ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

E.V. Shuryak

TOWARD THE QUANTITATIVE THEORY OF THE TOPOLOGICAL EFFECTS IN GAUGE FIELD THEORIES I. PHENOMENOLOGY AND THE METHOD OF COLLECTIVE COORDINATES

PREPRINT 87-54

НОВОСИБИРСК

Toward the Quantitative Theory of the Topological Effects in Gauge Field Theories I. Phenomenology and the Method of Collective Coordinates

E.V. Shuryak

Institute of Nuclear Physics
630090, Novosibirsk, USSR

ABSTRACT

This paper in the first in the series of works, applying the method of collective coordinates to the description of ensemble of interacting topological fluctuations, the so called «instanton liquid». We first present the results of the semiclassical theory of instantons and then proceed to a summary of known phenomenological facts, both coming from «real» experiments (for QCD) and from the lattice data (for the SU(2) gluodynamics). Then we describe the main ideas of the collective coordinate method (in the case when such coordinates are not based on a symmetry) and derive the main formulae for the resulting effective theory in the semiclassical approximation.

1. INTRODUCTION

Nonabelian gauge fields are the main ingredient of the theories, by means of which we hope to understand strong, weak and electromagnetic interactions (and may be even their unification). Although the famous «asymptotic freedom» has allowed many useful calculations at the perturbative level, the nonperturbative phenomena are much more complicated, and, in spite of multiple efforts, we are still far from their understanding.

Several years ago much excitement was raised by first successful numerical experiments [1] based on the lattice formulation of these theories [2]. Their attractive feature is the fact that information about the gauge vacuum and the lowest excitations is obtained directly from the first principles of the theory. During these years many works has been made in this direction, which have indeed provided a lot of interesting information.

And nevertheless, the main goals of this programm are still very far from being reached. First of all, experience have shown that the nonabelian gauge theories in four dimensions are indeed too difficult problem for such a straightforward approach. Even consideration of the simplest quantum mechanical examples (we consider one of them in details in paper IV of this series) clearly demonstrates, that in order to obtain an accurate description of the ground state wave function, the correlation functions etc. the number of lattice sites (per dimension) should be taken at least of the order of few hundreds, or even thousends. Obviously, it is impossible to do in four dimensions with available computers.

Moreover, experimental data indicate that in QCD there exist some nontrivial dependence of the behaviour of the correlation function on the quantum numbers of the «probes» used [3]. In particular, in the spin-zero channels the violation of the perturbative predictions takes place already at distances of the order of 1/20 fermi! It is possible only if the nonperturbative fields are very inhomogeneous, possessing nontrivial hierarhy of intrinsic scales [4]. Obviously these facts make straightforward numerical experiments on the lattice to be especially difficult.

Another problem is even deeper. Imagine that in some future all technical limitations are overcomed and some supercomputer will reproduce the exact values of hadronic masses etc. But we need not only the correct numbers, but rather some insight into the problem, which can only be reached by a development of some approximate but understandable models. The present series of papers is an attempt of this kind, in which the gauge field vacuum is approxmated by the «instanton liquid».

The main disadvantage of the lattice parametrization of the fields is the necessity to use too many parameters. (In practice, people use up to a million of them per one configuration!) Reduction in their number is badly needed, but this is possible only if one manage to select the minority of the «most important» coordinates from the majority of the «noninteresting» ones.

One known possibility is to use the renormalization group approach, based on the «smoothed fields» (or «block spins»). It proved to be useful in the perturbative context, as well as in the theory of critical phenomena. However, in both these problems the fluctuations are similar at all scales, therefore the effective theory for the «smoothed fields» differs with the original one only in small variations of the coupling constants. It is impossible to apply this method if the underlying physics is rapidly changes at some fixed scale, as it takes place for the gauge theories under consideration.

In this series of works we study another approach (also known in many physical contexts) based on the introduction of some «collective coordinates» for the gauge fields. For example, let me mention that the field in magnetics can be described in terms of the domain wall positions; the deformation in solids can be described by the positions of dislocations; the flow velocity in liquids can be reconstructed from the positions of the vortex lines, etc. As a more nontrivial example let me mention collective coordinates instroduced for the description of the solitons (in fact, being the «close relatives» to our instantons).

Instanton-type fluctuations take place in various places in (Euclidean) space-time, which are in fact correlated due to mutial interaction. Therefore, our effective theory reminds that of some liquid. Although for such models it is somewhat more difficult to write the efficient codes, but, comparing them with the lattice studies one can see that the economy in the number of parameters is enormous. Indeed, as we demonstrate in next papers of this series, for the description of the «instanton liquid» one needs the number of parameters per fixed space-time volume by the factor 10^4-10^5 smaller than it is used in current lattice works. That is why, with a modest computer we are able to study many phenomena which remains beyond reach of modern lattice calculations.

One more comment is that although in the present series of works we concentrate mainly on the topological fluctuations, in principle the methods developed can be used in more general context as well. For example, even in the simplest quantum mechanical problem (e.g. that to be considered in IV), apart of instantons (the tunneling events) there exist also other strong fluctuations (the «fluctons»), being events in which the particle spontaneously goes far into the classically forbidden region, and we show that they can indeed be studied quite similarly to instantons.

This work is structured as follows. It starts with a very brief summary of the present stage of the instanton physics (a detailed review can be found e.g. in Ref. [5]) in which the main formulae (Sect. 2), phenomenological facts (Sect. 3, 4) and the necessary references are presented. We turn to our own programm in sections 5—7, considering the general collective coordinate method in the case when these coordinates are not specified by some exact symmetry. Finally, in section 8 we outline the content of the subsequent works of this series.

2. INSTANTON DENSITY IN DILUTE GAS APPROXIMATION

The topologically nontrivial classical configurations of the nonabealian gauge fields, «pseudoparticles» (below PPs) or instantons and anti-instantons, were discovered in the classical paper [6] by Belavin, Polyakov, Schwartz and Tyupkin in 1975. It was immedeately recognized that this discovery has opened completely new perspectives in the gauge field theory, showing that the barriers separating the topologically distinct sets of gauge fields, are penetrable. Instanton is the path connecting the topologically distinct sectors, possessing the minimal possible action. Therefore it is a solution of the Yang—Mills equation.

Throughout this work we use it in the so called singular gauge

$$A^a_\mu = 2\bar{\eta}^a_{\mu\nu} x_\nu \rho^2/[x^2(x^2+\rho^2)] \tag{1}$$

where $\bar{\eta}$ are the so called t'Hooft symbols. (For anti-instantons one should put η instead of $\bar{\eta}$.) Our field normalization is such that the field strength and the gauge action are as follows

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu ,$$

$$S = -\frac{1}{4g^2} \int d^4x (G^a_{\mu\nu})^2 . \tag{2}$$

The semiclassical theory of instantons was developed in the classical paper by G. t'Hooft [7]. If one considers strong fluctuations of this kind, posessing strong field $G^a_{\mu\nu} \sim \rho^{-2}$ in the region of the small size ρ, he may be sure that it happens rarely and therefore it is possible to ignore all other fluctuations of such kind and to consider this fluctuation placed in the «perturbative» vacuum. Such approach, known as the «dilute gas approximation» (DGA), leads to the following expression for the PP density (instantons and anti-instantons together):

$$\frac{d^5 n_{PP}(\rho)}{d^4z\, d\rho} = (2C_{N_c}/\rho^5)[\beta(\rho)]^{2N_c - b'/2b} \left(\frac{b}{2}\right)^{b'/2b} \times$$

$$\times \exp\left[-\beta(\rho) + (2N_c - b'/2b)(b'/2b) \ln \beta(\rho)/\beta(\rho)\right] (1 + 0(1/\beta)), \tag{3}$$

where

$$C_{N_c} = \frac{4.66 \exp(-1.68 N_c)}{\pi^2 (N_c-1)!\,(N_c-2)!};$$

$$\beta(\rho) \stackrel{\text{def}}{=} b \ln(1/\rho\Lambda_{PV}).$$

The particular definition of lambda parameter here is made via the Pauli-Villars regularization method (for other definitions one should change the coefficient C_{N_c}). Note also that in (3) we have included the two-loop effects in the effective charge (as it was suggested in [8], following [9]). The constants b, b' here are the well-known coefficients of the Gell-Mann-Low function

$$b = \frac{11}{3} N_c - \frac{2}{3} N_f ,$$

$$b' = \frac{34}{3} N_c - \frac{13}{3} N_c N_f + \frac{N_f}{N_c} \tag{4}$$

with N_c, N_f being the number of colors and light quark flavors, respectively.

Expression (3) is valid in the semiclassical domain, where the action per pseudoparticle $\beta(\rho) \gg 1$. One may observe that, extrapolating this expression literally to small β, it leads to a peak in the distribution over radii and then even vanishes at $\beta \to 0$. Unfortunately, this peak is in the region where one cannot trust expression (3). Therefore, the integral density (and other integral effects) cannot be estimated in dilute gas approximation. Multiple attempts to use similar expressions for the evaluation of some observable effects (e.g. see [10]) have failed, and it was finally realized that in order to do so one has to go beyond this approximation and to face the question about the physical nature of the effects which may cut off the integral over ρ at large ρ.

Logically speaking there are two candidates: (i) mutial repulsive interaction of the PPs; (ii) «melting» of PPs due to large quantum fluctuations. Below we are going to show that in fact the former effect dominates, stabilizing the instanton density at the point where quantum corrections are still small and the semiclassical approach is quite reasonable.

3. PHENOMENOLOGY OF THE «INSTANTON LIQUID»

Soon after discovery of the instantons it was realized that they may be responsible for such important effects as the solution of the Weinberg U(1) problem [7], the spontaneous breaking of the chiral SU(N_f) symmetry [10], etc. (In other words, it was susspected that it was the instanton-induced forces between quarks which make the η' meson so heavy and the pion so light). Unfortunately, due to the shortcomings of the «dilute gas approximation» it was not possible to make any theoretical estimates of the magnitude of these effects, thus these suspicions have remained to be a plausible although unproved hypothesis.

New page in the development of the instanton physics was open with the development of the so called «QCD sum rules» [11], connecting the correlation function of various operators with hadronic phenomenology. The value of the so called «gluonic condensate» has set the scale for the nonperturbative fields. As a result, one had the following phenomenological upper limit on the *PP* density

$$n_{PP} \stackrel{\text{def}}{=} (N_+ + N_-)/V < n_{\max} = \frac{1}{32\pi^2} \langle G^2 \rangle \simeq (197\ \text{MeV})^4 . \quad (5)$$

Another important number came from the physics of the η' meson, it is the value of the so called «topological susceptibility» [12] (in fact, in the world without light quarks):

$$\chi_{top} = \lim_{(V\to\infty)} \langle (N_+ - N_-)^2 \rangle / V = (180-190\ \text{MeV})^4 \quad (6)$$

where N_+ and N_- is the number of instantons and anti-instantons in the (space-time) volume V considered.

Generally speaking, this quantity depends both on the *PP* density in vacuum and on the correlations of their positions in space-time. In particular, in the simplest case of independent Poisson distribution over N_+ and N_- one can see that this quantity is just equal to the *PP* density because

$$\langle (N_+ - N_-)^2 \rangle = \langle N_+ + N_- \rangle . \quad (7)$$

Thus it is tempting to ascribe two independent experimental numbers (5, 6) to instantons, taking only one free parameter, the *PP* density of the order of 1 fm^{-4}.

(It was noticed by Novikov et al [3] that the dilute instanton gas with its Poisson distribution over N_+, N_- cannot be the true picture, as it contradicts to the low energy theorem. Indeed, ascribing all nonperturbative field to instantons, one finds from it that the fluctuations in total *PP* number should have statistics different from the Poisson one because this theorem leads under these assumptions to

$$\langle (N_+ + N_-)^2 \rangle - \langle N_+ + N_- \rangle^2 = (4/b) \langle N_+ + N_- \rangle \quad (8)$$

and not just to $\langle N_+ + N_- \rangle$. However, (8) follows from very general arguments related to the correct renorminvariant dependence of the calculated vacuum energy, so it should hold in any selconsistent model. In particular, as it was shown by Dyakonov and Petrov [9], (8) does take place for their approximate theory of the «instanton liquid».)

The most important conclusion drown by Novikov et al [3] from the analysis of the QCD sum rules was the following one: in the spin-zero channels data indicate appearence of some nonperturbative corrections already at the momentum transfer of the order of $Q^2 = 20\ \text{GeV}^2$. Moreover, it seemed impossible to describe them by usual operator-product-expansion formulae. It means that the QCD vacuum does contain strong nonperturbative fields!

In a series of papers [4] I have attempted to fit all these data by some simple instanton-based model. It has just two free parameters, the *PP* density and the typical instanton radius. Not going into details I may just summarize that a lot of phenomenological facts were found to agree with their values

$$\bar{\rho} \simeq 0{,}3\ \text{fm}, \quad n_{PP} \simeq 1\ \text{fm}^{-4}, \quad (9)$$

If so, one may make four important observations:

(i) the vacuum is reasonably dilute:

$$n_{PP}\, \bar{\rho}^4 \ll 1 ; \quad (10)$$

(ii) instantons are nearly classical:

$$\beta(\bar{\rho}) = b \ln\left(\frac{1}{\bar{\rho}\,\Lambda_{PV}}\right) \sim 10 \gg 1 ; \quad (11)$$

(iii) interaction does not spoil instantons

$$\Delta\beta^{int} \sim 2 \ll \beta(\bar{\rho}) ; \quad (12)$$

(iv) interaction is not negligible, so we have some «liquid» rather than a dilute gas:

$$\exp |\Delta\beta^{int}| \gg 1 . \quad (13)$$

It is clear that this «instanton liquid» picture of the QCD vacuum, being a phenomenological observation, cannot shed any light on the gauge theories other than QCD. However, later studies made by Dyakonov and Petrov [9] (their variational approach is discussed below) have lead to essentially the same vacuum properties for quarkless gauge theories as well.

4. INSTANTONS ON THE LATTICE

The question of whether the ensemble of the gauge fields generated on the lattice does or does not contain essential fraction of the topologically nontrivial fluctuations was raized from the first days of these studies, but it turns out not so easy to answer it. Only rather recent studies have resulted in more or less definite conclusions on this issue. We first say some words on the «problems» and then come to the results.

Generally speaking, any studies of the topological effects on the lattice is rather tricky bisiness because in this case the groundstone of the topology is lost: all discretized configurations can be connected by a continuous transformations. As a result, all definitions of the topological charge on the lattice are some conventions, counting only the fluctuatons of the size much larger than the lattice spacing. (We are going to discuss this point in IV in details, using as a toy models the quantum-mechanical double-well system). As phenomenology suggest the typical instanton size in QCD to be around 0.3 fm, while the typical lattice spacing used is about 0.15—0.20 fm, such cut off may produce significant systematical errors.

Another problem (common to all lattice studies) is related to matching of the unites used. In principle, there is no question about the relation between the lattice parameter Λ_L (defined in terms of the lattice spacing and the bare lattice coupling g) and Λ_{PV} used through this work. In particular, for the simplest SU(2) gauge theory without fermions (for which most of the studies are made) the calculations of various physical effects can be compared for both regularizations, with the well-known result

$$\Lambda_{PV}=21.5\cdot\Lambda_L\,. \tag{14}$$

However, it is not quite clear whether the bare lattice coupling g is indeed small enough, so that all diagrams but the two-loop ones (used in the derivation of (14)) can indeed be neglected. The lattice data themselves demonstrate, that the so called «asymptotic scaling» is not yet very accurate. This means that so far the observables do depend not only on Λ_L (as they asymptotically should do), but on other details of the «numerical experiments» as well. Therefore, one should use «exact» expressions like (14) only keeping in mind that some systematic corrections to them are possible.

As it was mentioned above, the upper bound on the *PP* density follows from the value of the nonperturbative gluonic condensate. for the SU(2) theory the best statistical accuracy ($\simeq 5\%$) was reported by Ishikawa et al. [13]:

$$\langle G^2\rangle \simeq 8\cdot 10^8\,\Lambda_L^4 \tag{15}$$

which leads to

$$n_{PP} < n_{\max} \simeq 1{,}3\cdot 10^6\,\Lambda_L^4\,. \tag{16}$$

However, the lattice definition of the «nonperturbative» fields is based on the subtraction of large «perturbative» ones, and the systematic errors due to them are much larger than the given statistical errors. And nevertheless, one may hope that (15) holds at least up to the factor of two.

First attempts to measure net topological charge Q the gauge field configurations in the box [14] have lead to surprisingly small values of the topological susceptibility

$$\chi_{top} \simeq 2\cdot 10^3\,\Lambda_L^4\,. \tag{17}$$

However, later other definitions of the topological charge and essentially larger lattices have lead to much larger χ_{top}. In particular, the definition suggested by P. Woit was used in the experiment made by the Prinston group [15] with the result:

$$\chi_{top} \simeq 10^5\,\Lambda_L^4\,. \tag{18}$$

Using another definition of Q due to Lusher, DESY group [16] has found somewhat larger value

$$\chi_{top} \simeq 2.6\cdot 10^5\,\Lambda_L^4\,. \tag{19}$$

Both results (18, 19) demonstrate reasonably accurate scaling, so their descrepancy in absolute magnitude is presumably due to different cut off implied by these two definitions of Q. Thus, it is a measure of the systematic errors involved, showing that we know the order of magnitude of χ_{top}, at best.

New chapter in the «hunt for instantons» was connected with the aplication of the «cooling» method [17]. Starting with a configuration from the ensemble, the programm minimizes the action till some minimum is reached. It may be either the trivial one (zero field), or that posessing the nonzero topological chage. These minima have passed a number of tests, showing that the objects observed are indeed instantons: they do have about the needed action, exactly one fermion zero mode, etc. Thus, now we have also some clear lower bound for the *PP* density:

$$n_{PP} > n_{\min} \sim 2 \cdot 10^4 \Lambda_L^4 . \quad (20)$$

It is still much lower than the upper bound (16), but it clearly shows that the early estimates (17) were wrong.

Moreover, recent data reported by the ITEP group [18] show clear evidences that the «survived» instantons do have radii about 1/3 of the interparticle spacing R, and even strong evidences for the PP repulsion, from the distribution over R.

Of course, one may well criticize this method too. Say, these instantons are not really the action minimum if the standard Wilson action is used, therefore they have «finite lifetimes». Also, one may argue that close instanton — anti-instanton (and even instanton — instanton) pairs most probably annihilate during the minimization process, so the picture seen «a posteriori» is strongly distorted.

Summarizing this section we may say that although the real accuracy of the particular numbers mentioned is not yet good enough (and even is not yet well understood), the steady progress is observed. The numbers for the topological succeptibility have changed by two orders of magnitude and are definitely convergent. It was shown, that what is seen is, at least sometimes, the true instantons, possessing the continuous limit, and not just some topologically nontrivial lattice «defects». And what is the most important: due to these works the lattice-oriented people are becaming convinced, that the topological phenomena are indeed the important ingredients of the gauge field vacuum, providing significant fraction (or may be even the main part) of the nonperturbative fields.

5. THE GENERAL IDEA OF THE COLLECTIVE COORDINATE METHOD AND THE VARIATIONAL APPROACH

The main technical problems of any quantum field theory deal with evaluation of some statistical sum Z over all configurations of the system. Generally speaking, the collective coordinate method is just a selection of some subset of «interesting» variables $a_i (i=1, N)$ out of the (infinite) set of the rest («noninteresting») ones. The standard formal trick used is the incorporation of the Faddeev — Popov unite factor into the statistical sum:

$$1 = \int \prod_{i=1}^{N} da_i \quad J(A, a) \prod_{i=1}^{N} \delta(I_i(A, a)) . \quad (21)$$

Here $I_i(A_\mu^a(x), a)$ are N «conditions» and $J(A(x), a)$ is the corresponding Jacobian. Taking the integration over a_i apart and performing first integration over all other variables, one obtains the so called «effective theory» expressed in terms of the collective variables a:

$$z_{eH} = \int \prod_{i=1}^{N} da_i \exp(-S_{eH}(a)) ,$$

$$\exp(-S_{eH}(a)) = \int DA \exp\{-S[A(x)]\} \cdot J(A, a) \prod_{i=1}^{N} \delta(I_i(A, a)) . \quad (22)$$

Applications of this method are well known in the simplest case, in which the definition of all collective variables is governed by some exact symmetry. For example, if one has found one single-instanton solution, he may well construct the whole set of them using translational, rotational and dilatational invariance of the classical Yang — Mills theory. (For the SU(2) color group it leads to some «plane» in the configuration space, parametrized by 8 parameters.) For all directions (again, in the configuration space) transverse to this plane the action increases, thus the integrand in the statistical sum is peaked at this plane, which is the basis of t'Hooft semiclassical theory. The 8-dimensional integral over this plane is left intact: the action is constant on it, so the sum is proportional to its volume.

This lesson should teach us, that the basis for any similar calculation should be some n-dimensional manifold, to be called the «ansatz plane» below, which is able to absorbe the non-gaussian part of the functional integral. However, generally speaking it is not necessary to hold the action constant on it. What we actually want to do is to integrate safely over the «transverse» coordinates, remaining integration over the collective coordintes for separate investigation. Therefore, we have first face the question whether the integrand is indeed peaked in the transverse directions.

In the present series of papers we consider superpositions of instantons and anti-instantons, and these configurations do not possess any exact symmetries: the action in general depends notrivially on all coordinates. Moreover, in this case there are no classical configurations available: for any of them the first variation of the action, the current, is nonzero

$$j^a_\mu(x)=\frac{\delta S}{\delta A^a_\mu(x)}\Big|_{A=A^{\text{ansatz}}}=(D^{ab}_\mu\, G^b_{\mu\nu})\Big|_{A=A^{ansatz}}\neq 0\,. \qquad (23)$$

Therefore, writing the field potential as the sum of the «ansatz» and the «fluctuating» part

$$A^a_\mu(x)=A^{ansatz}_{a\mu}(x)+b_{a\mu}(x)\,. \qquad (24)$$

one generally has the so called linear term in the action expansion,

$$S(A)=S(A^{ansatz})+\int d^4x\, j^a_\mu(x)\, b_{a\mu}(x)+\ldots \qquad (25)$$

which «shifts» the mean values of A away from our ansatz.

Dyakonov and Petrov [9] have made quite a radical step, omitting this «linear term» (25). The physical idea behind it can be explained as follows. Suppose one has added some external current $j^{ext}(x)$ which exactly compensates the linear term, forcing the integrand to have its maximum just at our ansatz. However, this external current makes some work on the system, therefore its energy becomes greater than that in the unperturbed vacuum:

$$\varepsilon(j^{ext})>\varepsilon(j^{ext}=0)=\varepsilon_{vac}\,. \qquad (26)$$

This inequality (which they have regorously proved using the Feynman variational principle) is the essence of their work: they have suggested the variational approach to the problem. Indeed, taking better ansatz one find smaller current and, respectively, lower $\varepsilon(j^{ext})$.

However, this interesting work has left a lot of questions open. What ansatz is the best possible one, and how to find it? Is it possible, even in principle, to get rid of the current completely, so that the estimated quantity $\varepsilon(j^{ext})$ be arbitrarily close to ε_{vac}?

6. WHAT ANSATZ IS THE BEST?

Imagine the action distribution in the configuration space (we remind that its point is some field configuration $A(x)$) for a set of instantons and anti-instantons. The resulting «landscape» is similar to that in some mountain country, it posesses a series of «peaks» (extremely unprobable configurations) separated by a complicated

system of deep «valleys». If the *PPs* are well separated and nearly noninteracting, the action on the valley bottoms is nearly constant. If they approach each other, the slopes of the valleys increases, but it still may be much smaller on the bottoms than on the «walls».

Such structure suggests the idea of natural separation of the «longitudinal» coordinates along the valleys from the «transverse» ones mentioned above. Indeed, everybody who once was in the mountains knows that there exist some outstanding line in each valley: it is the «streamline» on its bottom. Formally speaking it is the line which is the minimum in the transverse directions. Another its definition: it is the line at which direction of the «driving force» (the first derivative) is tangent to it.

In the gauge theory this «driving force» is in fact the «current» j. It can be decomposed into two parts, the longitudinal and the transverse one, according to the chosen ansatz plane. Thus, the best ansatz is that which has the properties of the «streamline», for all point on this plane the current should be «tangent» to it, or in other words,

$$j^\perp_{a\mu}(x)=0\,. \qquad (27)$$

This is the answer to the questiones posed at the end of the preceeding section. Indeed, the current is nonzero everywhere, but its longitudinal part of the current is not dangerous for us, as the integration over the collective variables is presumably made accurately. If the transverse part is absent, the integration over transverse coordinates (leading to our effective action) can be made without any external currents etc.

Unfortunately, even for the simplest toy models it is not easy to find such «streamline» configurations analytically (see e. g. [19]). There exist simple numerical method suggested by myself [20] (see its discussion and application in paper IV of this series). The idea is quite simple: one should just start with two well separated pseudoparticles and follow the direction of the «driving force», which leads to the «streamline» and then «downstream».

However, for such a complicated system as the 4-dimensional gauge field the «streamline» configurations are not yet explicitely found and, as a first step, in the subsequent papers we approach this problem in the variational way, considering various trial functions which approximately describe this «streamline». The quantity which we are going to minimize is now clear, it is $j^\perp$. Thus, comparing various trial functions we calculate the current and make (so far rather crude) estimates of the corrections induced by it.

7. THE SEMICLASSICAL EFFECTIVE THEORY

In this section we are going to outline the efective theory which corresponds to gaussian (semiclassical) approximation. This is possible if the field fluctuates around our ansatz plane, and the quantum fluctuations are not too strong. All steps are in fact quite standard. First, one has to expand the action up to the second order in the fluctuating field

$$S(A)=S(A^{ansatz})+\int d^4x\, j^a_\mu(x)\, b_{a\mu}(x)+$$
$$+\frac{1}{2}\int b_{a\mu}(x)\,\Box_{a\mu,b\nu}(x,y)\, b_{b\nu}(y)\,dx\,dy+\dots \quad (28)$$

and then integrate over the «transverse» coordinates in the gaussian approximation. If it is possible, the simbolic result for the effective action can be written as follows

$$\exp(-S_{eH})=\int Db_\perp(x)\,e^{-S}=$$
$$=\exp(-S^{clas}-S^{current}-S^{quantum}-S^{Jacobian}), \quad (29)$$

(where $\int Db_\perp\dots$ means $\int Db\prod\delta(I_i)J\dots$).

Let us discuss four terms of the resulting effective action subsequently. The first «classical» one is just the action distribution in the ansatz plane

$$S^{clas}(a_i)=S[A^{ansatz}(x,a)]. \quad (30)$$

Obviously it is the simplest one, and it is quite easy to deal with it, provided the ansatz is defined. However, it is the main part of the effective action only if its «current part»

$$S^{current}=\frac{1}{2}\int j^\perp_{a\mu}(x)\,\Box^\perp_{a\mu,b\nu}(x,y)\,j^\perp_{b\nu}(y)\,dx\,dy \quad (31)$$

is reasonably small compared to it. Note, that this term is formally of the same magnitude as the «classical» term (in pertubative language it corresponds to the «zero loop» order), even if the ansatz field is strong enough to justify applications of the gaussian appro-

ximation. The sum of (30,31) is in this case ansatz independent, and the relative role of them depends on how close the ansatz used is to the «streamline».

The third «quantum» term in this action

$$S^{quantum}(a)=-\frac{1}{2}\log\det(\Box^\perp) \quad (32)$$

contains the functional determinant of the differential operator $\Box^\perp$. Such determinant was explicitely found for the instanton solution by t'Hooft [7], which can be used as a reference point for estimates of its magnitude.

Finally, there is the forth structure in the effective action, related with the «transverse» projectors and the corresponding jacobian

$$S^{Jacobian}=-\log J(A^{ansatz},a). \quad (33)$$

The most natural way to write our conditions I is to project them «transversly». This means that the fluctuating part of the field $b_{a\mu}(x)$ is orthogonal to all tangent vectors V^i of the ansatz plane

$$I_i=\int d^4x\, b_{a\mu}(x)\,V^{(i)}_{a\mu}(x). \quad (34)$$

For the non-gauge systems these vectors can be obtained by a differenciation of the ansatz, but for the gauge one have also take care about the projector P_{gauge}, which garantees that b is not just pure gauge transformation

$$V^{(i)}_{a\mu}(x)\propto P_{gauge}\frac{\partial A^{ansatz}_{a\mu}(x,a)}{\delta a_i}. \quad (35)$$

In particular, it is most convenient to take both $b_{a\mu}(x)$ and $V^{(i)}_{a\mu}(x)$ satisfying the background gauge condition

$$D^{ab}_\mu b_\mu(x)=0,\quad D^{ab}_\mu V^{(i)}_{b\mu}(x)=0. \quad (36)$$

The Jacobian in general can be written as

$$J=\det\frac{\int dx\,V^{(i)}_{a\mu}(x)\,P_{gauge}\,V^{(j)}_{a\mu}(x)}{\left(\int dx\,V^{(i)}_{a\mu}(x)\,P_{gauge}\,V^{(i)}_{a\mu}(x)\right)^{1/2}}. \quad (37)$$

In the semiclassical context it should be taken at the ansatz plane. Again, as for the «quantum» part of the effective action, in the first approximation it may be taken from the known expressions for the individual *PPs*.

7. SUMMARY AND OUTLINE OF THE SUBSEQUENT PAPERS

We have shown that at the moment all necessary ingredients for the creation of the theory of the topological phenomena in gauge theories are ready. Indeed, on one hand there is rather detailed picture of the «instanton liquid» [4], which is now supported by rather impressive set of phenomenological observations, both coming from «real» and lattice numerical experiments. On the other hand, they are in agreement with the first results of the variational approach due to Dyakonov and Petrov [9]. And the third, now there is much better understanding of how to find better trial functions, how one should naturally introduce collective variables and how to derive the effective theory.

To put it into practice is the main aim of this series of papers. Of course, practical realization of this programm needs a lot of work, many methodical problems should be solved etc., therefore we naturally proceed from the simpler theories to more realistic ones. In paper II we consider the SU(2) gluodynamics, being the simplest non-abelian gauge theory. We compare various trial functions, derive the interaction law for the pseudoparticles, perform numerical simulation of the resulting statistical mechanics of the «instanton liquid» and end up with some set of physical results, ranging from the integral instanton density to rather delicate measurements of the correlation functions.

The difficulty of the problem increases enormously when light quarks are involved, for they generate specific interaction between the *PPs*. Our studies of these phenomena are collected in the paper III. The central question discussed in it is whether the instantons do or do not generate the so called «quark condensate», manifasting spontaneous chiral symmetry breaking, and our answer to it is definitely positive. We show, that in the presence of light quarks the «instanton liquid» consists of essentially two components, the «polymer», creating the condensate all over the space, and the separate «molecules». Their relative role depends nontrivially on the number of light quark flavors.

The paper IV is devoted to much simpler «toy model», a quantum-mechanical motion in a double-well potential. In this case it is possible to obtain both very accurate «lattice» data for the ensemble of the paths, and to push our programm significantly further. The path containing tunneling events forth and bacl also can be considered as a one-dimentional «liquid of kinks», and although in this case their mutial interaction is much less important, its studies turns to be very instructive.

REFERENCES

1. *Creutz M.* Phys. Lett. 45B (1980) 313.
2. *Wilson K.G.* Phys. Rev. D10 (1974) 2445.
3. *Novikov V.A., M.A. Shifman, A.I. Vainstein and V.I. Zakharov*, Nucl. Phys. B191 (1981) 301.
4. *Shuryak E.V.*, Nucl. Phys. B203 (1982) 93, 214B (1983) 237.
5. *E.V. Shuryak.* Phys. Reports C115 (1984) 151.
6. *A.M. Polyakov*, Phys. Lett. 59B (1975) 82.
   A.A. Belav..., A.M. Polyakov, A.A. Schwartz and Yu.S. Tyupkin. Phys. Lett. 59B (1975) 85.
7. *t'Hooft G.*, Phys. Rev. 14d (1976) 3432, (e) 18d (1978) 2199. *Bernard C.*, Phys. Rev. D19 (1979) 3013.
8. *Novikov V.A., M.A. Shifman, A.I. Vainshtein and V.I. Zakharov*, Uspekhi Fiz. Nauk (Soviet Physics—Uspekhi) 136 (1982) 553.
9. *Dyakonov D.I. and V.YU. Petrov*, Nucl.Phys. B245 (1984) 259.
10. *Caldi D.G.*, Phys. Rev. Lett. 39 (1977) 121.
    Callan C.G., R. Dashen and D.J. Gross. Phys. Rev. D17 (1978) 2717, D19 (1979) 1826.
    Carlitz R.D. And Creamer D.B., Ann. Phys.116 (1979) 429.
    Callan C.G., R. Dashen and D.J. Gross. Phys. Rev. D20 (1979) 3279.
11. *M.A. Shifman, A.I. Vainshtein and V.I. Zakharov*, Nucl. Phys. B163 (1980) 43, B165 (165) 45; Phys. Lett. 76b (1978) 971.
12. *Veneziano G.* Nucl. Phys. B159 (1979) 213. *Witten E. Nucl. Phys. B156 (1979) 269.*
13. *Ishikawa K., G. Shierholtz, H. Schneider and M.Teper*, Nucl. Phys. B227 (1983) 221.
14. *P. Di Vecchia, K. Fabricius, C.G. Rossi and G. Veneciano* Nucl. Phys. B192 (1981) 392, Phys. Lett., 108B (1982) 323.
15. *Woit P.*, Phys. Rev. Lett. 51 (1983) 638, G. Bhanot et al. Nucl. Phys. B230 [FS10] (1984) 291.
16. *Fox I.A., J.P. Gilchrist, M.L. Laursen, G. Shierholtz*, Phys. Rev. Lett 54 (1984) 749.
17. *Berg B.* Phys. Lett. 104B (1981) 475.
    Iwasaki Y. and T. Yoshie, Phys. Lett. 131B (1983) 159, 131B (1984) 73, 143B (1984) 449.
    E.-M. Ilgenfritz et al. Nucl. Phys. B268 (1986) 693.
18. *M.I. Polikarpov, A.I. Veselov.* Instantons and Confinement in the SU(2) Lattice Gauge Theory. ITEP Preprint 41, Moscow 1987.
19. *I.I. Balitsky and A.V. Yung*, Phys. Lett 168B (1986) 113.
20. *E.V. Shuryak.* In: Proceedings of the Conference on Numerical Experiments in Quantum Field Theories, Alma-Ata 1985.
    A.A. Migdal, editor. (in Russian).

E.V. Shuryak

Toward the Quantitative Theory of the Topological Effects in Gauge Field Theories I. Phenomenology and the Method of Collective Coordinates

Э.В. Шуряк

К количественной теории топологических эффектов в калибровочных теориях поля. I. Феноменология и метод коллективных координат

Ответственный за выпуск С.Г.Попов

Работа поступила 27.04. 1987 г.
Подписано в печать 18.05. 1987 г. МН 08185
Формат бумаги 60×90 1/16 Объем 1,7 печ.л., 1,4 уч.-изд.л.
Тираж 250 экз. Бесплатно. Заказ № 54

Набрано в автоматизированной системе на базе фотонаборного автомата ФА1000 и ЭВМ «Электроника» и отпечатано на ротапринте Института ядерной физики СО АН СССР,
Новосибирск, 630090, пр. академика Лаврентьева, 11.