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PHOTONIC CHIRAL CURRENT AND ITS ANOMALY IN A GRAVITATIONAL FIELD

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Photonic Chiral Current and its Anomaly in a Gravitational Field

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ABSTRACT

The notion of chirality for electromagnetic field which is conserved in interactions with gravitons is formulated. The corresponding chiral current is the one-particle-state analogue of the Pauli—Lubansky vector. The anomaly of this current in an external gravitational field is found. The results obtained are used for the calculation of the electromagnetic radiative correction to the fermionic chiral anomaly in a gravitational field.

1. INTRODUCTION

At present chiralities of massless bosons and fermions appear on quite unequal footings. For massless fermions interacting with electromagnetic field there exists well defined U(1)-symmetry with respect to chiral rotations. This symmetry generates the Noether axial-vector current a_μ which is classically conserved but because of the famous triangle anomalies [1—5] the divergence of this current is nonvanishing

$$\nabla_\mu a^\mu = \frac{Q^2\alpha}{2\pi} F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{192\pi^2} R_{\mu\nu\varkappa\lambda}\tilde{R}^{\mu\nu\varkappa\lambda}, \qquad (1)$$

where $a^\mu = \bar\psi\gamma^\mu\gamma^5\psi$. ψ is the massless Dirac field with electric charge Q. ∇_μ is the covariant derivative

$$\nabla_\mu a^\mu = \frac{1}{\sqrt{-g}}\,\partial_\mu(\sqrt{-g}\;a^\mu),$$

$F_{\mu\nu}$ is the electromagnetic field strength tensor, $R_{\mu\nu\varkappa\lambda}$ is the Riemann tensor, and

$$\tilde{F}^{\mu\nu} = \frac{1}{2\sqrt{-g}}\,\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}, \qquad \tilde{R}^{\mu\nu\varkappa\lambda} = \frac{1}{2\sqrt{-g}}\,\varepsilon^{\mu\nu\rho\sigma}R_{\rho\sigma}{}^{\varkappa\lambda}.$$

To extend the notion of chirality to bosons let us consider an electromagnetic field in an external gravitational background. It is well known that the Maxwell equations both for free photons and for photons in a gravitational field are invariant under duality transfomation

$$F'_{\mu\nu}=\cos\alpha\, F_{\mu\nu}+\sin\alpha\, \tilde{F}_{\mu\nu}\,. \qquad (2)$$

This transformation, however, is not expressed in terms of the vector potential A_μ so the corresponding vector current cannot be obtained through the standard procedure.

This difficulty is avoided in the light-cone formalism. In this approach photons are described by a complex field A and the action is bilinear in A and A^*. In this terms the photonic chirality is defined completely along the same lines as the fermionic one.

Noncovariant Lagrangian density in the light-cone formulation leads to the conserved but noncovariant photonic axial-vector current. A Lorentz-covariant current which generates the same transformation can be written as

$$K^\mu=-\frac{1}{\sqrt{-g}}\,\varepsilon^{\mu\nu\varkappa\lambda}A_\nu\partial_\varkappa A_\lambda\,. \qquad (3)$$

This current is of course nonconserved, $\nabla_\mu K^\mu=-\frac{1}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}$. Nevertheless the chirality conservation results in «naive» vanishing of expectation value $\langle\nabla_\mu K^\mu\rangle$ in a gravitational field. But the triangle diagrams in this case also are anomalous and their contribution leads to

$$\langle\nabla_\mu K^\mu\rangle=-\frac{1}{2}\langle F_{\mu\nu}\tilde{F}^{\mu\nu}\rangle=-\frac{1}{96\pi^2}\,R_{\mu\nu\varkappa\lambda}\,\tilde{R}^{\mu\nu\varkappa\lambda}\,. \qquad (4)$$

This anomaly was found in our paper [6]. In a different form chiral anomaly for photons was independently obtained in Ref. [7].

Physically anomaly (4) seems to be very natural. Indeed the term proportional to $R\tilde{R}$ in the fermionic case (1) arises because of the fermion spin interaction with the gravitational field. This interaction is known to be universal, it exists both for bosons and fermions universally.

Eq. (4) presents the one-loop anomaly. We show in Sect. 5, however, that this relation allows to calculate the two-loop radiative correction of order α to the fermionic chiral anomaly in a gravitational field. The result can be obtained by taking the expectation value of expression (1) and the substitution of $\langle F_{\mu\nu}\tilde{F}^{\mu\nu}\rangle$ from eq. (4).

The paper is organized as follows. In the second section we present the definition of the photon chirality in the light-cone formalism and check the chirality conservation in the gravitational interactions of photons. In Sect. 3 the expression for the chiral current is considered and in Sect. 4 the anomalous current nonconservation is discussed. In a short Sect. 5 electromagnetic corrections to the fermionic chiral anomaly in a gravitational field are calculated. We conclude (Sect. 6) by the discussion of some unsolved problems and of earlier works on bosonic chiral anomalies.

2. CHIRALITY OF PHOTON FIELD

Let us discuss first the case of free electromagnetic field. The Maxwell equations being of the form

$$\partial_\mu F^\mu{}_\nu=0\,,\quad \partial_\mu\tilde{F}^{\mu\nu}=0$$

the symmetry with respect to duality transformation (2) is obvious.

The eigenstates of the duality transformation

$$F^{\pm}_{\mu\nu}=F_{\mu\nu}\mp i\tilde{F}_{\mu\nu} \qquad (6)$$

can be considered as the fields with positive and negative chirality.

Unfortunately transformation (2) is not formulated as a transformation over vector potential A_μ in terms of which the theory is quantized. Moreover, eq. (2) expressing A'_μ through A_μ can be satisfied only on mass shell (i. e. for A_μ satisfying the Maxwell equations). For orbitrary A_μ eq. (2) is not valid.

Our purpose is to introduce chirality as a consequence of U(1)-symmetry of the action for arbitrary functions A_μ. It proves to be possible in the light-cone formalism. As we shall see, the chiral transformation of the vector potential leads in this case to duality rotation for the field strength (2) only on mass shell.

Light-cone gauge was discussed in the literature in great detail. We present here necessary formulae using spinor notations. In these notations for each vector index two spinor indices are substituted through the relation

$$b_{\alpha\dot{\alpha}}=(\sigma^\mu)_{\alpha\dot{\alpha}}\,b_\mu\,,\quad \sigma^\mu=(1,\vec{\sigma})\,.$$

Indices are raised by antisymmetric tensors $\varepsilon^{\alpha\beta}$ and $\varepsilon^{\dot{\alpha}\dot{\beta}}$. In particular, four-vector x^μ is replaced by $x^{\alpha\dot{\alpha}}$. Let us choose as usually as a new time variable the quantity $x^{1\dot{1}}=(x^0-x^3)/\sqrt{2}$. The equation of motion for vector potential A_μ.

$$\partial_\nu\partial^\nu A_\mu-\partial_\mu\partial^\nu A_\nu=0 \qquad (7)$$

rewritten in terms of spinor indices looks as follows

$$\partial_{\beta\dot\beta}\partial^{\beta\dot\beta}A_{\alpha\dot\alpha}-\partial_{\alpha\dot\alpha}\partial^{\beta\dot\beta}A_{\beta\dot\beta}=0\,. \qquad (8)$$

One can easily check that these equations do not contain time derivative of $A_{1\dot{1}}$, i. e. $\partial_{1\dot1}A_{1\dot1}$. Hence, in the canonical quantization formalism this component is not a dynamical variable but a Lagrange multiplier. So it can be expressed through other components of $A_{\alpha\dot\alpha}$.

We impose the following gauge condition

$$A_{2\dot2}=0\,. \qquad (9)$$

Thus only components $A_{1\dot2}$ and $A_{2\dot1}$ are nonvanishing. The $2\dot2$-component of eq. (8) gives

$$\partial_{2\dot2}\partial^{\beta\dot\beta}A_{\beta\dot\beta}=\partial_{2\dot2}(\partial_{2\dot2}A_{1\dot1}-\partial_{2\dot1}A_{1\dot2}-\partial_{1\dot2}A_{2\dot1})=0\,. \qquad (10)$$

Assuming that the inverse operator $\partial_{2\dot2}^{-1}$ exists we obtain

$$\partial^{\beta\dot\beta}A_{\beta\dot\beta}=0\,,$$

$$A_{1\dot1}=\partial_{2\dot2}^{-1}(\partial_{2\dot1}A_{1\dot2}+\partial_{1\dot2}A_{2\dot1})\,. \qquad (11)$$

The equations for $A_{1\dot2}$ and $A_{2\dot1}$ are

$$\Box A_{1\dot2}=0\,, \quad \Box A_{2\dot1}=0\,, \qquad (12)$$

where $\Box=\partial_\mu\partial^\mu=\frac{1}{2}\partial_{\beta\dot\beta}\partial^{\beta\dot\beta}$.

The action S can be written as

$$S=-\frac{1}{4}\int d^4x\, F_{\mu\nu}F^{\mu\nu}=$$

$$=\frac{1}{2}\int d^4x[A_\nu\Box A^\nu+(\partial_\mu A^\mu)^2]=\frac{1}{2}\int d^4x\, A_{1\dot2}\Box A_{2\dot1}\,. \qquad (13)$$

Introducing the notation

$$A_{2\dot1}=\sqrt{2}\,A \qquad (14)$$

we get

$$S=\int d^4x\,\bar{A}\,\Box A\,, \qquad (15)$$

where $\bar{A}$ is the complex conjugate of A. This expression evidently possesses U(1)-symmetry. The corresponding global transformations are

$$A\to Ae^{i\varphi}, \quad \bar{A}\to\bar{A}e^{-i\varphi}, \qquad (16)$$

In gauge (9) the field strength tensor $F_{\mu\nu}$ is expressed through A and $\bar{A}$ as

$$F_{\alpha\dot\alpha,\beta\dot\beta}=(\sigma^\mu)_{\alpha\dot\alpha}(\sigma^\nu)_{\beta\dot\beta}F_{\mu\nu}\,, \quad F^+_{\alpha\dot\alpha,\beta\dot\beta}=\varepsilon_{\dot\alpha\dot\beta}f_{\alpha\beta}\,, \quad F^-_{\alpha\dot\alpha,\beta\dot\beta}=\varepsilon_{\alpha\beta}\bar{f}_{\dot\alpha\dot\beta}\,,$$

$$f_{\alpha\beta}=\frac{1}{2}(\partial_\alpha{}^{\dot\alpha}A_{\beta\dot\alpha}+\partial_\beta{}^{\dot\alpha}A_{\alpha\dot\alpha}), \quad f_{11}=\frac{1}{2}\partial_\xi^{-1}\partial^2A-\partial_\xi^{-1}\Box\bar{A}\,, \qquad (17)$$

$$f_{22}=\partial_\xi A/2\,, \quad f_{12}=f_{21}=\bar\partial A/2\,,$$

where the following notation are used

$$\partial=\sqrt{2}\,\partial_{2\dot1}\,, \quad \bar\partial=\sqrt{2}\,\partial_{1\dot2}\,, \quad \partial_\xi=\sqrt{2}\,\partial_{2\dot2}\,,$$

$$\partial_\tau=\sqrt{2}.\,\partial_{1\dot1}, \quad \Box=\partial_\mu\partial^\mu=\frac{1}{2}(\partial_\xi\partial_\tau-\partial\bar\partial)\,. \qquad (18)$$

Quantities $\bar{f}_{\dot\alpha\dot\beta}$ can be obtained from eqs (17) by complex conjugation. Using these expressions one can readily see that the chiral rotations of $F_{\mu\nu}$ correspond to that of A only for fields A satisfying the equations of motion.

Of course the presented here construction of the chirality for free photonic field is a rather commonplace exercise. Somewhat more complicated technically is its generalization to the case of interacting fields. A nontrivial example of such a generalization in the case of photons interacting with gravity.

This theory is described by the action

$$S=-\frac{1}{4}\int d^4x\,\sqrt{-g}\;g^{\mu\varkappa}g^{\nu\lambda}F_{\mu\nu}F_{\varkappa\lambda}\,. \qquad (19)$$

It is convenient to present symmetric matrix g in the form $g=e^H$ where H is also a real symmetric matrix. Since $\sqrt{-g}=$ $=\exp\left(-\frac{1}{2}\mathrm{Tr}\,H\right)$, action (19) depends only on the traceless part of matrix H, i. e. on $h=H-I\,\mathrm{Tr}\,H/4$,

$$S=\frac{1}{4}\int d^4x\,\mathrm{Tr}\,(e^hFe^hF) \qquad (20)$$

(this is due to conformal invariance of the action). Here summation is made with the flat space metric tensor $\eta_{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1)$. We will use perturbation theory and so expand S in powers of h:

$$S = \frac{1}{4}\int d^4x\, \mathrm{Tr}\,(F^2 + 2hFF + h^2F^2 + hFhF + \ldots)\,. \tag{21}$$

Using spinor notations for the field strength of definite chirality we obtain for the terms of the zeroth, first and second order in h the following expressions

$$S^{(0)} = -\frac{1}{8}\int d^4x\,(f_{\alpha\beta}f^{\alpha\beta} + \bar{f}_{\dot\alpha\dot\beta}\bar{f}^{\dot\alpha\dot\beta})\,,$$

$$S^{(1)} = \frac{1}{8}\int d^4x\; h^{\alpha\dot\alpha\beta\dot\beta}\, f_{\alpha\beta}\bar{f}_{\dot\alpha\dot\beta}\,, \tag{22}$$

$$S^{(2)} = -\int d^4x\,\Big[\frac{1}{48}\,(\mathrm{Tr}\,h^2)\,(f_{\alpha\beta}f^{\alpha\beta} + \bar{f}_{\dot\alpha\dot\beta}\bar{f}^{\dot\alpha\dot\beta}) +$$

$$+ \frac{1}{64}\,h^{\alpha\dot\alpha\beta\dot\beta}\,h^{\gamma}{}_{\dot\beta}{}^{\delta}{}_{\dot\alpha}\, f_{[\beta\gamma}\, f_{\delta\alpha]} + \frac{1}{64}\,h^{\alpha\dot\alpha\beta\dot\beta}\,h_{\beta}{}^{\dot\gamma}{}_{\alpha}{}^{\dot\delta}\,\bar{f}_{[\dot\beta\dot\gamma}\bar{f}_{\dot\delta\dot\alpha]}\Big]\,.$$

The first term $S^{(0)}$ looks as violating chirality. However $S^{(0)}$ vanishes on mass shell, and off mass shell it actually conserves chirality as follows from eq. (15) written in terms of the fields A and $\bar{A}$.

In the first order in h one can substitute into $S^{(0)}$ expressions (17) for $f_{\alpha\beta}$ and $\bar{f}_{\dot\alpha\dot\beta}$. In this order the terms proportional to $\Box A$ and $\Box\bar{A}$ can be neglected. Thus $f \sim A$ and $\bar{f} \sim \bar{A}$ and chirallity is conserved.

Term $S^{(2)}$ explicitly violates chirality. But in the second order in h there is some chirality violating piece in $S^{(1)}$ because in this order $\Box\bar{A}$ does not vanish and $f_{\alpha\beta}$ contains field $\bar{A}$ of the opposite chirality.

To check the cancellation of the chirality violating terms let us consider S-matrix in the interaction representation. Zeroth order term $S^{(0)}$ determines propagator of field A and hence the Wick contraction for $f_{\alpha\beta}$ and $\bar{f}_{\dot\alpha\dot\beta}$. Despite the noncovariant formalism used, one can check that as usually noncovariant terms in S-matrix are cancelled out and the following covariant time-ordered products can be used

$$\langle 0|\,TF_{\mu\nu}(x)\,F_{\varkappa\lambda}(y)\,|0\rangle = \frac{\partial}{\partial x^{\mu}}\,\frac{\partial}{\partial x^{\varkappa}}\,\langle 0|\,TA_{\nu}(x)\,A_{\lambda}(y)\,|0\rangle +$$

$$+ (\mu\leftrightarrow\nu,\, \varkappa\leftrightarrow\lambda) - (\mu\leftrightarrow\nu) - (\varkappa\leftrightarrow\lambda)\,. \tag{23}$$

where the propagator of A_{μ} is of the standard form

$$\langle 0|\,TA_{\mu}(x)\,A_{\nu}(y)\,|0\rangle = i\eta_{\mu\nu}\Box^{-1}\delta(x-y)\,.$$

Returning to $f_{\alpha\beta}$ and $\bar{f}_{\dot\alpha\dot\beta}$ we get

$$\langle 0|\,Tf_{\alpha\beta}(x)\,\bar{f}_{\dot\alpha\dot\beta}(y)\,|0\rangle = i\,(\partial_{\alpha\dot\alpha}\,\partial_{\beta\dot\beta} + \partial_{\alpha\dot\beta}\,\partial_{\beta\dot\alpha})\,\Box^{-1}\delta(x-y)\,,$$

$$\langle 0|\,Tf_{\alpha\beta}(x)\,f_{\gamma\delta}(y)\,|0\rangle = -i\,(\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta} + \varepsilon_{\alpha\delta}\varepsilon_{\beta\gamma})\,\delta(x-y)\,, \tag{24}$$

$$\langle 0|\,T\bar{f}_{\dot\alpha\dot\beta}(x)\,\bar{f}_{\dot\gamma\dot\delta}(y)\,|0\rangle = -i\,(\varepsilon_{\dot\alpha\dot\gamma}\varepsilon_{\dot\beta\dot\delta} + \varepsilon_{\dot\alpha\dot\delta}\varepsilon_{\dot\beta\dot\gamma})\,\delta(x-y)\,.$$

Of course contractions ff and $\bar{f}\bar{f}$ do not describe particle propagation and the corresponding graphs are point-like in fact. The nonvanishing of these contractions is a reflection of the «alien» chirality terms $\Box\bar{A}$ in $f_{\alpha\beta}$. In the second order in h these terms result in the following contribution into the action

$$\Delta S^{(2)} = \frac{i}{2}\int d^4x\, d^4y\, h^{\alpha\dot\alpha,\beta\dot\beta}\, f_{\alpha\beta}\,\bar{f}_{\dot\alpha\dot\beta}(x)\,\bar{f}_{\dot\gamma\dot\delta}(y)\,f_{\gamma\delta}h^{\gamma\dot\gamma\delta\dot\delta} + (f\leftrightarrow\bar{f})\,. \tag{25}$$

The substitution of contact T-ordered products (24) gives

$$\Delta S^{(2)} = -S^{(2)}\,. \tag{26}$$

Thus all the chirality violating amplitudes are cancelled out in this order. We have not found yet a simple proof of such a cancellation in an arbitrary order but this seems to be only a technical problem.

In this connection the construction of graviton vertices with the use of transversality condition [8] is of interest. Let us consider for example photon scattering in gravitational field in the second order of perturbation theory. The scattering amplitude is given by the sum of the pole diagrams and contact terms h^2F^2. If the initial and final photons have opposite chiralities, the diagram with the graviton pole vanishes and the diagrams with the photon poles contract and become contact. But it is impossible to satisfy transversality condition without pole terms. Hence the amplitude must vanish.

Let us mention also Ref. [9] where the effective Lagrangian of electromagnetic field with graviton exchange taken into account was calculated. The authors of this paper noted that the Lagrangian did not contain terms violating chirality.

3. THE CHIRAL CURRENT OF PHOTONS

We have demonstrated in the light-cone gauge that the chirality of photons interacting with gravitons is conserved. One can easily write down the chirality charge ($Q=+1$ for left-handed state and $Q=-1$ for right-handed one):

$$Q=\frac{i}{4}\int d^3x\,\bar{A}(x)\,\overleftrightarrow{\partial}_\zeta A(x)\,, \tag{27}$$

where $d^3x=d^2x_\perp d\zeta$, $\overleftrightarrow{\partial}=\overrightarrow{\partial}-\overleftarrow{\partial}$. Charge Q is the integral of ζ-component of the conserved current

$$j_\mu=\frac{i}{4}\bar{A}(x)\,\overleftrightarrow{\partial}_\mu A(x)\,. \tag{28}$$

This current is not however a Lorentz-vector because fields A and $\bar{A}$ are not scalars. Lorentz-covariant current which ζ-component coincides with j_ζ is K_μ (eq. (3)). We have to pay for its covariance by the loss of continuity equation, the current K_μ is not classically conserved

$$\nabla_\mu K^\mu=-\frac{1}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{29}$$

Nevertheless, matrix elements of $\nabla_\mu K^\mu$ in an external gravitational field naively vanish. Indeed the operator

$$F_{\mu\nu}\tilde{F}^{\mu\nu}=\frac{i}{2}(f_{\alpha\beta}f^{\alpha\beta}-\bar{f}_{\dot\alpha\dot\beta}\bar{f}^{\dot\alpha\dot\beta}) \tag{30}$$

changes the chirality by ± 2 and the gravitational interaction conserves chirality. So a nonzero expectation value of $\nabla_\mu K^\mu$ in an external gravitational field can be called chiral anomaly.

The calculation of this anomaly is presented in the next section and now we make several comments of the chiral current. First, current K^μ explicitly depends on vector potential A_μ and thus is not gauge invariant. The corresponding charge

$$Q=\int d^3x\,K^0$$

is known to be gauge independent however. Moreover, as we shall see in the next section, matrix elements of K^μ in a gravitational field are independent of the photon gauge.

In Refs [10—12] the explicitly gauge invariant current

$$S^\mu=\varepsilon^{\mu\nu\varkappa\lambda}\partial_\nu F_{\varkappa\rho}F_\lambda{}^\rho$$

was proposed. It is proportional to the number difference of left-handed and right-handed photons. This current is conserved for free fields. It has noncanonical dimension however and the corresponding charge is determined not only by the photon chirality but also by its energy.

Our last comment is about analogy between K^μ and the Pauli—Lubansky vector [13]. This vector is generally defined in terms of the generators of the Poincaré group

$$\Gamma^\mu=\varepsilon^{\mu\nu\varkappa\lambda}P_\nu M_{\varkappa\lambda}\,, \tag{31}$$

where P_ν and $M_{\varkappa\lambda}$ are the generators of translation and Lorentz-rotations respectively. For massless one-particle states with spin s

$$\Gamma_\mu|p_\mu,\lambda\rangle=\lambda p_\mu|p_\mu,\lambda\rangle, \tag{32}$$

where p_μ is the particle four-momentum, $p^2=0$ and $\lambda=\pm|s|$ is its helicity.

Our purpose is to construct the current which integrated density (its zeroth component) gives chirality generator Q. $Q|p_\mu,\lambda\rangle=\lambda|p_\mu,\lambda\rangle$. It is natural to choose for such a current the vector

$$j^\mu=-\frac{i}{3!}\varepsilon^{\mu\nu\varkappa\lambda}S_{\nu\varkappa\lambda}\,, \tag{33}$$

where $S_{\nu\varkappa\lambda}$ is the spin part of the total angular momentum density,

$$S_{\nu\varkappa\lambda}=-\frac{\partial\mathcal{L}}{\partial(\partial^\nu\varphi^a)}(\Sigma_{\varkappa\lambda})_{ab}\varphi^b \tag{33a}$$

and $\Sigma_{\varkappa\lambda}$ represents the Lorentz-rotation generators $M_{\varkappa\lambda}$ for field φ_a. For spinor field ψ $\Sigma_{\varkappa\lambda}=\frac{i}{2}\sigma_{\varkappa\lambda}$ and for vector field A_μ

$$(\Sigma_{\varkappa\lambda})_{ab}=i(g_{\varkappa a}g_{\lambda b}-g_{\varkappa b}g_{\lambda a})\,.$$

The current j_μ can be considered as a one-particle analogue of the

Pauli—Lubansky vector. It is easy to see that definition (33), (33a) gives $j_\mu = a_\mu/2$ for a Dirac field ψ and $j_\mu = \frac{3}{2} K_\mu$ for a vector field A_μ. Using this definition one can construct chiral currents for any other spin.

4. TRIANGLE DIAGRAMS

The calculation of anomalies presented here is based on the dispersion relation approach of Ref. [14]. To demonstrate the similarity of the bosonic and fermionic cases we consider them in parallel.

First, let us briefly discuss the general description of the triangle diagrams (Fig. 1). Note that because of the contact vertices the

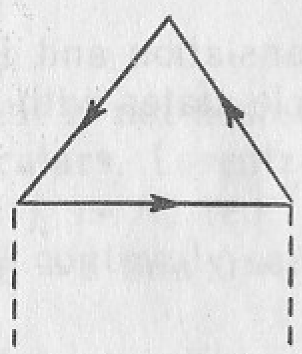

Fig. 1.

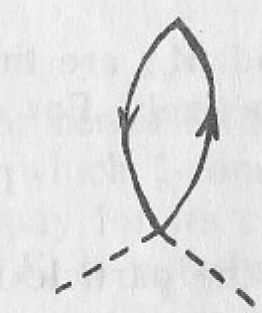

Fig. 2.

diagrams of Fig. 2 should be also taken into account. We consider two photon and two graviton production by currents a_μ and K_μ. Each matrix element is determined by a single formfactor:

$$\begin{aligned} \langle 0|a_\mu|2\gamma\rangle &= f_1(q^2)\, q_\mu F_{\varkappa\lambda}\tilde{F}^{\varkappa\lambda}, \\ \langle 0|a_\mu|2g\rangle &= f_2(q^2)\, q_\mu R_{\varkappa\lambda\rho\sigma}\tilde{R}^{\varkappa\lambda\rho\sigma}, \\ \langle 0|K_\mu|2g\rangle &= f_3(q^2)\, q_\mu R_{\varkappa\lambda\rho\sigma}\tilde{R}^{\varkappa\lambda\rho\sigma}, \end{aligned} \tag{34}$$

where q_μ is the four-momentum carried by the current. Such a structure of each matrix element is a consequence of the gauge invariance with respect to external fields. Note that the external photons and gravitons are on mass shell.

We calculate $f_{1,2,3}$ using dispersion relations. The imaginary parts of the amplitudes are given by the tree diagrams which at a first glance satisfy all invariance properties of the classical theory.

One could see, however, that if it were the case then $\mathrm{Im}\, f_{1,2,3} = 0$. Indeed the longitudinal parts of the currents in the Born approximation produce intermediate particles with the total chirality equal to ± 2, and the subsequent annihilation into photons or gravitons is possible only for zero-chirality states.

Anomalous properties of the amplitudes arise in dispersion relation approach because of the necessity of infrared regularization. If the masses of particles in the intermediate states are assumed to be zero from the very beginning, we encounter the problem of infrared definition of the imaginary parts because the singularities connected with the massless particle exchange are situated at the physical region boundary. To regularize the amplitude in this infrared region we prescribe a small mass to the intermediate particles. Such a regularization does not violate gauge invariance, both for external electromagnetic and gravitational fields, but breaks chirality conservation. A manifestation of this anomaly in this language is the nonvanishing imaginary part of $f(q^2)$. From the presented arguments it follows that $\mathrm{Im}\, f(q^2)$ can be nonzero only in the vanishing region $q^2 \sim m^2$.

The calculation of $\mathrm{Im}\, f_{1,2,3}$ proceeds through the usual lines and the final results looks as follows:

$$\begin{aligned} \mathrm{Im}\, f_1(q^2) &= \lim_{m\to 0} \left(-\frac{\alpha}{4q^2}\right)(1-v^2)\ln\frac{1+v}{1-v} = -\frac{\alpha}{2}\delta(q^2), \\ \mathrm{Im}\, f_2(q^2) &= \lim_{m\to 0} \frac{(1-v^2)^2}{128\pi q^2}\ln\frac{1+v}{1-v} = \frac{1}{192\pi}\delta(q^2), \\ \mathrm{Im}\, f_3(q^2) &= \lim_{m\to 0} \frac{v^2(1-v^2)}{128\pi^2 q^2}\ln\frac{1+v}{1-v} = \frac{1}{96\pi}\delta(q^2), \end{aligned} \tag{35}$$

where $v = \sqrt{1 - 4m^2/q^2}$ is the c.m. velocity of particles in the intermediate state.

Using dispersion relations to calculate the real parts of $f_{1,2,3}$ we come to eqs (1) and (4). Thus technically the bosonic and fermionic anomalies arise in the same way.

Now let us make some comments on the derivation of eqs (35). It is more convenient to consider instead of the matrix elements of the currents the matrix elements of their divergences. Since in each case there exists only one formfactor, the relation between these matrix elements is trivial.

Note also that contact diagram of Fig 2 gives a nonvanishing contribution into formfactor f_2.

Introduction of a nonzero photon mass deserves some discussion too. Addition of the mass terms

$$S_m = \frac{m^2}{2} \int d^4x \sqrt{-g}\, g^{\mu\nu} A_\mu A_\nu \qquad (36)$$

to action (19) corresponds to the Proca formalism for massive vector field. Of course, in this approach an extra degree of freedom appears which describes zero chirality states of the vector field.

It is worth emphasizing however that $\mathrm{Im}\, f_3$ defined by eq. (35) is a gauge invariant quantity, i. e. the same result as in the Proca formalism arises if we add to the action the term

$$\Delta S = -\frac{1}{2}\zeta \int d^4x \sqrt{-g}\, (\nabla_\mu A^\mu)^2 + S_m,$$

where S_m is the mass term (36) and ζ is the gauge parameter. Note that the same result for physical quantities is obtained also in noncovariant with respect to external field gauges

$$\Delta S = -\frac{1}{2}\zeta \int d^4x (\partial_\mu A^\mu)^2 + \frac{m^2}{2} \int d^4x\, A_\mu A^\mu. \qquad (37)$$

Gauge invariance of the result shows that the mass term ensures an infrared regularization of the theory but does not give a rise to the undesired extra degrees of freedom.

Our last comment refers to the derivation of the direct relation between expressions (35) for fermionic current ($\mathrm{Im}\, f_2$) and bosonic current ($\mathrm{Im}\, f_3$). To this end let us introduce fermionic current S_μ corresponding to the Pauli—Lubansky vector.

$$S_\mu = -\frac{1}{4m}\, \varepsilon_{\mu\nu\varkappa\lambda}\, \bar{\psi}\, \overleftrightarrow{\partial}_\nu \sigma_{\varkappa\lambda} \psi\,. \qquad (38)$$

Using equation of motion one can write this current in the form

$$S_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi + \frac{i}{2m}\, \partial_\mu(\bar{\psi}\gamma_5\psi)\,. \qquad (39)$$

First let us show that the matrix elements $\langle 0|S_\mu|2g\rangle$ and $\langle 0|K_\mu|2g\rangle$ differ only by sign. Using perturbation theory we presume that the gravitons have the same, say, left-handed helicity.

Hence, the Riemann tensor for each graviton is antiselfdual*) and right-handed connections $\omega_\mu{}^{\dot\alpha}{}_{\dot\beta}$ are equal to zero. (Here $\dot\alpha$ and $\dot\beta$ are tetrade indices.) Let us describe photon by the field $A_{\alpha\dot\alpha} = (\sigma^a)_{\alpha\dot\alpha} e^\mu_a A_\mu$ where e^μ_a is a tetrade. Since $\omega_\mu{}^{\dot\alpha}{}_{\dot\beta} = 0$, the dotted indices are sterile with respect to left-handed gravitons.

Thus the expression for $\langle K_\mu \rangle$ which describes the propagation of fields $A_{\alpha\dot 1}$ and $A_{\alpha\dot 2}$ in the loop is absolutely the same as in the case of two Weyl fields. The latter case just corresponds to the calculation of $\langle S_\mu \rangle$. Consequently the matrix elements of K_μ and S_μ differ only by sign because of the anticommutativity of fermionic operators.

On the other hand a direct calculation of imaginary part of $\langle \partial_\mu S^\mu \rangle$ gives

$$\mathrm{Im} \int d^4x\, e^{-iqx} \langle \partial_\mu S^\mu(x) \rangle = \left(1 - \frac{q^2}{4m^2}\right) \mathrm{Im} \int d^4x\, e^{-iqx} \langle \partial_\mu a^\mu(x) \rangle =$$

$$= -\frac{v^2}{1 - v^2}\, \mathrm{Im}\, f_2(q^2)\,. \qquad (40)$$

Using the explicit expression for $\mathrm{Im}\,(f_2)$ and changing sign in accordance with the presented arguments we come to obtained above result for $\mathrm{Im}\, f_3$ (see eqs (35)).

5. ELECTROMAGNETIC CORRECTIONS TO THE FERMIONIC CHIRAL ANOMALY IN A GRAVITATIONAL FIELD

Eq. (4) allows to find easily electromagnetic correction to the well-known triangle fermionic anomaly in an external gravitational field. To do that let us evaluate expectation value of eq. (1) in a gravitational field. Using expression (4) for $\langle F\tilde{F} \rangle$ we obtain

$$\nabla_\mu a^\mu = -\frac{1}{192\pi^2}\left(1 - \frac{2\alpha Q^2}{\pi}\right) R_{\mu\nu\varkappa\lambda} \tilde{R}^{\mu\nu\varkappa\lambda}. \qquad (41)$$

where Q is the electric charge of the fermion.

This result corresponds to a specific infrared regularization of two-loop diagrams, namely to the case when infrared photon mass m_γ is assumed to be much larger then fermion mass m_f

*) This procedure is equivalent to the choice of an antiselfdual external field in the Euclidean approach. This field, however, cannot be weak and the perturbation theory is not applicable. For gravitons this limitation is absent.

$$m_\gamma \gg m_f. \qquad (42)$$

In this case only two-photon intermediate state (Fig. 3) contributes into two-loop result for $\mathrm{Im}\, f_2$.

Indeed let us consider three-particle intermediate state (Fig. 4). The corresponding imaginary part is unambiguously defined for

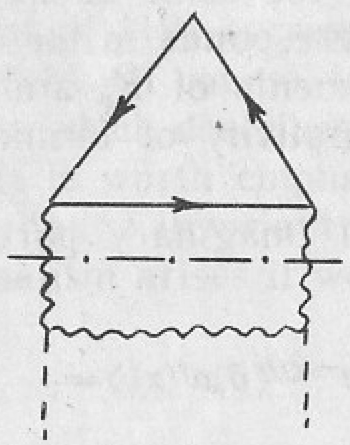

Fig. 3.

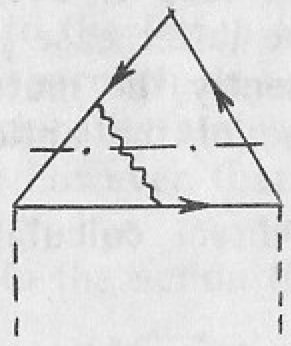

Fig. 4.

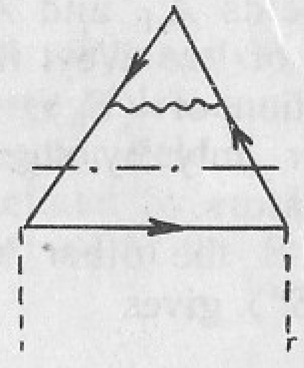

Fig. 5.

$m_\gamma \neq 0$ and $m_f \to 0$ and is equal to zero because a finite photon mass does not break conservation of a_μ when $m_f = 0$. This argument is an evident analogue of the Adler—Bardeen theorem [15].

Thus only two-particle intermediate states should be considered. As for fermionic intermediate state (Fig. 5) its contribution is equal to zero. Indeed the dispersion integral is dominated by $q^2 \sim m_f^2 \ll m_\gamma^2$. So for the determination of the imaginary part one has in fact to calculate the renormalization of the chiral current vertex at $q=0$ (up to terms $\sim q^2/m_\gamma^2$). Within our regularization scheme when $m_f = 0$ and $m_\gamma \neq 0$ this renormalization coincides with the renormalization of the vector current vertex and is cancelled out by Z-factors of external lines.

At last let us consider the two-photon intermediate state (Fig. 3). It is essential that the first loop gives a polinomial contribution into $\partial_\mu a^\mu$, $\partial_\mu a^\mu = \frac{\alpha Q^2}{2\pi} F\tilde{F}$, and hence the two-loop calculation actually reduces to one-loop one.

6. CONCLUSION

Although the existence of bosonic chiral anomaly seems to be firmly established, some questions need further clarification.

First of all the definition of bosonic chiral current does not look as satisfactory as in fermionic case. The problem is that no field-theoretical formalism has been constructed which respects both Lorentz-covariance and the chiral invariance of the action. In the light-cone approach a conserved chiral current exists but there is no explicit Lorentz-invariance. On the opposite in the usual covariant framework current K_μ is not conserved.

The way out might be an introduction of auxiliary fields. Let us recall that for instance in supersymmetry the introduction of auxiliary fields via superfields makes the symmetry manifest.

The naturalness of such a reformulation of the theory is suggested by the interpretation [16] of anomaly (4) in terms of zero modes. In Euclidean space with nontrivial topology, such that $\int d^4x \sqrt{g} R\tilde{R}$ is nonzero, the coefficient in anomaly (4) is expressed through the difference of left-handed and right-handed zero modes of antisymmetric tensor field $\varphi_{\mu\nu}$. Recall that vector field A_μ does not have zero modes [17].

Note an interesting observation due to M. Duff. For massless Weyl fermions with spin s chiral anomaly is given by [18]

$$\int d^4x \sqrt{g}\ \nabla_\mu J^\mu = (2s^3 - s)\frac{1}{96\pi^2}\int d^4x\ \sqrt{g}\ R\tilde{R}. \qquad (43)$$

Photonic anomaly (4) happens to satisfy this formula, multiplied by the natural factor $(-1)^{2s+1}$.

Let us mention in conclusion some other works about chiral anomalies for bosons. The first example of this kind refers to an antisymmetric tensor field [19].

Another example is given by a nonabelian vector field. Generalization of the expression (3) for K_μ is well known

$$K_\mu = -\varepsilon_{\mu\nu\varkappa\lambda}\left(A^a_\nu \partial_\varkappa A^a_\lambda + \frac{2}{3} f^{abc} A^a_\nu A^b_\varkappa A^c_\lambda\right).$$

As it was noted in Ref. [20] the expectation value of K_μ in an external nonabelian vector field, found by the calculation of the triangle diagrams, coincides (up to factor (-2)) with the corresponding fermionic diagrams for current a_μ in the same background field. Thus the calculation shows that an anomaly exists for $\partial_\mu K^\mu$. It is noteworthy that in the field of instantons this anomaly counts the number of bosonic zero modes in analogy with $\partial_\mu a^\mu$. The status of this anomaly is not quite clear however because the corresponding «naive» Ward identities are not yet found.

The anomaly of nonabelian vector field is of interest also because the analogous anomaly for graviton chiral current should exist. This one has not yet been calculated.

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and its Anomaly in a Gravitational Field

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и его аномалия в гравитационном поле

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