12

ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

A.V. Blinov, I.B. Khriplovich, L.A. Kondratyuk

DEUTERON QUADRUPOLE MOMENT, ASYMPTOTIC D/S RATIO AND np SCATTERING AMPLITUDE AT THRESHOLD

PREPRINT 86-134

НОВОСИБИРСК
1986

A.V. Blinov[*], I.B. Khriplovich, L.A. Kondratyuk[*]

DEUTERON QUADRUPOLE MOMENT, ASYMPTOTIC D/S RATIO AND np SCATTERING AMPLITUDE AT THRESHOLD

Abstract

Extrapolating *np* scattering amplitude into the deuteron pole we relate it to the deuteron asymptotic *D/S* ratio $\eta = A_D/A_S$. It gives a useful restriction on the phenomenological *NN* amplitudes. The phase shifts by Arndt et al. lead to η in good agreement with other approaches. In this way we find also the deuteron quadrupole moment which in the zero range approximation is expressed directly through the parameters of *np* amplitude.

*) Institute of Theoretical and Experimental Physics, Moscow, USSR

In recent years the deuteron static characteristics are often discussed, and among them the parameter $\eta = A_D/A_S$ which defines the ratio of *D*- and *S*-wave deuteron functions at $r \to \infty$. It can be found from the stripping data and its value is considered to be less model-dependent than the deuteron quadrupole moment Q_d (see, e. g., Refs. [1, 2] and references therein).

Here we derive the model- independent relation between η and the parameters of spin-dependent *np* amplitudes at threshold. This relation is similar to the well-known relation between the asymptotics of the deuteron *S*-wave function and the triplet *np* scattering length. Its existence is quite natural since the *D*-wave admixture in deuteron is closely related to the *D*-wave *np* scattering parameters (*D*-wave phase shift and $^3S_1 - ^3D_1$ mixing parameter). We express also the deuteron quadrupole moment directly through the *np* amplitudes.

To derive these relations we shall use the zero range approximation for tensor forces which is equivalent to the extrapolation of spin-spin *np* scattering amplitudes into the deuteron pole. Our approach is very close to the method suggested by Danilov [3] for the investigation of *P*-violating effects in *np* system, but differs in technical details. Note that the wave function found in Ref. [3] was used afterwards in Ref. [4] for the calculation of the deuteron anapole moment, its *P*-odd electromagnetic characteristic.

We start from the standard expression for the *np* scattering amplitude valid under the assumption of *P*- and *T*-invariance as well as of charge independence of nuclear forces (see Ref. [5])

$$f(\vec{k}, \vec{k}') = a + b(\vec{\sigma}_1\vec{n})(\vec{\sigma}_2\vec{n}) + c(\vec{\sigma}_1 + \vec{\sigma}_2)\vec{n} +$$
$$+ (g+h)(\vec{\sigma}_1\vec{n}_+)(\vec{\sigma}_2\vec{n}_+) + (g-h)(\vec{\sigma}_1\vec{n}_-)(\vec{\sigma}_2\vec{n}_-) \quad (1)$$

Here $\vec{k}$ and $\vec{k}'$ are the initial and final relative momenta of nucleons in cms, $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the proton and neutron spin operators, $\vec{n}_\pm$ are the unit vectors directed along $\vec{k}_\pm = \vec{k} \pm \vec{k}'$, $\vec{n}$ is the unit vector directed along $\vec{n}_+ \times \vec{n}_-$. Using the evident relation $(\vec{\sigma}_1\vec{n})(\vec{\sigma}_2\vec{n}) = \vec{\sigma}_1\vec{\sigma}_2 - (\vec{\sigma}_1\vec{n}_+)(\vec{\sigma}_2\vec{n}_+) - (\vec{\sigma}_1\vec{n}_-)(\vec{\sigma}_2\vec{n}_-)$ we eliminate the structure $(\vec{\sigma}_1\vec{n})(\vec{\sigma}_2\vec{n})$ from Eq. (1) and then introduce instead of the unit vectors $\vec{n}_\pm$ the momenta $\vec{k}$ and $\vec{k}'$. Since we are only interested in the triplet scattering we pass from the operators $\vec{\sigma}_1$ and $\vec{\sigma}_2$ to the total spin operator $\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$. Finally, the triplet *np* scattering amplitude can be represented as

$$f_t(\vec{k}, \vec{k}') = -\alpha + c'\vec{S}\cdot[\vec{k}' \times \vec{k}]/m^2 + g_1(\vec{S}(\vec{k}' + \vec{k}))^2/m^2 +$$
$$+ g_2(\vec{S}(\vec{k}' - \vec{k}))^2/m^2 \quad (2)$$

where

$$\alpha = -(a+b), \quad c' = 2cm^2/k^2 \sin\theta,$$

$$g_1 = (g-b+h)m^2/2k^2\cos^2\frac{\theta}{2} \quad g_2 = (g-b-h)m^2/2k^2\sin^2\frac{\theta}{2}.$$

Here m is the nucleon mass, θ is the scattering angle. Eq. (2) can be treated as the expansion of the scattering amplitude in powers of momenta up to the second order in k/m. Then $\alpha = (m\varepsilon)^{-1/2}$ is the triplet scattering length and c', g_1, g_2 are the constants independent of θ. Note that here and below we omit the terms of the order of $(k/m)^2$ independent of S.

To construct the deuteron wave function at large r we derive at first the effective δ-type potential $\hat{U}(\vec{r})$ (pseudopotential) which reproduces the scattering amplitude (2) in the Born approximation (see, e.g., Ref. [6]). Using the correspondence

$$k_i' k_j' \to -\nabla_i\nabla_j\delta(\vec{r}), \quad k_i' k_j \to -\nabla_i\delta(\vec{r})\nabla_j, \quad k_i k_j \to -\delta(\vec{r})\nabla_i\nabla_j$$

we get

$$\hat{U}(\vec{r}) = \frac{4\pi}{m}\Big\{\alpha\delta(\vec{r}) + \frac{c'}{m_2}\varepsilon_{ijk}S_i\nabla_j\delta(\vec{r})\nabla_k +$$

$$+ \frac{1}{m^2}\hat{S}_{ij}[(g_1+g_2)(\nabla_i\nabla_j\delta(\vec{r}) + \delta(\vec{r})\nabla_i\nabla_j) +$$
$$+ (g_1-g_2)(\nabla_i\delta(\vec{r})\nabla_j + \nabla_j\delta(\vec{r})\nabla_i)]\Big\} \quad (3)$$

where $\hat{S}_{ij} = \frac{1}{2}(S_iS_j + S_jS_i) - \frac{2}{3}\delta_{ij}$. Then the outgoing scattered wave can be written as

$$-\frac{m}{4\pi}\int d^3r' \frac{e^{ik|r-r'|}}{|\vec{r}-\vec{r}'|}\hat{U}(\vec{r}')\psi_0(\vec{r}'). \quad (4)$$

As an unperturbed solution ψ_0 we take the *S*-wave one. Now expression (4) can be transformed to

$$\left(\alpha + \frac{(g_1+g_2)}{m^2}\hat{S}_{ij}\nabla_i\nabla_j\right)\frac{e^{ikr}}{r}. \quad (5)$$

Performing the analytic continuation of the wave function (5) to the point $k = i/\alpha$, which corresponds to the bound state, we find the following expression for the deuteron wave function at $r \to \infty$

$$\psi_d(\vec{r}) = \frac{1}{\sqrt{4\pi}}A_S\left(1 + \frac{g_1+g_2}{\alpha m^2}\hat{S}_{ij}\nabla_i\nabla_j\right)\frac{e^{-r/\alpha}}{r}. \quad (6)$$

From (6) we get immediately the following result for $\eta = A_D/A_S$

$$\eta = \frac{\sqrt{2}\,(g_1+g_2)}{3\alpha^3m^2}. \quad (7)$$

According to Ref. [1] the average value of η found from the elastic dp scattering and neutron stripping reactions equals 0.0271(4). Using this number we get from (7)

$$g_1 + g_2 = 105 \text{ fm} \quad \text{at } k \to 0 \quad (8)$$

Let us determine now the value of g_1+g_2 at $k \to 0$ using the available phase shift data. At Fig. 1 we show the dependence of $\mathrm{Re}(g_1+g_2)$ and $\mathrm{Im}(g_1+g_2)$ on T_p parametrized by the formulas

$$\mathrm{Re}(g_1+g_2) = (1 + T_p/T_0)^{-1}\sum_{n=0}^{2} a_n T_p^n,$$

$$\mathrm{Im}(g_1+g_2)=(T_p/T_0)^{1/2}(1+T_p/T_0)^{-1}\sum_{n=0}^{2} b_n T_p^n, \quad (9)$$

$$T_0=2\varepsilon+\frac{3\varepsilon^2}{2m}$$

with coefficients $a_{0,1,2}$ and $b_{0,1,2}$ calculated using the phase shifts by Arndt et al. [7] at $T_p=1$, 5 and 10 MeV. It can be seen that $\mathrm{Im}(g_1+g_2)$ is independent of the scattering angle for all the energy interval considered. The real part of g_1+g_2, which is of interest to us, still depends on θ at $T_p=10$ and 5 MeV and becomes constant only at $T_p=1$ MeV. It gives some error in the coefficient a_0 which should be determined by the extrapolation of g_1+g_2 to $T=0$. Thus we get $a_0=94$ fm or 109 fm for $\theta=0°$ or 180°. The values of a_0 for other angles lie in this interval 94—109 fm. Therefore, using Eq. (7) we find from the data [7]

$$0.024\leqslant\eta\leqslant0{,}028 \quad (10)$$

in good agreement with the results found by other methods.

Note that the error introduced by the extrapolation from $k=0$ to the deuteron pole is not large, $\sim(r_0/\alpha)^2$, where r_0 is the range of nuclear forces.

Using the relation between η and Q_d given in Ref. [8] (see also Ref. [2])

$$Q_d/A_S^2=13{,}9\eta$$

where Q_d and A_S^2 are measured in fm^2 and fm^{-1} respectively, we get at $A_S=0.885$

$$Q_d=0.26-0.30\ \mathrm{fm}^2 \quad (11)$$

in reasonable agreement with the experimental value 0.286 fm^2.

Now we make a step further and in the spirit of the zero range approximation use the function (6) to compute the quadrupole moment. Then we find the following closed expression for Q_d ($A_S=(2/\alpha)^{1/2}$ in this approximation)

$$Q_d^{(0)}=\frac{(g_1+g_2)}{3\alpha m^2}=\frac{\eta\alpha^2}{\sqrt{2}}. \quad (12)$$

In this way we get $Q_d^{(0)}=0.32-0.37$ fm^2, by 10—30% exceeding the experimental value of Q_d. The accuracy of this result is lower, $\sim|\psi_d(r_0)|^2 r_0^3\sim r_0/\alpha$.

In conclusion we note that since the value of η can be determined with good accuracy from elastic scattering or stripping data, the model-independent relation (7) can be used for the reliable estimation of the amplitude g_1+g_2 and for the check of the selfconsistency of phenomenological analysis of low energy *np* scattering.

We are grateful to P.N. Isaev, I.M. Narodetsky, Yu.A. Simonov and V.G. Zelevinsky for useful discussions.

REFERENCES

1. *T.E.O. Ericson, M. Rosa-Clot,* Nucl. Phys. A405 (1984) 493.
2. *D.W.L. Sprung,* in: Few body problems in physics, ed. L. Faddeev et al. (World Scientific, 1985).
3. *G.S. Danilov,* Phys. Lett. 18 (1965) 40.
4. *V.V. Flambaum, I.B. Khriplovich,* Zh. Eksp. Teor. Fiz. 79 (1980) 1656 (Sov.Phys. JETP 52 (1980) 835).
5. *L. Wolfenstein, J.Ashkin,* Phys. Rev. 85 (1952) 947.
6. *L.D. Landau, E.M. Lifshitz,* Quantum Mechanics (Nauka, Moscow, 1974).
7. *R.A. Arndt et al.,* Phys. Rev. D28 (1983) 97.
8. *M.P. Butler, D.W.L. Sprung,* Can. J. Phys. 62 (1984) 65.

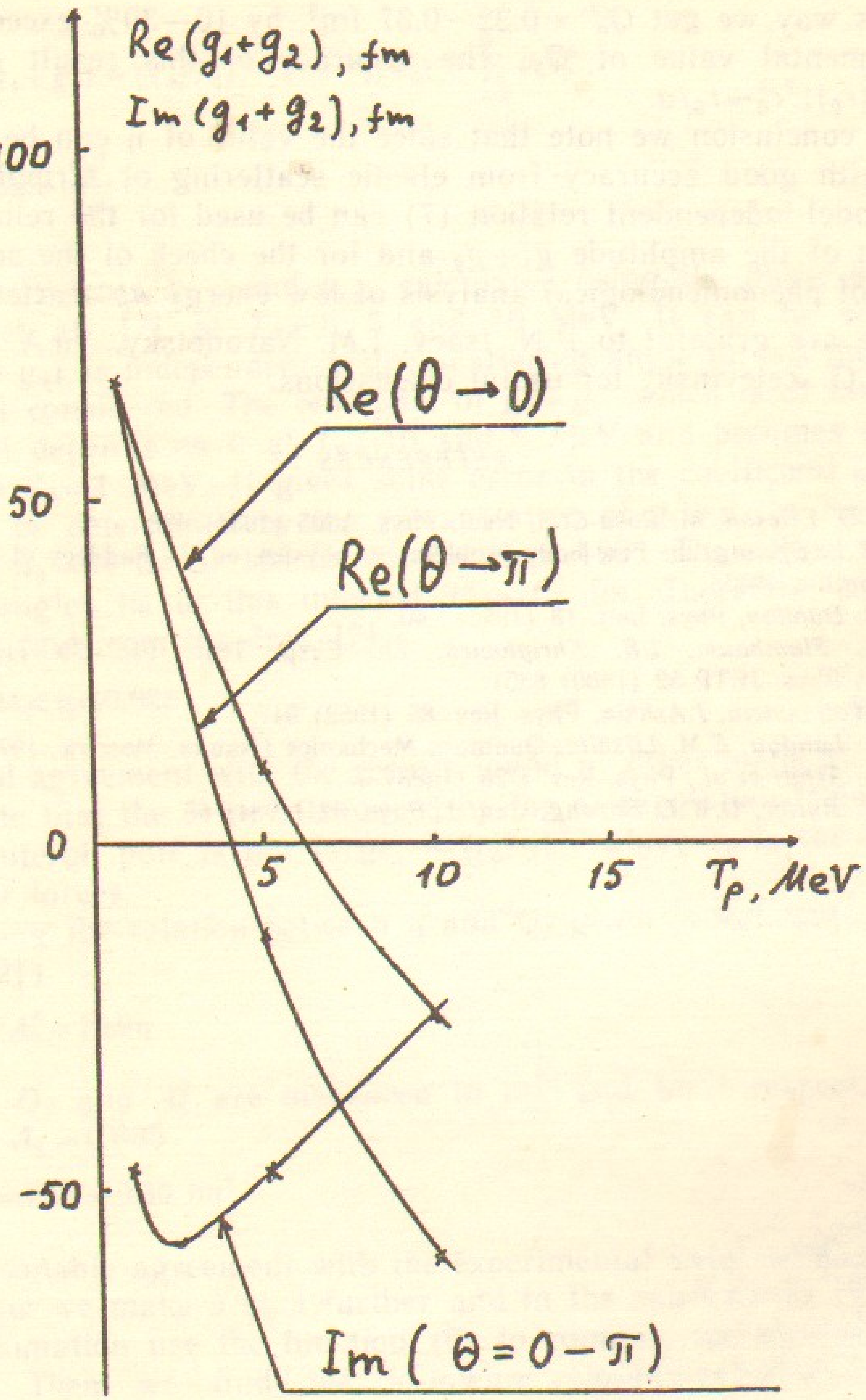

Fig. 1.

A.V. Blinov, I.B. Khriplovich, L.A. Kondratyuk

**DEUTERON QUADRUPOLE MOMENT,
ASYMPTOTIC D/S RATIO AND
np SCATTERING AMPLITUDE AT THRESHOLD**

А.В. Блинов, И.Б. Хриплович, Л.А. Кондратюк

**Асимптотическое отношение DS для дейтрона
и амплитуда пр рассеяния вблизи порога**

Ответственный за выпуск С.Г.Попов

Работа поступила 11 июля 1986 г.
Подписано к печати 4 сентября 1986 г. МН 11810
Формат бумаги 60×90 1/16 Объем 0,8 печ.л., 0,7 уч.-изд.л.
Тираж 290 экз. Бесплатно. Заказ № 134

*Набрано в автоматизированной системе на базе фотонаборного автомата ФА1000 и ЭВМ «Электроника» и отпечатано на ротапринте Института ядерной физики СО АН СССР.
Новосибирск, 630090, пр. академика Лаврентьева, 11.*