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SELF-SIMILAR LANGMUIR COLLAPSE

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НОВОСИБИРСК

Self-Similar Langmuir Collapse

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ABSTRACT

The first example of a three-dimensional self-similar solution of Zakharov equations for a supersonic Langmuir wave collapse is constructed. The parameters of this solution differ essentially from those predicted by the model of a highly oblate cavity, in particular, the maximum depth of the cavity exceeds the expected one by nearly four times. An explanation of this discrepancy is given.

The interest to self-similar regimes of a supersonic Langmuir wave collapse [1] is explained by the important role, they are known to play in a strong Langmuir turbulence (see Ref. [2], for example). Numerical computations yielded some evidence for the establishment of self-similar regimes (see, e. g., [3]), but the compression coefficients obtained for the Langmuir waves trapped in the cavity are still insufficient for reliable conclusions. The identification of solutions of the initial value problem with the self-similar solutions has also difficulty in a poverty of available information concerning the latter those. Up to now, only self-similar solutions of one-dimensional equations, which presumably give an approximate description of an electric field on the «short» axis of a highly oblate cavity [4], and a self-similar solution with an almost centrally symmetric cavity, which contains a slightly split triplet of identically populated ground states (corresponding to the unit «orbital momentum» $l=1$) [5] have been found. No valid examples of single-mode self-similar regimes of supersonic collapse which are especially interesting from the point of view of the theory of a strong Langmuir turbulence have been obtained so far.

Self-similar regimes of a supersonic Langmuir collapse are described by the following equations:

$$\Delta(\Delta-1)\,\psi=\bar{\nabla}(u\,\bar{\nabla}\,\psi)\,, \tag{1}$$

$$\left(\frac{7}{3}+\frac{2}{3}\,r\,\frac{\partial}{\partial r}\right)\left(\frac{4}{3}+\frac{2}{3}\,r\frac{\partial}{\partial r}\right)u=\Delta\Phi\,,\quad \Phi\equiv|\,\bar{\nabla}\,\psi\,|^2. \tag{2}$$

Here ψ is the time envelope of a high-frequency electric potential, u is the perturbation of the ion density, Φ is the pressure of Langmuir waves. Eq. (2) allows one to express explicitly the function u in terms of Φ:

$$u(\mathrm{r}) = \frac{3}{2r^2}\int_0^r dr_1\, r_1 \left(1 - \frac{r_1^{3/2}}{r^{3/2}}\right) \Delta\Phi(\mathrm{r}_1)\,, \tag{3}$$

and thus reduce (1) to a closed equation for the function ψ. Axially symmetric and odd with respect to the reflection of a specified direction z, the solution of this equation can be expanded in Legendre polynomials with odd numbers:

$$\psi(\mathrm{r}) = \sum_{l=2k-1} R_l(r)\, P_l(\cos\theta)\,. \tag{4}$$

The «radial wave functions» $R_l(r)$ satisfy the equations

$$\Delta_l(\Delta_l - 1)\, R_l = \sum_{l_1, l_2} F_{l,l_1,l_2}\,, \tag{5}$$

$$F_{l,l_1,l_2} = c_{l,l_1,l_2}\left[\frac{1}{r^2}\frac{d}{dr} r^2 u_{l_2}\frac{d}{dr} + \frac{l_2(l_2+1) - l(l+1) - l_1(l_1+1)}{2r^2} u_{l_2}\right] R_{l_1}\,, \tag{6}$$

$$u_{l_2} = \frac{3}{2r^2}\int_0^r dr_1\, r_1\left(1 - \frac{r_1^{3/2}}{r^{3/2}}\right)\Delta_{l_2}\sum_{l, l_1}\Phi_{l_2,l,l_1}(r_1) \tag{7}$$

$$\Phi_{l_2,l,l_1} = c_{l_2,l,l_1}\left[\frac{dR_{l_1}}{dr}\frac{dR_l}{dr} + \frac{l(l+1) + l_1(l_1+1) - l_2(l_2+1)}{2r^2} R_l R_{l_1}\right]. \tag{8}$$

Here the signs

$$\Delta_l = \frac{1}{r}\frac{d^2}{dr^2} r - \frac{l(l+1)}{r^2}\,, \quad c_{l,l_1,l_2} = \left(l + \frac{1}{2}\right)\int_{-1}^{1} dx\, P_l(x)\, P_{l_1}(x)\, P_{l_2}(x)$$

are introduced for the l-th Laplacian and Clebsch—Gordan coefficients.

A finite-dimensional approximation of equations (5) — (8), used below, is obtained by a formal replacing of all Clebsch—Gordan coefficients with max $(l, l_1) > L \equiv 2K-1$ in (6), (8) by zeros. Such an approximation corresponds to the retention of first K terms of the electrical potential expansion (4). The corresponding expansion of the density perturbation u contains $2K$ harmonics. When $K=1$ in

the expansion of electric potential only a dipole term is retained, and the regular in the centre of the cavity solution of reduced equations (5) — (8) depends on two parameters A and a:

$$R_1(r)\,|_{r\to 0} = Ar + ar^3 + O(r^5)\,.$$

The values of A and a are determined, as in [5], from the conditions of the singularity

$$R_1(r)\,|_{r\approx r_s} \approx \frac{2\sqrt{10}}{3\sqrt{3}}\, r_s \ln\left|1 - \frac{r}{r_s}\right|$$

shifted to the infinity and the coefficient C turned to zero in the asymptotics $R_1(r)$:

$$R_1(r)\,|_{r\to\infty} = C\left[r + O\left(\frac{1}{r}\right)\right] + O\left(\frac{1}{r^2}\right).$$

which valid for $r_s = \infty$. As the solution of the K-approximated equations (5) — (8) is known, one can find the solution of the $K+1$-approximation in the following way. One should formally multiply the Clebsch—Gordan coefficients with max $(l, l_1) = 2K+1$ in (6), (8) by ε. The solution of the equations, obtained in this manner depends parametrically on ε. It coincides with the already known solution of K-approximation when $\varepsilon = 0$ and coincides with a sought for solution of $K+1$-approximation when $\varepsilon = 1$. The derivatives $\frac{\partial R_l}{\partial\varepsilon}(l = 1, 3, \ldots, 2K+1)$ satisfy the linear nonuniform set of $K+1$ integro-differential equations. The regular in the centre of the cavity solution of this set depends on $2(K+1)$ parameters a_1, $a_2, \ldots, a_{2(K+1)}$:

$$\left.\frac{\partial R_l}{\partial\varepsilon}\right|_{r\to 0} = a_l r^l + a_{l+1} r^{l+2} + O(r^{l+4})\,, \qquad l = 1, 3, \ldots, 2K+1\,.$$

The values of the parameters $a_1, \ldots, a_{2(K+1)}$ are unambigously determined by the condition of decreasing of all functions $\frac{\partial R_l}{\partial\varepsilon}$ at $r\to\infty$, i. e. of turn to zeros all coefficients attached to increasing at the infinity asymptotics, a number of which is exactly equal to $2(K+1)$. As $\frac{\partial K_l}{d\varepsilon}$ is known, one can calculate the difference between solutions of $K+1$- and K-approximated equations (5) — (8). The

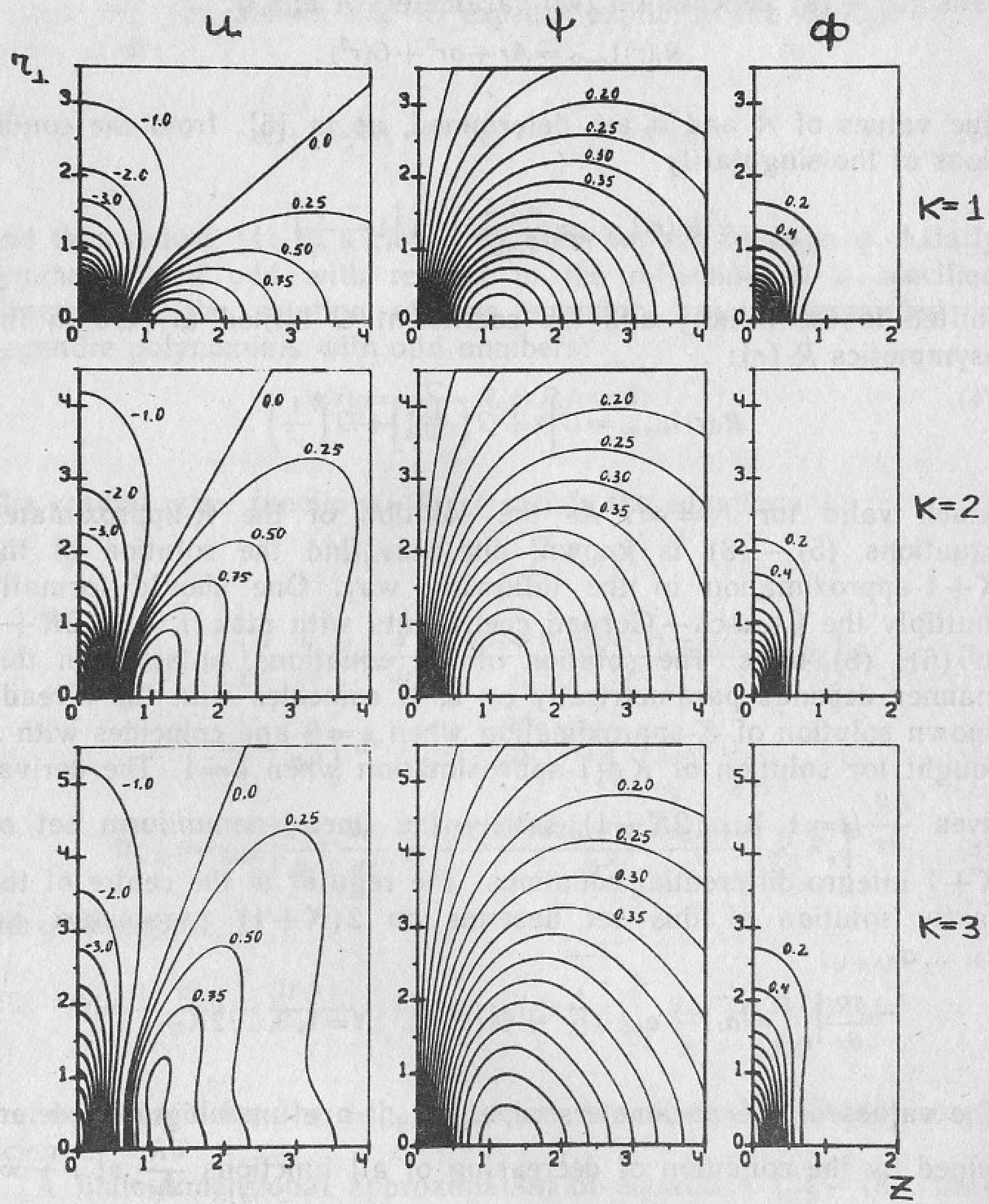

Fig. 1.

results of numerical solution of reduced equations (5) — (8) for $K=1$, 2, 3 are plotted in the pictures. The $K=3$-approximation turn out to be quite sufficient so far as the further increase of K does not change the picture practically. Near the centre of the cavity the isolines of functions $u(\mathbf{r})=u(r_\perp, z)$, $\Phi(\mathbf{r})=\Phi(r_\perp, z)$ are highly oblate in z direction ellipses: the axis ratio is equal to six for $u(r_\perp, z)$ and is equal to four for $\Phi(r_\perp, z)$. As the $r_\perp$-derivatives are negligible, one can simplify equations (1), (2) on the axis z near the origin:

$$\left(\frac{\partial^2}{\partial z^2}-1\right)E=uE+B,\quad E\equiv\frac{\partial\psi}{\partial z},$$

$$\left(\frac{7}{3}+\frac{2}{3}z\frac{\partial}{\partial z}\right)\left(\frac{4}{3}+\frac{2}{3}z\frac{\partial}{\partial z}\right)u=\frac{\partial^2}{\partial z^2}|E|^2. \qquad (9)$$

The constant B is simply expressed in terms of the values $E(0)\approx 1.5$; $u(0)\approx -10$:

$$B=\left[\frac{14}{9}-E^2(0)\right]\frac{u(0)}{E(0)}-E(0)\approx 3.1\,.$$

If equations (9) were applicable for all the values of z, then the condition $B=0$, which was assumed in the model of a highly oblate cavity [4], is inevitably follow from the conditions on the infinity. Really the longitudinal and transverse scales of variation of the function $\psi(r_\perp, z)$ turn out to be approximately equal, when $|z|\geqslant 1$. This very fact explains the essential difference between the parameters of the self-similar solution presented here and their values calculated earlier under assumption that model of a highly oblate cavity is applicable uniformly with respect to z.

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