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MACROSCOPIC MANIFESTATIONS OF THE CHIRAL ANOMALY IN GRAVITATIONAL FIELD

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Macroscopic Manifestations of the Chiral Anomaly in Gravitational Field

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ABSTRACT

The chiral anomaly is shown to result in some macroscopic effects for a rotating gravitating body. In particular, vacuum condensate $\langle \mathbf{EH} \rangle$ is formed, where $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic field strength respectively. The condensate value decreases as a power of the distance from the gravitating body. The corresponding vacuum currents are found. We present also arguments that the anomaly gives rise to production of massless particles (neutrinos, photons and gravitons) in the gravitational field of rotating black holes. This process results in the angular momentum loss by black holes.

The current of massless Weyl fermions in an external gravitational field is known to be nonconserved due to the chiral anomaly [1, 2]:

$$\partial_\mu j^\mu = -\frac{1}{384\pi^2} R\tilde{R}, \qquad (1)$$

where $R_{\mu\nu\alpha\beta}$ is the curvature tensor and $\tilde{R}_{\mu\nu\alpha\beta} = \frac{1}{2}\varepsilon_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma\alpha\beta}$. The simplest case when this anomaly does not vanish for a classical external field is the gravitational field of a rotating body. In what follows we discuss macroscopic consequences of the chiral anomaly. We consider the following three phenomena:

1) the formation of the condensate $\langle F\tilde{F} \rangle$ around (even electrically neutral) rotating bodies, here $F_{\mu\nu}$ is the electromagnetic field strength tensor and $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu}{}^{\rho\sigma} F_{\rho\sigma}$;
2) dipole vacuum currents;
3) massless particle radiation by rotating black holes leading to the slowing down of the rotation.

1. The existence of the condensate $\langle F\tilde{F} \rangle$ follows from the analogue of eq. (1) for photons [3]

$$\partial_\mu K^\mu = -\frac{1}{96\pi^2} R\tilde{R}. \qquad (2)$$

The current $K^\mu = -\varepsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta$ can be considered as the photon axial current since the expectation value of the operator $\int \mathbf{dr}\, K^0$ in one particle states equals to $(+1)$ of (-1) for left-handed or right-handed photon respectively (here A_μ is the photon vector potential). Due to the operator identity $\partial_\mu K^\mu = -\frac{1}{2} F\tilde{F}$ eq. (2) gives

$$F\tilde{F} = \frac{1}{48\pi^2} R\tilde{R}. \qquad (3)$$

For a homogeneous rotating body with radius r_0 we obtain

$$R\tilde{R} = \begin{cases} 0, & r < r_0; \\ 36 r_g^2 (\mathbf{ra})/r^8, & r > r_0. \end{cases} \qquad (4)$$

Here r_g is the gravitational radius of the body, $\mathbf{a} = \mathbf{M}/m$, $\mathbf{M}$ and m are respectively its angular momentum and mass.

Of course relation (3) is quantitatively essential only for black holes with very small radius, $r_g \lesssim 10^{-13}$ cm.

The relation analogous to (3) is valid for the gluon condensate also. Thus the induced CP-odd θ-term is generated. However $R\tilde{R}$ by itself leads to a considerable CP violating effects around rotating mini black holes.

2. Let calculate now not the divergence but the current itself. Note first that eqs (1) and (2) can be written in a unified way

$$\partial_\mu j^\mu_{(s)} = A_{(s)}(\mathbf{r}), \qquad (5)$$

where the anomalous term $A_{(s)}(\mathbf{r})$ for neutrinos and photons is $A_{(s)}(\mathbf{r}) = -\frac{s^2}{96\pi^2} R\tilde{R}$ and s is the particle helicity. In a static case eq. (5), div $\mathbf{j}_{(s)} = A_{(s)}(\mathbf{r})$ looks exactly like the electrostatic equation div $\mathbf{E} = \rho$.

The general solution of eq. (5) can be written in the form

$$\mathbf{j}_{(s)}(\mathbf{r}) = -\nabla \frac{1}{4\pi} \int \frac{\mathbf{dr}' A_{(s)}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} + \mathbf{j}_{(0s)}(\mathbf{r}), \qquad (6)$$

where $\mathbf{j}_{(0s)}$ is a conserved current. Neglecting it for the moment and using $R\tilde{R}$ given by eq. (4) we find

$$\mathbf{j}_{(s)}(\mathbf{r}) = \frac{s^2}{48\pi^2} r_g^2 \begin{cases} \mathbf{a}/r_0^6, & r < r_0; \\ \frac{2}{r_0^3} \frac{r^2\mathbf{a} - 3\mathbf{r}(\mathbf{ra})}{r^5} - \frac{r^2\mathbf{a} - 6\mathbf{r}(\mathbf{ra})}{r^8}, & r > r_0. \end{cases} \qquad (7)$$

Thus around rotating bodies the vacuum current is generated which falls down for large r as r^{-3}.

Note however that the analogy with electrostatics is not complete since there is no condition rot $\mathbf{j} = 0$. In particular there exists the solution of eq. (5) which falls down at large r as r^{-2}:

$$\mathbf{j}_{(s)} = -\frac{3s^2}{32\pi^2} r_g^2 \frac{\mathbf{r}(\mathbf{ra})}{r^4} \left(\frac{1}{r_0^4} - \frac{1}{r^4}\right). \qquad (8)$$

This solution describes the particle radiation. As $\mathbf{j}_{(s)}$ is the difference between the currents of left-handed and right-handed particles, the radiation of left-handed particles dominates in the lower hemisphere $(\mathbf{ra} < 0)$ and that of right-handed particles dominates in the upper hemisphere $(\mathbf{ra} > 0)$.

Thus the anomaly equation by itself does not permit to see whether the particle radiation exists without additional physical arguments. When the external field is weak, the radiation is most probably absent. Indeed the problem resembles the problem of vacuum stability in the Coulomb field. Instability in the Dirac equation appears only for $Z_\alpha > 1$ independently of the produced particle mass m. This condition can be easily understood. The electrostatic energy of a particle localized in the region of the size λ around the Coulomb centre, $U \sim -Z\alpha/\lambda$, should exceed by absolute value the kinetic energy $p \sim \frac{1}{\lambda}$ $(\lambda^{-1} > m)$. Exactly in the same way the gravitational interaction between the particle spin and the rotation of the massive body $U \sim r_g a/r^3$ should exceed $1/\lambda$. In the weak field case when $r \gg r_g$ this condition surely is not valid.

3. Thus we naturally come to the problem of physical consequences of the chiral anomaly in the field of a rotating black hole which characteristic size r_0 is of the order of r_g. Here we encounter the problem of the boundary condition for the current at $r = r_g$ and the more technical one of taking into account the space-time curvature. We consider a very crude model of the phenomenon. To formulate it return to expression (7) for the vacuum current. It corresponds to the isotropic particle source with intensity $A(\mathbf{r}')$ at every point where anomaly $A(\mathbf{r}')$ is nonvanishing. The vector summation of currents flowing out from different points gives the dipole result (7) when $A(\mathbf{r}') \sim \mathbf{ar}'$. Correspondingly no radiation exists.

In our simplified model of the black hole the total absorption of the current on the surface $r = r_g$ is assumed. In other words when

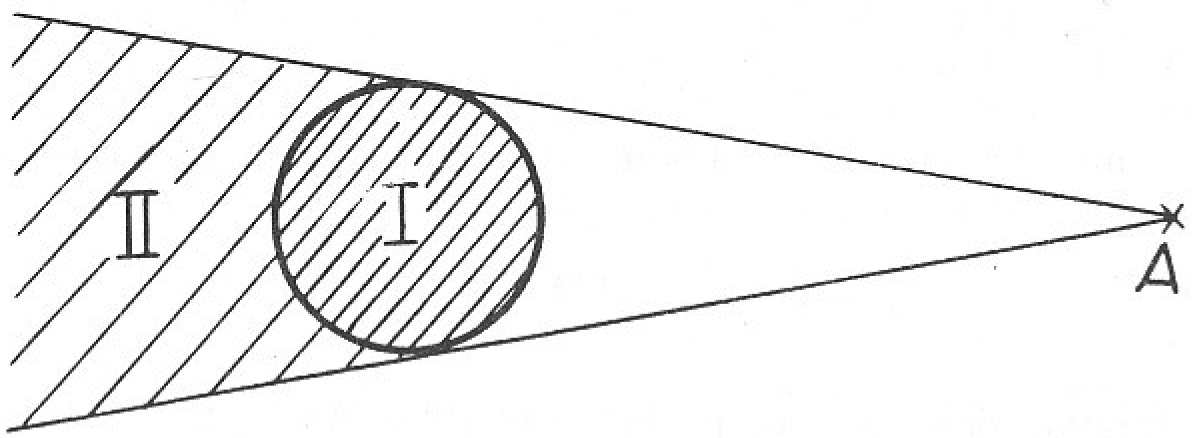

Fig. 1. The integration region in expression (6) for the case of a black hole. A is an observation point, I is the interior of the Schwarzschild sphere, II is the shadow region.

integrating over $\mathbf{r}'$ not only the region $r<r_g$ but also the shadow region is excluded (see Fig. 1). Of course we are aware that the characteristic wave length is of the order of r_g so the geometric approximation can be used only as an order of magnitude estimate. Besides, for $r>r_g$ the space-time is considered as flat, the curvature being taken into account only in the anomaly which is proportional to $R\tilde{R}$.

The exact expression for the anomaly in the Kerr metric is

$$\frac{1}{2}\frac{\varepsilon^{\varkappa\lambda\rho\sigma}}{\sqrt{-g}} R_{\mu\nu\varkappa\lambda} R^{\mu\nu}{}_{\rho\sigma} = 12 r_g^2\, ar\cos\theta\, \frac{(3r^2-a^2\cos^2\theta)(r^2-3a^2\cos^2\theta)}{(r^2+a^2\cos^2\theta)^6}. \qquad (9)$$

To derive this formula it is convenient to use the expressions for the Riemann tensor in the tetrad formalism presented in book [4]. In what follows we neglect terms of higher orders in a in fact using eq. (4) when $r>r_g$. With these simplifications the current is

$$\mathbf{j}_{(s)} = \frac{1}{4\pi}\int d\mathbf{r}' \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} A_{(s)}(\mathbf{r}'). \qquad (10)$$

Asymptotically for large r it tends to

$$\mathbf{j}_{(s)} = -\frac{3s^2}{128\pi^2}\frac{1}{r_g^2}\frac{\mathbf{r}(\mathbf{ra})}{r^4}. \qquad (11)$$

where only the term decreasing as r^{-2} is retained.

Thus in this model there exists a nonzero flow of neutrinos and photons at infinity. The right-handed particles are emitted mostly

along the direction of the black hole angular momentum whereas the left-handed ones go in the opposite direction. It is evident that the same phenomenon takes place also for gravitons but the coefficient at the corresponding anomaly has not been up to now calculated explicitly.

The arguments presented here are formally applicable to any rotating gravitating body possessing no horizon if one takes into account the particle absorption due to their nongravitational interaction with the matter of the body. We cannot exclude that the radiation exists even in this case.

Both the right-handed particles emitted along the angular momentum of the star and the left-handed ones emitted in the opposite direction carry the spin angular momentum of the same sign. This leads to the slowing down of the star rotation.

The angular momentum loss by a black hole due to the Hawking radiation of spinning particles is considered in Refs [5, 6]. The phenomenon discussed here by an order of magnitude or parametrically is comparable to the Hawking radiation. Nevertheless one can think that these two effects are different. Indeed at least in the approximation used the particle production rate connected with the chiral anomaly depends upon the particle spin trivially, only through the coefficient in eq. (5). In the case of the Hawking radiation the spin dependence is quite different [7]. The particle production due to the anomaly does not also coincide with the superradiance because the latter is absent for neutrino [4, 8]. Note in this connection that according to the calculations of Ref. [7] the momentum loss due to neutrino radiation does not vanish even in the limiting case $a=r_g/2$ when the Hawking radiation disappears since the black hole temperature tends to zero. It may be a manifestation of the effect we are discussing.

The connection between the chiral anomaly and the fermion radiation was considered in Refs [9, 10] for the case of a dyon with magnetic charge g and electric charge Q. As a result of the particle emission the dyon radiates away its electric charge turning into magnetic monopole. Analogously, the crude model considered above leads to the conclusion of the angular momentum loss by the black hole. Let follow this analogy in some more detail.

For radiation from the dyon it is essential that both classical magnetic and electric fields **H** and **E** are nonvanishing. Correspondingly the product **EH** which defines the anomaly does not vanich too. The interaction of magnetic monopole with a fermion is strong because the charge g is large. Hence there exists either only incoming or only outgoing solution in the S-wave depending upon the magnetic moment orientation of the fermion relative to the direction of the magnetic field. There is no dependence upon the fermion electric charge because particles with opposite charges and opposite chiralities have the same magnetic moment. This degeneracy is destroyed by the electric charge of the dyon. Correspondingly the Coulomb energy of the dyon decreases because of the radiation. The strong magnetic interaction is important for this process because just this interaction binds the particle near the monopole (compare with the pure Coulomb case discussed above).

Considering this problem in terms of the anomaly one sees that the latter is vanishing for the monopole and is nonvanishing and of definite sign for the dyon. So in the last case the particles of definite chirality are emitted. However the anomaly is quadratic in fermion charges and the statement that the particles of definite charge are radiated follows from additional physical arguments.

In the case of a black hole its strong gravitational attraction is analogous to the magnetic interaction since just the former leads to the particle capture at the Schwarzschild sphere. Relatively weak interaction of the particle spin with the black hole angular momentum makes it energetically more favourable to absorb particles with a definite spin projection on the rotation axis. For a rotating black hole the chiral anomaly is nonzero, but using only this it is impossible to make any statement about particle radiation because the anomaly does not distinguish between the absorption and emission of particles of opposite chiralities. If however the radiation follows from some additional arguments, then the anomaly results in the angular momentum loss. The correspondence between a dyon and a Kerr black hole is illustrated by Table.

Dyon	Kerr black hole
$F\tilde{F}\neq 0$	$R\tilde{R}\neq 0$
$(\int \mathbf{dr}\ F\tilde{F}\neq 0)$	$(\int \mathbf{dr}\ R\tilde{R}=0)$
Magnetic interaction	Gravitational interaction
Fall to the centre	Capture at the horizon
Coulomb interaction	Interaction of particle spin with the black hole rotation

It is noteworthy that besides current (11) in higher orders in r_g/r there exist vacuum currents which like current (7) do not correspond to particle radiation. These are peculiar quantum «hairs» of a rotating black hole which fall off as a power of distance. Of course other quantum effects are khown resulting in field condensates which also fall off as a power of r. In particular quantum corrections to the energy-momentum tensor of massless fields decrease as r^{-4}. Note that vacuum currents discussed here behave as r^{-3}.

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