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A b s t r a c t

Beam-beam effects are considered for the case of electron-positron storage rings with a monochromator scheme, when the interaction of the beams is taking place in the presence of a large vertical energy dispersion at the interaction point. A limitation of luminosity in monochromatic experiments due to the decrease of a monochromaticity factor under the influence of the beam-beam effects is obtained.

INTRODUCTION

In the Institute of Nuclear Physics at Novosibirsk the works are continued on the upgrade project of the storage ring VEPP-4 for the monochromatic experiments in Ψ, Υ -mesons energy region. The important question for these studies is the estimate of the maximum luminosity, for which the effects of electromagnetic interaction of electrons and positrons(beam-beam effects) don't significantly affect the monochromaticity of the interaction energy of the particles. It should be noted, that according to a suggested in the work /2/ scheme, the monochromaticity is obtained through the special method of performing the electron and positron beams collision. The beams at the interaction point are decomposed relative to the energy in the vertical direction, so that the size of decomposition is much larger than the r.m.s. betatron size. The gain of energy resolution due to the monochromator is determined for the unperturbed motion, when the particle distribution is gaussian, by a factor $\lambda = |\Psi_z| \sigma_\varepsilon / \sigma_{z\beta}$, where Ψ_z , $\sigma_{z\beta}$ are vertical dispersion function and betatron beam size at the interaction point, and σ_ε is the relative energy spread in the beam. The interaction of particles of one beam with the space charge field of an opposite beam changes the distribution of particles. Even for the standard regime of electron-positron collision the number of particles in the distribution tails and the average size of the beam are growing with the increase of the beams intensity (see, for example, /3/). In our case one should expect a stronger manifestation of the effect because of a larger vertical dispersion at the interaction point. The broadening of the particles distribution function in z -direction leads to a stronger intra-beam mixing of different energies and monochromaticity deterioration.

The first studies of mechanisms of the possible loss of monochromaticity because of beam-beam effects were carried out in a one-dimensional model in the work /4/. The conclusions therein are resuming basically to the prediction of a relatively high value of a threshold space charge parameter ξ for a particle distribution function broadening, and the identification of the effect of a vertical dispersion function perturbation by an opposite beam field as the most dangerous one. In a whole, this allowed to obtain a well enough optimistic prediction of a beam-beam effect influence on a monochromaticity, moreover that the compensation of a linear part of an opposite beam "force" (see /1,4/) allows to considerably supress the phenomenon.

The problem of a magnitude of a beam-beam effect influence on a monochromaticity requires, however, a further investigation, where the two dimensional character of motion should be taken into account. Such an investigation was carried out and the results are given in a present paper.

The first paragraph containts the analysis of a magnitude of a vertical betatron amplitude oscillation for the nonlinear betatron and synchro betatron resonances, arising from the particle interaction with the opposite beam of elliptical cross--section. The dependence of this quantity on the monochromaticity parameter λ and the information about the relative "strengths" of different resonances are obtained.

In the second paragraph the model and results of simulation are given.

The analytic estimates of a resonance size in monochromatic regime

Let us consider the case of elliptical beams with a large aspect ratio $\varkappa = \sigma_x/\sigma_z \gg 1$. Then the oscillations of a betatron amplitudes on the coupling resonances $\ell\nu_x + m\nu_z + n\nu_s = K$ occur basically, in the region of a moderate amplitudes A_x, A_z, in the direction Z : $\Delta A_z/\Delta A_x \ll 1$.
The quantity ΔA_z in a general case is given by /5/:

$$\Delta A_z = \frac{2m}{\nu A_z} \frac{\xi_z}{\xi_x} \sqrt{\left| \frac{V_{\ell mn}}{\alpha} \right|} \qquad (1.1)$$

where the nonlinearity α is

$$\alpha = 2\pi \left(\frac{\ell^2}{A_x} \frac{\partial(\Delta\nu_x)}{\partial A_x} + 2\ell m \frac{\xi_z}{\xi_x} \frac{1}{A_x} \frac{\partial(\Delta\nu_z)}{\partial A_x} + m^2 \left(\frac{\xi_z}{\xi_x}\right)^2 \frac{\varkappa}{A_z} \frac{\partial(\Delta\nu_z)}{\partial A_z} \right) \qquad (1.2)$$

harmonic amplitude $V_{\ell mn}$ has the form:

$$V_{\ell mn} = \frac{\nu A_z}{m(2\pi)^2} \iiint_0^{2\pi} d\theta_x d\theta_z d\theta_s \cdot \zeta_z(A_x\cos\theta_x, A_z\cos\theta_z + A_s\cos\theta_s) \cdot \qquad (1.3)$$

$$\cdot \cos\ell\theta_x \sin\theta_z \sin m\theta_z$$

and normalized tune shifts $\Delta\nu_{x,z}$ can be calculated through the formulas:

$$\Delta\nu_x = \frac{1}{16\pi^4 A_x} \iiint_0^{2\pi} d\theta_x d\theta_z d\theta_s \cdot \zeta_x(A_x\cos\theta_x, A_z\cos\theta_z + A_s\cos\theta_s) \cdot \cos\theta_x$$

$$\Delta\nu_z = \frac{1}{16\pi^4 A_z} \iiint_0^{2\pi} d\theta_x d\theta_z d\theta_s \cdot \zeta_z(A_x\cos\theta_x, A_z\cos\theta_z + A_s\cos\theta_s) \cdot \cos\theta_z \qquad (1.4)$$

and equal unity for zero amplitudes A_x, A_z.

In the formulas above the following notations were implemented: $\nu = 1/\varkappa$ - aspect ratio parameter, ξ_x, ξ_z -linear tune shifts, ζ_x, ζ_z - forces of beam-beam interaction, normalized through the condition $\zeta_x \xrightarrow[x,z\to 0]{} 4\pi X$, $\zeta_z \xrightarrow[x,z\to 0]{} 4\pi Z$, A_x, A_z - dimensionless amplitudes $A_x = \sqrt{X^2+P_x^2}$, $A_z = 1/\lambda \sqrt{Z^2+P_z^2}$, normalized to σ_x and σ_z (the full vertical size of the beam) respectively.

As it was shown in /6/, for the case $\varkappa \gg 1$ and $|X|,|Z| \sim 1$ the forces ζ_x, ζ_z can be well enough approximated with the expressions:

$$\zeta_x = 2\pi\sqrt{\tfrac{2}{\pi}}\, F_D(X/\sqrt{2})$$

$$\zeta_z = 2\pi\sqrt{2\pi}\, e^{-X^2/2} \operatorname{erf}(Z/\sqrt{2}) \qquad (1.5)$$

where $F_D(y)$ is the Dawson function $F_D(y) = e^{-y^2} \int_0^y e^{x^2} dx$ (see /7/), and $\operatorname{erf}(y)$ is the error integral. It should be noted here, that the value of a betatron amplitude A_z in monochromatic regime is much smaller, than the synchrotron amplitude A_s. Making use of this relationship, we can use the power decomposition of the force ζ_z in the integrals (1.3), (1.4) and obtain explicite analytic expressions for the nonlinearity, harmonic amplitude, and the rezonance width itself. Thus, taking into account the relations:

$$\frac{\partial \zeta_z}{\partial Z} = 4\pi e^{-X^2/2} e^{-Z^2/2}$$

$$\frac{\partial^2 \zeta_z}{\partial Z^2} = -4\pi e^{-X^2/2} e^{-Z^2/2} \qquad (1.6)$$

$$\frac{\partial^3 \zeta_z}{\partial Z^3} = 4\pi e^{-X^2/2} e^{-Z^2/2}$$

we can get the function $\Delta\nu_z$ and the derivative $\frac{\partial(\Delta\nu_z)}{\partial A_z}$:

$$\Delta\nu_z = I_0\left(\frac{A_x^2}{4}\right) e^{-A_x^2/4} e^{-A_s^2/4} \left[I_0\left(\frac{A_s^2}{4}\right) - \frac{A_z^2}{8}\left(I_0\left(\frac{A_s^2}{4}\right) - \frac{A_s^2}{2}\left\{ I_0\left(\frac{A_s^2}{4}\right) + I_1\left(\frac{A_s^2}{4}\right) \right\} \right) \right] \qquad (1.7)$$

$$\frac{\partial(\Delta\nu_z)}{\partial A_z} = \frac{A_z}{4} I_0\left(\frac{A_x^2}{4}\right) e^{-A_x^2/4}\left[I_0\left(\frac{A_s^2}{4}\right) - \frac{A_s^2}{2}\left(I_0\left(\frac{A_s^2}{4}\right) + I_1\left(\frac{A_s^2}{4}\right)\right)\right] e^{-A_s^2/4} \tag{1.8}$$

where I_0 and I_1 are the modified Bessel functions of zero and first orders.

It should be noted, that for the case $\varkappa\left(\frac{\xi_z}{\xi_x}\right)^2 \frac{m^2}{\ell^2} >> 1$ the last, proportional to $\frac{\partial(\Delta\nu_z)}{\partial A_z}$, term in the expression for the nonlinearity α (1.2) is prevailing, so that in such a situation we don't need $\Delta\nu_x$ for the calculation of nonlinearity α.

Another characteristic feature of nonlinear resonances in monochromatic regime, following from the expression (1.7), is also important: since we have $A_z << 1$ (full vertical betatron beam size normalization), the quantity $\Delta\nu_z$ is weakly depending on $A_{z\beta}$, what in its turn means, that in the A_x, A_z plane the resonance lines are nearly parallel to the A_z axis (with $1\sigma_x$ and $1\sigma_z$ taken for a unity length in corresponding direction). For the harmonic amplitude $V_{\ell m n}$ calculation we can use, for each given m, a corresponding power of ξ_z decompozition in $A_z \cos\theta_z$, what gives:

$$m=1 \qquad V_{\ell 1 n} = \frac{\nu\pi}{n} A_z A_s e^{-A_x^2/4} I_{\ell/2}\left(\frac{A_x^2}{4}\right)\left[I_{\frac{n-1}{2}}\left(\frac{A_s^2}{4}\right) + I_{\frac{n+1}{2}}\left(\frac{A_s^2}{4}\right)\right] e^{-A_s^2/4}$$

where n is odd

$$m=2 \qquad V_{\ell 2 n} = \frac{\nu\pi}{2} A_z^2 e^{-A_x^2/4} I_{\ell/2}\left(\frac{A_x^2}{4}\right) I_{\frac{n}{2}}\left(\frac{A_s^2}{4}\right) \cdot e^{-A_s^2/4} \tag{1.9}$$

where n is even

$$m=3 \qquad V_{\ell 3 n} = \frac{\nu\pi}{24} A_z^3 e^{-A_x^2/4} I_{\ell/2}\left(\frac{A_x^2}{4}\right) A_s \left[I_{\frac{n-1}{2}}\left(\frac{A_s^2}{4}\right) + I_{\frac{n+1}{2}}\left(\frac{A_s^2}{4}\right)\right] e^{-A_s^2/4}$$

where n is odd.

Making use of expressions (1.9) and of an approximate relation $\alpha \approx 2\pi m^2 \left(\frac{\xi_z}{\xi_x}\right)^2 \frac{\varkappa}{A_z} \frac{\partial(\Delta\nu_z)}{\partial A_z}$ with $\frac{\partial(\Delta\nu_z)}{\partial A_z}$ from (1.8), we can have, introducing normalized to the vertical betatron size amplitude $A_{z\beta} = \lambda A_z$, $\lambda = \sigma_z / \sigma_{z\beta}$, the expression for the resonance width:

$$\Delta A_{z\beta} = G_\ell(A_x) \cdot F_m(A_{z\beta}, A_s, \lambda) \tag{1.10}$$

where

$$G_\ell(A_x) = \sqrt{\frac{I_{\ell/2}(A_x^2/4)}{I_0(A_x^2/4)}} \tag{1.11}$$

and F_m for the lowest values of $|m|$ has explicit form:

$$F_1 = \frac{2\lambda^{3/2}}{\sqrt{A_{z\beta}}} \sqrt{\left|\frac{\frac{2}{n} A_s\left[I_{\frac{n-1}{2}}\left(\frac{A_s^2}{4}\right) + I_{\frac{n+1}{2}}\left(\frac{A_s^2}{4}\right)\right]}{I_0\left(\frac{A_s^2}{4}\right) - \frac{A_s^2}{2}\left(I_0\left(\frac{A_s^2}{4}\right) + I_1\left(\frac{A_s^2}{4}\right)\right)}\right|}$$

$$F_m \sim \lambda \left(\frac{A_{z\beta}}{\lambda}\right)^{\frac{m}{2}-1} \qquad F_2 = 2\lambda \sqrt{\frac{I_{n/2}(A_s^2/4)}{\left|I_0\left(\frac{A_s^2}{4}\right) - \frac{A_s^2}{2}\left(I_0\left(\frac{A_s^2}{4}\right) + I_1\left(\frac{A_s^2}{4}\right)\right)\right|}} \tag{1.12}$$

$$F_3 = 2\sqrt{\lambda A_{z\beta}} \sqrt{\left|\frac{A_s}{12} \frac{I_{\frac{n-1}{2}}\left(\frac{A_s^2}{4}\right) + I_{n/2}\left(\frac{A_s^2}{4}\right)}{I_0\left(\frac{A_s^2}{4}\right) - \frac{A_s^2}{2}\left(I_0\left(\frac{A_s^2}{4}\right) + I_1\left(\frac{A_s^2}{4}\right)\right)}\right|}$$

Obtained thus formulas for the resonance width $\Delta A_{z\beta}$ at the point A_x, A_z show a number of peculiarities. The first of those is the same as in a conventional non-monochromatic regime independence of $\Delta A_{z\beta}$ either on $\varkappa$, or on ξ_x, ξ_z (as it was pointed out earlier, this is valid under the assumption $\xi_x^2 \ell^2 / \varkappa \xi_z^2 m^2 << 1$). The absence of any influence of the parameters $\varkappa$ and ξ_x on the quantity $\Delta A_{z\beta}$ arise from the assymptotic character of a beam field for a large $\varkappa >> 1$ (1.5), so that the amplitude oscillations occur predominantly in z direction. The independence of $\Delta A_{z\beta}$ of ξ_z is a standard consequence of linearity of an unperturbed hamiltonian of betatron oscillations, so that the value of stabilizing nonlinearity and the resonance harmonic amplitude are proportional both to the parameter of the perturbation intensity ξ_z (see /8/). The second peculiarity is the specific dependence of a relative magnitude of different harmonics resonances on the parameter λ. It is clear from the expressions (1.12), that the resonance widths for $|m| = 1,2,3$ and arbitrary ℓ and n are growing with the increase of λ, while the resonances width with $|m| = 4$ doesn't change, and the resonance widths with $|m| > 4$ are decreasing. The third peculiarity of the formulas of discussion is a zero value of denominators in the expressions (1.12) (that is, $\frac{\partial(\Delta\nu_z)}{\partial A_z}$ from (1.8)) for the argument value $A_s \approx 1.35$. At the same time the resonance widths (1.12) are formaly infinite. In the reality the oscillation amplitude will be stabilized by a higher power of nonlinearity. Moreover, it is reasonable to assume, that the value $A_s \approx 1.35$ doesn't have a significant affect on the average beam sizes, because the direction of a resonance line, for a decreasing nonlinearity α, is approaching the direction of amplitude oscillation on the resonance, so that the resonance stripe width in the plane A_x, A_z for the fixed amplitude A_z tends to zero when $\frac{\partial(\Delta\nu_z)}{\partial A_z} \to 0$. $(A_s \to 1.35\lambda)$

The applicability condition of the formulas (1.10)-(1.12) is the inequality $\Delta A_{z\beta} \ll A_{z\beta}$, which is fulfilled for $A_x, A_{z\beta}, A_s \sim 1$. only for high enough indices $|\ell|, |m|, |n|$. In spite of this limitation, however, the formulas (1.10)-(1.12) can be helpful as well for a comparison of relative magnitudes of different, and even low, order resonances (because a larger value of harmonic amplitude leads to a larger resonance size). Furthermore, in a conventional situation of having the working point in the tune region with a minimal beam "blow up", essential are high harmonic resonances, for which the formulas (1.10)-(1.12) are valid.

The information about the relative strength of different harmonic ℓ, m, n resonances is presented in the table I, showing the resonance width $\Delta A_{z\beta}$ for the lowest possible value of $|n|$ (zero for even m and 1 for odd m) and the first $|n|$, following it - 2 for even m and 3 for odd m). The width was calculated from the formulas (1.10)-(1.12) for $A_x = A_{z\beta} = A_s = 1$. Resonance widths ΔA_z for the standard, non-monochromatic regime $\lambda = 0$ are shown for comparison. These quantities are nonzero only for even ℓ and m, and for zero n, because synchrotron modulations are absent in this case. The data shows, that the magnitudes of $\Delta A_{z\beta}(\lambda = 10)$ are considerably larger, than $\Delta A_z(\lambda = 0)$ magnitudes.

An important question of the optimal regime choice in the ratio ξ_x/ξ_z is the dependence of individual resonance widths on this parameter. In the formulas (1.10) the resonance width $\Delta A_{z\beta}$ doesn't depend on the parameters ξ_x, ξ_z, but the formulas themselves are valid only for $\frac{\xi_x^2}{\xi_z^2}\frac{\ell^2}{\varkappa m^2} \ll 1$. When the ratio ξ_x/ξ_z is of the order

$$\xi_x/\xi_z \sim \sqrt{\varkappa}\,\left|\frac{m}{\ell}\right| \qquad (1.13)$$

then one has to take into account, besides the third, the first term in the nonlinearity α (1.2) (we can always

neglect the second term if we have $\varkappa \gg 1$). The quantity $\Delta A_{z\beta}$ for this situation is decreasing relative to the value (1.10)-(1.12) and it's characteristic dependence on A_x, A_s as a product of a function of A_x and a function of A_z is violated. The decrease of $\Delta A_{z\beta}$ in respect to (1.10) can be estimated, however, for $A_x, A_{z\beta} \sim 1$, by the following expression:

$$\Delta A_{z\beta} \sim \frac{\Delta A^{\circ}_{z\beta}}{\sqrt{1 + \xi_x^2 \ell^2/\xi_z^2 m^2 \varkappa}} \qquad (1.14)$$

In the estimate (1.14) the smallness of the second term in the nonlinearity α (1.2) in respect to the first or the third was utilized, and the amplitudes and derivatives $\frac{\partial(\Delta\nu_z)}{\partial A_x}, \frac{\partial(\Delta\nu_z)}{\partial A_z}$ were considered to be of the order of unity. Thus, the magnitude of the amplitude oscillation on the individual resonances, is decreasing for increasing ratio ξ_x/ξ_z, what lead us to the conjecture of the advantage of a regime with $\xi_x \gg \xi_z$ in respect to the conventional regime $\xi_z \gtrsim \xi_x$.

Table I

The value of resonance widths $\Delta A_{z\beta}$ for monochromatic and standard regimes

1) $\lambda = 10$; $A_x = A_{z\beta} = A_s = 1$.
2) $\lambda = 0$; $\Delta A_{z\beta} = \Delta A_z$; $A_x = A_z = 1$.

$\|m\|$ \ $\|\ell\|$		2		4		6	
		I	II	I	II	I	II
I	n = ±I	50.	0.	14.	0.	3.	0.
	n = ±3	10.	0.	3.	0.	0.6	0.
2	n = 0	10.	0.72	3.	0.18	0.6	0.03
	n = ±2	1.	0.	0.33	0.	0.06	0
3	n = ±1	1.	0.	0.33	0.	0.06	0.
	n = ±3	0.33	0.	0.11	0.	0.02	0.

Simulation

In the model of computer simulation of beam-beam effects the following features were included: betatron and synchrotron oscillations in transverse and longitudinal directions, noise and damping in all coordinates, beam-beam kick. The position of the opposite beam centre was modulated with a synchrotron oscillations of a particle according to the formula:

$$\bar{\rho} = \rho + \xi \cdot \delta(X, Z_s) \quad (2.1)$$

where the full displacement Z_s is the sum of synchrotron and betatron displacements, normalized to the full vertical size of the beam:

$$Z_s = \frac{Z + \lambda E}{\sqrt{1+\lambda^2}} \quad (2.2)$$

$\delta(a,b)$ are the normalized forces of the interaction with the opposite beam, depending on the normalized to the corresponding sizes of the beam coordinates; X - the coordinate x, normalized to the horizontal size of the beam; Z - the coordinate z, normalized to the vertical betatron size of the beam; E - the energy coordinate, normalized to the energy spread magnitude; λ - the ratio of synchrotron and betatron beam sizes in vertical direction.

Moreover, in our model the affect of the longitudinal oscillations of the particles on the tune ν_z and linear tune shift ξ_z was taken into account (see /9/):

$$\nu_z = \nu_{z0} + \delta\nu_z \cdot E \qquad \xi_z = \xi_{z0}\sqrt{1+A^2E^2} \quad (2.3)$$

where the modulation amplitudes are given by

$$\delta\nu_z = \nu_s A$$
$$A = \ell/\beta_z \quad (2.4)$$

with ℓ standing for the beam length, and β_z - for the beta function at the interaction point. The forces from the opposite beam δ_x, δ_z were computed with a linear interpolation from the grid in X, Z plane, and the values of forces on the grid were calculated with a numerical integration. The main results of simulation were the values of the "monochromaticity

factor" K_m and specific luminosity L_{sp}, defined by the expressions

$$L_{sp} = \frac{1}{N_L}\iint \rho_s(X,Z_s)\, e^{-X^2/2} \cdot e^{-Z_s^2/2}\, dx\, dz \quad (2.5a)$$

$$K_m = \frac{1}{N_K}\iint \rho(X,Z)\, e^{-Z^2/2}\, dx\, dz \quad (2.5b)$$

where $\rho(X,Z)$, $\rho_s(X,Z_s)$ are the equilibrium distribution functions in (X,Z) and (X,Z_s) planes, and normalization constants N_L, N_K are determined from the condition, that in the absence of an opposite beam, when $\xi_x = \xi_z = 0$ and $\rho(X,Z) = exp(-X^2/2 - Z^2/2)$, $\rho_s = exp(-X^2/2 - Z_s^2/2)$, the quantities L_{sp} and K_m equal unity. Thus defined "monochromaticity factor" K_m doesn't depend on the horizontal beam size, and it's dependence on the vertical beam size is the same, as the corresponding dependence of specific luminosity, so that $K_m \sim 1/\sigma_z$ (for a gaussian distribution $\rho(X,Z) \sim exp(-Z^2/2\sigma_z^2)$). Therefore, the real energy resolution gain due to monochromatization will equal, with the beam-beam effects in account, to $K_m\lambda$. The convolution of the distribution function $\rho(X,Z)$ in the integral (2.5) with the unperturbed gaussian distribution corresponds to the "weak-strong" situation of our simulation. The distribution functions $\rho(X,Z)$ and $\rho_s(X,Z_s)$ were computed in the simulation programs as the density distribution of all the particles of the simulation at each iteration step.

The values of the model parameters, constant in all the simulation runs (and corresponding to the planned monochromatic regime of VEPP-4), were: synchrotron frequency $\nu_s = 0.02$; frequency modulation amplitude $\delta\nu_z = 0.06$; beam intensity modulation amplitude $A = 1$; aspect ratio of the opposite beam $\varkappa = 30$; damping time, measured in the number of collizions $N = 3000$; synchrotron/ betatron beam sizes ratio $\lambda = 10$; collizions number in the ring equals one.

For the specific luminosity and monochromaticity factor computation one need to have a comparatively large iterations number in the simulation. Correspondingly, the typical statistic error of the calculated quantity K_m for the absence of an opposite beam $(\xi_{x,z} = 0)$ was 2% for 150 damping times of a total iterations number, and was drastically increasing, when the vertical beam size was growing under the influence of the opposite beam. So, for a 40% increase of a vertical beam size, the typical statistic error of the quantity K_m , calculated from the K_m values of each of ten different initial conditions particles of simulation, was 8% for the same iteration number.

The goal of the simulation was the determining of the specific luminosity and monochromaticity factor dependence on the opposite beam current for the optimum tunes ν_x, ν_z and ratio ξ_x/ξ_z . It doesn't seam possible, however, to carry out a direct optimization in a large number parameters in consideration with a direct computation of a necessary quantities because of an amount of computations required. So the problem of optimization in ξ_x/ξ_z was solved with a help of a special methodics of fast computation of an auxiliary quantity, connected qualitatively with a monochromaticity factor.

It is known, that the main reason of a beam size growth in beam-beam effects is the appearance of nonlinear resonances in a phase space, so that for the trajectories, close enough to these resonances, the amplitudes A_x, A_z oscillate. It is important also, that the libration frequency of resonances, affecting the beam sizes (that is, localed not too far from the coordinate centre and having the libration amplitude $\gtrsim \sigma$), is always much smaller, than the damping time (this condition was discussed in /10/, and its relevance was shown either in /5/). We presume, that the libration amplitude at the moderate amplitude values $A_x, A_{z\beta} \leq 1$ has to be qualitatively connected to the beam size growth, so we choose this quantity for the beam size enlargement estimate.

To avoid the semi-integer resonances $2\nu_{x,z} = K$ influence, leading to a nonresonant oscillation of a corresponding amplitudes A_x, A_z in a rather wide regions, close to these resonances in a tune plane (what isn't leading to a beam size growth), we computed the value of the oscillations of the coordinate Z at $P_z \approx 0$. moments, rather than the oscillations of amplitude themselves. Thus, we computed an auxiliary quantity Δ_z :

$$\Delta_z = \max_i \left[(Z_{max})\Big|_{\frac{P_z}{\sqrt{Z^2+P_z^2}}<0.1} - (Z_{min})\Big|_{\frac{P_z}{\sqrt{Z^2+P_z^2}}<0.1} \right] \qquad (2.6)$$

where Z_{max} and Z_{min} refer to the maximal and minimal values of Z under the condition $P_z/\sqrt{Z^2+P_z^2} < 0.1$ and for Z , belonging to a single trajectory i , and $\max$ means taking a maximum from different trajectories (initial conditions). We had, in our computations, 10 particles with different initial conditions (with $A_{x0} = 1$, $A_{z\beta 0} = 1$ and randomly taken phases θ_{x0}, θ_{z0}) and each particle was interated for 600 steps. It was checked, that the quantity Δ_z didn't increase with further increase of a particle number, or a steps number for each particle. Since we want the quantity Δ_z to characterize a hamiltonian motion, the damping and noise in the process of its computation were switched off.

Examples of level curves of the quantity Δ_z in the plane ν_x, ν_z are shown at the Fig.1,2. The pictures were got with a help of a standard program, with a use of a 40 x 40 grid in the tune plane ν_x, ν_z for a level curves construction. Comparing Fig. 1 and 2, one sees the principal difference of a standard ($\lambda = 0$, Fig.1) and monochromatic ($\lambda = 10$, Fig.2) regimes. So, the strongest in Fig.1 are the coupling resonances $\nu_x = \nu_z$ and $\nu_x + \nu_z = 1$, while the strongest in Fig.2 are the double lines (synchrotron sidebands $n = \pm 1$) of synchrotron resonances $2\nu_x + 2\nu_z = \pm \nu_s$ and $2\nu_x - \nu_s = \pm \nu_s$, what agrees well with a Table 1 data. The pictures of Fig.3 and 4 illustrate the assertion of a $\xi_x/\xi_z \gg 1$ regime preferability, distinct from conventional regime $\xi_z/\xi_x \sim 3$. . The reduction of a ($\ell = 2$, $m = -2$) resonance for $\xi_x/\xi_z \sim 3$ is clearly observed. Moreover, the background height of Δ_z , generated presumably by nonresonance oscillations, is also lower for a larger ξ_x/ξ_z . So, with the data at hand, we choose the ratio ξ_x/ξ_z to be equal to 5. This can be achieved at VEPP-4 facility with the help of a β_x -function at the interaction point increase to 30 m with β_z = 0.05 m. The efforts of a further ξ_x/ξ_z increase via β_x enlargement require an overcomplicated optics of the interaction point region.

To check the methodics of a qualitative beam size estimate with a fast computed quantity Δ_z and to have an exact data about the monochromaticity factor K_m , a numerical simulation was conducted in a model with a noise and damping switched on. The quantity K_m was computed from the formula

(2.5b) with a help of numerical integration. Simultaniously the specific luminosity L_{sp} was computed from the formula (2.5a). To decrease the total computation time the iteration number in each point ν_x, ν_z was chosen in dependence on the value of K_m in this point. Thus, for $K_m > 0.7$ the iteration number was $N_t = 150\,\tau$, where τ is the damping time, with 10 particles; for $0.5 < K_m < 0.7$ we had $N_t = 40\,\tau$ with 6 particles and $N_t = 25\,\tau$ with 6 particles for $K_m < 0.5$. The scheme of iteration number distribution was motivated by a desire to have a better statistical accurasy for a larger values of K_m. The estimate of K_m value, by which a choice of N_t was made, was obtained with the shortest time $N_t = 25\,\tau$.

The results of the computations are presented in Fig.5, where the level curves of K_m are shown in a plane ν_x, ν_z, and the regions with $K_m < 0.38$ and $0.38 < K_m < 0.68$, in contrast to the region $K_m > 0.68$, are marked with a shading. The picture consists of six smaller ones, each computed from a 40 x 40 grid, so that the full picture has a resolution, corresponding to a 120 x 80 grid.

A good qualitative agreement of a Fig.5 picture with a bottom half of a Fig.2 picture confirms the validness of a conjecture of a qualitative connection of K_m decrease with a Δ_z growth and justify the Δ_z -methodics implementation.

The most powerful in a Fig.5, the same as in Fig.2, are the synchrobetatron resonances $\ell = 2$, $m = \pm 1$, $n = \pm 1$, while the following, and approximately equal in strength are the resonances $\ell = 4$, $m = \pm 1$, $n = \pm 1$ and $\ell = 2$, $m = \pm 2$, $n = 0$. The resonances $\ell = 4$, $m = \pm 1$ are more clearly seen in Fig.5, than in Fig.2, and this is presumably related to a higher tune resolution of the first (120 x 80 for Fig.5 with 40 x 20 for Fig.2).

With a help of an additional analysis of L_{sp} values, which were computed simultaneously with K_m, it was cleared out, that the horizontal beam size was strongly enlarged in all the ν_x, ν_z regions with high K_m values in Fig.5, except the regions, close to the integer and semi-integer resonances $\nu_x = 0.5$ and $\nu_x = 1$. This leads to a luminosity and (possibly) beam lifetime deterioration, but doesn't affect the

monochromaticity factor K_m value. Furter on it should be noted, that in a close to the integer resonance region of tunes, as it is known from the experiment, the beam size grows under the strong influence of a "machine" resonance, the existence of which is not taken into account in our model.

Thus, in monochromatic regime, the best in respect to beam-beam effects is the region of close to semi-integer resonce $\nu_x = 0.5$ and not too high tunes $\nu_z < 0.7$. An example of monochromaticity factor K_m and specific luminosity L_{sp} dependence on the opposite beam space charge parameter ξ_z in the considered to be the best working point $\nu_x = 0.525$, $\nu_z = 0.63$ is given in Fig.6. One sees, that with an increase of ξ_z the monochromaticity factor K_m rapidly decreases. The specific luminosity L_{sp} is decreasing at the same time much more slowly. This is quite natural, because the lumunosity decrease occurs only in a situation, when the vertical betatron beam size is of the order of magnitude of the synchrotron size and, consequently, has to be enlarged many times (~ 10).

The "monochromaticity factor" cutoff in a storage rings with a monochromator scheme impose a severe limitation of a maximum ξ, and, consequently, a maximum luminosity, values. The value of ξ_z in monochromatic experiments, according to Fig.6, if we wouldn't allow the energy resolution gain to decrease more, than 20%, can not exceed 0.01 (for $\xi_x/\xi_z = 5$).

Conclusion remarks

Let us repeat in conclusion the main results of the work.

For nonlinear resonances $\ell\nu_x + m\nu_z + n\nu_s = k$ in a monochromatic regime it was shown, that a vertical amplitude oscillation magnitude for the resonances with $m < 4$ is growing with the monochromaticity parameter λ growth, and can substantially exceed the oscillation magnitude in a standard nonmonochromatic regime. The vertical amplitude oscillations for a large aspect ratio parameters don't depend on the linear tune shifts ξ_x, ξ_z values, but decrease, when the ratio ξ_x/ξ_z is getting larger, than $\left|\frac{m}{\ell}\right|\sqrt{\varkappa}$

It was found in a simulation, that the vertical betatron beam size in a monochromatic regime is growing with the opposite beam current increase much faster, than it happens in a nonmonochromatic regime. Thus for the optimized tunes and ratio $\xi_x/\xi_z = 5.$, the vertical beam size is getting 20% larger when $\xi_z = 0.01$. This cause a monochromaticity factor decrease and impose a fundamental limitation of the luminosity in a storage rings with a monochromator scheme.

In conducting the numerical simulation of beam-beam effects one adways meet the problem of how well is the computation model, and a real beam-beam interaction in some particular machine, correspondence. The question isn't simple and the answer can not be given explicitly. For the problem of investigation of a present paper, we have, however, a substantial simplification in this aspect. More particularly, there is a single effect, namely the large synchrotron modulation of an opposite beam space charge field, which is so strong, that it presumably dominates all the others. It is reasonable to assume therefore, that in spite of an obvious incompleteness of the model (such effects, as a machine nonlinearity, a residual beam separation at the interaction point, the second interaction point with not fully separated beams, coherent beam-beam effects, weren't included), the results of a present work will prove to be wright in a future experiments.

Over a considerable period of time the authors frequently discussed the problems of a present study with F.M.Izrailev and G.M.Tumaikin. A substantial help in a simulation methodies choise was given by A.B.Temnikh. To all of them we wish to express our sincere gratitude.

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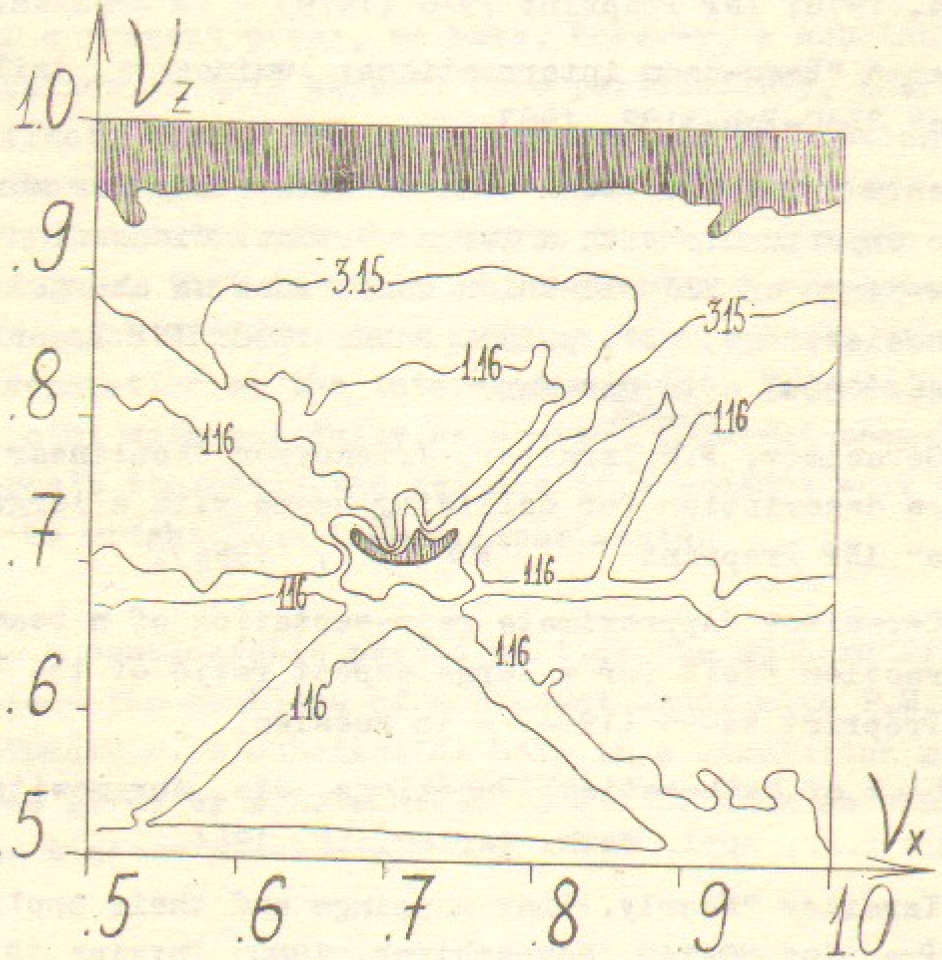

Fig. 1. Level curves of the quantity Δ_z in a plane ν_x, ν_z for a standard regime: $\lambda = 0$, $\xi_x = 0.05$, $\xi_z = 0.10$. The regions with $\Delta_z > 10$ are marked with a shading.

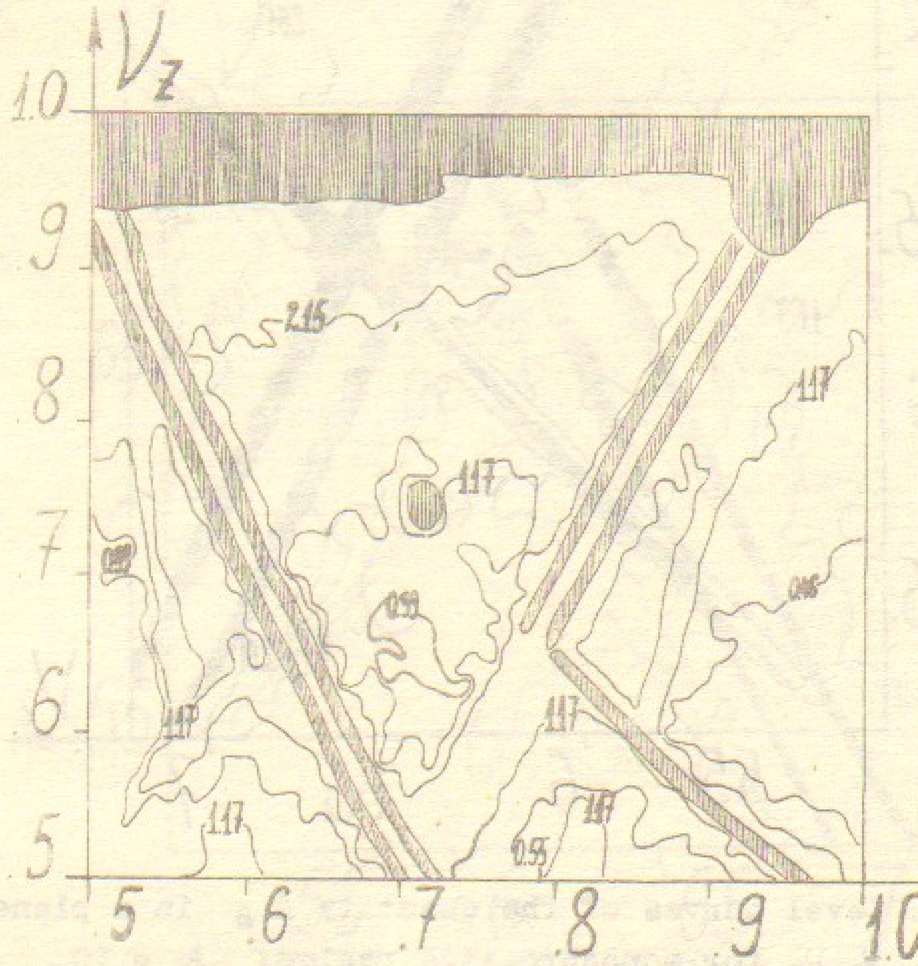

Fig. 2. Level curves of the quantity Δ_z in a plane ν_x, ν_z for monochtomatic regime: $\lambda = 10$, $\xi_x = 0.05$, $\xi_z = 0.01$. The regions with $\Delta_z > 10$ are marked with a shading.

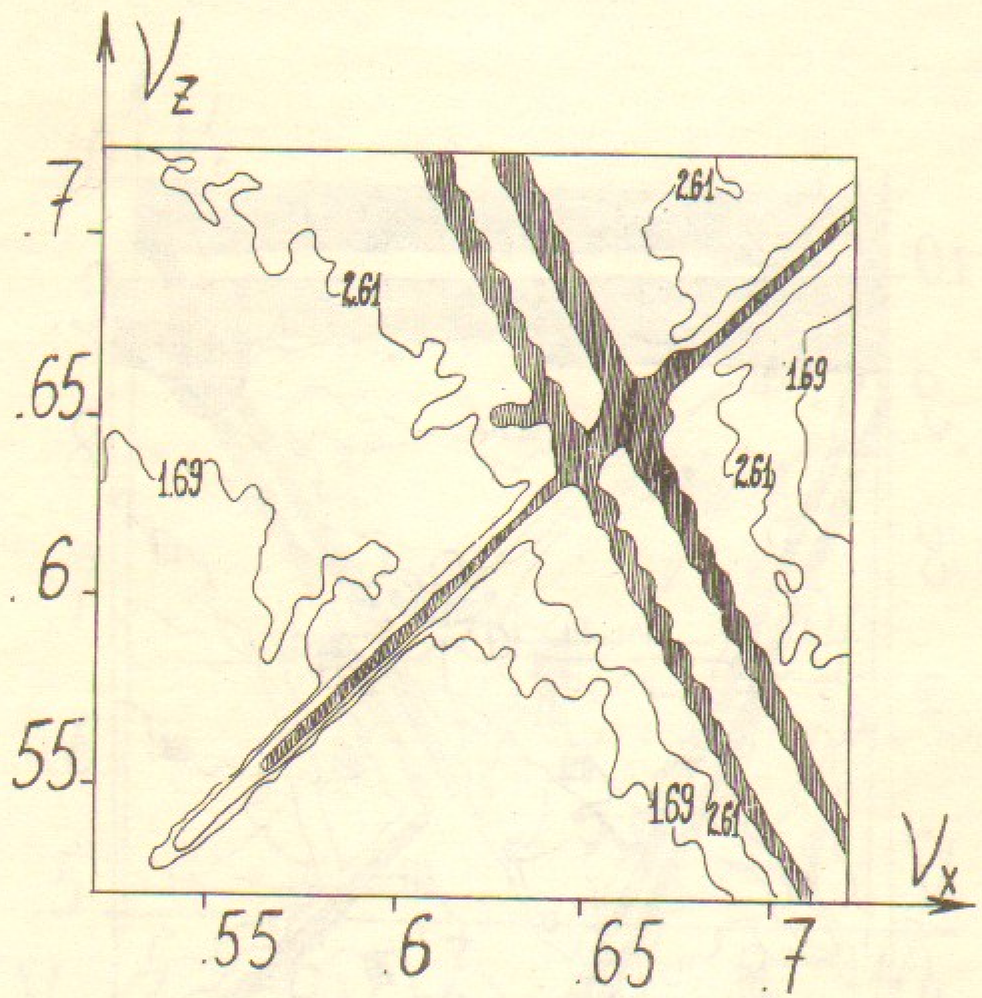

Fig. 3. Level curves of the quantity Δ_z in a plane ν_x, ν_z for monochromatic regime: $\lambda = 10$, $\xi_x = 0.015$, $\xi_z = 0.04$. The regions with $\Delta_z > 10$ are marked with a shading.

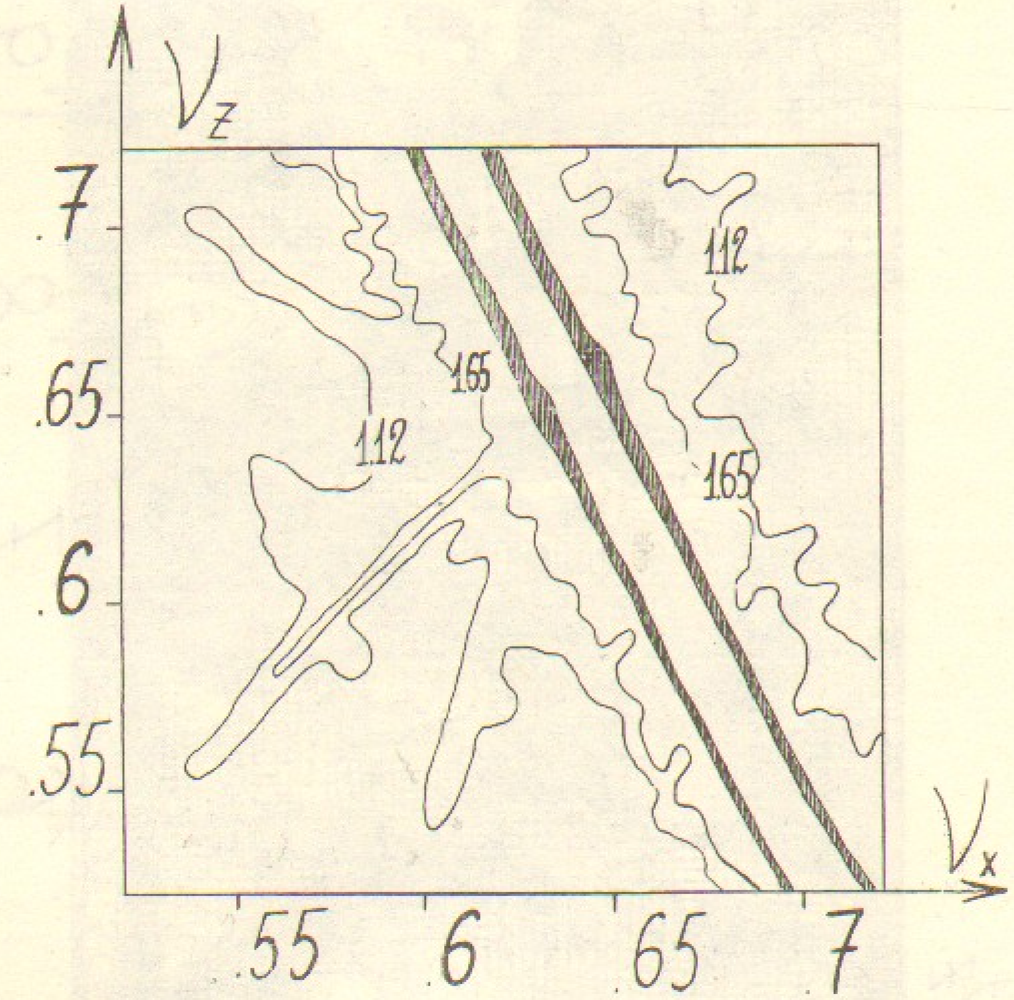

Fig. 4. Level curves of the quantity Δ_z in a plane ν_x, ν_z for monochromatic regime: $\lambda = 10$, $\xi_x = 0.04$, $\xi_z = 0.015$. The regions with $\Delta_z > 10$ are marked with a shading.

Fig. 5. Level curves of the quantity K_m in a plane ν_x, ν_z. The regions with $K_m < 0.38$ are marked with a dense shading, the regions with $0.38 < K_m < 0.68$ - with a sparce shading. Regions with $K_m > 0.68$ are blank.

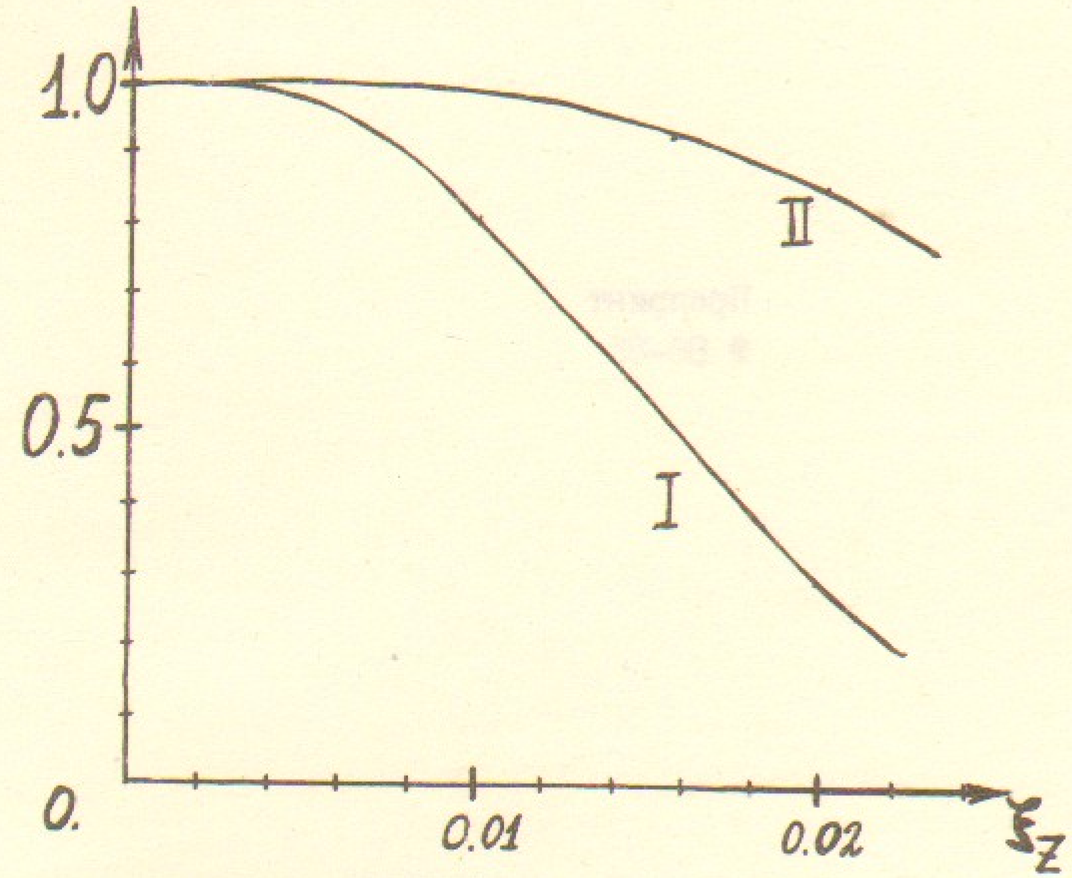

Fig. 6. The dependence of the monochromaticity factor K_m (curve I) and specific luminosity L_{sp} (curve II) on the parameter ξ_z for a constant ratio $\xi_x/\xi_z = 5.$ and tunes $\nu_x = 0.525, \nu_z = 0.63.$

А.Л.Герасимов, А.А.Жоленц

ЭФФЕКТЫ ВСТРЕЧИ В НАКОПИТЕЛЬНЫХ КОЛЬЦАХ
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