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ON THE TRANSITION TO APERIODIC MOTION IN LINEAR SYSTEMS WITH STRONG RELAXATION

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ABSTRACT

The Bloch equations are shown to possess purely aperiodic solutions at intermediate values of the transverse relaxation coefficient only. We present also a simple example of a mechanical system where the regions of aperiodic motion are separated by the region of damped oscillations.

It is well-known that the motion of a linear oscillator described by the equation

$$\ddot{x}+\eta\dot{x}+\omega_0^2x=0$$

with the increase of damping η changes qualitatively at the point $\eta=2\omega_0$ from damped oscillations to purely aperiodic motion. In the present note we wish to turn attention to the fact that such a situation is not a universal one. In more complicated linear system aperiodic motion can take place also at intermediate values of relaxation coefficients, passing again with the further increase of the latter to damped oscillations. We shall demonstrate this assertion by two examples.

We start from the Bloch equations which describe the motion of a magnetization vector **M** in an alternating magnetic field

$$\mathbf{B}(t)=(B\cos\nu t,\,B\sin\nu t,\,B_0)\;,$$

$$\frac{d}{dt}\begin{pmatrix}M_x\\M_y\\M_z\end{pmatrix}=-\hat{\Gamma}\begin{pmatrix}M_x\\M_y\\M_z\end{pmatrix}+\eta_1\begin{pmatrix}0\\0\\M_z^{(e)}\end{pmatrix} \qquad (1)$$

$$\hat{\Gamma}=\begin{pmatrix}\eta_2 & \Delta & 0\\ -\Delta & \eta_2 & \omega\\ 0 & -\omega & \eta_1\end{pmatrix}$$

These equations are written in the frame rotating around z axis with the frequency ν. Here $\Delta=\gamma B_0-\nu$, $\omega=\gamma B$, γ is the gyromagnetic ratio, η_1 and η_2 are the longitudinal and transverse relaxation coefficients, $\mathbf{M}^{(e)}$ is the equilibrium value of magnetization in the absence of the rotating field.

We remind that eqs. (1) describe equally the behaviour of a two-level system interacting with a resonant radiation field.

If at the initial moment $t=0$ the magnetization vector is equal to $\mathbf{M}^{(0)}$, then its further evolution is described in virtue of eq. (1) by the expression

$$\mathbf{M}(t) = \exp(-\hat{\Gamma}t)\,(\mathbf{M}^{(0)}-\eta_1\hat{\Gamma}^{-1}\mathbf{M}^{(e)}) + \eta_1\hat{\Gamma}^{-1}\mathbf{M}^{(e)} \,. \tag{2}$$

The second term in the rhs. of eq. (2) gives the equilibrium value of the magnetization, and the first one describes the relaxation process.

It is clear that the decrements of the motion are the eigenvalues of the matrix $\hat{\Gamma}$. The motion is purely aperiodic if all the three decrements are real. It is necessary for this that the discriminant of the corresponding cubic equation should be negative. This discriminant looks as follows:

$$D(\cos\alpha, z) = \frac{\Omega^6}{27}\left[1 + \frac{1}{4}(27\cos^4\alpha - 18\cos^2\alpha - 1)z + \cos^2\alpha z^2\right], \tag{3}$$

where $\Omega=\sqrt{\Delta^2+\omega^2}$ is the Rabi frequency, $z=\frac{(\eta_2-\eta_1)^2}{\Omega^2}$, $\cos\alpha=\Delta/\Omega$.

It is seen immediately that not only at small z, but at large z as well the discriminant is positive, $D>0$. In other words, in the limit of strong relaxation just the damped precession (more exactly, nutation) takes place in general. Only at exact resonance when $\cos\alpha=0$, the situation is analogous to the case of linear oscillator: at all $z>4$ $(\eta_2-\eta_1>2\Omega)$ the motion is aperiodic. In general case in the plane $(\cos\alpha, z)$ there is a region of purely aperiodic motion. In this region $D(\cos\alpha, z)<0$ and its boundary is given by the equation

$$D(\cos\alpha, z) = 0. \tag{4}$$

At fixed α the roots cf eq. (4) are

$$z_\pm = \frac{1+18\cos^2\alpha - 27\cos^4\alpha \pm \sqrt{\sin^2\alpha(1-9\cos^2\alpha)^3}}{8\cos^2\alpha}. \tag{5}$$

From eq. (5) it is seen that aperiodic motion is possible only at $\cos^2\alpha<1/9$. In this case both the roots (5) are positive and the

allowed values of z lie between them. It is clear also that the region of aperiodic motion lies symmetrically in respect to z axis. At Fig. 1 this region is shaded. To avoid misunderstandings we note that after the transition with increase of z to the region of damped precession the damping continues to grow.

Another example is the system of two coupled oscillators with friction. After the diagonalization of the matrix of friction coefficients, the equations of motion are:

$$\begin{cases} \ddot{x}_1+\eta_1\dot{x}_1+\omega_1^2x_1+\omega_{12}x_2=0 \\ \ddot{x}_2+\eta_2\dot{x}_2+\omega_2^2x_2+\omega_{12}x_1=0 \end{cases}$$

Now case the secular equation is of fourth order and its investigation becomes rather tedious. As an example we present at Fig. 2 the result of the numerical investigation at the following values of the parameters: $\omega_1/\omega_2=10$, $\omega_{12}^2/\omega_1^2\omega_2^2=0.3$. In the non-shaded region all the four decrements are complex. In the lightly shaded region only two of them are complex. And at last the densely shaded regions correspond to the purely aperiodic motion. One of them is an island surrounded by the regions of damped oscillations.

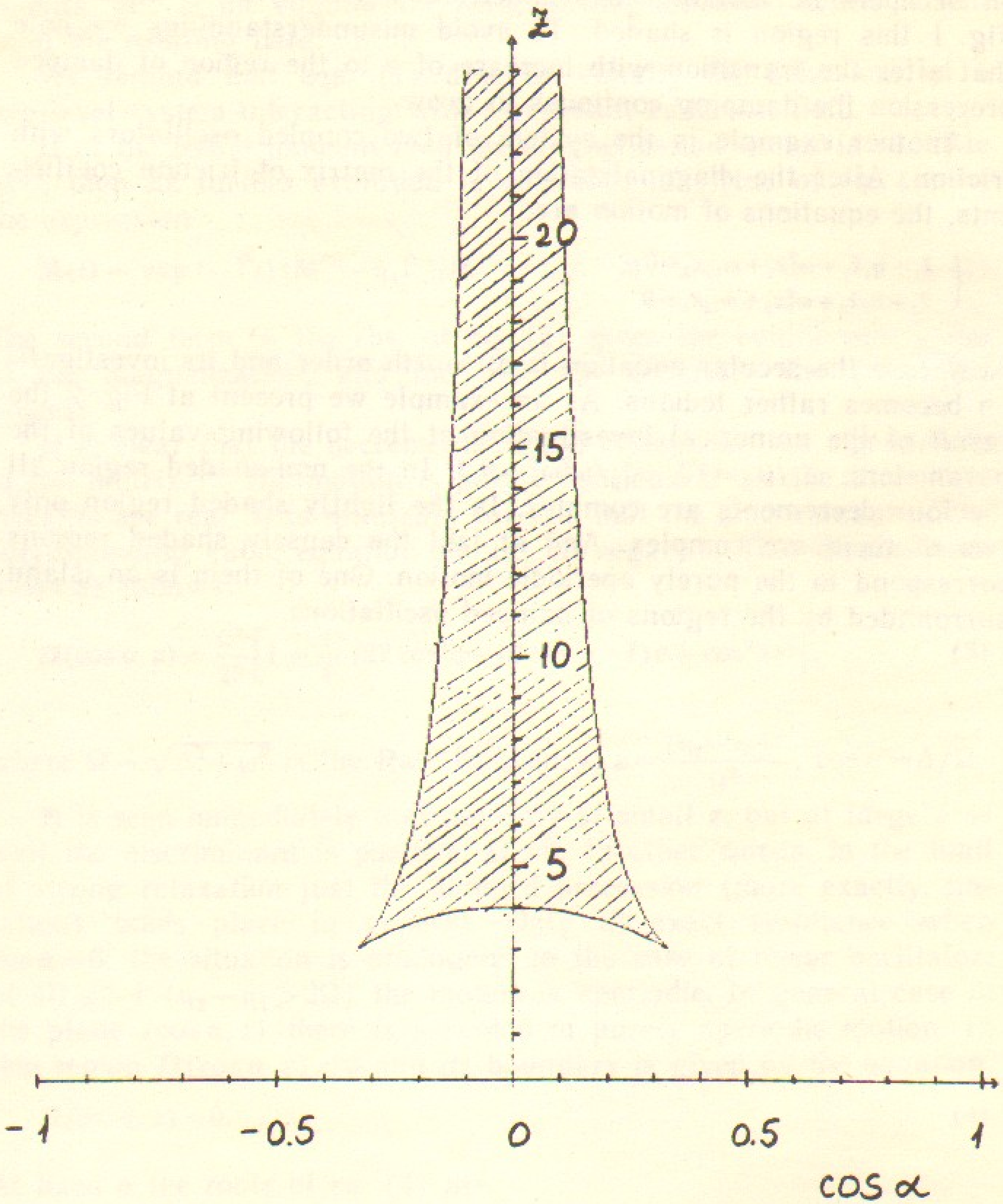

Fig. 1.

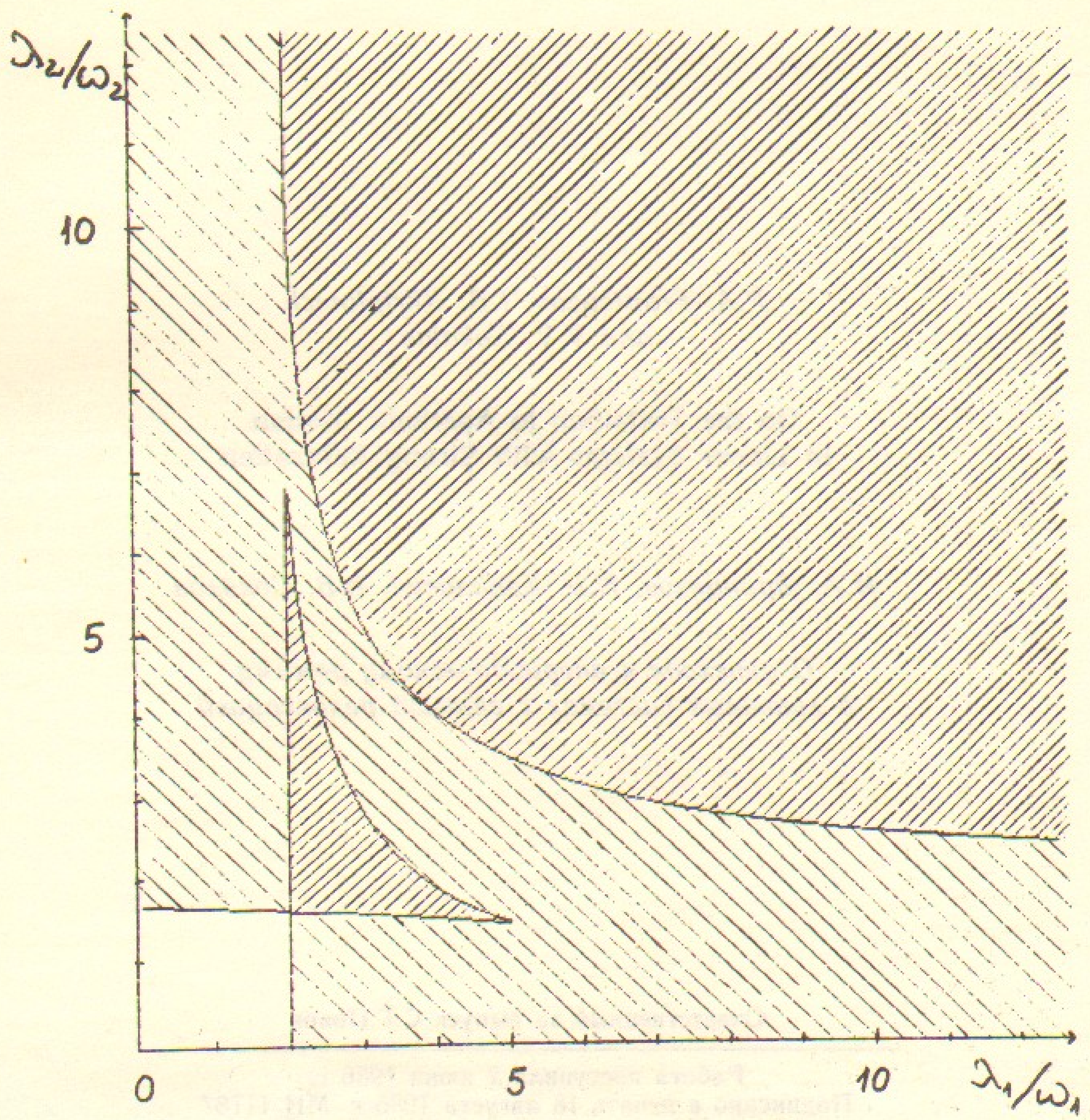

Fig. 2.

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