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ELECTROWEAK RADIATIVE CORRECTIONS TO THE Z^0 LINE SHAPE IN THE PROCESSES OF $\mu^+\mu^-$ PRODUCTION IN e^+e^- COLLIDING BEAMS

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of $\mu^+\mu^-$ Production in e^+e^- Colliding Beams

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ABSTRACT

The analytical formulae for electroweak radiative corrections to the total cross section of $\mu^+\mu^-$ pair production in e^+e^- collisions are derived for the various inclusive experiments. Calculations are performed around the Z^0 peak within the terms of order $\frac{\delta\sigma}{\sigma} \simeq O\left(\frac{\alpha}{\pi}\right)$.

1. INTRODUCTION

As it is well known the Standard $SU(3)_C \times SU(2)_L \times U(1)$ Model (SM) of the strong and electromagnetic interactions works exceedingly well phenomenologically in describing a large body of experimental data, see e. g. Ref. [1]. However, the theorists remain unhappy even in the face of such a success of the SM, because it is not a fundamental theory and is universally thought to be incomplete. The SM involves too many parameters and does not provide satisfactory answers to the famous gentleman set of the «grand questions». Any extension of the SM (extended gauge models supersymmetry, compositeness, etc.) in one way or another leads to the appearance of new dynamical degrees of freedom.

Besides the direct experimental searches for the new fundamental elements, the manifestations of possible new physics may be discovered by the precise measurements of some observables whose values are predicted unambiguously in the framework of the SM. The credibility of such an indirect approach rests strongly on one's ability to perform the calculations in a reliable way and with the high enough accuracy.

An important role in the realization of the program of stringent testing of the SM should be played by the high energy e^+e^- colliders SLC and LEP, and in the first place in the experiments at the «Z^0 factory», see e. g. Refs [2, 3]. In particular the measurements in the resonance region provide very detailed information on the properties of the neutral intermediate vector boson Z^0 itself.

Thus, owing to the expected high precision of the measurements of the Z^0 parameters [2, 3]

$$\frac{\delta M_Z}{\Gamma_Z} \sim \frac{\delta\Gamma_Z}{\Gamma_Z} = \varepsilon \simeq (0.5 \div 1)\,\%$$

the possibility arises to perform the comparison with the theory at the level of one-loop electroweak radiative corrections (RC). These corrections are sensitive both to the gauge structure of the theory and to the effects of the new fundamental objects (quarks, leptons, standard and nonstandard Higgs bosons, SUSY partners, new W', Z', etc). Therefore, their experimental analysis (even before the direct observation of the three-gauge WWZ vertices and new objects) opens up a new area of testing the theory beyond the tree level.

In order to supply the precision measurements with the theoretical predictions of equivalent accuracy it is necessary to take into account consistently the RC and in particular the QED effects much deforming the shape of the resonance line, see e. g. Refs [2, 4, 5]. Recall that these effects decrease, in particular, the height of the resonance maximum, shift its position to the right by the quantity $\Delta M_Z \sim \beta(M_Z)\Gamma_Z$, where

$$\beta = \frac{4\alpha}{\pi}\left(\ln\frac{W}{m_e} - \frac{1}{2}\right), \quad \beta(M_Z) = 0.108$$

and the so-called radiative tail arises on the right side of the Z^0.

Leptonic process

$$e^+e^- \to \mu^+\mu^- + X. \tag{1}$$

(X represents any number of photons, extra pairs of leptons may or may not be included depending on the experimental conditions) near the Z^0 is of prime interest since here the ambiguities due to strong interaction effects are minimal, and the cleanest test of the electroweak theory can be performed.

In this paper we calculate for the minimal SM the RC to the total cross-section of the process (1) under the different experimental conditions in the energy range of the incoming beams $2E = W$

$$\left|\frac{W - M_Z}{M_Z}\right| = \varkappa \leqslant 0.1\,, \tag{2}$$

corresponding to the primary experiments at SLC and LEP, see Refs [2, 3, 6].*)

*) As it is shown below in this energy region expressions for the RC are essentially simplified (for details, see Ref. [7]).

The plan of the paper is as follows. In Sect. 2 we present the general formula for the inclusive cross-section of $\mu^+\mu^-$ production. This formula is written in terms of the so-called electron structure functions. The so-called hard cross-section σ_{hard} is calculated in Sect. 3. In Sect. 4 we present the final analytical formulae for the cross-sections for the different experimental conditions.

The explicit expressions for some quantities used in the paper are collected in Appendix (for details, see Ref. [7]).

2. INCLUSIVE CROSS-SECTION OF THE $\mu^+\mu^-$ PAIR PRODUCTION

To provide the required accuracy in RC calculations we need to account for the four types of contributions:

1) the sum of the so-called double logarithmic terms $\sim(\beta \ln E/\Delta E)^n$, where ΔE is one of the quantities: σ — the beam energy spread, Γ_Z, $|W - M_Z|$ or the other characteristic of the experimental energy resolution;
2) terms of order β^2;
3) terms of order β;
4) terms of order $\alpha/\pi \simeq 0.23\%$ (neglecting the terms $\sim\varkappa\alpha/\pi$).

Electroweak RC to the process (1) were discussed in many papers adopting different strategies and various degrees of accuracy, see e.g. Refs [8—20]. However, up to now the combined account for the all four types of contributions has not been performed. The above-mentioned papers are naturally subdivided into two basic groups: the papers [8—14] where only the QED effects are considered and those [15—20] where all the electroweak RC are calculated but only at the one-loop level.

Calculations of the RC of types 1—3, connected with the emission from the initial legs, can be performed analogously to the paper by Kuraev and one of the authors [21], see also Ref. [11]. Using the factorization of the electron mass singularities one may present the total cross-section corresponding to Fig. 1 in the form:

$$\sigma(W^2) = \int_{\varepsilon_1}^{1}\int_{\varepsilon_2}^{1} dx_1\,dx_2\,D(x_1, W^2)\,D(x_2, W^2)\,\sigma_{hard}(x_1x_2W^2)\,, \tag{3}$$

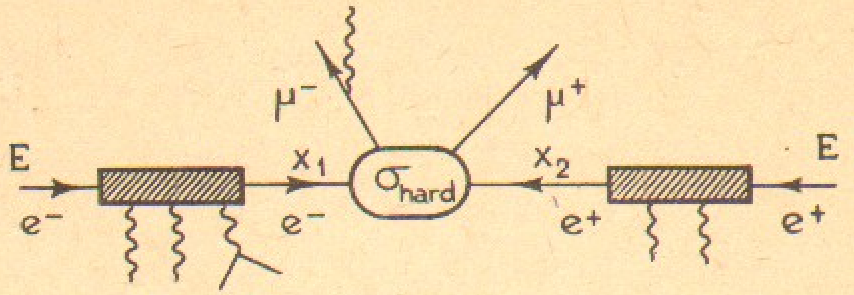

Fig. 1.

where $D(x, W^2)$ is the so-called electron structure function, being calculated in Ref. [21] with the required accuracy.*)

Variables x_1, x_2 are connected with the total energy $\mathscr{P}_0$ of particles produced in the hard process and the component $\mathscr{P}_\parallel$ of their momentum parallel to the electron beam direction by the relation

$$x_{1,2}=\frac{1}{2}\,\frac{\mathscr{P}_0\pm\mathscr{P}_\parallel}{E}, \qquad (4)$$

the limits of integration ε_1, ε_2 are determined by the experimental conditions.

$\sigma_{hard}(q^2)$ is the so-called hard cross-section. In the explicit form of $\sigma_{hard}(q^2)$ there are no electron mass logarithmic singularities,**) if one represents this cross-section in terms of the «running» coupling

$$\alpha(q^2)=\frac{\alpha}{1-P(q^2)},$$

($P(q^2)\equiv \mathrm{Re}\,\Pi_{\gamma\gamma}(q^2)$ is the real part of the photon vacuum polarization operator).

It is convenient to write down the cross-section $\sigma^{(1)}_{hard}(q^2)$ in the one-loop approximation in the form:

$$\sigma^{(1)}_{hard}(q^2)=\sigma_0(q^2)\,(1+2P(q^2)+\delta^{(1)})\,, \qquad (5)$$

where $\sigma_0(q^2)$ is the Born cross-section

*) Function $D(x, W^2)$ includes the effects of soft photon bremmstrahlung in all orders of the perturbation theory. It is described by the simple enough analytical expression unlike the formulae presented in Ref. [11] where the emission was accounted for only up to the second order. Analogously to Ref. [21] we have neglected the production of extra $\mu^+\mu^-$, $\tau^+\tau^-$ pairs and hadrons.

**) Strictly speaking, σ_{hard} may contain the logarithmic terms connected with the final particles, see Sect. 4. In the total inclusive cross-section for the muon pair production such terms are absent.

$$\sigma_0(q^2)=\frac{4\pi\alpha^2}{3q^2}\,[\,1+2v_e^2\ \mathrm{Re}\,(q^2\Delta(q^2))+(a_e^2+v_e^2)^2q^4|\Delta(q^2)|^2]. \qquad (6)$$

Here $\Delta(q^2)$ is the resonance propogator,*)

$$v_e=\frac{4s^2-1}{4sc},\quad a_e=-\frac{1}{4sc}, \qquad (7)$$

where

$$c=M_W/M_Z\,,\quad s=\sqrt{1-c^2} \qquad (8)$$
$$(c\equiv\cos\theta_W\,,\quad s\equiv\sin\theta_W)\,,$$

The relation (5) is presented in such a form that the correction $\delta^{(1)}$ does not contain the electron mass singularity. Since $1+2P(q^2)$ is the first terms in the expansion of $\alpha^2(q^2)/\alpha^2$ and terms $\sim\alpha P(q^2)$ are here neglected, the hard cross-section σ_{hard} can be rewritten in the form:

$$\sigma_{hard}(q^2)=\frac{\sigma_0(q^2)}{(1-P(q^2))^2}\,(1+\delta^{(1)})\,. \qquad (9)$$

Thus the problem reduced to the calculation of the one-loop correction $\delta^{(1)}$.

*) We use the following definition of $\Delta(q^2)$ (for details, see Ref. [7]): $\Delta^{-1}(q^2)=(q^2-M_Z^2)+\frac{(\mathrm{Im}\,\Sigma_T^{\gamma Z}(M_Z^2))^2}{M_Z^2}+iM_Z\,\Gamma_Z(q^2)$, where $\Gamma_Z(q^2)=\Gamma_Z+(q^2-M_Z^2)\times\frac{\mathrm{Im}\,\Sigma_T^{Z'}(M_Z^2)}{M_Z}$. Other definitions of M_Z, Γ_Z (see e. g. Ref. [19]) differ by quantities $\delta M_Z/M_Z\sim\delta\Gamma_Z/\Gamma_Z\sim\alpha^2$. The explicit forms of $\mathrm{Im}\,\Sigma_T^{\gamma Z}(M_Z^2)$, $\mathrm{Im}\,\Sigma_T^{Z'}(M_Z^2)$ are given in Ref. [7]. At $M_Z=92$ GeV$<2m_t$ and $\sin^2\theta_W=0.23$ we have $\frac{\mathrm{Im}\,\Sigma_T^{\gamma Z}(M_Z^2)}{M_Z^2}\simeq-5.6\cdot10^{-3}$, $\mathrm{Im}\,\Sigma_T^{Z'}(M_Z^2)\equiv C_1\simeq2.51\cdot10^{-2}$, and with the required accuracy the second term in $\Delta^{-1}(q^2)$ can be dropped (corrections $\delta\sigma/\sigma\sim0.1\%$). The width Γ_Z will be determined from the fit of the experimental curves and should be compared with the SM predictions, see Refs [17, 19, 30, 31].

Remind that an extra sequential neutrino increases Γ_Z by about 170 MeV and the total width measurement is a simple and effective probe of the new physics.

Note also that it is of importance to compare the top mass measurements with the precise results on M_Z. The point is that in the framework of SM the allowed regions of m_t and M_Z are correlated and the deviations may indicate the presence of new degrees of freedom.

3. CALCULATION OF THE HARD CROSS-SECTION σ_{hard}

Using eq. (3) it is easy to see that for finding $\sigma^{(1)}_{hard}$ one should calculate the one-loop order RC to the total cross-section $\sigma(q^2)$ and subtract from it the right-hand part of eq. (3), where the σ_{hard} and one of the D-functions are taken in Born approximation ($\sigma_{hard}=\sigma_0$, $D^{(0)}=\delta(1-x)$) and another D-function is taken in the one-loop approximation.

Note that owing to the condition (2) the one-loop RC to the total cross-section of the process (1) reduced only to the self-energies and the vertex corrections and to the contribution of the emission from the initial and final particles without account for their interference. The point is that the contributions from the interference between the lowest-order diagrams and the so-called box diagrams as well as the other possible interference terms prove to be of the order of $\varkappa\alpha/\pi$ or $v_e\alpha/\pi$ ($v_e \ll 1$), and can be neglected here, see Refs [7, 19, 22].

Thus we have with the required accuracy

$$\delta^{(1)}=\frac{2\alpha}{\pi}(A_\mu+A_e)+\delta^{(1)}_{virt}-2P(q^2)\,, \tag{10}$$

where

$$A_\mu=-\frac{1}{4}\Lambda_{1(\mu)}+\frac{3}{8},\quad A_e=-\frac{1}{4}(\Lambda_{1(e)}+1)+\frac{\pi^2}{6}, \tag{11}$$

$$\Lambda_{1(l)}=-2\ln\frac{q^2}{\lambda^2}(L_l-1)+L_l^2+L_l+4\left(\frac{\pi^2}{3}-1\right),\quad l=e,\mu, \tag{12}$$

$L_l=\ln\frac{q^2}{m_l^2}$, λ is the fictious photon mass introduced to regularize the infrared divergences. The term $\delta^{(1)}_{virt}$ in the right-hand side of eq. (10) corresponds to the account for the virtual one-loop RC. The term $\frac{2\alpha}{\pi}A_\mu$ is connected with the emission from the final muons, $\frac{2\alpha}{\pi}A_e$ is the contribution of the initial particle emission without the part that has been already included in the structure functions $D(x_i, W^2)$.

Each term in eq. (10) taken separately contains both the infrared and the collinear singularities. However, in the sum (10) the contributions proportional to $\Lambda_{1(\mu)}$, $\Lambda_{1(e)}$ are cancelled together with

the electromagnetic vertex parts included in $\delta^{(1)}_{virt}$. The term $(-2P)$ in the right-hand side of eq. (10) is cancelled with the vacuum polarization contributions included in $\delta^{(1)}_{virt}$. As a result, the cross-section σ_{hard} (see eqs (9), (10)) contains only the hard noncollinear contributions.

The calculations of the self-energies and the vertex corrections in various renormalization schemes have been performed by several working groups, see Refs [15, 23—30]. We shall adopt here the on-shell renormalization scheme (see Ref. [24]) where the masses M_W, M_Z, M_H, m_f of the gauge and Higgs bosons and of fermions are used as the physical parameters together with the electromagnetic fine structure constant α.

In this scheme, by definition, the tree level relation for the weak mixing parameter $\sin^2\theta_W=1-\frac{M_W^2}{M_Z^2}$ remains. Inside the on-shell scheme itself we fix the renormalization procedure adopted in Ref. [28] to have the possibility of comparing the intermediate results with other papers.

With δ_{virt} calculated and substituted into eq. (10), the final expression for the hard cross-section is given by

$$\sigma_{hard}(q^2)=\sigma_0(q^2)(1+\delta)=\frac{\sigma_0(q^2)}{(1-P(M_Z^2))^2}(1+\delta^{(1)})\,, \tag{13}$$

where $\delta^{(1)}=\delta_V+\delta_z$.

$$\delta_V=\frac{\alpha}{\pi}\left[\frac{1}{16s^2c^2}\Lambda_2(M_Z^2,M_Z)-\frac{1}{2s^2}\Lambda_2(M_Z^2,M_W)+\right.$$
$$\left.+\frac{3c^2}{s^2}\Lambda_3(M_Z^2,M_W)+\frac{\pi^2}{3}+\frac{1}{4}\right], \tag{14}$$

$$\delta_z=-2\,\mathrm{Re}\left[\Sigma_T^{Z'}(M_Z^2)-\frac{\Sigma_T^{\gamma}(M_Z^2)}{M_Z^2}-\right.$$
$$\left.-\frac{(1-2s^2)}{s^2}\left(\frac{\Sigma_T^W(M_W^2)}{M_W^2}+\frac{2s}{c}\frac{\Sigma_T^{\gamma Z}(0)}{M_Z^2}-\frac{\Sigma_T^Z(M_Z^2)}{M_Z^2}\right)\right]. \tag{15}$$

Here and below $\Sigma_T^Z(q^2)$, $\Sigma_T^\gamma(q^2)$, $\Sigma_T^W(q^2)$, $\Sigma_T^{\gamma Z}(q^2)$ are the nonrenormalized self-energies for the $Z-Z$, $\gamma-\gamma$, $W-W$, $\gamma-Z$ propogators respectively, determined analogously to Ref. [28]; $\Sigma_T^{Z'}(q^2)\equiv\frac{d\Sigma_T^Z}{dq^2}$. Λ_2, Λ_3 are the functions originated from the calculation of the vertex corrections, see Ref. [28]. $\Lambda_2(M_Z^2, M_Z)$ is connected with the diagram of Fig. 2,a, $\Lambda_2(M_Z^2, M_W)$ with the diagram of Fig. 2,b and $\Lambda_3(M_Z^2, M_W)$ with the diagram of Fig. 2,c.

The term $\frac{\alpha}{\pi}\left(\frac{\pi^2}{3}+\frac{1}{4}\right)$ in the right-hand side of eq. (14) is of the pure electromagnetic origin, cf. eqs (10), (11). The term δ_z arises owing to the renormalization of M_W, M_Z, α from their bare values to the physical ones: M_W, M_Z and $\alpha(M_Z^2)$.

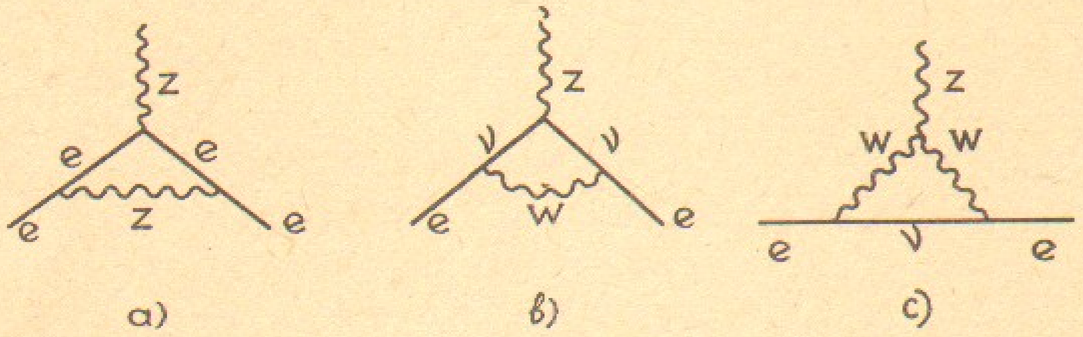

Fig. 2.

The functions Λ_2, Λ_3, δ_z are given in the Appendix. Some deviations from the results of Refs [16, 28] are indicated in Ref. [7].

The expression for $P(M_Z^2)$ includes the known contributions from the leptons, W-bosons and hadrons

$$P = P_l + P_W + P_h. \tag{16}$$

In the minimal SM

$$P_l = \frac{\alpha}{3\pi}\sum_{l=e,\mu,\tau}\left(\ln\frac{M_Z^2}{m_l^2}-\frac{5}{3}\right), \tag{17}$$

$$P_W = \frac{\alpha}{4\pi}\left[(3+4c^2)\left(2-2\sqrt{4c^2-1}\ \mathrm{arctg}\,\frac{1}{\sqrt{4c^2-1}}\right)-\frac{2}{3}\right]. \tag{18}$$

Unlike the terms P_l, P_W, the hadronic contribution P_h can not be calculated analytically because of the strong interaction ambiguities. This contribution has been considered in details by several authors, see e. g. Refs [26, 30]. We shall use here the value presented in Ref. [30]

$$P_h = 0.0286. \tag{19}$$

The quantities P_l, P_W, Λ_2, Λ_3 do not depend on the unknown (for the time being) parameters m_t, M_H. We shall present here the numerical results calculated for

$$M_Z = 92\ \text{GeV}, \quad \sin^2\theta_W = 0.23. \tag{20}$$

$$\frac{\alpha}{16\pi s^2c^2}\Lambda_2(M_Z^2, M_Z) = 0.09\cdot10^{-2}, \quad -\frac{\alpha}{2\pi s^2}\Lambda_2(M_Z, M_W) = -0.6\cdot10^{-2},$$

$$\frac{3\alpha c^2}{\pi s^2}\Lambda_3(M_Z^2, M_W) = -0.68\cdot10^{-2}, \quad P_l = 3.15\cdot10^{-2}, \quad P_W = 0.05\cdot10^{-2}. \tag{21}$$

Then

$$P = 6.06\cdot10^{-2}, \quad \delta_V = -0.36\cdot10^{-2}. \tag{22}$$

As it follows from eqs (21), (22) and formula (13) the cross-section σ_{hard} is sensitive within the required accuracy to the corrections connected with P only.

The correction δ_z can be represented as the sum of three terms:

$$\delta_z = \delta_z^{(0)} + \delta_z^{(H)} + \delta_z^{(t)}. \tag{23}$$

Here $\delta_z^{(0)}$ is the contribution of the known leptons, quarks and gauge bosons, the term $\delta_z^{(H)}$ is connected with the Higgs boson contribution, the term $\delta_z^{(t)}$ originates from the (t, b) dublet.

For the parameters (20)

$$\delta_z^{(0)} = 1.88\cdot10^{-2}. \tag{24}$$

The corrections $\delta_z^{(H)}$ and $\delta_z^{(t)}$ as the functions of M_H, m_t are plotted in Figs 3, 4 respectively. Fig. 5 illustrates the dependence of δ on m_t at $M_H = 100$ and 800 GeV.

4. INCLUSIVE MEASUREMENTS AND FORMULAE FOR THE CROSS-SECTIONS

As it has been already mentioned, theoretical ambiguities due to strong interactions are minimal for the leptonic process (1). Besides, this process has the important advantages connected with the background situation and with the high triggering efficiency, see e. g. Ref. [32].

The inclusive measurements of the cross-section of the process (1) near the Z^0 can be made under various experimental conditions.

Let us at first discuss the case when only events without extra charged leptons are selected experimentally.

Using the variable x via the relation

$$x_1x_2 = 1 - x \tag{25}$$

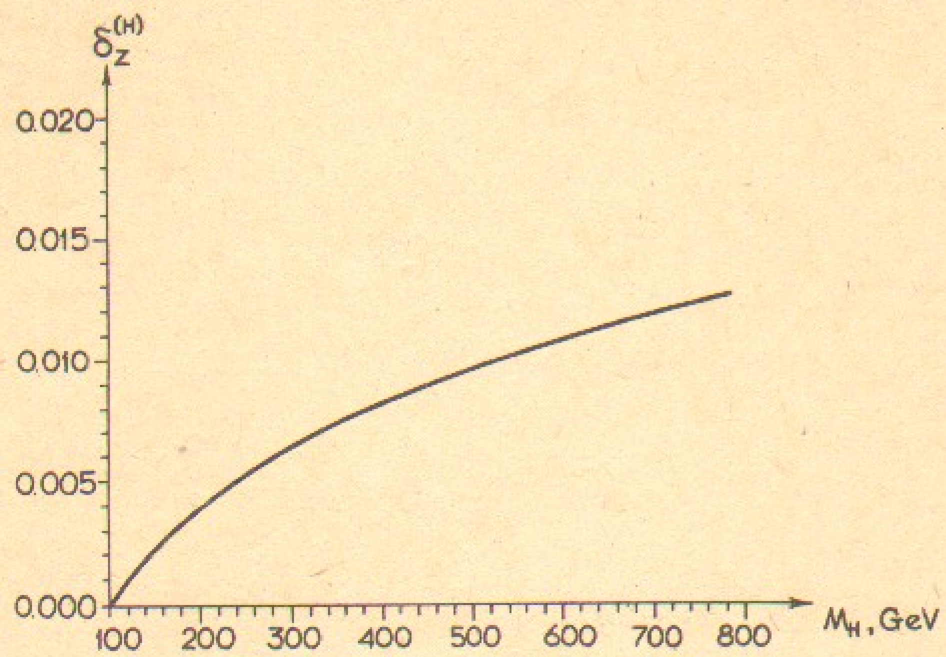

Fig. 3. $\delta_Z^{(H)}$ as a function of the Higgs boson mass M_H.

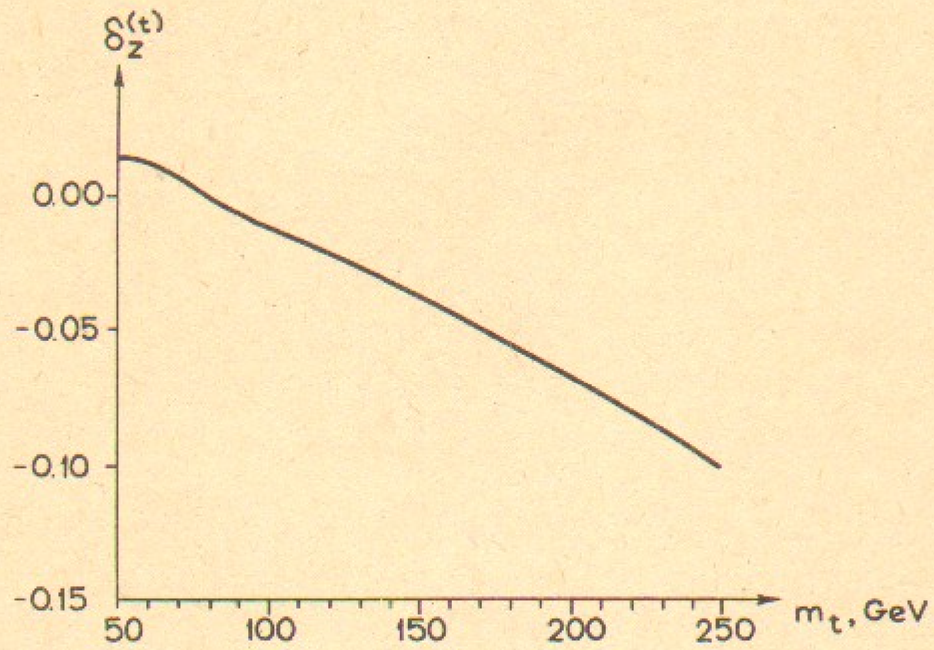

Fig. 4. $\delta_Z^{(t)}$ as a function of m_t.

after the integration of eq. (3) at fixed x one obtains the expression for an inclusive (over photons) cross-section

$$\sigma^{(\gamma)}(W^2) = \int\limits_0^{x_{max}} dx\, \sigma_{hard}((1-x)\, W^2)\, \tilde{F}_1(x, W^2)\,, \tag{26}$$

where

$$\tilde{F}_1(x, W^2) = \int\limits_0^1 \int\limits_0^1 dx_1\, dx_2\, \delta(1-x-x_1 x_2)\, D^{(\gamma)}(x_1, W^2)\, D^{(\gamma)}(x_2, W^2)\,. \tag{27}$$

Here $D^{(\gamma)}(x_i, W^2)$ are the electron structure functions without regard to the contribution of the real e^+e^--pairs. The integral (27) has been calculated in Ref. [21], the explicit form for $\tilde{F}_1(x, W^2)$ is given by

$$\tilde{F}_1(x, W^2) = \beta x^{\beta-1}\left[1 + \frac{3}{4}\beta - \frac{1}{24}\beta^2\left(\frac{L_e}{3} + 2\pi^2 - \frac{37}{4}\right)\right] - \beta\left(1 - \frac{x}{2}\right) +$$

$$+ \frac{\beta^2}{8}\left[4(2-x)\, \ln\frac{1}{x} + \frac{1}{x}(1+3(1-x)^2)\, \ln\frac{1}{1-x} - 6 + x\right]. \tag{28}$$

The term $\sim\beta^2 L_e$ in the right-hand side of eq. (28) arises from the electron contribution to the vacuum polarization. In the totally inclusive cross-section the terms of such a type should cancel each other.

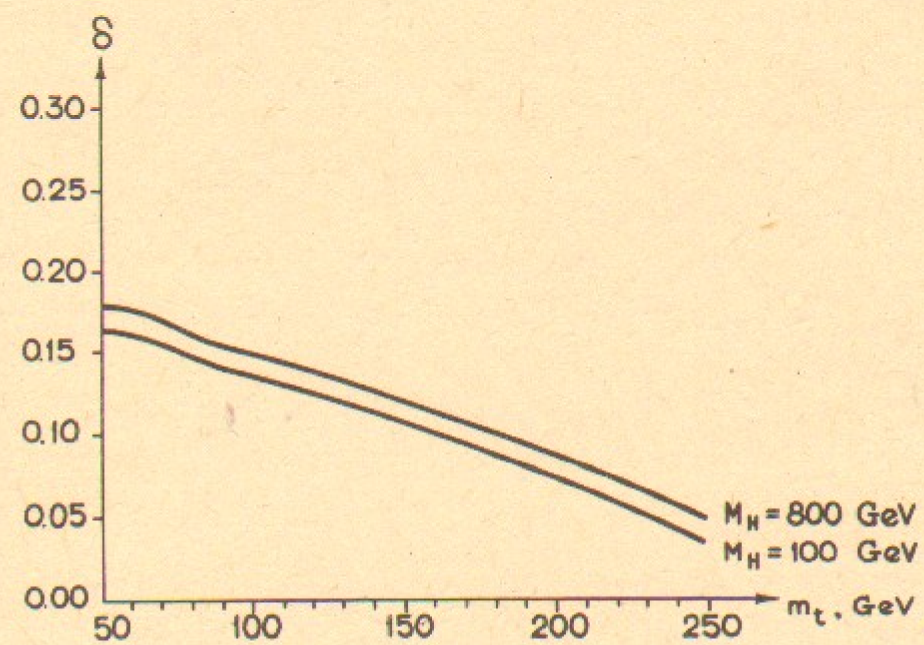

Fig. 5. Total electroweak correction δ as a function of m_t for $M_H = 100$ GeV and $M_H = 800$ GeV.

The upper integration limit x_{max} and the magnitude of the hard cross-section σ_{hard} in eq. (26) depend on the experimental conditions.

If the experimental constraints are absent x_{max} is determined by kinematics only: $(1-x_{max})W^2=4m_\mu^2$.

Owing to the prohibition of the real e^+e^--pair production by the final muons the logarithmic corrections might appear in the expression for σ_{hard}. However, numerically these corrections turn out to be of the order of $\frac{\delta\sigma}{\sigma}\leqslant 0.1\%$ and we shall drop them here.

Nevertheless, the application of σ_{hard} in the form of eq. (13) leads to the relative error roughly equal to 2% near the lower limit of the energy interval (2), where this error is maximal (in the cross-section maximum the error is of the order 0.1%). The error appears since eq. (26) contains integration over energies but the expression (13) gives σ_{hard} only for the energy range near the Z^0.

To make this error lower than 0.1% it is enough to account for the energy dependence of the photon vacuum polarization $P(q^2)$ and for the low energy behaviour of the photon contribution to the total cross-section, see Ref. [7].

Thus the inclusive (over photons) cross-section is given by formula (26), where

$$x_{max}=1-\frac{4m_\mu^2}{W^2} \qquad (29)$$

and

$$\sigma_{hard}(q^2)=\frac{\sigma_0(q^2)}{(1-P(q^2))^2}\left(R(q^2)+\delta^{(1)}\right), \qquad (30)$$

$$R(q^2)=\left(1+\frac{2m_\mu^2}{q^2}\right)\sqrt{1-\frac{4m_\mu^2}{q^2}}. \qquad (31)$$

The cross-section $\sigma_0(q^2)$ is given by formula (6), where

$$\Delta^{-1}(q^2)=q^2-M_Z^2+iM_Z\left(\Gamma_Z+\frac{q^2-M_Z^2}{M_Z}C_1\right),$$

$$C_1\simeq 2.51\cdot 10^{-2}\quad \left(\text{for}\quad m_t>\frac{M_Z}{2}\right), \qquad (32)$$

see the footnote in Sect. 2.

The above version of the inclusive measurement of the cross-section of the process (1) is not free from shortcomings.

First of all, the essential contribution to the cross-section originates from the region of energies in σ_{hard} much lower than M_Z where the uninteresting photon mechanism dominates. Secondly, there are purely experimental difficulties since one should tune out from the various QED background processes with the production of extra e^+e^--pairs.

The most important is the reaction of $\mu^+\mu^-$ pair electroproduction due to the so-called double-photon mechanism (see Fig. 6,*a*) or

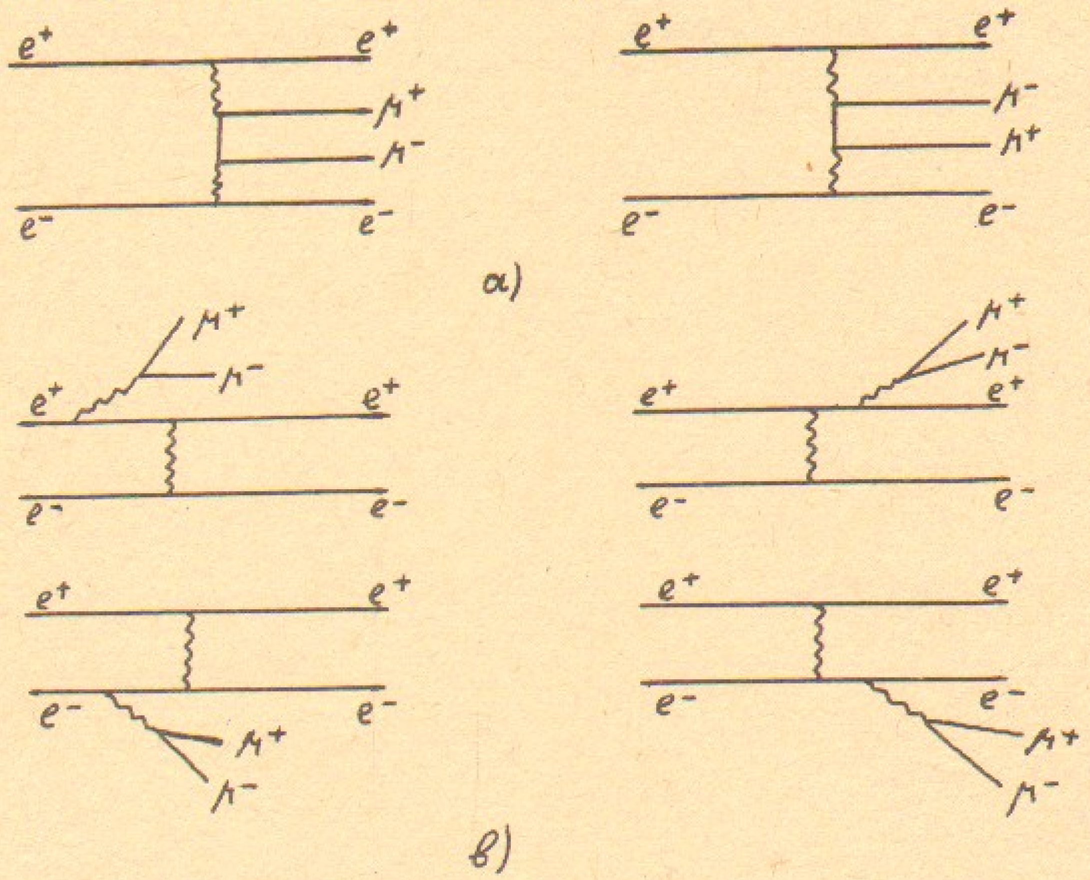

Fig. 6.

due to the so-called bremsstrahlung mechanism (see Fig. 6,*b*). The total cross-sections of these processes are very large. Thus the cross-section σ_{DP} corresponding to the double-photon mechanism is two order of magnitude higher than the cross-section of the process (1) in the resonance peak, see e. g. Ref. [5]. The bulk of the background QED contributions comes from the kinematical region where the final electron and positron move along the direction of the incoming beams, thereby impeding their detection. The final muon pair has the low effective mass: $S_\mu=(p_{\mu^-}+p_{\mu^+})^2\sim 4m_\mu^2$. These kinematical configurations differ essentially from that corresponding to the process (1) near the Z^0.

The contribution from the electroproduction processes can be suppressed, e. g. by imposing the experimental cut on S_μ. Such a cut can also remove the first shortcoming. The additional possibilities appear from the direct subtraction of the QED backgrounds using the data obtained in the measurements below the Z^0.

If one chooses the events corresponding to the experimental contraint $S_\mu > S_0$, then the integration in formula (26) should be performed up to $x_{\max} = 1 - \frac{S_0}{W^2}$, and it is necessary to subtract from the σ_{hard} the terms $\Delta\sigma_\gamma$ and $\Delta\sigma_{\gamma\gamma}$, corresponding to the photon emissions by muons in the frequency range that is not allowed due to the S_μ cut.

The contribution $\Delta\sigma_\gamma$, connected with the single photon emission, can be obtained using the results of Ref. [33]

$$\Delta\sigma_\gamma(q^2) = \frac{\sigma_0(q^2)}{(1-P(q^2))^2}\, \delta_\gamma(S_0/q^2)\,, \tag{33}$$

$$\begin{aligned}\delta_\gamma(t) &= \frac{2\alpha}{\pi} \int_{1-t}^{1} \frac{dx}{x}\left(1 - x + \frac{x^2}{2}\right)\left(\ln \frac{S_0(1-x)}{t m_\mu^2} - 1\right) = \\ &= \frac{\alpha}{\pi}\Big[\left(\ln\left(\frac{S_0}{m_\mu^2}\right) - 1\right)\left(-2\ln(1-t) - t - \frac{t^2}{2}\right) + t + \frac{t^2}{4} + \\ &\quad + 2\int_0^t \frac{dx}{x}\ln(1-x)\Big].\end{aligned} \tag{34}$$

With the required accuracy it is necessary to remain in the quantity $\Delta\sigma_{\gamma\gamma}$ only the terms of order $\sigma_0(q^2)\left(\frac{\alpha}{\pi} L_\mu\right)^2$, see Ref. [7]:

$$\begin{aligned}\delta_{\gamma\gamma} &= 2\left(\frac{\alpha}{\pi} L_\mu\right)^2 \Big[-\ln^2(1-t) + \ln t \ln(1-t) + \\ &\quad + t(2+t)\left(\frac{3}{8}\ln t - \frac{1}{2}\ln(1-t)\right) - \frac{t}{2} - \frac{t^2}{16} - \int_0^t \frac{dx}{x}\ln(1-x)\Big].\end{aligned} \tag{35}$$

The final result is as follows: for the experimental constraint $S_\mu \geqslant S_0$ the inclusive (over photons) cross-section of the process (1) in the energy region (2) is given by eq. (26), where

$$x_{\max} = 1 - \frac{S_0}{W^2},$$

$$\sigma_{hard}(q^2) = \frac{\sigma_0(q^2)}{(1-P(q^2))^2}\left[R(q^2) + \delta^{(1)} - \delta_\gamma\left(\frac{S_0}{q^2}\right) - \delta_{\gamma\gamma}\left(\frac{S_0}{q^2}\right)\right]. \tag{36}$$

For $S_0 \ll M_Z^2$ one may neglect the terms δ_γ, $\delta_{\gamma\gamma}$ in eq. (36), and formula (30) is restored. As has already been noted, to suppress the QED backgrounds one should put $S_0 \sim M_Z^2$. Then one can substitute into eg. (36) the values $P(q^2) = 6.0\cdot 10^{-2}$, $R(q^2) = 1$.

In Fig. 7 we have plotted the total cross-section as a function of W for $M_Z = 92$ GeV, $\sin^2\theta_W = 0.23$, $\Gamma_Z = 2.55$ GeV, $m_t = 100$ GeV, $M_H = 200$ GeV, $S_0 = \frac{3}{4} M_Z^2$.

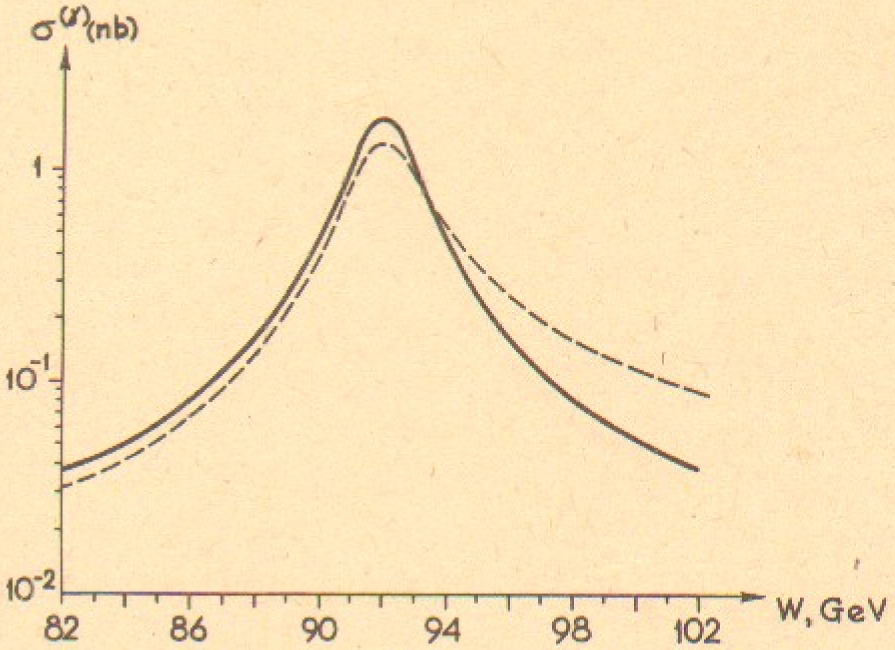

Fig. 7. Total cross-section for $e^+e^- \to \mu^+\mu^- +$ photons near Z^0 as a function of W for $M_Z = 92$ GeV, $\sin^2\theta_W = 0.23$, $\Gamma_Z = 2.55$ GeV, $m_t = 100$ GeV, $M_H = 200$ GeV, $S_0 = \frac{3}{4} M_Z^2$. The dashed line corresponds to $\sigma_0(W^2)$.

Let us discuss now the case of inclusive measurements when extra e^+e^--pairs are included in the definition of the process (1). Here the using of the S_0 cut gains the principal importance because of the necessity to suppress the QED electroproduction contributions. Emphasize that in such a case the contribution of the bremsstrahlung electroproduction mechanism is automatically incorporated into the general formula (3) (the so-called singlet part of the structure function $D^{(s)}(x, W^2)$, see Ref. [21])*).

Using the cut on $S_\mu (S_\mu > S_0 \gg 4m_\mu^2)$ we can rewrite the cross-sec-

*) The cross-section σ_{DP} for the double-photon mechanism can be naturally incorporated into formula (3), that is easily generalized, see Ref. [7].

tion of the process (1) in the form

$$\sigma(W^2, S_0)=\int_0^{1-S_0/W^2} dx \frac{\sigma_0(S_x)}{(1-P(S_x))^2}\left(1+\delta^{(1)}-\delta_\gamma\left(\frac{S_0}{S_x}\right)-\delta_{\gamma\gamma}\left(\frac{S_0}{S_x}\right)\right)\times$$

$$\times[(\tilde{x}, W^2)+(\tilde{x}, W^2)]+\sigma_{DP}(W^2, S_0)\,, \quad S_x\equiv W^2(1-x)\,. \qquad (37)$$

Here with the required accuracy (cf. Ref. [21])

$$(\tilde{x}, W^2)=\left(\frac{\alpha}{\pi}\right)^2\left\{\frac{2}{3}x^{\beta-1}\left(\ln\frac{Wx}{m_e}\right)^2(2-2x+x^2)+\right.$$

$$\left.+\frac{L_e^2}{2}\left[\frac{2}{3}\,\frac{(1-(1-x)^3)}{1-x}+(2-x)\ln(1-x)+\frac{x}{2}\right]\right\}\theta\left(x-\frac{4m_e}{W}\right). \qquad (38)$$

The double-photon contribution can be found e. g. in Ref. [5], see eqs (4.10), (4.11).

For $S_0\ll M_Z^2$ it is possible to neglect in (37) the terms δ_γ, $\delta_{\gamma\gamma}$. For $S_0\sim M_Z^2$ one may put $P(q^2)=6.0\cdot10^{-2}$ and omit the contribution connected with $\tilde{F}_2$ (in the energy region (2) this contribution does not exceed 0.5% for $S_0>0.1M^2$).

At last for $S_0>0.4M^2$ the contribution from the double-photon mechanism in the region (2) is also less than 0.5%.

CONCLUSION

We have presented in this paper the analytical formulae for the $O\left(\frac{\alpha}{\pi}\right)$ RC to cross-section of the process (1) calculated for the minimal SM. The numerical results are given for $M_Z=92$ GeV, $\sin^2\theta_W=0.23$, $\Gamma_Z=2.55$ GeV. The presented formulae correcsponding to the different experimental conditions permit one in principle to provide the measurements of the Z^0 parameters with the accuracy $\frac{\delta M_Z}{M_Z}, \frac{\delta\Gamma_Z}{\Gamma_Z}\sim O$ (1%), which is comparable with the theoretical uncertainties in Γ_Z due to the strong interaction effects.

The approach used here for calculation of the QED effects (based on the use of the electron structure functions) may be applied also to calculations of the RC to the other processes, e. g. $e^+e^-\to e^+e^-$, $e^+e^-\to\nu\bar{\nu}\gamma$ as well as to the analysis of the various asymmetries in the leptonic reactions near the Z^0.

In the framework of this approach one may extend the applicability of the results obtained and in particular to eliminate the constraint (2).

The results may be effectively used when discussing the line shapes of the possible new heavy resonances (toponium, Z', etc.) where the double-logarithmic parameter $\beta\ln\frac{M}{\Gamma}$ might happen to be of the order 1.

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Appendix

The functions $\Lambda_2(K^2, m)$, $\Lambda_3(K^2, m)$ introduced in eq. (14) for $0<K^2<4m^2$ are given by (see also Refs [7, 28])

$$\Lambda_2(K^2, m)=-\frac{7}{2}-\frac{2}{t}+\left(3+\frac{2}{t}\right)\ln t-2\left(1+\frac{1}{t}\right)^2\int_0^t\frac{dx}{1+x}\ln x\,; \qquad (A1)$$

$$\Lambda_3(K^2, m)=\frac{5}{6}-\frac{2}{3t}+\frac{2}{3}\left(1+\frac{2}{t}\right)\sqrt{\frac{4-t}{t}}\operatorname{arctg}\sqrt{\frac{t}{4-t}}-$$

$$-\frac{8}{3t}\left(2+\frac{1}{t}\right)\left(\operatorname{arctg}\sqrt{\frac{t}{4-t}}\right)^2, \qquad (A2)$$

where $t=K^2/m^2$.

The quantity δ_z is the sum of fermion and boson terms.

Each fermion doublet leads to the contribution (we neglect here the mixing):

$$\delta_z^{(D_+,D_-)}=\frac{\alpha}{2\pi}\operatorname{Re}\sum_{f=\pm}\left\{\mathcal{K}_2(\eta_f,\eta_f,1)\left(\frac{1}{6s^4}-\frac{2fQ_f}{3s^2}\right)-\right.$$

$$-\eta_f[\mathcal{K}_2(\eta_f,\eta_f,1)-\ln\eta_f]\left[\frac{8s^2Q_f^2}{3c^2}+\frac{4fQ_f(1-2s^2)}{3s^2c^2}+\frac{(1-2s^2)}{6s^4c^2}\right]-$$

$$-\mathcal{K}_2(\eta_f,\eta_{-f},c^2)\left(\frac{1-2s^2}{6s^4}\right)\left(1-\frac{\eta_f}{c^2}-\frac{\eta_f(\eta_f-\eta_{-f})}{c^4}\right)+$$

$$+\mathcal{K}_2'(\eta_f,\eta_f,1)\left[\frac{4}{3}(1+2\eta_f)\left(\frac{s^2}{c^2}Q_f^2-\frac{fQ_f}{2c^2}+\frac{1}{8s^2c^2}\right)-\frac{\eta_f}{2s^2c^2}\right]-$$

$$\left.-\frac{(1-2s^2)}{6s^4c^2}\left[\eta_f\ln\eta_f+\frac{(\eta_f-\eta_{-f})}{c^2}\,\eta_f(\ln\eta_f-1)\right]\right\}. \qquad (A3)$$

Here one sums over the doublet components, Q_f is the charge of the corresponding component, $\eta_f = \frac{m_f^2}{M_Z^2}$, $\mathcal{K}_2(x, y, z)$ was introduced in Ref. [16]

$$\mathcal{K}_2(x,y,z)=\int_0^1 dt \ln(xt+y(1-t)-zt(1-t)) =$$

$$= -2+\frac{1}{2}\ln(xy)+\frac{(x-y)}{2z}\ln\left(\frac{x}{y}\right)+$$

$$+\begin{cases} -\frac{\sqrt{a_+a_-}}{z}\ln\left(\frac{\sqrt{a_+}+\sqrt{a_-}}{\sqrt{a_+}-\sqrt{a_-}}\right), & a_-\geqslant 0 \\ +2\frac{\sqrt{a_+(-a_-)}}{z}\,\mathrm{arctg}\,\frac{\sqrt{-a_-}}{\sqrt{a_+}}, & a_-<0,\ a_+>0 \\ \frac{\sqrt{a_+a_-}}{z}\left(\ln\left(\frac{\sqrt{-a_+}+\sqrt{-a_-}}{\sqrt{-a_-}-\sqrt{-a_+}}\right)-i\pi\right), & a_+\leqslant 0 \end{cases} \qquad \text{(A4)}$$

where $a_\pm = (\sqrt{x}\pm\sqrt{y})^2 - z$, $\mathcal{K}_2'(x, y, z) \equiv \frac{\partial}{\partial z}\mathcal{K}_2(x, y, z)$.

For $\eta_\pm \ll 1$

$$\delta_z^{(D_+,D_-)} \simeq \frac{2\alpha}{3\pi c^2}\left[s^2(Q_+^2+Q_-^2)+\frac{(1-2s^2)}{4s^2}\left(1-\frac{c^2\ln c^2}{s^2}\right)\right]. \qquad \text{(A5)}$$

If $|\eta_+ - \eta_-| \gg 1$, then

$$\delta_z^{(D_+,D_-)} \simeq \frac{\alpha}{4\pi}\frac{(1-2s^2)}{s^4c^2}\left[\frac{\eta_+\eta_-}{(\eta_+-\eta_-)}\ln\left(\frac{\eta_+}{\eta_-}\right)-\frac{(\eta_++\eta_-)}{2}\right]. \qquad \text{(A6)}$$

It is convenient to separate in the bosonic term the Higgs boson part

$$\delta_z^{(H)} = \frac{\alpha}{2\pi}\,\mathrm{Re}\left\{\mathcal{K}_2(1,\eta_H,1)\frac{1}{s^4c^2}\left[1-2s^2+\frac{\eta_H}{6}(5s^2-2)-\frac{\eta_H^2}{12}(3s^2-1)\right]-\right.$$

$$-\mathcal{K}_2(c^2,\eta_H,c^2)\frac{(1-2s^2)}{s^4}\left(1-\frac{\eta_H}{3c^2}+\frac{\eta_H^2}{12c^4}\right)+$$

$$+\mathcal{K}_2'(1,\eta_H,1)\frac{1}{3s^2c^2}\left(3-\eta_H+\frac{\eta_H^2}{4}\right)-\frac{\eta_H}{12s^2c^2}\left(1-\eta_H\left(3-\frac{1}{c^2}\right)\right)\ln\eta_H-$$

$$\left.-\frac{\eta_H}{12s^4c^2}(1-2s^2)\ln c^2+\frac{\eta_H}{6s^2c^2}-\frac{\eta_H^2}{4s^2c^2}\left(1-\frac{1}{3c^2}\right)\right\}. \qquad \text{(A7)}$$

The remaining bosonic contribution is

$$\delta_z^{(b)} = \frac{\alpha}{2\pi}\,\mathrm{Re}\left\{\mathcal{K}_2(c^2,c^2,1)\left(4s^2-\frac{64}{3}+\frac{27}{s^2}-\frac{33}{4s^4}\right)-\right.$$

$$-\mathcal{K}_2(1,c^2,c^2)\frac{(1-2s^2)}{s^2}\left(4+\frac{17}{12c^2}-\frac{33}{4s^2}+\frac{1}{12c^4}\right)+$$

$$+\mathcal{K}_2'(c^2,c^2,1)\left(\frac{1}{12c^2}+4c^2-10\frac{c^2}{s^2}+\frac{7}{4s^2}-\frac{1}{3}\right)-$$

$$-\left(4s^2-\frac{40}{3}-\frac{1}{12c^2}+\frac{115}{12s^2}-\frac{17}{12s^4}\right)\ln c^2+$$

$$\left.+16-\frac{26}{3s^2}+\frac{1}{3c^2}+\frac{1}{12c^4}\right\}. \qquad \text{(A8)}$$

δ_z is given by eq. (23), where

$$\delta_z^{(0)} = \delta_z^{(b)} + 3(\delta_z^{(u,d)}+\delta_z^{(c,s)}) + \delta_z^{(\nu_e,e)} + \delta_z^{(\nu_\mu,\mu)} + \delta_z^{(\nu_\tau,\tau)},$$

$$\delta_z^{(t)} = 3\delta_z^{(t,b)}. \qquad \text{(A9)}$$

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Electroweak Radiative Corrections to the Z^0 Line Shape in the Processes of $\mu^+\mu^-$ Production in e^+e^- Colliding Beams

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Электрослабые радиационные поправки к кривой возбуждения Z_0 бозона в процессе образования пары $\mu^+\mu^-$ во встречных e^+e^- пучках

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