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A b s t r a c t

The method is suggested to classify quark operators in QCD sum rules. The coefficients of expansion of all the $d \leq 8$ bilinear quark condensates in gluon condensates are calculated in the background field formalism. The coefficient functions of the gluon operators with $d \leq 8$ in the polarization operator $\Pi(q^2)$ of the light-quark vector current are obtained. A comparison is performed with the calculations in the instanton and covariantly constant field. Rigorous relations between condensates in an arbitrary self-dual fields are proved. The vanishing of the $1/q^8$ contribution to $\Pi(q^2)$ in these field follows from these relations in the one-loop approximation. This is consistent with the Dubovikov-Smilga theorem. The results obtained can be used in the sum rules for the ρ, ω and φ families.

1. In ref. /1/ we have started with the calculation of a contribution from dimension-8 vacuum condensates to the sum rules for light quarks using the method suggested in ref. /2/.

To find the coefficient functions in the operator product expansion (OPE) polarization operator for an auxiliary quark of mass $m^2 \gg \nu^2$ (ν^2 is the characteristic virtuality of quark fields in the vacuum) in the region $Q^2 = -q^2 \gg m^2$ is first calculated. In this case the polarization operator can be on the one hand parametrized (in the lowest order in α_s), just as for light quarks, by means of the vacuum expectation values (VEV's) G_k of gluon operators and $Q_k = \langle \Psi O_k[\mathcal{P}_\mu]\Psi \rangle$ of bilinear quark operators, where $\mathcal{P}_\mu = i\partial_\mu + B_\mu$:

$$\Pi(q^2) = \Pi_G(q^2) + \Pi_Q(q^2),$$
$$\Pi_G(q^2) = \sum_k a_k(q^2) G_k \ , \quad \Pi_Q(q^2) = \sum_k b_k(q^2, m) Q_k \ . \tag{1}$$

Here $\Pi_G(q^2)$ corresponds to the contribution from the region where the virtualities of a quark and an antiquark $p^2 \sim Q^2$ (fig.1), and $\Pi_Q(q^2)$ corresponds to the contribution of the region where either the quark or the antiquark has the virtuality $p^2 \sim m^2$. This contribution is usually depicted using the diagrams of fig. 2.

On the other hand, quark condensates may be expessed via gluon ones (fig. 3) by virtue of the fact that the quark and antiquark are heavy:

$$Q_k = \sum_n C_{kn}(m) G_n \ . \tag{2}$$

With the expansion (2) substituted into (1) we obtain a correlator of heavy-quark currents expressed only via gluon condensates:

$$\Pi(q^2)\big|_G = \sum_k \tilde{a}_k(q^2, m)\, G_k \,. \tag{3}$$

This have to coincide with the limit $Q^2 \gg m^2$ of the polarization operator of heavy quarks. The coefficient functions at the dimension-4 gluon condensate for the correlators of heavy-quark currents have been derived in ref. /2/, and at the condensates of dimension d = 6,8 in ref. /3/.

If one finds $\Pi_Q(q^2)$ and obtains the expansions (2), then one can find $\Pi_G(q^2)$ as

$$\Pi_G(q^2) = \Pi(q^2)\big|_G - \Pi_Q(q^2)\big|_G = \sum_k \tilde{a}_k(q^2, m)\, G_k - \sum_{k,n} b_k(q^2, m)\, C_{kn}(m)\, G_n \,. \tag{4}$$

The coefficients $\tilde{a}_k(q^2, m)$ have singularities at $m \to 0$ and these singularities arise from the contribution from the region $p^2 \sim m^2$. However, these singularities have to cancel in (4) since the quark mass m can result in merely m^2/Q^2 corrections to (4) as it is clear from the definition of $\Pi_G(q^2)$. The possibility of checking of some results of ref. /3/ follows from such a cancellation. Unlike (3), the $\Pi_G(q^2)$ contribution (4) admits taking the limit $m \to 0$, and this limit gives the correct coefficient functions $a_k(q^2)$ at gluon operators in OPE of light-quark currents.

In ref. /2/ the method described above was used for operators of dimension 4 and in ref. /4/ it was applied to dimension-6 operators.

According to the sense of the operator product expansion, VEV's include only soft fields with momenta $p^2 < \mu^2$, where μ^2 is the normalization point. To the lowest order in α_s, the normalization point of α_s and of gluon condensates cannot be fixed since taking into account the dependence on μ^2 would

result in terms of the next order in α_s. Quark condensates are calculated in the one-loop approximation in eq. (2) and hence we need their careful definition and an exact specification of the normalization point. When calculating $Q_k(\mu^2)$, the loop integral should be cut at $p^2 < \mu^2$. This cutting eliminates ultraviolet divergences contained in those coefficients C_{kn}, for which the dimension of the gluon condensate G_n is less than or equal to that of the quark condensate Q_k. In other coefficients the integrals converge at $p^2 \sim m^2$ and the cutting has no influence on them in the case when $\mu^2 > m^2$.

A convenient way of imposing the cutting is dimensional regularization and minimal subtraction ($\overline{MS}$): in the loop integral in fig.3 we make a substitution $\frac{d^4p}{(2\pi)^4} \to d_n p \equiv \left(\frac{\mu^2 e^{\gamma}}{4\pi}\right)^{\varepsilon} \frac{d^n p}{(2\pi)^n}$ where $n = 4 - 2\varepsilon$ is the dimension of space, and define the quark condensate $Q_k(\mu^2)$ as a sum of series (2) from which all the pole terms of the form $1/\varepsilon$ are discarded and the limit $\varepsilon \to 0$ is then taken.

With such a way of definition we are free to use the operator equations of motion (which are valid in a space any dimension) to reduce the arising operators to the simplest form. Then, since Q_k are finite at $\varepsilon \to 0$, the coefficient functions are needed only with an accuracy $O(\varepsilon^0)$.

For quark operators of dimension 8 and higher there arises an interesting phenomenon similar to the axial anomaly, namely, at n $\neq$ 4 we can construct a condensate

$$A = \langle \bar{\psi}\, \mathcal{P}_\mu \mathcal{P}_\nu \mathcal{P}_\rho \mathcal{P}_\sigma \mathcal{P}_\lambda \Gamma^{(5)}_{\mu\nu\rho\sigma\lambda} \psi \rangle = -\frac{1}{4} \langle \bar{\psi}\, G_{\mu\nu} G_{\rho\sigma} \mathcal{P}_\lambda \Gamma^{(5)}_{\mu\nu\rho\sigma\lambda} \psi \rangle \,. \tag{5}$$

Here $\Gamma^{(m)}_{\mu_1 \dots \mu_m} = \frac{1}{m!} \sum_\delta P(\delta)\, \gamma_{\mu_{\delta_1}} \dots \gamma_{\mu_{\delta_m}}$; the summation is carried out over different permutations of the numbers 1,2,...,m and

$P(\sigma)$ is the parity of a permutation. This condensate is not equal to zero with the above definition. When deriving the expansion (2) for this condensate, in all the coefficients there arise the traces vanishing in four-dimensional space and hence proportional to ε. All the terms of this expansion with the $d > 8$ gluon condensates G_k contain convergent integrals and therefore disappear in the limit $\varepsilon \to 0$, while the coefficients at G_k with $d = 8$ contain divergences $\sim 1/\varepsilon$. As a result, the quark condensate A is expressed via a combination of the dimension-8 gluon condensates, and what's more this equality is exact. In ref. /1/ we did not take such a contributions into account because they reduce to gluon condensates. Denoting the contribution from condensate A to Π_Q (fig. 2) as $\Pi_A = -2A/q^8$ we may join it with the contribution Π_G from the gluon condensates:

$$\Pi = \Pi'_Q + \Pi'_G , \qquad (6)$$

$$\Pi'_Q = \Pi_Q - \Pi_A , \quad \Pi'_G = \Pi_G + \Pi_A = \sum_k a'_k(q^2) G_k ,$$

where Π'_Q is the contribution of the diagrams in fig. 2, disregarding condensate A.

In ref. /1/ we have calculated the contribution Π'_Q of bilinear quark condensates with $d \leq 8$ to the two-point correlator of the light-quark vector current $j_\mu = \bar{\psi}\gamma_\mu \tau \psi$ (τ is a flavour matrix). Just as in that paper, here we restrict ourselves to the case when τ commutes with the mass matrix m, then the current is conserved and the correlator contains only the transverse tensor structure. The results for the case when $[m,\tau]_- \neq 0$ will be presented elsewhere. Each contribution in fig. 2a corresponds to that in fig. 2b, which are different

from each other by a permutation of τ and τ^+ and by the C-conjugation. Then, the answer can be written just as in the case of one flavour ($N_f = 1$) if one assumes $\frac{1}{2}[\tau,\tau^+]_+$ between $\bar{\psi}$ and ψ and m be a matrix inserted at the same place. In ref. /1/ we have also derived the expansion (2) for the quark condensate $\langle\bar{\psi}\psi\rangle$, using the results of ref. /3/. In addition to bilinear quark operators we have taken into account four-quark operators with $d \leq 8$, which according to ref. /2/ can play an important role in the sum rules due to the absence of loop in the corresponding diagrams.

In the present paper we suggest a regular method of the QCD sum rules quark operators classifying and their reducing to basis ones. We obtain the expansions (2) for all the $d \leq 8$ bilinear quark condensates and, using them, calculate the coefficient functions $a'_k(q^2)$ at the $d \leq 8$ gluon condensates in the OPE of the correlator of the light-quark vector currents. The results obtained are applicable to the quarks both with zero and non-zero mass if $m^2 \ll Q^2$, i.e. to the currents involving u, d and s quarks provided that $[m,\tau]_- = 0$.

We make a comparison with the known analytical expressions for polarization operators in an instanton /5,6/ and in a covariantly constant field /7/. Our results are consistent with the known theorem /6/ about the vanishing of all $1/q^{2k}$ terms ($k \neq 2$) in an arbitrary self-dual fields.

2. When calculating the coefficient functions in the OPE it is necessary to choose the basis of operators. It is desirable that it would include a minimum number of operators and be convenient in calculations. For $d \leq 8$ gluon operators this problem has been discussed in refs. /2,3/. Here we choose

$$G^{(4)} = \langle Sp\, G_{\mu\nu} G_{\mu\nu} \rangle,\ G_1^{(6)} = i \langle Sp\, G_{\mu\nu} G_{\nu\lambda} G_{\lambda\mu} \rangle,\ G_2^{(6)} = \langle Sp\, J_\mu J_\mu \rangle,$$

$$G_1^{(8)} = \langle Sp\, G_{\mu\nu} G_{\mu\nu} G_{\alpha\beta} G_{\alpha\beta} \rangle,\ G_2^{(8)} = \langle Sp\, G_{\mu\nu} G_{\alpha\beta} G_{\mu\nu} G_{\alpha\beta} \rangle,$$

$$G_3^{(8)} = \langle Sp\, G_{\mu\alpha} G_{\alpha\nu} G_{\nu\beta} G_{\beta\mu} \rangle,\ G_4^{(8)} = \langle Sp\, G_{\mu\alpha} G_{\alpha\nu} G_{\mu\beta} G_{\beta\nu} \rangle, \tag{7}$$

$$G_5^{(8)} = i \langle Sp\, J_\mu G_{\mu\nu} J_\nu \rangle,\ G_6^{(8)} = i \langle Sp\, J_\lambda [\mathcal{D}_\lambda G_{\mu\nu}, G_{\mu\nu}]_- \rangle,\ G_7^{(8)} = \langle Sp\, J_\mu \mathcal{D}^2 J_\mu \rangle$$

as basis gluon condensates, where $G_{\mu\nu} = G^a_{\mu\nu} t^a$, $J_\mu = J^a_\mu t^a$, $J^a_\mu = \mathcal{D}^{ab}_\nu G^b_{\mu\nu}$. The set (7) essentially coincides with that accepted in ref. /3/, except for the condensate $G_6^{(8)}$.

With the purpose of classifying and reducing bilinear quark condensates to a given set of basis condensates, we apply the following regular procedure. It is easy to write down any VEV of dimension $d = m + 3$ as $\langle \bar{\psi} \mathcal{P}_{\mu_1} \dots \mathcal{P}_{\mu_m} \Gamma_{\mu_1 \dots \mu_m} \psi \rangle$, where $\Gamma_{\mu_1 \dots \mu_m}$ is constructed using γ_μ and $g_{\mu\nu}$. Choosing the terms with the largest number of γ-matrices in $\Gamma_{\mu_1 \dots \mu_m}$, we permute them in such a way that their induces are in the same order as in $\mathcal{P}_{\mu_1} \dots \mathcal{P}_{\mu_m}$. The arising additional terms have by 2 γ-matrices less. Then, in going to terms with a smaller number of the γ-matrices, we repeat this procedure until we reach terms with one γ-matrix or without them at all. As a result, the VEV considered reduces to a linear combination of terms of the form $\langle \bar{\psi} O_i \psi \rangle$, where O_i are constructed from $\hat{\mathcal{P}}$ and $\mathcal{P}_\mu$. By virtue of the equations of motion those terms in which $\hat{\mathcal{P}}$ is adjacent to ψ or $\bar{\psi}$ reduce to lower-dimensional VEV's multiplied by m.

Having written all the condensates $B_i^{(d)} = \langle \bar{\psi} O_i \psi \rangle$ in which O_i are constructed from $\hat{\mathcal{P}}$ and $\mathcal{P}_\mu$ ($\hat{\mathcal{P}}$ is not adjacent to ψ and $\bar{\psi}$), we obtain a certain set of condensates of dimension d. As it is clear from the said above, any

condensate of dimension d can be systematically expressed via $B_i^{(d)}, m B_i^{(d-1)}, \dots$. With the basis chosen from the most convenient condensates $Q_j^{(d)}$, we express them via $B_i^{(d)}, m B_i^{(d-1)}, \dots$ and solve the set of equations obtained. As a result, we obtain the expressions for $B_i^{(d)}$ via $Q_j^{(d)}, m Q_j^{(d-1)}, \dots$. After that, it is easy to express any given condensate via the basis ones $Q_j^{(d)}$, $m Q_j^{(d-1)}, \dots$.

For dimensions $d \leq 6$ we have

$$B^{(3)} = \langle \bar{\psi} \psi \rangle,\ B^{(5)} = \langle \bar{\psi} \mathcal{P}^2 \psi \rangle,\ B^{(6)} = \langle \bar{\psi} \mathcal{P}_\mu \hat{\mathcal{P}} \mathcal{P}_\mu \psi \rangle \tag{8}$$

and the known assertion that all condensates with $d \leq 6$ are expressed via only three condensates $\left(\sigma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu]_- \right)$

$$Q^{(3)} = \langle \bar{\psi} \psi \rangle,\ Q^{(5)} = i \langle \bar{\psi} \sigma_{\mu\nu} G_{\mu\nu} \psi \rangle,\ Q^{(6)} = \langle \bar{\psi} \hat{J} \psi \rangle \tag{9}$$

immediately follows. For $d = 7$

$$B_1^{(7)} = \langle \bar{\psi} \mathcal{P}^4 \psi \rangle,\ B_2^{(7)} = \langle \bar{\psi} \mathcal{P}_\mu \mathcal{P}^2 \mathcal{P}_\mu \psi \rangle, \tag{10}$$

$$B_3^{(7)} = \langle \bar{\psi} \mathcal{P}_\mu \mathcal{P}_\nu \mathcal{P}_\mu \mathcal{P}_\nu \psi \rangle,\ B_4^{(7)} = \langle \bar{\psi} \mathcal{P}_\mu \hat{\mathcal{P}}^2 \mathcal{P}_\mu \psi \rangle$$

and 4 basis condensates are therefore added:

$$Q_1^{(7)} = \langle \bar{\psi} G_{\mu\nu} G_{\mu\nu} \psi \rangle,\ Q_2^{(7)} = i \langle \bar{\psi} G_{\mu\nu} G^*_{\mu\nu} \gamma_5 \psi \rangle, \tag{11}$$

$$Q_3^{(7)} = \langle \bar{\psi} G_{\mu\lambda} G_{\lambda\nu} \sigma_{\mu\nu} \psi \rangle,\ Q_4^{(7)} = i \langle \bar{\psi} \mathcal{D}_\mu J_\nu \sigma_{\mu\nu} \psi \rangle.$$

Similarly, for $d = 8$

$$B_1^{(8)} = \langle \bar{\psi} (\mathcal{P}_\mu \hat{\mathcal{P}} \mathcal{P}_\mu \mathcal{P}^2 + \mathcal{P}^2 \mathcal{P}_\mu \hat{\mathcal{P}} \mathcal{P}_\mu) \psi \rangle,\ B_2^{(8)} = \langle \bar{\psi} (\mathcal{P}_\mu \hat{\mathcal{P}} \mathcal{P}^2 \mathcal{P}_\mu + \mathcal{P}_\mu \mathcal{P}^2 \hat{\mathcal{P}} \mathcal{P}_\mu) \psi \rangle,$$

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$$B_4^{(8)} = \langle \bar{\psi} \mathcal{P}^2 \hat{\mathcal{P}} \mathcal{P}^2 \psi \rangle,\ B_5^{(8)} = \langle \bar{\psi} \mathcal{P}_\mu \mathcal{P}_\nu \hat{\mathcal{P}} \mathcal{P}_\nu \mathcal{P}_\mu \psi \rangle,$$

$$B_6^{(8)} = \langle \bar{\psi} \mathcal{P}_\mu \mathcal{P}_\nu \hat{\mathcal{P}} \mathcal{P}_\mu \mathcal{P}_\nu \psi \rangle,\ B_7^{(8)} = \langle \bar{\psi} \mathcal{P}_\mu \hat{\mathcal{P}}^3 \mathcal{P}_\mu \psi \rangle.$$

Operators similar to those in $B_{1+3}^{(8)}$ but with a minus sign bet-

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as basis gluon condensates, where $G_{\mu\nu} = G^a_{\mu\nu} t^a$, $J_\mu = J^a_\mu t^a$, $J^a_\mu = \mathcal{D}^{ab}_\nu G^b_{\mu\nu}$. The set (7) essentially coincides with that accepted in ref. /3/, except for the condensate $G_6^{(8)}$.

With the purpose of classifying and reducing bilinear quark condensates to a given set of basis condensates, we apply the following regular procedure. It is easy to write down any VEV of dimension $d = m + 3$ as $\langle \bar{\psi} \mathcal{P}_{\mu_1} \dots \mathcal{P}_{\mu_m} \Gamma_{\mu_1 \dots \mu_m} \psi \rangle$, where $\Gamma_{\mu_1 \dots \mu_m}$ is constructed using γ_μ and $g_{\mu\nu}$. Choosing the terms with the largest number of γ-matrices in $\Gamma_{\mu_1 \dots \mu_m}$, we permute them in such a way that their induces are in the same order as in $\mathcal{P}_{\mu_1} \dots \mathcal{P}_{\mu_m}$. The arising additional terms have by 2 γ-matrices less. Then, in going to terms with a smaller number of the γ-matrices, we repeat this procedure until we reach terms with one γ-matrix or without them at all. As a result, the VEV considered reduces to a linear combination of terms of the form $\langle \bar{\psi} O_i \psi \rangle$, where O_i are constructed from $\hat{\mathcal{P}}$ and $\mathcal{P}_\mu$. By virtue of the equations of motion those terms in which $\hat{\mathcal{P}}$ is adjacent to ψ or $\bar{\psi}$ reduce to lower-dimensional VEV's multiplied by m.

Having written all the condensates $B_i^{(d)} = \langle \bar{\psi} O_i \psi \rangle$ in which O_i are constructed from $\hat{\mathcal{P}}$ and $\mathcal{P}_\mu$ ($\hat{\mathcal{P}}$ is not adjacent to ψ and $\bar{\psi}$), we obtain a certain set of condensates of dimension d. As it is clear from the said above, any

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For dimensions $d \leq 6$ we have

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$$Q_3^{(7)} = \langle \bar{\psi} G_{\mu\lambda} G_{\lambda\nu} \sigma_{\mu\nu} \psi \rangle,\ Q_4^{(7)} = i \langle \bar{\psi} \mathcal{D}_\mu J_\nu \sigma_{\mu\nu} \psi \rangle.$$

Similarly, for $d = 8$

$$B_1^{(8)} = \langle \bar{\psi} (\mathcal{P}_\mu \hat{\mathcal{P}} \mathcal{P}_\mu \mathcal{P}^2 + \mathcal{P}^2 \mathcal{P}_\mu \hat{\mathcal{P}} \mathcal{P}_\mu) \psi \rangle,\ B_2^{(8)} = \langle \bar{\psi} (\mathcal{P}_\mu \hat{\mathcal{P}} \mathcal{P}^2 \mathcal{P}_\mu + \mathcal{P}_\mu \mathcal{P}^2 \hat{\mathcal{P}} \mathcal{P}_\mu) \psi \rangle,$$

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$$B_4^{(8)} = \langle \bar{\psi} \mathcal{P}^2 \hat{\mathcal{P}} \mathcal{P}^2 \psi \rangle,\ B_5^{(8)} = \langle \bar{\psi} \mathcal{P}_\mu \mathcal{P}_\nu \hat{\mathcal{P}} \mathcal{P}_\nu \mathcal{P}_\mu \psi \rangle,$$

$$B_6^{(8)} = \langle \bar{\psi} \mathcal{P}_\mu \mathcal{P}_\nu \hat{\mathcal{P}} \mathcal{P}_\mu \mathcal{P}_\nu \psi \rangle,\ B_7^{(8)} = \langle \bar{\psi} \mathcal{P}_\mu \hat{\mathcal{P}}^3 \mathcal{P}_\mu \psi \rangle.$$

Operators similar to those in $B_{1+3}^{(8)}$ but with a minus sign bet-

ween two terms are C-odd, and their VEV's are zero. The condensate $A = \langle \bar\psi \mathcal{P}_\mu \mathcal{P}_\nu \mathcal{P}_\rho \mathcal{P}_\delta \mathcal{P}_\lambda \Gamma^{(5)}_{\mu\nu\rho\delta\lambda}\psi\rangle$ can be expressed via $m^5 B^{(3)}$, $m^3 B^{(5)}$, $m^2 B^{(6)}$, $m B^{(7)}_{1\div 4}$, $B^{(8)}_{1\div 7}$. According to our definition, A is non-zero dimension-8 gluon condensate. Taking it as one of 7 elements of the basis, we can choose 6 proper quark condensates with $d = 8$ in the form:

$$Q_1^{(8)} = \langle \bar\psi [[G_{\mu\lambda}, G_{\lambda\nu}]_+, \mathcal{P}_\mu]_+ \gamma_\nu \psi\rangle, \quad Q_2^{(8)} = i\langle \bar\psi [[G_{\mu\lambda}, G^*_{\lambda\nu}]_-, \mathcal{P}_\mu]_+ \gamma_\nu \gamma_5 \psi\rangle,$$
$$Q_3^{(8)} = i\langle \bar\psi [\hat{\mathcal{D}} G_{\mu\nu}, G_{\mu\nu}]_- \psi\rangle, \quad Q_4^{(8)} = \langle \bar\psi \mathcal{D}^2 \hat{J} \psi\rangle, \tag{13}$$
$$Q_5^{(8)} = i\langle \bar\psi [G_{\mu\nu}, J_\mu]_- \gamma_\nu \psi\rangle, \quad Q_6^{(8)} = i\langle \bar\psi [G^*_{\mu\nu}, J_\mu]_+ \gamma_\nu \gamma_5 \psi\rangle.$$

The presented classification of bilinear quark condensates is valid in a space of any dimension. The expressions with $G^*_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} G_{\alpha\beta}$ and γ_5 or $\gamma_\nu\gamma_5$ should be understood only as a convenient form of writing the appropriate expressions involving $\Gamma^{(4)}$ or $\Gamma^{(3)}$ respectively.

The quark-condensate basis $Q_k^{(d)}$ chosen here proves to be convenient when calculating the coefficients $C_{kn}(m)$ in eq.(2), in particular, owing to the minimum amount of $\mathcal{P}_\mu$ acting on ψ. That offers the possibility of expanding $S(p)$ to the lower-dimension operators. Most of $Q_k^{(d)}$ coincide, while the remaining are related to the condensates in ref./1/ by linear relations:

$$i\langle \bar\psi [G_{\nu\lambda} G^*_{\lambda\mu}, \mathcal{P}_\mu]_+ \gamma_\nu \gamma_5 \psi\rangle = \tfrac{1}{2} Q_2^{(8)} - \tfrac{1}{2} Q_3^{(8)} - Q_5^{(8)} + 2m Q_3^{(7)},$$
$$i\langle \bar\psi [G_{\mu\lambda} G^*_{\lambda\nu}, \mathcal{P}_\mu]_+ \gamma_\nu \gamma_5 \psi\rangle = \tfrac{1}{2} Q_2^{(8)}, \tag{14}$$
$$\langle \bar\psi [\mathcal{D}^2 G^*_{\mu\nu}, \mathcal{P}_\mu]_+ \gamma_\nu \gamma_5 \psi\rangle = Q_3^{(8)} - Q_4^{(8)} + Q_5^{(8)} - 4m Q_3^{(7)} - 2m Q_4^{(7)},$$
$$\langle \bar\psi [J_\mu, \mathcal{P}_\mu]_+ \psi\rangle = -Q_4^{(7)} + 2m Q^{(6)}.$$

3. If the mass m of an auxiliary quark is large enough, $m^2 \gg \nu^2$, for VEV's involving quark fields it is possible to use OPE in terms of gluon operators only. In particular, for local quark VEV's the expansion proceeds in inverse powers of m, as in eq.(2), since the heavy quark has a time to propagate only at short distances of the order of Compton wavelength $1/m$ (fig.3).

To derive the expansions (2), it is convenient to work in the momentum representation in which (fig.3)

$$Q_k = \langle \bar\psi O_k [\mathcal{P}_\mu] \psi\rangle = -i \int d_n p \langle Sp\, O_k [p_\mu + \mathcal{B}_\mu]\, S(p)\rangle, \tag{15}$$

where in the Fock-Schwinger gauge /8,9/ (fig.4)

$$\mathcal{B}_\mu = -i \sum_{\ell=0}^{\infty} \frac{(-i)^\ell}{\ell!(\ell+2)} (\mathcal{D}_{\alpha_1} \dots \mathcal{D}_{\alpha_\ell} G_{\nu\mu})^a t^a \frac{\partial}{\partial p_{\alpha_1}} \dots \frac{\partial}{\partial p_{\alpha_\ell}} \frac{\partial}{\partial p_\nu},$$
$$S(p) = \sum_{k=0}^{\infty} S_k(p), \tag{16}$$
$$S_k(p) = [-S_0(p)\hat{\mathcal{B}}]^k S_0(p), \quad S_0(p) = \frac{1}{\hat p - m}.$$

We have calculated the VEV's (15) by means of two methods: 1) using the computer algebraic program DIRAC /10/ and 2) by hand, using some of the properties of this gauge.

In the first method S(p) is calculated directly according to formula (16) (fig.4). For $S_k(p)$ we use the recurrent relations:

$$S_k(p) = \sum_{\ell=0}^{\infty} \frac{S_{k,\ell}(p)}{\ell+2}, \quad S_{k,0}(p) = i S_0(p) G_{\mu\nu} \frac{\partial}{\partial p_\mu} \gamma_\nu S_{k-1}(p),$$
$$S_{k,\ell}(p) = -\frac{i}{\ell} \mathcal{D}_\mu \frac{\partial}{\partial p_\mu} S_{k,\ell-1}(p). \tag{17}$$

Note that in the last of them $\mathcal{D}_\mu$ acts on the nearest $G_{\mu\nu}$ only. Interrupting the summation on the terms of the required dimension, acting on the ontained $S(p)$ by O_k, then calculating the trace and averaging over p directions, we obtain the

expansion (2) for Q_k whose coefficients are expressed via the integrals $\int d_n p\,(p^2)^a/(p^2-m^2)^b$.

Another method of calculations is based on the representation of the propagator $S(p)$ from eq.(15) in the form:

$$S(p)=\frac{1}{\hat{\mathcal{P}}-m}=\frac{1}{H}\left(\mathcal{P}^2-m^2-\frac{i}{2}\sigma\cdot\mathcal{G}\right)\left(\hat{\mathcal{P}}+m\right),$$
$$H=\left(\mathcal{P}^2-m^2\right)^2+\frac{i}{2}\left[\mathcal{P}^2,\sigma\cdot\mathcal{G}\right]_-+\frac{1}{4}\left(\sigma\cdot\mathcal{G}\right)^2=H_0+\Delta H, \tag{18}$$
$$H_0=\left(p^2-m^2\right)^2,$$

where $\mathcal{P}_\mu=p_\mu+\mathcal{B}_\mu$ (see eq.(16)) and $\sigma\cdot\mathcal{G}=\sigma_{\mu\nu}\mathcal{G}_{\mu\nu}$, $\mathcal{G}_{\mu\nu}=-i\left[\mathcal{P}_\mu,\mathcal{P}_\nu\right]_-$. The OPE for $S(p)$ is resulted from the expansion in ΔH. For the simplification of the calculations it turns out to be important that the only term having the dimension-2 contribution in ΔH in eq.(18) is the operator $\left[p^2-m^2,\mathcal{B}_\mu p_\mu\right]_+$; the remaining terms have dimension 3 and higher. This operator acting on $1/H_0$ yields zero. This property of the Fock-Schwinger gauge (see/9/) enables us to diminish the number of iterations over ΔH in $S(p)$ to obtain the operator contributions up to a given dimension.

Using both of the described methods we obtain the coinciding results for the expansions (2) of the quark condensates:

$$Q^{(3)}=-\frac{1}{24\pi^2}\Big[-6m^3(\ell+1)N_c+\frac{1}{m}G^{(4)}-\frac{1}{15m^3}\left(G_1^{(6)}-6G_2^{(6)}\right) \tag{19.1}$$
$$+\frac{1}{210m^5}\left(19G_1^{(8)}+16G_2^{(8)}-20G_3^{(8)}-78G_4^{(8)}+84G_5^{(8)}-4G_6^{(8)}-18G_7^{(8)}\right)\Big],$$

$$Q^{(5)}=-\frac{1}{12\pi^2}\Big[3m\ell\,G^{(4)}-\frac{1}{m}\left(G_1^{(6)}-G_2^{(6)}\right) \tag{19.2}$$
$$+\frac{1}{10m^3}\left(3G_1^{(8)}+2G_2^{(8)}-G_3^{(8)}-9G_4^{(8)}+9G_5^{(8)}-\frac{5}{4}G_6^{(8)}-G_7^{(8)}\right)\Big],$$

$$Q^{(6)}=-\frac{1}{12\pi^2}\Big[\ell\,G_2^{(6)}+\frac{1}{20m^3}\left(18G_5^{(8)}-\frac{1}{2}G_6^{(8)}-4G_7^{(8)}\right)\Big],$$

$$Q_1^{(7)}=-\frac{1}{12\pi^2}\Big[-3m^3(\ell+1)G^{(4)}+\frac{1}{2m}G_1^{(8)}\Big], \tag{19.4}$$

$$Q_2^{(7)}=\frac{1}{16\pi^2 m}\left(G_1^{(8)}+G_2^{(8)}-4G_4^{(8)}\right), \tag{19.5}$$

$$Q_3^{(7)}=\frac{1}{4\pi^2}\Big[-m\ell\,G_1^{(6)}+\frac{1}{6m}\left(G_3^{(8)}-G_4^{(8)}+2G_5^{(8)}-\frac{1}{2}G_6^{(8)}\right)\Big], \tag{19.6}$$

$$Q_4^{(7)}=-\frac{1}{4\pi^2}\Big[m\ell\,G_2^{(6)}+\frac{1}{6m}\left(4G_5^{(8)}-\frac{1}{2}G_6^{(8)}-G_7^{(8)}\right)\Big], \tag{19.7}$$

$$Q_1^{(8)}=-\frac{1}{12\pi^2}\Big[3m^4\left(\ell+\frac{3}{2}\right)G^{(4)}+\ell\left(G_1^{(8)}-2G_3^{(8)}-2G_4^{(8)}\right)\Big], \tag{19.8}$$

$$Q_2^{(8)}=\frac{1}{24\pi^2}\Big[12m^2(\ell+1)G_1^{(6)}-\ell\left(G_1^{(8)}-G_2^{(8)}-2G_3^{(8)}+2G_4^{(8)}-4G_5^{(8)}+G_6^{(8)}\right)\Big], \tag{19.9}$$

$$Q_3^{(8)}=-\frac{\ell}{12\pi^2}G_6^{(8)}, \tag{19.10}$$

$$Q_4^{(8)}=-\frac{\ell}{12\pi^2}G_7^{(8)}, \tag{19.11}$$

$$Q_5^{(8)}=\frac{\ell}{6\pi^2}G_5^{(8)}, \tag{19.12}$$

$$Q_6^{(8)}=0, \tag{19.13}$$

where $\ell=\ln(\mu^2/m^2)$ and $N_c=3$ is the number of colours. The terms not taken into account in expansions (19) are of the order of $O(1/m^{10-d})$, where d is the dimension of a condensate.

According to our definition of the condensates $Q_k^{(d)}$ (see section 1) the term of the order $m^3\ln\frac{\mu^2}{m^2}$ (it is sometimes refered to the coefficient function of a unit operator) is retained, e.g., in $Q^{(3)}$. This corresponds to the usual sence of the OPE when soft fields with momenta $p^2<\mu^2$ are included in condensates, while the coefficient functions absorb only hard

fields with $p^2 > \mu^2$. For a quark of non-zero mass m the normalization point should be chosen such that $\mu^2 > m^2$. Otherwise, at $\mu^2 < m^2$, condensates will be power-dependent on the normalization point μ^2, say, as $O(\mu^2/m^2)$ because in this case a contribution from the fluctuations typical for them is excluded (for heavy quarks with $m^2 > \nu^2$ the typical momenta are $p^2 \sim m^2 > \mu^2$, and for light quarks with $m^2 < \nu^2$ they are $p^2 \sim \nu^2 > m^2 > \mu^2$). Then it is not worth introducing any condensates at $\mu^2 < m^2$. Whereas at $\mu^2 > m^2$ the condensates are weakly dependent on μ^2, according to the renormalization group equations.

In expansions (19) the terms with positive powers of m lead to the contributions to the current correlators of the same order as m-corrections to the contributions of gluon condensates. However it is erroneous to refer them to the latters and to exclude them from the definition of quark condensates since they are connected with small virtualities $p^2 < \mu^2$. Consequently, there is no uncertainly allowing one to take or not to take into account such terms in the definition of a quark condensate; the only ambiguity is associated with a choice of the normalization point $\mu^2 > m^2$.

For heavy quarks, if we even fix $\mu^2 = m^2$, some quark condensates grows as the powers of m. This means that in this case it would be more adequate not to use the concept of a quark condensate at all, but to speak of gluon condensates only (heavy quarks will contribute to the coefficient functions at the gluon operators since they appear in the loops and are taken into account by the effective lagrangian).

Formulae (19) are not only used in the obtaining of $\Pi_G(q^2)$, but may be useful for estimations of the values of corresponding condensates $Q_k^{(d)}$ in the method of expansion in inverse

powers of the quark mass m /2/.

The expression for the condensate A via gluon condensates is obtained in a similar way as expansions (19). A series for A contains only the first term (see section 1):

$$A = -\frac{1}{32\pi^2}\left(G_{T1}^{(8)} + G_{T2}^{(8)} - 4G_{T4}^{(8)}\right). \tag{20}$$

Formula (19.1) has been derived in ref./1/; the coefficient at $G^{(4)}$ is well-known /2/, the coefficients at $G_{T1,2}^{(6)}$ have been obtained in refs./11,4,12/ and those at $G_{T1\div4}^{(8)}$ are obtained in ref./11/. The coefficients at the $d \leq 6$ operators in eqs. (19.2) and (19.3) have been considered in refs./13,4/. The terms not involving current J_μ in formula (19.6) (as well as the analogues (19.4-5) for operators having the different colour structure) are presented in ref./14/.

It is interesting that many of the results (19) and (20) can be obtained using simple independent arguments. In addition to the two derivations of eq.(19.1), presented in ref./1/, there are at least three more. We shall describe them and the ways how to check eqs.(19.2) and (19.3) in the next section.

When calculating the d = 8 contribution to $\langle\bar\psi G_{\mu\nu}G_{\mu\nu}\psi\rangle$ (19.4) the same terms from $S(p)$ in eq.(15) are singled out as those in the calculation of the d = 4 contribution to $\langle\bar\psi\psi\rangle$. Therefore it is possible to derive eq.(19.4) easily if one inserts $G_{\mu\nu}G_{\mu\nu}$ under the trace sign in eq.(19.1). Similarly, if one calculates the d = 8 contribution to $\langle\bar\psi G_{\mu\lambda}G_{\lambda\nu}\sigma_{\mu\nu}\psi\rangle$ in eq.(19.6) and $i\langle\bar\psi \mathcal{D}_\mu J_\nu \sigma_{\mu\nu}\psi\rangle$ in eq.(19.7), the same terms from $S(p)$ in eq.(15) are singled out as those in the calculation of the d = 6 contribution to $i\langle\bar\psi\sigma_{\mu\nu}G_{\mu\nu}\psi\rangle$ in eq.(19.2), but now this term is multiplied, correspondingly, by $\frac{1}{2}[G_{\mu\lambda}, G_{\lambda\nu}]$.

and $\frac{i}{2}(\mathcal{D}_\mu J_\nu - \mathcal{D}_\nu J_\mu)$ in place of $G_{\mu\nu}$. Consequently, eqs.(19.6) and (19.7) can readily be found from eq.(19.2) using the proper substitutions. This prescription is unambiguous because all $G_{\mu\nu}$ enter on equal rights in the d = 6 term in eq.(19.2). Lastly, when calculating the d = 8 contribution to $i\langle\bar\psi[\hat{\mathcal{D}}G_{\mu\nu}, G_{\mu\nu}]_-\psi\rangle$ in eq.(19.10), $\langle\bar\psi\mathcal{D}^2\hat{J}\psi\rangle$ in eq.(19.11) and $i\langle\bar\psi[G_{\mu\nu}, J_\mu]_-\gamma_\nu\psi\rangle$ in eq.(19.12), the same terms from $S(p)$ in eq.(15) are singled out as those in the calculation of the d = 6 contribution to $\langle\bar\psi\hat{J}\psi\rangle$ in eq.(19.3), but now it is multiplied by $i[\mathcal{D}_\lambda G_{\mu\nu}, G_{\mu\nu}]_-$, $\mathcal{D}^2 J_\lambda$ and $i[G_{\mu\lambda}, J_\mu]_-$, respectively, instead of J_λ. It enables one to readily derive (19.10)-(19.12) from (19.3). In the case of the d = 8 contribution to $i\langle\bar\psi[G^*_{\mu\nu}, J_\mu]_+\gamma_\nu\gamma_5\psi\rangle$ in eq.(19.13) there is no dimension-3 axial gluon operator by which the operator $i[G^*_{\mu\nu}, J_\mu]_+$ could be multiplied ($\mathcal{D}_\nu G^*_{\mu\nu} = 0$, unlike the vector operator $J_\mu = \mathcal{D}_\nu G_{\mu\nu}$).

Formula (19.5) for $i\langle\bar\psi G_{\mu\nu}G^*_{\mu\nu}\gamma_5\psi\rangle$ can be derived from the expression for the axial current anomaly:

$$\langle\partial_\lambda(\bar\psi G_{\mu\nu}G^*_{\mu\nu}\gamma_\lambda\gamma_5\psi)\rangle = \langle\bar\psi\,\partial_\lambda(G_{\mu\nu}G^*_{\mu\nu})\gamma_\lambda\gamma_5\psi\rangle + \langle(G_{\mu\nu}G^*_{\mu\nu})^{ij}\partial_\lambda(\bar\psi_i\gamma_\lambda\gamma_5\psi_j)\rangle = 2im\langle\bar\psi G_{\mu\nu}G^*_{\mu\nu}\gamma_5\psi\rangle + \frac{1}{8\pi^2}\langle Sp(G_{\mu\nu}G^*_{\mu\nu})^2\rangle = 0. \tag{21}$$

The second equality holds with an accuracy of $O(1/m^2)$ since there is no d = 3 axial gluon operator by which $\partial_\lambda(G_{\mu\nu}G_{\mu\nu})$ could be multiplied. With regard for $\langle Sp(G_{\mu\nu}G^*_{\mu\nu})^2\rangle = -G_1^{(8)} - G_2^{(8)} + 4G_4^{(8)}$, we obtain (19.5). Futher, it is easy to see that

$$mQ_2^{(7)} = -2A - i\langle\bar\psi[\mathcal{D}_\mu, [G_{\mu\lambda}, G^*_{\lambda\nu}]_+]_-\gamma_\nu\gamma_5\psi\rangle, \tag{22}$$

where the second term is equal to zero with the required accuracy for the same reason and then we easily obtain eq.(20).

4. Reducing, according to (14), the result of ref./1/ to the basis described above we have

$$\Pi'_Q(q^2) = \frac{1}{q^4}2mQ^{(3)} + \frac{4}{9q^6}\left[Q^{(6)} + 6m^3Q^{(3)}\right] + \frac{2}{9q^8}\left[Q_1^{(8)} + \frac{1}{2}Q_2^{(8)} - \frac{1}{4}Q_3^{(8)} + \frac{1}{2}Q_4^{(8)} + Q_5^{(8)} + m\left(7Q_1^{(7)} + 6Q_2^{(7)} + Q_3^{(7)} + Q_4^{(7)}\right) - 3m^2Q^{(6)} - 3m^3Q^{(5)} + 27m^5Q^{(3)}\right]. \tag{23}$$

With the expansions (19) of quark condensates in gluon ones substituted into eq.(17), we obtain the expression for $\Pi'_Q(q^2)|_G$ in the case of heavy quarks at $Q^2 \gg m^2$ via gluon condensates. It is evident that in this expression there are mass singularities, in accordance with the said in section 1. It is not difficult to verify that the corresponding formula from ref./3/ for the correlator $\Pi(q^2)|_G$ of heavy-quark vector currents has the same mass singularities (with equal coefficients).

The terms of dimensions 6 and 8 in eq.(19.1) are responsible, correspondingly, for the mass singularities $1/(q^4m^2)$ and $1/(q^4m^4)$ in $\Pi'_Q(q^2)|_G$, that makes it possible to check their correctness. Moreover, in the correlator of vector currents the coefficient at $Q^{(5)}$ vanishes when $m \to 0$; the coefficient at $1/(q^6m^2)$ in $\Pi'_Q(q^2)|_G$ is therefore determined by the already verified dimension-8 terms in (19.1) and the dimension-8 terms in (19.3) respectively. In view of that, the latter terms can be verified comparing the coefficients at $1/(q^6m^2)$ in $\Pi'_Q(q^2)|_G$ and $\Pi(q^2)|_G$. As it is obvious from the very beginning, the expansion (19.3) includes only the condensates $G^{(8)}_{5+7}$ with the current J_μ. It is not hard to obtain the analogues of the formulae for $\Pi_Q(q^2)|_G$ and $\Pi(q^2)|_G$ for the correlators of scalar and pseudoscalar currents with the accuracy $1/q^6$ (the ex-

pressions for $\Pi_Q(q^2)$ coincide with those derived in /4/). Having compared the coefficients at $1/(q^4 m^2)$ and $1/(q^4 m^4)$ in them, we obtain two more independent derivations of the coefficients at the dimension-8 operators in eq.(19.1); the comparison of the coefficients at $1/(q^6 m^2)$ enables one to obtain two independent derivations for the d = 8 terms in (19.2) taking into account the already verified expansions (19.1) and (19.3).

The difference $\Pi'_G(q^2) = \Pi(q^2)|_G - \Pi'_Q(q^2)|_G$ has no mass singularities and is associated with the region of large virtual momenta $p^2 \sim Q^2$:

$$\begin{aligned}\Pi'_G(q^2) = \frac{1}{18\pi^2}\Big\{ & -\frac{N_c}{2}(3L-5) + \frac{1}{q^2}9N_c m^2 + \frac{1}{q^4}\Big[\frac{3}{4}G^{(4)} + 9(L-\tfrac{1}{2})N_c m^4\Big] + \\ & + \frac{1}{q^6}\Big[-\Big(\frac{2}{3}L+\frac{1}{9}\Big)G_2^{(6)} - m^2 G^{(4)} + 4(3L-5)N_c m^6\Big] + \\ & + \frac{1}{q^8}\Big[-\Big(\frac{5}{12}L-\frac{143}{144}\Big)G_1^{(8)} + \Big(\frac{1}{12}L-\frac{31}{144}\Big)G_2^{(8)} + \Big(\frac{5}{6}L-\frac{131}{72}\Big)G_3^{(8)} + \\ & + \Big(\frac{1}{2}L-\frac{5}{8}\Big)G_4^{(8)} + \Big(L-\frac{5}{4}\Big)G_5^{(8)} + \frac{1}{12}G_6^{(8)} - \Big(\frac{1}{6}L-\frac{7}{72}\Big)G_7^{(8)} - \\ & - m^2\Big(\frac{5}{2}G_1^{(6)} + \frac{2}{3}G_2^{(6)}\Big) + (9L-22)m^4 G^{(4)} + 9\Big(3L-\frac{23}{4}\Big)N_c m^8\Big]\Big\},\end{aligned} \tag{24}$$

where $L = \ln Q^2/\mu^2$. For $N_f \neq 1$ eq.(24) should be multiplied by $\tau\tau^+$ and then the trace over flavour indices should be taken.

Formulae (12) (or (23) in the present paper), (17) in ref./1/ and (24) in the present paper solve the problem of calculating the coefficient functions at higher condensates with $d \leq 8$ in the correlators of vector currents involving u,d,s quarks (when $[\tau, m]_- = 0$). In particular, it is possible to study the role of higher vacuum condensates in sum rules for the ρ, ω and φ families, proceeding from the results obtained. The corresponding sum rules will be analysed by us in a separate paper. The calculations are readily generalized for

the other channels, using the expansions (19).

In deriving (23) and (24) we have neglected the radiative corrections to the coefficient functions. This is valid provided that $\frac{\alpha_s(\mu)}{\pi}\ln\frac{Q^2}{\mu^2} \ll 1$. Under the same condition we may not to take into account the anomalous dimensions of the operators. Note that due to the equations of motion the gluon condensates $G_5^{(8)}$, $G_6^{(8)}$ and $G_7^{(8)}$, which contain the current J_μ, are of the same structure as the bilinear quark condensates $Q_5^{(8)}$, $Q_3^{(8)}$ and $Q_4^{(8)}$ respectively, and their contributions to $\Pi(q^2)$ has an extra factor $\frac{\alpha_s(\mu)}{\pi}\ln\frac{Q^2}{\mu^2}$, in comparison with the quark ones. Consequently, the contribution of these gluon condensates is of the same order of magnitude as that given by radiative corrections to the coefficient functions of bilinear quark condensates.

The expression for $\Pi(q^2) = \Pi'_Q(q^2) + \Pi'_G(q^2)$ can be compared with the results of the analytical calculations (without the use of the OPE) of the correlators in fixed fields. Two field configurations are known for which relativistic polarization operators are calculated exactly with respect to an external field (in one-loop approximation). It is the case of massless quarks in the instanton field /5,6/ and the case of quarks with arbitrary mass in the covariantly constant fields /7/. Besides that the theorem about the vanishing of all the $1/q^{2k}$ terms ($k \neq 2$) in $\Pi(q^2)$ in self-dual fields is known /6/.

We have succeeded in proving rigorous relations between various d = 8 condensates, which are valid for arbitrary self-dual fields. In the Minkowskian space designations se (anti-self-dual) fields are defined by the condition $G^*_{\mu\nu} = \mp i G_{\mu\nu}$. By virtue of the Bianchi identity the condensates containing the current J_μ are zero in these fields. In the accepted approximation with respect to the number of loops we

can consider the operator $G_{\mu\nu}$ in gluon condensates (7) as external field. Then we replace in eqs.(7) an even number of $G_{\mu\nu}$ by $i G^*_{\mu\nu}$ and express $\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\varepsilon_{\nu_1\nu_2\nu_3\nu_4}$ via the products of $g_{\mu_i\nu_j}$. This yields the relations between gluon condensates. For d = 8 they have the form

$$G_3^{(8)} = \frac{1}{2} G_1^{(8)} - \frac{1}{4} G_2^{(8)}, \qquad G_4^{(8)} = \frac{1}{4} G_2^{(8)} . \tag{25}$$

The quark condensates Q_k in the present paper are considered in the one-loop approximation (see (15)) using $\overline{MS}$ scheme. We represent Q_k in the identical form

$$Q_k = \left[Q_k - Q_k\big|_{d_G \le d} \right] + Q_k\big|_{d_G \le d} . \tag{26}$$

The part of the quark condensate expansion in gluon operators for which $d_G \le d$ (d_G is the dimension of gluon operator) is added and subtracted here. Then the difference in the square brackets in eq.(26) has no ultraviolet divergences and may be calculated in the four-dimensional space. Then

$$Q_k = Q_k^{n=4} + \Delta Q_k , \quad \Delta Q_k = Q_k\big|_{d_G \le d} - Q_k^{n=4}\big|_{d_G \le d} , \tag{27}$$

where $Q_k^{n=4}$ may be calculated using, e.g., the gauge invariant point-splitting method /15/. In Fock-Schwinger gauge, $(\xi - y)_\mu B_\mu(\xi) = 0$, we have

$$Q_k^{n=4} = -i \left\langle Sp\, O_k\, S(x,y) \right\rangle\Big|_{x \to y} . \tag{28}$$

To find the relations between Q_k (as well as between Q_k and G_n), it is necessary to know the properties of the quark propagator S in self-dual fields. In ref./16/ it has been proved that in this case S (in four-dimensional space) can be expressed via the scalar particle propagator $\Delta(x,y)$ =

$\langle x | \frac{1}{\hat{p}^2 - m^2} | y \rangle$ $\left(\Delta_\pm(x,y) = \Delta(x,y) \frac{1 \pm \gamma_5}{2} \right)$:

$$S(x,y) = -\frac{1}{m} \left[\frac{1 \mp \gamma_5}{2} \delta^{(4)}(x-y) + \hat{\vec{\mathcal{P}}}_{(x)} \Delta_\pm(x,y) \hat{\overleftarrow{\mathcal{P}}}_{(y)} \right] \tag{29}$$
$$+ \hat{\vec{\mathcal{P}}}_{(x)} \Delta_\pm(x,y) - \Delta_\pm(x,y) \hat{\overleftarrow{\mathcal{P}}}_{(y)} + m\, \Delta_\pm(x,y) ,$$

where $\vec{\mathcal{P}}_\mu^{(x)} = \mathcal{P}_\mu^{(x)} = i \frac{\partial}{\partial x_\mu} + B_\mu(x)$, $\overleftarrow{\mathcal{P}}_\mu^{(y)} = i \frac{\overleftarrow{\partial}}{\partial y_\mu} - B_\mu(y)$. The 1/m term is related to the zero fermion modes in the field. In ref./6/ it has been shown that at $\delta = x - y \to 0$ (m = 0) in the gauge $(\xi - y)_\mu B_\mu(\xi) = 0$

$$\Delta(x,y) = \frac{i}{4\pi^2} \left[\frac{1}{\delta^2} + S(y) + V_\mu(y)\delta_\mu + \right. \tag{30}$$
$$\left. + T_{\mu\nu}(y)\delta_\mu\delta_\nu + \frac{\delta^2}{128} G_{\mu\nu}(y) G_{\mu\nu}(y) + O(\delta^3) \right] ,$$

where $T_{\mu\mu}(y) = 0$, and the dependences of S, V_μ and $T_{\mu\nu}$ on y are determined by the large distances and are not fixed by the self-duality condition (in (30) the coefficient at δ^2 is different from that in /6/). For obtaining $S(x,y)$ according to eq.(29) it is necessary to differentiate both with respect to x and y. Consequently, it is necessary for us to know the expression for the propagator $\Delta(x,y)$ in the gauge, which is independent on x and y, e.g., $(\xi - z)_\mu B_\mu(\xi) = 0$ (z is arbitrary). This expression differs from eq.(30) on the factor $P\exp i \int_x^z d\xi_\mu B_\mu(\xi)$, multiplied on the left (here the gauge of B_μ is $(\xi - y)_\mu B_\mu(\xi) = 0$). Substituting the obtained scalar propagator into the eq.(29), differentiating it with respect to y, we put z = y after that and obtain the quark propagator $S(x,y)$ in the gauge $(\xi - y)_\mu B_\mu(\xi) = 0$. Neglecting the total derivatives which vanish after the averaging over y as well as the $1/\delta^2$ terms which are canceled by the same terms in ΔQ_k, for quark condensates Q_k we have ~~with~~ the use of obtained $S(x,y)$:

$$Q^{(3)} = -\frac{G^{(4)}}{16\pi^2 m}, \quad Q^{(5)} = \frac{G_1^{(6)}}{12\pi^2 m} - \frac{\langle Sp(S\, G_{\mu\nu} G_{\mu\nu})\rangle}{\pi^2 m},$$

$$Q_1^{(7)} = -\frac{G_1^{(8)}}{16\pi^2 m} - \frac{\langle Sp(V_\lambda [\mathcal{D}_\lambda G_{\mu\nu}, G_{\mu\nu}]_+)\rangle}{2\pi^2 m}, \quad Q_2^{(7)} = -Q_1^{(7)},$$

$$Q_3^{(7)} = \frac{G_1^{(8)} - G_2^{(8)}}{48\pi^2 m} - \frac{i\langle Sp(S\, G_{\mu\nu} G_{\nu\lambda} G_{\lambda\mu})\rangle}{\pi^2 m} - \frac{\langle Sp(V_\lambda [\mathcal{D}_\lambda G_{\mu\nu}, G_{\mu\nu}]_-)\rangle}{2\pi^2 m},$$

$$Q_1^{(8)} = -m Q_1^{(7)} + \Delta Q_1^{(8)}, \tag{31}$$

$$Q_2^{(8)} = -\frac{7(G_1^{(8)} - G_2^{(8)})}{48\pi^2} + \frac{2i\langle Sp(S G_{\mu\nu} G_{\nu\lambda} G_{\lambda\mu})\rangle}{\pi^2} + \frac{\langle Sp(V_\lambda [\mathcal{D}_\lambda G_{\mu\nu}, G_{\mu\nu}]_-)\rangle}{2\pi^2} + \Delta Q_2^{(8)},$$

$$Q_3^{(8)} = -\frac{\langle Sp(V_\lambda [\mathcal{D}_\lambda G_{\mu\nu}, G_{\mu\nu}]_-)\rangle}{\pi^2} + \Delta Q_3^{(8)},$$

where ΔQ_k may be represented in the form

$$\Delta Q_k = \left(\lim_{\varepsilon\to 0}\lim_{\delta\to 0} - \lim_{\delta\to 0}\lim_{\varepsilon\to 0}\right)(-i)\int d_n p\, e^{-ip\cdot\delta}\langle Sp\, O_k S(p)\rangle\big|_{d_G \le d}. \tag{32}$$

Here the role of the terms with $d_G < d$ is only cancellation of the singularities $1/\delta^2$; the terms with $d_G = d$ give

$$\langle Sp\, O_k S(p)\rangle\big|_{d_G = d} = \frac{1}{(p^2 - m^2)^2}\sum_\ell (C'_\ell + \varepsilon C''_\ell) G_\ell,$$

$$\Delta Q_k = \left(\lim_{\varepsilon\to 0}\lim_{\delta\to 0} - \lim_{\delta\to 0}\lim_{\varepsilon\to 0}\right) I(\varepsilon,\delta)\sum_\ell (C'_\ell + \varepsilon C''_\ell) G_\ell, \tag{33}$$

$$I(\varepsilon,\delta) = -i\int d_n p\, \frac{e^{-ip\delta}}{(p^2 - m^2)^2}.$$

It is clear that the absence of logarithmic divergences at $\delta \to 0$ in (30) results in the absence of any divergences for the condensates Q_k in the case m = 0. In accordance to this fact, all the coefficients at $(m)^0 \ln\frac{\mu^2}{m^2}$ in expansions (19) vanish in self-dual fields because of (25) and $J_\mu = 0$. This point confirms the correctness of the expansions (19) once more. Then

$\sum_\ell C'_\ell G_\ell = 0$ and

$$\Delta Q_k = \lim_{\varepsilon\to 0} I(\varepsilon, 0)\, \varepsilon \sum_\ell C''_\ell G_\ell = \frac{1}{16\pi^2}\sum_\ell C''_\ell G_\ell. \tag{34}$$

For obtaining C''_ℓ it is necessary to know $O_k S(p)\big|_{d_G = d}$ with regard for the terms of order of ε . Calculating these terms we have

$$\Delta Q_1^{(8)} = -\frac{G_1^{(8)}}{24\pi^2}, \quad \Delta Q_2^{(8)} = \frac{7(G_1^{(8)} - G_2^{(8)})}{48\pi^2} \tag{35}$$

($\Delta Q_3^{(8)} = 0$ because a non-zero gluon operator of dimension 3 is absent in self-dual fields). After elimination of the "large distance" contributions in (31), containing S and V_μ , we obtain the general relations between condensates in self-dual fields (in $\overline{MS}$ scheme)

$$Q^{(3)} = -\frac{G^{(4)}}{16\pi^2 m}, \quad Q_2^{(7)} = -Q_1^{(7)},$$

$$Q_1^{(8)} = -\frac{G_1^{(8)}}{24\pi^2} - m Q_1^{(7)}, \tag{36}$$

$$Q_2^{(8)} = \frac{G_1^{(8)} - G_2^{(8)}}{24\pi^2} + \frac{1}{2} Q_3^{(8)} - 2m Q_3^{(7)}.$$

The first relation in (36) has previously been derived in /6/, proceeding from the requirement of transverseness of the polarization operator of vector current; the latter is evidently insufficient to obtain the other relations.

With the use of (25) and (36) it is easy to show that the $1/q^8$ contribution to $\Pi(q^2)$ in (23) and (24) vanishes, in accordance with the theorem in /6/.

In the field of the instanton

$$S(y) = \frac{\rho^2}{2[\rho^2 + (y - x_0)^2]^2},$$

$$V_\mu(y) = \frac{\rho^2}{[\rho^2 + (y - x_0)^2]^3}\left[(y - x_0)_\mu + \frac{i}{3}\eta^{(\pm)}_{\mu\nu a}(y - x_0)_\nu \tau_a\right], \tag{37}$$

where X_0 is a centre of instanton and $\eta^{(\pm)}_{\mu\nu a}$ are the t'Hooft symbols. Then

$$G^{(4)} = 32\pi^2\left\langle\frac{1}{\rho^4}\right\rangle, \quad mQ^{(3)} = -2\left\langle\frac{1}{\rho^4}\right\rangle,$$

$$G^{(6)}_1 = -\frac{192}{5}\pi^2\left\langle\frac{1}{\rho^6}\right\rangle, \; mQ^{(5)} = -8\left\langle\frac{1}{\rho^6}\right\rangle,$$

$$\left[G^{(8)}_1, G^{(8)}_2, G^{(8)}_3, G^{(8)}_4\right] = \left[\frac{1536}{7}, -\frac{512}{7}, 128, -\frac{128}{7}\right]\pi^2\left\langle\frac{1}{\rho^8}\right\rangle, \tag{38}$$

$$\left[mQ^{(7)}_1, mQ^{(7)}_2, mQ^{(7)}_3, Q^{(8)}_1, Q^{(8)}_2, Q^{(8)}_3\right] =$$

$$\left[-\frac{32}{5}, \frac{32}{5}, \frac{64}{5}, -\frac{96}{35}, -\frac{1664}{105}, -\frac{512}{105}\right]\left\langle\frac{1}{\rho^8}\right\rangle,$$

where $\left\langle\frac{1}{\rho^n}\right\rangle = \int\frac{d\rho}{\rho}\mathcal{D}(\rho)\frac{1}{\rho^n}$, $\mathcal{D}(\rho)$ is the instanton density. Gluon condensates with $d \leq 8$ and some of the quark odd-dimension condensates with $d \leq 7$ in the field of a single instanton have been considered in /17,2,6,3,14/. The quark condensates considered in these papers are determined only by zero modes and, hence, do not require the choice of the renormalization scheme and can easily be calculated at $n = 4$.

In the case of the covariantly constant field defined by the condition $\mathcal{D}_\lambda G_{\mu\nu} = 0$ the exact expression for $\Pi(q^2)$ has been obtained in ref./7/. Its expansion up to the terms $1/q^8$ has been performed in ref./18/, and the result has been expressed via VEV's in this field. VEV's containing commutators or the current J_μ vanish in this case. The expressions for quark condensates, which were necessary to derive the OPE in ref. /18/, had been obtained in refs./7,18/ using the Green function of a fermion of mass m in a constant field /15/ (it is not hard to generalize it for the case of $n \neq 4$ dimensions):

$$Q^{(3)} = N_c m^3\frac{\ell+1}{4\pi^2} - \frac{m}{4\pi^2}Sp\, I_0, \quad Q^{(5)} = -m\frac{\ell}{4\pi^2}G^{(4)} - \frac{m}{2\pi^2}Sp\, I_1, \tag{39}$$

$$Q^{(7)}_1 = m^3\frac{\ell+1}{4\pi^2}G^{(4)} - \frac{m}{2\pi^2}Sp\left[(\mathcal{E}^2+\mathcal{H}^2)I_0\right], \quad Q^{(7)}_2 = \frac{1}{8\pi^2 m}\left(G^{(8)}_1 - 2G^{(8)}_3\right),$$

$$Q^{(8)}_1 = -mQ^{(7)}_1 - \frac{m^4}{8\pi^2}G^{(4)} - \frac{\ell+\frac{1}{2}}{12\pi^2}G^{(8)}_1 + \frac{\ell}{3\pi^2}G^{(8)}_3 -$$

$$-\frac{1}{2\pi^2}Sp\left[(\mathcal{E}^2-\mathcal{H}^2)I_2\right],$$

where

$$I_0 = \int_0^\infty ds\, e^{-sm^2}\left(\mathcal{E}\mathcal{H}\,cth\,\mathcal{E}s\cdot cth\,\mathcal{H}s - \frac{1}{s^2}\right),$$

$$I_1 = \int_0^\infty ds\, e^{-sm^2}\left(\mathcal{E}^2\mathcal{H}\,cth\,\mathcal{H}s + \mathcal{E}\mathcal{H}^2\,cth\,\mathcal{E}s - \frac{\mathcal{E}^2+\mathcal{H}^2}{s}\right), \tag{40}$$

$$I_2 = \int_0^\infty ds\, e^{-sm^2}\left(\mathcal{E}^2\mathcal{H}\frac{cth\,\mathcal{H}s}{sh^2\mathcal{E}s} - \mathcal{E}\mathcal{H}^2\frac{cth\,\mathcal{E}s}{sh^2\mathcal{H}s} + \frac{2}{3}\,\frac{\mathcal{E}^2-\mathcal{H}^2}{s}\right);$$

and colour matrices $\mathcal{E}$ and $\mathcal{H}$ are defined by equations

$$G_{\mu\nu}G_{\mu\nu} = 2\left(\mathcal{E}^2+\mathcal{H}^2\right), \quad \left(G_{\mu\nu}G^*_{\mu\nu}\right)^2 = -16\,\mathcal{E}^2\mathcal{H}^2. \tag{41}$$

The gluon condensates with $d \leq 8$ have the form /7,18/

$$G^{(4)} = 2\,Sp\left(\mathcal{E}^2+\mathcal{H}^2\right), \tag{42}$$

$$G^{(8)}_1 = G^{(8)}_2 = 4\,Sp\left(\mathcal{E}^2+\mathcal{H}^2\right)^2, \quad G^{(8)}_3 = G^{(8)}_4 = 2\,Sp\left(\mathcal{E}^4+\mathcal{H}^4\right).$$

We convinced ourselves that eqs.(23), (24) of the present paper reduce to the results obtained previously in ref./18/ in the case of the covariantly constant field. The formulae (39) can easily be expanded in a series in 1/m, and the general expansions (19) reduce to these ones in the case considered (up to $d = 8$).

The case when $\mathcal{E} = \mathcal{H}$ corresponds to the self-dual covariantly constant field in which all expressions for the condensates (39), (42) are greatly simplified: $\left(Q^{(5)} = 0\right)$

$$G^{(4)} = 4\,Sp(\mathcal{H}^2),\quad G_1^{(8)} = G_2^{(8)} = 4G_3^{(8)} = 4G_4^{(8)} = 16\,Sp(\mathcal{H}^4),$$

$$Q^{(3)} = -\frac{Sp(\mathcal{H}^2)}{4\pi^2 m},\quad Q_1^{(7)} = -Q_2^{(7)} = -\frac{Sp(\mathcal{H}^4)}{\pi^2 m}, \tag{43}$$

$$Q_1^{(8)} = \frac{Sp(\mathcal{H}^4)}{3\pi^2}.$$

As it is easy to see, eq.(43) is consistent with eq.(31) $\left(S(y) = -\mathcal{H}/4\ ,\ V_\mu(y) = T_{\mu\nu}(y) = 0 \right)$.

In ref./18/ the sum rules of the type /19/ have been considered also, but which take into account the whole subsequence of any-dimension operators, not vanishing in the covariantly constant field ("Abelian" operators). Note that the calculation of VEV's in covariantly constant fields usually results in the values close or similar to those obtained by means of the factorization hypothesis of ref./2/.

When the present paper has been completed on the whole, we have got to know the paper /20/ which deals with the contributions of gluon and bilinear quark condensates of dimension d = 8 to the sum rules for light quarks. That paper is not concerned with the contribution of four-quark operators, which we have calculated in ref./1/; according to a factorization-based estimates at standard values of condensates, it is considerably larger than the contribution of gluon operators. With various ways of estimating of VEV's, the contribution of bilinear quark condensates is, as a rule, of the same order as that of gluon ones. The basis set of bilinear quark condensates Q_k with d = 8 consists of 10 condensates in ref./20/ instead of 6 condensates in our paper. The operators Q_1, Q_6 and Q_9 in ref./20/ are C-odd and, hence, have zero VEV's; the Q_3 and Q_4 operators have both C-even and C-odd parts. In addition, the

VEV of the d = 8 operator Q_5 /20/ is a combination of the d = = 7 quark condensate $m\,Q_2^{(7)}$ and the gluon condensate A of the present paper. Reducing the operators in ref./20/ to our basis and transferring the contribution of A to the gluon part $\Pi'_G(q^2)$, we convince ourselves that the results of ref./20/ agree with the limiting case m = 0 of our results (23) and (24).

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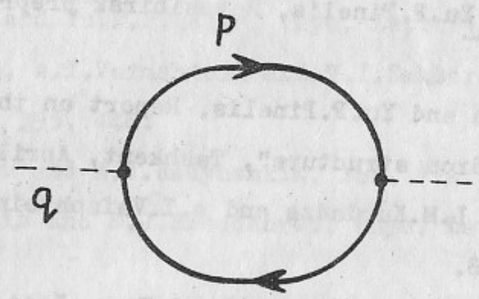

Fig. 1.

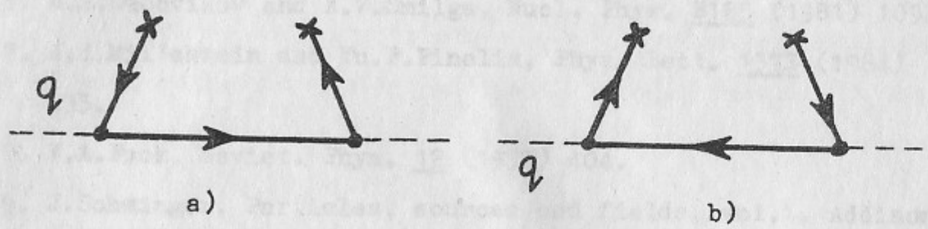

Fig. 2.

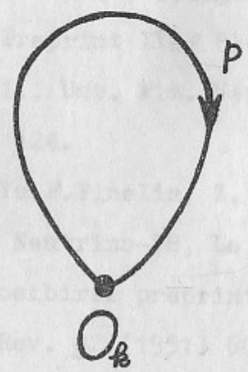

Fig. 3.

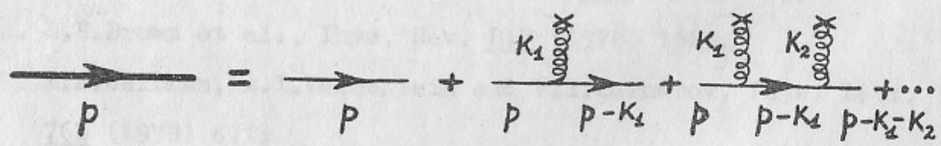

Fig. 4.

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ВКЛАД ВЫСШИХ ГЛЮОННЫХ КОНДЕНСАТОВ В ПОЛЯРИЗАЦИЮ ВАКУУМА ЛЕГКИМИ КВАРКАМИ

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