\rho\sigma} * \gamma_\rho * \gamma_\sigma}{q_1^2 q_2^2} + \frac{\gamma_\rho * O^b_{\rho\sigma} * \gamma_\sigma}{sq_2^2} + \frac{\gamma_\rho * \gamma_\sigma * O^c_{\rho\sigma}}{sq_1^2}.$$

Performing the contraction on vector index ρ,σ in the calculation of $\mathcal{M}^{++}$ introduces in r.h.s. (6.4) the factor (which is unity)

$$1 = \bar{U}(p_-)\hat{p}_+\omega_- U(q_{1-}) * \bar{V}(q_{1+})\hat{p}_-\omega_- U(q_{2-}) \cdot \bar{V}(q_{2+})\hat{p}_-\omega_- V(p_+) \cdot$$

$$\frac{\bar{U}(q_{1-})\hat{p}_+ U(p_-) \cdot \bar{U}(q_{2-})\hat{p}_-\omega_- V(q_{1+}) \cdot \bar{V}(p_+)\hat{p}_-\omega_- V(q_{2+})}{s^2(2p_+q_{1-})(2p_-q_{2-})(2p_-q_{1+})(2p_-q_{2+})}. \qquad (6.6)$$

The modulus $|\mathcal{M}^{++}|$ takes the form: $|\mathcal{M}^{++}| = \frac{1}{s}|S_p^{++}|\,[2p_+q_{1-} \cdot 2p_-q_{2+} \cdot 2p_-q_{2-} \cdot 2p_-q_{1+}]^{-1/2}$,

$$S_p^{++} = \frac{1}{q_1^2 q_2^2}\,\mathrm{Sp}\, \hat{p}_+ O^a_{\rho\sigma}\hat{p}_-\hat{p}_+\hat{q}_{1-}\gamma_\rho\hat{q}_{1+}\hat{p}_-\hat{q}_{2-}\gamma_\sigma\hat{q}_{2+}\hat{p}_-\omega_- +$$

$$+ \frac{1}{sq_2^2}\,\mathrm{Sp}\,\hat{p}_+\gamma_\rho\hat{p}_-\hat{p}_+\hat{q}_{1-}O^b_{\rho\sigma}\hat{q}_{1+}\hat{p}_-\hat{q}_{2-}\gamma_\sigma\hat{q}_{2+}\hat{p}_-\omega_- + \frac{1}{sq_1^2}\,\mathrm{Sp}\,\hat{p}_+\gamma_\rho\hat{p}_-\hat{p}_+\hat{q}_{1-}\gamma_\sigma\hat{q}_{1+}\hat{p}_-\hat{q}_{2-} \cdot \qquad (6.7)$$

$$\cdot O^c_{\rho\sigma}\hat{q}_{2+}\hat{p}_-\omega_- \equiv 8\varepsilon s^2\left\{\frac{A^{++}}{q_1^2 q_2^2} + \frac{B^{++}}{sq_2^2} + \frac{C^{++}}{sq_1^2}\right\}.$$

Instead of (6.6) the relation

$$1 = \bar{U}(p_-)\omega_+ U(q_{1-}) \cdot \bar{V}(q_{1+})\hat{p}_-\omega_+ U(q_{2-}) \cdot \bar{V}(q_{2+})\omega_- V(p_+)\,\frac{\bar{U}(q_{1-})\omega_- U(p_-)}{2p_-q_{1-}} \cdot \qquad (6.8)$$

$$\frac{\bar{U}(q_{2-})\hat{p}_-\omega_+ V(q_{1+}) \cdot \bar{V}(p_+)\omega_+ V(q_{2+})}{2p_-q_{1+} \cdot 2p_-q_{2-} \cdot 2p_+q_{2+}},$$

is convenient in calculation of amplitude $\mathcal{M}^{--}$. For its modulus one has:

$$|\mathcal{M}^{--}| = [2p_-q_{1-} \cdot 2p_-q_{1+} \cdot 2p_-q_{2-} \cdot 2p_+q_{2+}]^{-1/2}\, |S_p^{--}|, \qquad (6.9)$$

$$\mathrm{Sp}^{--} = 8\varepsilon s\left(\frac{A^{--}}{q_1^2 q_2^2} + \frac{B^{--}}{q_2^2 s} + \frac{C^{--}}{q_1^2 s}\right) = \frac{1}{q_1^2 q_2^2}\,\mathrm{sp}\, P_+ O^a_{\rho\sigma} P_- q_{1-} \gamma_\rho q_{1+} P_- q_{2-} \gamma_\sigma q_{2+} \omega_- +$$

$$+\frac{1}{s q_2^2}\,\mathrm{sp}\, P_+ \gamma_\rho P_- q_{1-} O^b_{\rho\sigma} q_{1+} P_- q_{2-} \gamma_\sigma q_{2+} \omega_- + \frac{1}{s q_1^2}\,\mathrm{sp}\, P_+ \gamma_\rho P_- q_{1-} \gamma_\sigma q_{1+} P_- q_{2-} O^c_{\rho\sigma} q_{2+} \omega_- .$$

The identity

$$1 = \bar{u}(P_-)\omega_+ u(q_{1-}) \cdot \bar{v}(q_{1+})\omega_- u(q_{2-}) \cdot \bar{v}(q_{2+}) P_- \omega_- v(P_+) \cdot \frac{\bar{u}(q_{1-})\omega_- u(P_-)}{s \cdot 2P_- q_{1-}} \cdot \tag{6.10}$$

$$\frac{\bar{u}(q_2)\omega_+ v(q_{1+}) \cdot \bar{v}(P_+) P_- \omega_- v(q_{2+})}{2P_- q_{2+} \cdot 2q_{1+} q_{2-}} ,$$

helps one in the calculation of amplitude M^{-+}. Its module has the form $|M^{-+}| = [s \cdot 2P_- q_{1-} \cdot 2P_- q_{2+} \cdot 2q_{1+} q_{2-}]^{-1/2} \cdot |\mathrm{Sp}^{-+}|$,

$$\mathrm{Sp}^{-+} = 8\varepsilon s\left(\frac{A^{-+}}{q_1^2 q_2^2} + \frac{B^{-+}}{s q_2^2} + \frac{C^{-+}}{s q_1^2}\right) = \frac{1}{q_1^2 q_2^2}\,\mathrm{sp}\, P_+ O^a_{\rho\sigma} P_- q_{1-} \gamma_\rho q_{1+} q_{2-} \gamma_\sigma q_{2+} P_- \omega_-$$

$$+ \frac{1}{s q_2^2}\,\mathrm{sp}\, P_+ \gamma_\rho P_- q_{1-} O^b_{\rho\sigma} q_{1+} q_{2-} \gamma_\sigma q_{2+} P_- \omega_- + \frac{1}{s q_1^2}\,\mathrm{sp}\, P_+ \gamma_\rho P_- q_{1-} \gamma_\sigma q_{1+} q_{2-} O^c_{\rho\sigma} q_{2+} P_- \omega_- . \tag{6.11}$$

At least the relation

$$1 = \bar{u}(P_-) P_+ \omega_- u(q_{1-}) \cdot \bar{v}(q_{1+})\omega_+ u(q_{2-}) \cdot \bar{v}(q_{2+})\omega_- v(P_+) \cdot \frac{\bar{u}(q_{1-}) P_+ \omega_- u(P_-)}{s \cdot 2P_+ q_{2+}} \cdot \tag{6.12}$$

$$\frac{\bar{u}(q_{2-})\omega_- v(q_{1+}) \cdot \bar{v}(P_+)\omega_+ v(q_{2+})}{2P_+ q_{1-} \cdot 2q_{1+} q_{2-}} ,$$

is useful in calculation of amplitude M^{+-}. Its module has the form: $|M^{+-}| = [s \cdot 2P_+ q_{1-} \cdot 2q_{1+} q_{2-} \cdot 2P_+ q_{2+}]^{-1/2}\, |\mathrm{Sp}^{+-}|$,

$$\mathrm{Sp}^{+-} = 8\varepsilon s\left(\frac{A^{+-}}{q_1^2 q_2^2} + \frac{B^{+-}}{s q_2^2} + \frac{C^{+-}}{s q_1^2}\right) = \frac{1}{q_1^2 q_2^2}\,\mathrm{sp}\, P_+ O^a_{\rho\sigma} P_- P_+ q_{1-} \gamma_\rho q_{1+} q_{2-} \gamma_\sigma q_{2+} \omega_-$$

$$+ \frac{1}{s q_2^2}\,\mathrm{sp}\, P_+ \gamma_\rho P_- P_+ q_{1-} O^b_{\rho\sigma} q_{1+} q_{2-} \gamma_\sigma q_{2+} \omega_- + \frac{1}{s q_1^2}\,\mathrm{sp}\, P_+ \gamma_\rho P_- P_+ q_{1-} \gamma_\sigma q_{1+} q_{2-} \cdot \tag{6.13}$$

$$O^c_{\rho\sigma} q_{2+} \omega_- .$$

Further calculation can be directly performed using the results of Appendix B. The result is:

$$A^{++} = \frac{q_{1+-} \cdot q_{2-\perp} Z^*(q_{2+}, P_- - q_{1+}, q_{1-})}{(P_- - q_1)^2} - \frac{q_{1-+} q_{2+-} Z_u(q_{2-}, P_- - q_{2+}, q_{1+})}{(P_- - q_2)^2} ;$$

$$B^{++} = \frac{-q_{1+-}[Z(q_{2-}, q_{1-}, q_2) q^*_{2+\perp} + (2\varepsilon - q_{1++}) Z_u(q_{2-}, q_{1-}, q_{2+})]}{(q_{1-} + q_2)^2} + \frac{q_{1-+} Y^*(q_{1+}, q_{2+}) Y(q_{1+} + q_2, q_{2-})}{(q_{1+} + q_2)^2} ;$$

$$C^{++} = \frac{q_{2+-} Z(q_{1+}, q_{2-} + q_1) Z^*(q_{2-}, q_{1-})}{(q_{2-} + q_1)^2} + \frac{q_{2-\perp}[q^*_{1+\perp} Z_u(q_{2+} + q_1, q_{1-}, q_{2+}) - q^*_{2+\perp} Z_u(q_{2+} + q_1, q_{1-}, q_{1+})]}{(q_{2+} + q_1)^2} . \tag{6.14}$$

$$A^{--} = \frac{q_{1--} q_{2++} Z_u(q_{1+}, P_- - q_1, q_{2-})}{(P_- - q_1)^2} + \frac{q_{2--}[q_{2++} Z_u(q_{1+}, P_- - q_2, q_{1-}) - Z^*(q_{1+}, q_{2+}) Z(q_{1-}, P_- - q_2)]}{(P_- - q_2)^2} ;$$

$$B^{--} = \frac{q_{1+\perp} Y^*(q_{1-}, q_{2-}) Z^*(q_{1-} + q_2, q_{2+})}{(q_{1-} + q_2)^2} + \frac{q_{1--} Z(q_{2-}, q_{1+} + q_2) Z^*(q_{1+}, q_{2+})}{(q_{1+} + q_2)^2} ;$$

$$C^{--} = \frac{q_{2++} Y^*(q_{2-}, q_{1-}) Y(q_{2-} + q_1, q_{1+})}{(q_{2-} + q_1)^2} + \frac{q_{2--} Z(q_{1-}, q_{2+} + q_1) Z^*(q_{1+}, q_{2+})}{(q_{2+} + q_1)^2} . \tag{6.15}$$

$$A^{+-} = \frac{-q_{2++}}{(P_- - q_1)^2} Z(q_{1+}, q_{2-}, P_- - q_1, q_{1-}) + \frac{q_{1-+}}{(P_- - q_2)^2} Z(q_{2-}, q_{1+}, P_- - q_2, q_{2+}) ;$$

$$B^{+-} = \frac{X^*(q_{2+}, q_{1-})}{(q_{1-} + q_2)^2} Z(q_{1+}, q_{2-}, q_{1-} + q_2) + \frac{2q_{1-+}(q_{1+} q_{2-}) Z^*(q_{1+} + q_2, q_{2+})}{(q_{1+} + q_2)^2} ;$$

$$C^{+-} = \frac{2(q_{1+} q_{2-}) \cdot q_{2++} Z^*(q_{2-} + q_1, q_{1-})}{(q_{2-} + q_1)^2} + \frac{Z(q_{2-}, q_{1+}, q_{2+} + q_1) X^*(q_{1-}, q_{2+})}{(q_{2+} + q_1)^2} .$$

(6.16)

$$A^{-+} = \frac{-q_{1--} Z^*(q_{2+}, P_- - q_1, q_{1+}, q_{2-})}{(P_- - q_1)^2} + \frac{q_{2+-} Z^*(q_{1-}, P_- - q_2, q_{2-}, q_{1+})}{(P_- - q_2)^2} ;$$

$$B^{-+} = \frac{q_{1--} Z^*(q_{2+}, q_1, q_{2-}, q_{1+}) - q_{2+-} Z^*(q_{1-}, q_2, q_{2-}, q_{1+})}{(q_{1-} - q_2)^2} + \frac{2q_{1--}(q_{1+} q_{2-})}{(q_{1+} + q_2)^2} Z(q_{2+}, q_{1+} - q_2) ;$$

$$C^{-+} = \frac{-2\, q_{1+} q_{2-} \cdot q_{2+-}}{(q_{2-}+q_1)^2} Z(q_{1-}, q_{2-}+q_1) + \frac{Z(q_{2+}+q_1, q_{1+}, q_{2-})\, Y^*(q_{2+}, q_{1-})}{(q_{2+}+q_1)^2} . \tag{6.17}$$

The cross section of process (6.1) has the form:

$$d\sigma^{e^+e^- \to \mu^+\mu^- q\bar{q}} = \frac{\alpha^4 Q^2}{64\pi^4 s} \left[|M^{++}|^2 + |M^{--}|^2 + |M^{-+}|^2 + |M^{+-}|^2 \right] \cdot \frac{d^3q_{1+}\, d^3q_{1-}\, d^3q_{2+}\, d^3q_{2-}}{q_{1+0}\, q_{1-0}\, q_{2+0}\, q_{2-0}} \delta^{(4)}(P_+ + P_- - q_{1+} - q_{1-} - q_{2+} - q_{2-}), \tag{6.18}$$

where the quantities Q , M^{ij} are defined in (6.3), (6.7), (6.9), (6.11), (6.13-6.17). It should be noted that in the region (1.1) the main contribution arises from the Feynman diagram in which both heavy photons q_1 , q_2 are emitted by initial particles, since its contribution contains the small denominator $\left[(P - q_{1,2})^2 - M_e^2 \right]$. So the kinematical constraint

$$\vec{q}_{1-} + \vec{q}_{1+} = \vec{q}_{2-} + \vec{q}_{2+}$$

for the process (6.1) must be fulfilled.

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Appendix A

We use the following metric and reference frame

$$ab = a_0 b_0 - \vec{a}\vec{b}, \quad P_- = (\overset{z}{\varepsilon}, \overset{x}{\varepsilon}, \overset{y}{0}, 0), \quad P_+ = (\varepsilon, -\varepsilon, 0, 0). \qquad (A.1)$$

The parametrization of momenta used through all the paper is:

$$a_\pm = a_0 \pm a_z, \quad a_\perp = a_x + i a_y, \quad 2ab = a_+ b_- + a_- b_+ - a_\perp b_\perp^* - a_\perp^* b_\perp, \qquad (A.2)$$
$$P_{-+} = P_{+-} = 2\varepsilon, \quad P_{--} = P_{++} = P_{+\perp} = P_{-\perp} = 0.$$

Dirac matrices (standard representation)

$$\gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma_i = \begin{pmatrix} 0 & \vec{\sigma}_i \\ -\vec{\sigma}_i & 0 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$
$$\gamma_0 \gamma_\mu^+ \gamma_0 = \gamma_\mu, \quad \gamma_\mu \gamma_{\alpha_1} \cdots \gamma_{\alpha_{2k+1}} \gamma^\mu = -2\gamma_{\alpha_{2k+1}} \cdots \gamma_{\alpha_1}, \quad \gamma_\mu \gamma_\alpha \gamma_\beta \gamma^\mu = 4 g_{\alpha\beta}, \qquad (A.3)$$
$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2 g_{\mu\nu} = 2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}_{\mu\nu}.$$

We construct using (A.3) the projection operators and the charge-conjugation matrix

$$\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad C = \gamma_2\gamma_0 = i \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad C = C^{-1}, \; \gamma_5^2 = 1.$$

$$\frac{1}{4} Sp\, \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5 = i\varepsilon_{\mu\nu\rho\sigma}, \quad \varepsilon_{0123} = +1, \quad \gamma_5 \gamma_\mu = -\gamma_\mu \gamma_5, \qquad (A.4)$$
$$C \gamma_\mu C = -\gamma_\mu^T, \quad \omega_\pm = \frac{1}{2}(1 \pm \gamma_5), \quad \omega_+ + \omega_- = I, \; \omega_\pm^2 = \omega_\pm, \; \omega_+ \omega_- = 0.$$

The useful identity

$$\gamma^\mu (ac \cdot b_\mu - bc \cdot a_\mu + i\lambda \varepsilon_{\mu\alpha\beta\gamma} a^\alpha b^\beta c^\gamma) = \hat{c}(\hat{a}\hat{b} - ab)\omega_\lambda - (\hat{a}\hat{b} - ab)\hat{c}\omega_{-\lambda}.$$

Chiral states of fermions are described by spinors:

$$U_\pm(P) = \omega_\pm U(P), \quad V_\pm(P) = \omega_\mp V(P), \qquad (A.5)$$
$$\bar{V}_\pm = \bar{V}\omega_\pm, \quad \bar{U}_\pm = \bar{U}\omega_\mp.$$

As chiral states of the initial electron and positron we choose the explicit expressions $U(P_-) \equiv U$, $V(P_+) \equiv V$:

$$U_+ = \sqrt{\varepsilon}\begin{pmatrix} W \\ -W \end{pmatrix}, \quad U_- = \sqrt{\varepsilon}\begin{pmatrix} W' \\ W' \end{pmatrix}, \quad \bar{U}_+ = \sqrt{\varepsilon}(W^T, W^T), \quad \bar{U}_- = \sqrt{\varepsilon}(W'^T, -W'^T);$$

$$V_+ = \sqrt{\varepsilon}\begin{pmatrix} W \\ W \end{pmatrix}, \quad V_- = \sqrt{\varepsilon}\begin{pmatrix} -W' \\ W' \end{pmatrix}, \quad \bar{V}_+ = \sqrt{\varepsilon}(W^T, -W^T),$$
$$\bar{V}_- = \sqrt{\varepsilon}(-W'^T, -W'^T); \quad W = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad W^T = (0, 1) \qquad (A.6)$$
$$W' = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad W'^T = (1, 0),$$

where we omit the arguments of initial spinors. These spinors satisfy the Dirac equation

$$\hat{P}_+ V_\pm = \bar{V}_\pm \hat{P}_+ = \hat{P}_- U_\pm = \bar{U}_\pm \hat{P}_- = 0, \qquad (A.7)$$

the completeness condition

$$U_\pm \bar{U}_\pm = \omega_\pm \hat{P}_-, \quad V_\pm \bar{V}_\pm = \omega_\mp \hat{P}_+, \qquad (A.8)$$
$$\bar{U}_\pm U_\pm = \bar{V}_\pm V_\pm = 0,$$

and to the charge-conjugate conditions

$$\bar{V}_\pm^T C = i\bar{V}_\mp, \quad U_\pm^T C = i\bar{U}_\mp, \quad V_\pm^T \gamma^0 = \bar{V}_\pm, \quad U_\pm^T \gamma^0 = \bar{U}_\pm, \qquad (A.9)$$
$$C\bar{V}_\pm^T = iV_\mp, \quad C\bar{U}_\pm^T = iU_\mp, \quad \gamma_0 U_\pm^T = U_\pm, \quad \gamma_0 \bar{V}_\pm^T = V_\pm.$$

Final state fermions with definite chirality satisfy (A.5), and the Dirac equation:

$$\hat{q}_+ V_\pm(q_+) = \bar{V}_\pm(q_+)\hat{q}_+ = \hat{q}_- U_\pm(q_-) = \bar{U}_\pm(q_-)\hat{q}_- = 0. \qquad (A.10)$$

We choose them in the form

$$U_+(q_-) = \sqrt{\varepsilon_-}\begin{pmatrix} W_+ \\ -W_+ \end{pmatrix}, \quad \bar{U}_+(q_-) = \sqrt{\varepsilon_-}(W_+^+, W_+^+), \quad W_+ = \begin{pmatrix} -\sin\frac{\theta_-}{2} \\ e^{i\varphi_-}\cos\frac{\theta_-}{2} \end{pmatrix}; \qquad (A.11)$$
$$U_-(q_-) = \sqrt{\varepsilon_-}\begin{pmatrix} W_- \\ W_- \end{pmatrix}, \quad \bar{U}_-(q_-) = \sqrt{\varepsilon_-}(W_-^+, -W_-^+), \quad W_- = \begin{pmatrix} e^{-i\varphi_-}\cos\frac{\theta_-}{2} \\ \sin\frac{\theta_-}{2} \end{pmatrix},$$

where the polar angles $\theta_\pm$ are counted out from the direction of montion of the initial electron

$$\theta_- = \widehat{\vec{q}_- \vec{P}_-}, \quad \theta_+ = \widehat{\vec{q}_+ \vec{P}_-} \qquad (A.12)$$

and the azimuthal ones are the angles between the some plane containing the beam axis and the plane containing the beam axis and the 3-momentum of final fermion.

$$V_+(q_+)=\sqrt{\varepsilon_+}\begin{pmatrix} w'_+ \\ w'_+ \end{pmatrix},\ \bar{V}_+(q_+)=\sqrt{\varepsilon_+}\left(w'^{+}_+,-w'^{+}_+\right),\ w'_+=\begin{pmatrix} e^{-i\varphi_+}\cos\frac{\theta_+}{2} \\ \sin\frac{\theta_+}{2} \end{pmatrix}, \tag{A.13}$$

$$V_-(q_+)=\sqrt{\varepsilon_+}\begin{pmatrix} -w'_- \\ w'_- \end{pmatrix},\ \bar{V}_-(q_+)=\sqrt{\varepsilon_+}\left(-w'^{+}_-,-w'^{+}_-\right),\ w'_-=\begin{pmatrix} -\sin\frac{\theta_+}{2} \\ e^{i\frac{\varphi_+}{2}}\cos\frac{\theta_+}{2} \end{pmatrix}.$$

The spinors (A.11) and (A.13) satisfy the charge-conjugation relations, similar to (A.9)

$$V^T_\pm(q_+)C=-i\bar{V}_\mp(q_+),\quad U^T_\pm(q_-)C=i\bar{U}_\mp(q_-), \tag{A.14}$$

$$C\bar{V}^T_\pm(q_+)=-iV_\mp(q_+),\quad C\bar{U}^T_\pm(q_-)=iU_\mp(q_-).$$

Appendix B

We have expressed the chiral amplitudes of processes $2 \to 4$ in terms of bilinear combinations of initial spinors. We will refer to them here as the averaged of matrix products. These averaged products are divided into classes, counting the number of Dirac matrices in product. Note the useful relations between the averaged products of the same class, which follows from (A.3) and (A.9):

$$\bar{U}_{\lambda_1}a_1\cdots a_nV_{\lambda_2}=(-1)^{n+1}\bar{V}_{-\lambda_2}a_n\cdots a_1U_{-\lambda_1},\ \bar{U}_{\lambda_1}a_1\cdots a_nU_{\lambda_2}=(-1)^{n+1}\bar{U}_{-\lambda_2}a_n\cdots a_1U_{-\lambda_1}, \tag{B.1}$$

$$\left(\bar{U}_{\lambda_1}a_1\cdots a_nV_{\lambda_2}\right)^*=\bar{V}_{\lambda_2}a_n\cdots a_1U_{\lambda_1},\ \left(\bar{U}_{\lambda_1}a_1\cdots a_nU_{\lambda_2}\right)^*=\bar{U}_{\lambda_2}a_n\cdots a_1U_{\lambda_1}. \tag{B.2}$$

From (A.6) one obtains for the zero-class:

$$\bar{U}_+V_+=\bar{V}_+U_+=-\bar{U}_-V_-=-\bar{V}_-U_-=2\varepsilon. \tag{B.3}$$

Calculating the average of $\hat{a}=\gamma_\mu a^\mu=\begin{pmatrix} a_0, & -\vec{a}\vec{\sigma} \\ \vec{a}\vec{\sigma}, & -a_0 \end{pmatrix}$ with the explicit form of spinors (A.6) we obtain for the averaged of the 1-class:

$$\bar{V}_+aU_-=\bar{U}_+aV_-=(\bar{U}_-aV_+)^*=(\bar{V}_-aU_+)^*=-2\varepsilon a_\perp, \tag{B.4}$$

$$\bar{U}_+aU_+=\bar{U}_-aU_-=2\varepsilon a_-,\ \bar{V}_+aV_+=\bar{V}_-aV_-=2\varepsilon a_+. \tag{B.5}$$

Average of product of two Dirac matrices $\hat{a}\hat{b}$ multipliers are:

$$\bar{U}_+abV_+=(\bar{V}_+baU_+)^*=-(\bar{U}_-abV_-)^*=-\bar{V}_-baU_-=2\varepsilon z^*(a,b), \tag{B.6}$$

$$\bar{U}_-abU_+=-(\bar{U}_+abU_-)^*=2\varepsilon Y^*(a,b);\ \bar{V}_+abV_-=-(\bar{V}_-abV_+)^*=-2\varepsilon X(a,b), \tag{B.7}$$

where we use the notations

$$z(a,b)=a_-b_+-a^*_\perp b_\perp,\ Y(a,b)=a_-b_\perp-b_-a_\perp,\ X(a,b)=a_+b_\perp-b_+a_\perp. \tag{B.8}$$

Averaged $\hat{a}\hat{b}\hat{c}$ of third class are

$$\bar{U}_+abcV_-=(\bar{U}_-abcV_+)^*=\bar{V}_+cbaU_-=(\bar{V}_-cbaU_+)^*=2\varepsilon z(a,b,c); \tag{B.9}$$

$$\bar{u}_+ abc\, u_+ = (\bar{u}_- abc\, u_-)^* = 2\varepsilon z_u(a,b,c);\ \bar{V}_+ abc V_+ = (\bar{V}_- abc V_-)^* = 2\varepsilon z_v(a,b,c), \tag{B.10}$$

where

$$z(a,b,c) = C_+ Y(a,b) - C_\perp z^*(a,b)\ ;\ z_u(a,b,c) = C_- z^*(a,b) - C_\perp^* Y(a,b);$$

$$z_v(a,b,c) = C_+ z(b,a) - C_\perp^* X(a,b). \tag{B.11}$$

The average of product of four Dirac matrices $\hat{a}\hat{b}\hat{c}\hat{d}$ are:

$$\bar{u}_+ abcd V_+ = -\bar{V}_- dcba\, u_- = -(\bar{u}_- abcd V_-)^* = (\bar{V}_+ dcba\, u_+)^* = 2\varepsilon z(a,b,c,d); \tag{B.12}$$

$$\bar{u}_+ abcd\, u_- = (\bar{u}_- dcba\, u_+)^* = 2\varepsilon z_u(a,b,c,d);$$

$$\bar{V}_+ (a,b,c,d) V_- = (\bar{V}_- dcba V_+)^* = 2\varepsilon z_v(a,b,c,d), \tag{B.13}$$

where

$$z(a,b,c,d) = Y(a,b) X^*(c,d) + z^*(a,b) z^*(c,d);$$

$$z_u(a,b,c,d) = Y(b,a) z^*(d,c) + Y(d,c) z^*(a,b); \tag{B.14}$$

$$z_v(a,b,c,d) = X(b,a) z(c,d) + X(d,c) z(b,a).$$

Finally, the five matrix $\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}$ average are :

$$\bar{V}_+ abcde V_+ = (\bar{V}_- abcde V_-)^* = 2\varepsilon z_v(a,b,c,d,e); \tag{B.15}$$

$$\bar{u}_+ abcde\, u_+ = (\bar{u}_- abcde\, u_-)^* = 2\varepsilon z_u(a,b,c,d,e), \tag{B.16}$$

where

$$z_v(a,b,c,d,e) = C_+ z(b,a) z^*(d,e) + C_- X(b,a) X^*(d,e) + C_\perp^* X(b,a) z^*(d,e) + C_\perp z(b,a) X^*(d,e); \tag{B.17}$$

$$z_u(a,b,c,d,e) = C_- z^*(a,b) z(e,d) + C_+ Y(b,a) Y^*(d,e) + C_\perp^* Y(b,a) z(e,d) + C_\perp z^*(a,b) Y^*(d,e).$$

Note useful relations, following from (B.1), (B.2) and the definitions (B.10), (B.13), (B.14), (B.17):

$$z_{v,u}(a,b,c) = z^*_{v,u}(c,b,a)\ ;\ z_{u,v}(a,b,c,d) = -z_{u,v}(d,c,b,a). \tag{B.18}$$

$$z_{v,u}(a,b,c,d,e) = z^*_{v,u}(e,d,c,b,a);$$

we give the particular case also

$$z(P_-,a) = z(b,P_+) = Y(P_-,b) = X(a,P_+) = 0\ ;\ z(a,b) + z(b,a) = 2ab;$$

$$X(a,b) + X(b,a) = Y(a,b) + Y(b,a) = 0;\ |z(a,b)|^2 = \frac{1}{S} Sp\, P_- a b P_+ b a \omega_+ . \tag{B.19}$$

In the case when one of matrices $a, \ldots e$ coincides with P_+ or P_- the corresponding bilinear combination can be presented as a product of two bilinear combinations. For example,

$$z_u(a,b,P_+,c,d) = \frac{1}{2\varepsilon} \bar{u}_+ a b P_+ c d \omega_+ u_+ = \frac{1}{2\varepsilon} \bar{u}_+ a b \omega_- P_+ c d\, u_+ =$$

$$= \frac{1}{2\varepsilon} \cdot \bar{u}_+ a b V_+ \cdot \bar{V}_+ c d\, u_+ = 2\varepsilon z^*(a,b) \cdot z(d,c).$$

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