ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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MODELS OF STRONG LANGMUIR TURBULENCE

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НОВОСИБИРСК

1. The three-dimensional models of strong Langmuir turbulence (SLT), discussed below, came into light after the prediction of Langmuir waves (LW) collapse [1]. The first model was suggested in [2]. The main hypothesis of [2] can be formulated in the following way. The LW energy is transferred from the power-containing long-wave range $k \sim k_0$ to the dissipative short-wave one $k \gtrsim k_f \gg k_0$ by the collapsing cavities. The energy of LW trapped in the cavity does not change in the process of collapse until Landau damping becomes essential. The collapse is of supersonic self-similar character. All the formed cavities are approximately the same in size $a_0 \sim k_0^{-1}$ and in energies of trapped waves E_0. Under these assumptions the spectrum of waves in the «inertial» range $k_0 \ll k \ll k_f$ is defined easily (and identically in all the SLT models). Indeed, the estimate

$$\tau^{-1}(k) \sim \gamma_{mod}(k) \sim \omega_{pi}[W_{cav}(k)/(n_0 T)]^{1/2}$$

is valid for the typical time $\tau(k)$ of cavity deepening. Here ω_{pi} is the ion plasma frequency, n_0 and T are the plasma concentration and temperature, $W_{cav}(k)$ is the density of the LW energy in the centre of cavity of size k^{-1} (in the «inertial» range $W_{cav}(k) \sim E_0 k^3$). Taking into account that the number of cavities passing through the scale k per unit time is independent of k, one can estimate the concentration $N_{cav}(k)$ of the cavities of size k^{-1} and the mean energy density $W(k) \sim N_{cav}(k) E_0$ of LW of length k^{-1}:

$$W(k) \propto \tau(k) \propto k^{-3/2}, \qquad k_0 \ll k \ll k_f.$$

In the dissipative range $k \gtrsim k_f$ the authors of [2] equated the time of LW transfer through the spectrum with the time of Landau damping:

$$\tau^{-1}(\kappa) \sim \gamma_L(k) \sim \omega_{pe} n(\omega_{pe}/k)/n_0$$

(here ω_{pe} is the electron plasma frequency, $n(v)$ is the concentration of electrons with velocities of order v). This condition was considered together with the quasilinear diffusion equation for the electron distribution function $f(v) = n(v)/v^3$:

$$\frac{\partial f}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} v^2 D \frac{\partial f}{\partial v}, \qquad D(v) \sim \frac{\omega_{pe}}{n_0 m_e v^2} W(\omega_{pe}/v).$$

Assuming that the quasistationary state with the independent of v electron flux

$$J = v^2 D \frac{\partial f}{\partial v}$$

is established in the range $v \ll v_f = \omega_{pe}/k_f$, the authors of [2] obtained the following distribution of electrons and waves:

$$n(v) \propto v^{-3/2}, \quad v \ll v_f; \qquad W(k) \propto k^{-7/2}, \quad k \gg k_f.$$

In this treatment the estimate $\tau(k) \propto k^{-3/2}$ was used, which does not take into account the LW absorption. The absorption was taken into account in [3], where the expression for the transfer rate in terms of the spectrum:

$$W(k) \sim N_{cav}(k) W_{cav}(k) k^{-3} \propto \tau^{-1}(k) k^{-3}$$

was added to the basic relations of [2]. As a result, the formulas for the principal values in the dissipative range take the form:

$$\tau(k) \propto k^{-1/2}, \quad W(k) \propto k^{-5/2}, \quad n(v) \propto v^{-1/2}.$$

This result was admitted correct in the review [4]. However, for the spectrum $W(k) \propto k^{-5/2}$, obtained in [3], the typical time of electron diffusion $t_d(v) \sim v^2/D(v)$ grows when v decreases and, consequently, in spite of the assumption made in the works under discussion, the quasistationary electron distribution with the v-independent flux $J \neq 0$ has no time to be established in the range $v \ll v_f$.

Another SLT model was suggested in [5], where it was assumed that all trapped LW have been absorbed in the cavities of

scale k_f^{-1}, and $W(k)$ decreases exponentially in the range $k \gg k_f$. Then the electron diffusion in the range $v \ll v_f(t)$ is negligible and the distribution $n(v)$ remains the same, as just after acceleration, which took place at the times t', when $v_f(t') = v$. The concentration of accelerated (and thereby all other) electrons is defined from the condition that at $k \sim k_f$ the modulational instability increment coincides with the decrement of Landau damping:

$$n(v_f) \sim n_0 \gamma_{mod}(\omega_{pe}/v_f)/\omega_{pe} \propto v_f^{-3/2}; \quad n(v) \propto v^{-3/2}, \quad v \ll v_f.$$

For this distribution the decrement of Landau damping in the range $k \gg k_f$ approximately coincides with their modulational instability increment, calculated in the absence of dissipation. Since the dissipation slows down the collapse, the assumption of authors [5] about the absorption of all trapped waves at $k \sim k_f$ is justified. The law of increasing the velocity v_f is determined from the condition $t_d(v_f) \sim t$ and turns out to be the following: $v_f \propto t^2$.

One more SLT model was suggested in [6]. It was assumed there that the minimum velocity $v_{\min}$ of electrons taking part in energy absorption is practically time-independent and equal to several thermal velocities. The formula $W(k) \propto k^{-3/2}$ was used in the whole range $k \ll \omega_{pe}/v_{\min}$, i. e. the influence of absorption on the collapse dynamics was neglected up to the scale of cavities $r_{\min} \sim v_{\min}/\omega_{pe}$ (which is equal to several Debye lengths r_D). Further, the self-similar solution of the diffusion equation has been constructed, which satisfies the condition of constant power supply to plasma:

$$\frac{d}{dt} \int_0^\infty dv v^4 f(v) = \text{const}.$$

As one can show, this solution looks as

$$f(v, t) = t^{-9} F(v/t^2).$$

For the spectrum $W(k) \propto k^{-3/2}$ the typical diffusion time decreases with diminishing of the electron velocity, and the quasistationary distribution with the constant flux J is established in the range $v \ll v_f(t)$:

$$f(v, t) \propto t^{-9}(v/t^2)^{-5/2} = t^{-4} v^{-5/2}.$$

When $t \to \infty$, then $f(v, t) \to 0$, which confirms the assumption about the weakness of Landau damping in the range $v \gg v_{\min}$. The contradiction, however, reveals in another place: for the solution of the assumed kind, the number of electrons with velocities greater than some fixed value v from the range $(v_{\min}, v_f)$ decreases in time (as t^{-3}), but the flux of electrons is directed from the range $v'<v$ to the range $v'>v$, rather than vice versa. Consequently, such a solution does not exist.

2. The analysis presented above solves the problem of SLT model choice in the limits of conventional set of hypotheses, but it does not touch upon the question of their own correctness. Some base for the doubt about the latter can be found already in [7], where the existence of an infinite number of bounded states in a self-similar cavity was proved for the spherically-symmetric scalar collapse and the possibility of extending this theorem to the case of an asymmetrical scalar collapse and even to the case of LW collapse was claimed. The complete proof has never been published. Probably, therefore in the more recent paper [8] it was claimed still unknown, if an infinite number of bounded states exists in a self-similar cavity. The interest of the authors of [8] to this question was excited due to the noticed by them «effect of funnel», which consists in the following: in the remaining after a collapse the concentration «funnel», when it contains an infinite number of bounded states, the «fall on the centre» [9] of untrapped plasmons and, consequently, the additional energy absorption should take place. This idea did not receive further development. Meanwhile, it is possible to show that the theorem [7] may be extended in fact to the case of the Langmuir collapse and the «effect of funnel» in fact takes place. I can not discuss that in more details here, because the main object of the present communication is another effect, which related to the «effect of funnel», but is stronger and exists irrespectively of the correctness of theorem [7]. As it is known, the macroscopic movement of ions, accelerated in the collapse process, continues also after absorption of LW initially trapped by the cavity. The estimates show that the kinetic energy of this movement is sufficient for a cavity deepening by approximately

$$A=\left(\frac{W^0}{W^0_{th}}\frac{k_f}{k_0}\right)^{1/2}\gg 1$$

times, where $W^0=W(k_0)$ is the mean energy density of Langmuir turbulence, $W^0_{th}\sim n_0T(k_0r_D)^2$ is the lowest value of W^0 compatible with the condition of modulational instability of the main scale wa-

ves. When cavity deepens by A times, the number of bounded in it states grows approximately by $A^{1/2}$ times and becomes of order*) $A^{1/2}$. In each new bounded state some number of plasmons is trapped. This may influence the ion motion, absorption of trapped waves and acceleration of the resonance electrons. I have obtained the self-consistent solution of the problem, which takes into account all the process mentioned above. The character of this solution depends on the parameters of the problem, i. e. there are several regimes of SLT. It turns out that one of the most interesting is the threshold ($W^0\sim W^0_{th}$) regime of SLT with not too wide inertial range, for which every new bounded state traps the same energy as the first one. In this case, the SLT spectrum looks as follows: in the inertial range the relation $W(k)\propto k^{-3/2}$ remains correct (but the coefficient differs essentially from the one obtained in the previous models); in the range $k\sim k_f$ $W(k)$ decreases exponentially by $(k_f/k_0)^{3/28}$ times; when $k_f\ll k\ll k_f(k_f/k_0)^{1/4}$, then $W(k)\propto k^{-11/7}$, and when $k\gg k_f$, then $W(k)$ decreases exponentially again. The electron distribution is described in the range $v\ll v_f$ by the relation $n(v)\propto v^{-9/5}$, while $n(v)$ remains exponentially small in the range $v\gg v_f$. Velocity v_f increases with the time as t^4. The power, absorbing in the each cavity, turns out to be much larger than in the previous SLT models and the threshold regime of SLT takes place in a far wider range of power supplies to plasma, than it was assumed earlier.

*) «An infinite number of bounded states» mentioned in the theorem [7] transforms into $\ln(k_f/k_0)$, when the finiteness of inertial range is taken into account. Factor $\ln(k_f/k_0)$ is not very essential compared to $A^{1/2}$ and, therefore, it is omitted here and further.

REFERENCES

1. *Zakharov V.E.* Zh. Eksp. Teor. Fiz., 1972, v.62, p.1745.
2. *Galeev A.A., Sagdeev R.Z., Sigov Yu.S., Shapiro V.D., Shevchenko V.I.* Fiz. Plazmy, 1975, v.1, p.10.
3. *Pelletier G.* Phys. Rev. Lett., 1982, v.49, p.782.
4. *Shapiro V.D., Shevchenko V.I.* Sil'naja Turbulentnost' plazmennykh kolebanii (Strong Turbulence of Plasmas Oscillations). In: Osnovy fiziki plazmy (Basic Plasma Physics).—M., 1984, v.2, p.143—144.
5. *Gorev V.V., Kingsep A.S., Yan'kov V.V.* Zh. Eksp. Teor. Fiz., 1976, v.70, p.921.
6. *Zakharov V.E.* Kollaps i samofokusirovka lengmjurovskikh voln (Collaps and Self-Focusing of Langmuir Waves). In: Osnovy fisiki plazmy (Basic Plasma Physics).—M., 1984, v.2, p.110.
7. *Friman G.M.* Pis'ma Zh. Eksp. Teor. Fiz., 1979, v.30, p.557.
8. *Zakharov V.E., Shchur L.M.* Zh. Eksp. Teor. Fiz., 1981, v.81, p.2019.
9. *Landau L.D., Lifshitc E.M.* Kvantovaja mekhanika (Quantum Mechanics).—M., 1974, § 35.

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