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Functional Method for
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ABSTRACT

The representation for the generating functional of quantum Heisenberg ferromagnet as an integral over two *c*-number valued fields, charged and neutral, obeying the initial conditions (instead of commonly used periodic boundary conditions) is obtained. With help of this representation the long-time dynamics of the longitudinal spin component at low temperatures is studied.

1. INTRODUCTION

For a given quantum system it is suitable to represent the partition function and generating functionals as an integral over the space of real or complex functions. Such representation allows one to use the saddle-point method and is useful to analyze perturbative or nonperturbative effects. The attempts to obtain the functional representation for the quantum Heisenberg ferromagnet had been undertaken by several authors [1, 2]. Their results do not seem to be acceptable due to the absence of explicit closed expressions [1], or, as in the case of [2], the failure of trivial identities (see section 2). In the work of one of present authors [3] the method for rewritting the partition function of the magnet in nonsymmetric phase as the integral over two number-valued fields, charged and neutral, was proposed. The quasi-particles corresponding to the charged field behaved as the ordinary bosons (magnons). The perturbation theory expansion of the functional integral [3] reproduced the operator diagramm technic results [4, 5]. It is worth noting that the results indicative of faulty of developed perturbation theory was reproduced as well. (The frozen longitudinal spin component fluctuations; see section 3 for more details.)

In the present work we show that the functional representation obtained in [3] is erroneous due to incorrectly treated some global obstructions. Using as before the method of [3] we derive the correcting expression. The several ways to verify the validity of the new functional representation are also presented. (Note that the mentioned obstructions are irrelevant in the case of high temperature dynamic studied in [6] since the representations of the works [3] and [6] have quite different structures.) With the help of the derived functional integral we consider the longitudinal fluctuations and get conclude that they do have the nontrivial dynamics. The physical meaning of the construction is similar with the instanton effects discussed by Polyakov, but our variables allows one to extract them without trying to find some specific field configurations.

The fields of integration in our integral (2.12) retain the same as in [3]. However, the charged field no longer describe bosons inspite of their number nature. This result is in accordance with the fact that magnons are not bosons when treated rigorously.

2. FUNCTIONAL REPRESENTATION

1. Let us recall the derivation of the functional integral for the partition function of the Ising model [7]:

$$H_I = -\frac{1}{2} J_{ij}\sigma_i\sigma_j, \quad \sigma_i = \pm 1, \tag{2.1}$$

$$Z_I = \mathrm{Tr}\,(e^{-\beta H_I}) = \int \prod_i d\varphi_i \exp\left(-\frac{1}{2}\beta\varphi_i\varphi_j J_{ij}^{-1}\right)\times$$

$$\times \mathrm{Tr}\exp(\beta\varphi_i\sigma_i) = \int\prod_i d\varphi_i \exp\left(-\frac{1}{2}\beta\varphi_i\varphi_j J_{ij}^{-1} + \sum_i \ln \mathrm{ch}\,\beta\varphi_i\right). \tag{2.2}$$

Here J_{ij} is the energy of exchange interaction of the spins on lattice sites $\vec r_i$ and $\vec r_j$, J_{ij}^{-1} is the matrix inverse of J_{ij}, and summation over repeated indices is assumed. The aim of the Gaussian trick used in (2.2) is to reduce the trace over the set of magnet states to the product of the one-spin traces. The direct generalization of (2.2) for the case of quantum Heisenberg ferromagnet does not take place because of noncommutativity of spin operators. But for the operator $\exp(-\beta\varepsilon H_{ex})$, where

$$H_{ex} = -\frac{1}{2} J_{ij}\vec S_i \vec S_j \tag{2.3}$$

and $\varepsilon\to 0$, the Gaussian transformation like (2.2) can be performed

with the precision up to the terms $\sim\varepsilon^2$. Thus, writing $e^{-\beta H_{ex}}$ as $(e^{-\beta\varepsilon H_{ex}})^{1/\varepsilon}$, we come to the expression for the generating functional of temperature Green's functions of spin operators $Z(\vec h)$:

$$Z(\vec h) = \mathrm{Tr}\,T\exp\left(-\beta H_{ex} + \int_0^\beta \vec h_i(t)\,\vec S_i\,dt\right) \tag{2.4}$$

in the form [1, 8]:

$$Z(\vec h) = \int\prod_i D\vec\varphi_i(t)\exp\left(-\frac{1}{2}\int_0^\beta dt\,\vec\varphi_i(t)\,J_{ij}^{-1}\,\vec\varphi_j(t)\right)\times$$

$$\times\,\mathrm{Tr}\left[T\exp\left(\int_0^\beta dt\,[\vec\varphi_i(t)+\vec h_i(t)]\,\vec S_i\right)\right]. \tag{2.5}$$

The symbol T denotes a chronological product and $\vec h_i(t)$ is the external field on the lattice site $\vec r_i$.

Path integral (2.5) is understood as a limit of finite-dimensional approximations:

$$D\vec\varphi_i(t) = \prod_{\alpha=x,y,z}\ \prod_{n=1}^{N} d\varphi_i^\alpha(\beta n/N), \quad N\to\infty. \tag{2.6}$$

Let us rewrite (2.2) in more convenient form shifting the variables $\vec\varphi_i(t)$ by $-\vec h_i(t)$:

$$Z(\vec h) = \int\prod_i D\vec\varphi_i(t)\exp\left(-\frac{1}{2}\int_0^\beta dt\,\vec\varphi_i\,J_{ij}^{-1}\,\vec\varphi_j + \int_0^\beta dt\,\vec h_i\,J_{ij}^{-1}\,\vec\varphi_j -\right.$$

$$\left. -\frac{1}{2}\int_0^\beta dt\,\vec h_i\,J_{ij}^{-1}\,\vec h_j\right)\prod_i \mathrm{Tr}\left[T\exp\left(\int_0^\beta dt\,\vec\varphi_i(t)\,\vec S_i\right)\right]. \tag{2.5'}$$

Time-ordered operator exponent

$$A(t) = T\exp\left(\int_0^t dt'\,\vec\varphi(t')\,\vec S\right) \tag{2.7}$$

is defined by the equation

$$\dot A(t) = (\vec\varphi(t)\,\vec S)\,A(t) \tag{2.8}$$

and initial condition $A(0)=1$. The operator $A(t)$ can not be expressed explicitly as a functional of $\vec{\varphi}(t)$. However, the substitution does exist which recast T-ordered exponent into the product of usual ones (see also [3]). Indeed, let consider operator given in the explicit form:

$$B(t)=\exp(S^+\psi^-(t))\ \exp\left(S^z\int_0^t \rho(t')\,dt'\right)\times$$

$$\times\exp\left(S^-\int_0^t \psi^+(t')\exp\left(\int_0^{t'} \rho(t'')\,dt''\right)dt'\right)\exp(-S^+\psi^-(0)), \qquad (2.9)$$

where $S^\pm=S^x\pm iS^y$ and $\psi^\pm$, ρ are some new functions of t. Using the commutar relations of spin operators one can be convinced that the operator $B(t)$ obeys the equation

$$\dot{B}(t)=\{S^+(\dot{\psi}^- -\rho\psi^- -\psi^+(\psi^-)^2)+S^-\psi^+ + S^z(\rho+2\psi^+\psi^-)\}\,B(t). \qquad (2.10)$$

The last factor in (2.9) provides the equality $B(0)=1$. Thus, the substitution

$$\varphi^z=\rho+2\psi^+\psi^-,$$
$$\varphi^+=\psi^+, \qquad (2.11)$$
$$\varphi^-=\dot{\psi}^- -\rho\psi^- -\psi^+(\psi^-)^2$$

where $\varphi^\pm=\frac{1}{2}(\varphi^x\pm i\varphi^y)$, recasts $A(t)$ into the form (2.9).

Thus, considering ρ and $\psi^\pm$ as the new fields of integration, we can calculate the trace of T-exponent explicitly and obtain a closed functional representation for $Z(\vec{h})$. However, the change of variables (2.11) contains $\dot{\psi}^-$ on the right-hand side and either boundary or initial conditions should be imposed. It would be naturally to impose periodic boundary conditions (as it was done in [3]), but in that case the mapping (2.11) becomes noninvertible. In the present work we use the Cauchy-like condition:

$$\psi^-(0)=0. \qquad (2.12)$$

2. We may consider the fields $\varphi^\pm$, φ^z in the measure $D\varphi^z D\varphi^+ D\varphi^-$ as independent complex variables with the constraints $\operatorname{Im}\varphi^z=0$, $\varphi^+=(\varphi^-)^*$ fixing the surface of integration. The variables ρ, $\psi^\pm$ are also treated as independent when evaluating the Jacobian $J[\rho,\psi^+,\psi^-]$,

$$D\varphi^z D\varphi^+ D\varphi^- = J[\rho,\psi^+,\psi^-]\,D\rho\, D\psi^+ D\psi^-. \qquad (2.13)$$

The Jacobian $J=\det\hat{f}$ depends on a regularization of the differential $\hat{f}$ of the transformation (2.11). Expressions for J obtained under different regularization prescriptions can distinquish from each other by the factor $\exp\left(\alpha\int_0^\beta \rho\,dt\right)$, where α is an arbitrary real number. (See e. g. [9], where the determinant of similar operator is calculated.) In our case the regularization is fixed by the following evident condition. The partition function $Z(\vec{h}=0)$ calculated with the help of the functional integral with $J_{ij}=c\cdot\delta_{ij}$ must be equal to the expression

$$Z(\vec{h}=0)=\exp\left(\beta M\left(c\frac{1}{2}S(S+1)+\text{const}\right)\right),$$
$$J_{ij}=c\cdot\delta_{ij}, \quad M=\sum_i 1, \qquad (2.14)$$

being the trivial consequence of the kinematic identity $\vec{S}^2=S(S+1)$. It will be shown that the Jacobian corresponding to (2.14) is equal to

$$J[\rho,\psi^+,\psi^-]=\text{const}\cdot\exp\left(-\frac{1}{2}\int_0^\beta \rho\,dt\right). \qquad (2.15)$$

(The lattice site index is temporarily omitted.) This value of J is provided by the following discretization of transformation (2.11).

$$(\rho_n\equiv\rho(t_n),\ldots,\quad t_n=n\beta/N,\quad \Delta=\beta/N,\quad N\to\infty),$$

$$\varphi_n^z=\rho_n+\psi_n^+(\psi_n^-+\psi_{n-1}^-),\quad \varphi_n^+=\psi_n^+, \qquad (2.11')$$

$$\varphi_n^-=\frac{1}{\Delta}(\psi_n^- -\psi_{n-1}^-)-\frac{1}{2}\rho_n(\psi_n^-+\psi_{n-1}^-)-\frac{1}{4}\psi_n^+(\psi_n^-+\psi_{n-1}^-)^2.$$

Indeed, let the one more transformation be performed: $\rho\to\tilde{\rho}=\rho+2\psi^+\psi^-$, $\psi^\pm\to\psi^\pm$, having the unity Jacobian. The Jacobian of transformation from $\psi^\pm$, φ^z to $\tilde{\rho}$, $\psi^\pm$ is simply $\det(\partial_t-\tilde{\rho}+2\psi^+\psi^-)=\det(\partial_t-\rho)$. Thus, we can conclude that

$$\det\hat{f}=\det(\partial_t-\rho). \qquad (2.16)$$

In the discretization (2.11) with the condition (2.12) the right-hand side of (2.16) coincides with the Jacobian of one-field transformation

$$\varphi_n^- = \frac{1}{\Delta}(\psi_n^- - \psi_{n-1}^-) - \frac{1}{2}\rho_n(\psi_n^- + \psi_{n-1}^-),$$

where $n=1, \ldots, N$, $\psi_0^- = 0$. The last one can be calculated easily and is equal to

$$\det \hat{J} = \prod_{n=1}^{N}\left(\frac{1}{\Delta} - \frac{1}{2}\rho_n\right). \tag{2.17}$$

In the limit $\Delta \to 0$ we get the expression (2.15) where $\text{const} = 1/\Delta^N$. It is worth noting that the regularization (2.11′) leads to the coincidence of the operators $A(t)$ and $B(t)$ up to the terms $\sim\Delta$ inclusive. (The corresponding discretization of equations (2.5) and (2.7) is assumed.)

3. The trace of operator $B(\beta)$ can be calculated easily for an arbitrary value of spin S. However, we restrict ourselves with the case $S=1/2$ in order to avoid unwieldy expressions:

$$\operatorname{Tr} A(\beta) = \operatorname{Tr} B(\beta) = \exp\left(\frac{1}{2}\int_0^\beta \rho\, dt\right) +$$

$$+\exp\left(-\frac{1}{2}\int_0^\beta \rho\, dt\right)\left(1 + \psi^-(\beta)\int_0^\beta \psi^+(t)\exp\left(\int_0^t \rho\, dt'\right)dt\right). \tag{2.18}$$

Hence $Z(\vec{h})$ is represented as the following functional integral:

$$Z(\vec{h}) = \int D\rho\, D\psi^+\, D\psi^-\, e^{-\Gamma}\prod_i \Big[1 + \psi_i^-(\beta)\times$$

$$\times\int_0^\beta \psi_i^+(t)\exp\left(\int_0^t \rho_i\, dt'\right)dt + \exp\left(\int_0^\beta \rho_i\, dt\right)\Big],$$

$$\Gamma = \int_0^\beta dt\left(\frac{1}{2}\rho_i J_{ij}^{-1}\rho_j + 2\psi_i^+ J_{ij}^{-1}\dot{\psi}_j^- -\right.$$

$$-2\rho_i J_{ij}^{-1}(\psi_i^-\psi_j^+ - \psi_j^-\psi_i^+) + 2\psi_i^+\psi_i^- J_{ij}^{-1}\psi_j^+\psi_j^- -$$

$$\left.-2\psi_i^+ J_{ij}^{-1}(\psi_j^-)^2\psi_j^+\right) + \sum_i \int_0^\beta \rho_i\, dt - \int_0^\beta \vec{h}_i J_{ij}^{-1}\vec{\varphi}_j\, dt. \tag{2.19}$$

Here the quadratic in $\vec{h}$ term is omitted because it does not contribute into nonsimultaneous correlators which we are interested in. The field $\vec{\varphi}_i$ in product with the source $\vec{h}(t)$ implies the expression in terms of ρ, $\psi^{\pm}$ according to (2.11). We can deform the initial surface of integration to the standard one:

$$\operatorname{Im}\rho = 0, \quad \psi^+ = (\psi^-)^* \tag{2.20}$$

if the integral converges over any intermediate surface. For the case of ferromagnetic exchange interaction this convergence is provided by the kinetic term in Lagrangian $\psi_i^+ J_{ij}^{-1}\dot{\psi}_j^-$ and the mentioned deformation is possible.

4. It is crucial that the conditions imposed on the fields ψ are initial (not the boundary) ones: $\psi^-(0) = 0$. It means in particular that the excitations described by the field ψ (magnons) do not obey in strict sense the Bose—Einstein statistics. The last sentence is the obvious consequence of the spin operators boundedness though.

The validity of equality (2.14) is the necessary condition for the representation (2.19) to be correct. When $J_{ij} = c\delta_{ij}$ the action Γ is equal to:

$$\Gamma = c^{-1}\sum_i \int_0^\beta dt\left(\frac{1}{2}\rho_i^2 + 2\psi_i^+\dot{\psi}_i^- + \rho_i\right), \tag{2.21}$$

and $Z(\vec{h}=0)$ reduces to the product of one-lattice-site Gaussian integrals being easily calculated. The result is just (2.14). It can be verified by explicit calculations that the change of regularization would destroy the equality (2.14). (For example, the free energy would be nontrivial function of β instead of the constant.)

5. In the paper [3] the periodic boundary conditions $\psi^-(0) = \psi^-(\beta)$ are used instead of the present work initial conditions. When the periodic boundary conditions are imposed and the regularization (2.11′) is used the Jacobian of (2.11) is equal to (compare with (2.17); $\psi_0^- = \psi_N^-$):

$$\det \hat{J}_{per} = \prod_{n=1}^{N} \left(\frac{1}{\Delta} - \frac{1}{2}\rho_n \right) +$$

$$+(-1)^{N-1} \prod_{n=1}^{N} \left(-\frac{1}{\Delta} - \frac{1}{2}\rho_n \right) \xrightarrow[\Delta\to 0]{} \text{const} \cdot \text{sh} \left(\frac{1}{2} \int_0^\beta \rho \, dt \right). \quad (2.22)$$

In contrast with (2.15) the Jacobian (2.22) vanishes on the hypersurfaces $\int_0^\beta \rho \, dt = i\pi m$ where m is an arbitrary integer. It means that the holomorphic mapping of complex spaces $(\rho_n, \psi_n^+, \psi_n^-) \to (\varphi_n^z, \varphi_n^+, \varphi_n^-)$ is multivalued: a single configuration of fields $(\varphi^z, \varphi^\pm)$ corresponds to some set of $(\rho, \psi^\pm)$-configurations (see e. g. [10]).

When the segment $(0, \beta)$ is compactified in a circle then the zeros of Jacobian in the space of fields $(\rho, \psi^\pm)$-configurations do not disappear under modifications of transformation (2.11) keeping the number of derivatives in the right-hand side. The regularization providing the equality (2.14) does not exist in this case. The same one is valid e. g. in the case of antiperiodic boundary conditions. The passage from boundary to initial conditions as a method of getting rid of zero modes was proposed by S.N. Vergeles in his work [11] devoted to SU(2)-anomaly.

Refined method proposed in [2] leads to the functional integral which does not obey the relation (2.14). Apparently, this fact is connected with impossibility of unambiguous gauge—fixing in the approach of the work [2]. (The gauge of [2] suffers from the Gribov ambiguity.)

6. There exists another method to check the correctness of our representation (2.18). It is also a good illustration of great importance of boundary conditions in the functional integral.

Let us consider the vacuum expectation value of operator $e^{-\beta H_{ex}}$:

$$Z_0 = \langle 0 | e^{-\beta H_{ex}} | 0 \rangle = \exp \left(-\frac{M}{2} S^2 \beta J(0) \right) \quad (2.23)$$

where $J(0) = \sum_j J_{ij}$ and $M = \sum_i 1$. On the other hand, the functional representation of Z_0 is derived from (2.19) by the replacement of $\text{Tr}\, B(\beta)$ (2.18) by $\langle 0 | B(\beta) | 0 \rangle = \exp\left(-S \int_0^\beta \rho \, dt \right)$:

$$Z_0 = \int D\rho \, D\psi^+ D\psi^- \exp \left(-\Gamma + (-S + 1/2) \sum_i \int_0^\beta \rho_i \, dt \right). \quad (2.24)$$

The condition $\psi^-(0) = 0$ allows us to evaluate (2.24) exactly in spite of nonlinear interactions of fields $\psi^\pm$ (which prevents (2.24) from exact evaluation when periodic boundary conditions are imposed). Indeed, the bare propagator of field ψ is

$$\langle \psi_i^-(t_1) \, \psi_j^+(t_2) \rangle_b = \frac{1}{2} J_{ij} \, \theta(t_1 - t_2) \quad (2.25)$$

and when the integration over $\psi^\pm$ is performed, all the contributions to effective action (functional of $\rho_i(t)$) containing more than one $\psi\psi\psi\psi$ and $\psi\psi\rho$ vertexes vanish. As a result the effective action W_0 is the linear functional of $\rho_i(t)$:

$$\exp\{-W_0[\rho_i(t)]\} \equiv \int D\psi^+ D\psi^- \exp(-\Gamma_\psi) =$$

$$= \text{const} \cdot \exp \left(-\frac{1}{2} \sum_i \int_0^\beta \rho_i \, dt \right). \quad (2.26)$$

(Here Γ_ψ denotes ψ-dependent part of Γ.) Thus, the integration over $\rho_i(t)$ is Gaussian and performing it we come to (2.23). (Note, that the value of step function $\theta(0) = 1/2$ corresponds to our regularization (2.11′).)

7. In the recent work [12] the functional integral having local action has been derived for the cases $S = 1/2$, 1. Together with c-number-valued fields, similar to our ρ, ψ, this integral contains fermionic ghosts. The integration over these additional fields can not be performed exactly and in this sense the difference between the representations [12] and (2.19) is radical.

It is worth elucidating the origin of our set of fields (ρ, ψ). Classical spin is a vector of fixed length and its states form a sphere. It is well known that the sphere is covered by two complex planes. Thus, in order to define the state one complex number ψ and the number of the plane must be given. The last two-valued variable is similar with the Ising degree of freedom and can be described by the real field ρ as is shown by the formula (2.2). The quantum noncommutativity fills this picture with the nontrivial dynamics.

3. DYNAMICS AT LOW TEMPERATURES

1. The low temperatures limit means that $\beta J_0 \gg 1$ and $\bar{n} = \langle S^+ S^- \rangle \ll 1$. Besides that we are using one more small parameter which is defined as R^{-1}, where R is the length of exchange interaction. This parameter has been introduced for the first time for Heisenberg ferromagnet in the works [4]. When the lattice size a is less than R: $a/R \ll 1$, the Fourier transform of $J(\vec{r}_i - \vec{r}_j) \equiv J_{ij}$, $J_{\vec{k}}$, has the order of magnitude of J_0 in the domain with the linear size $\sim 1/R$ around the point $\vec{k} = 0$ and is about $J_0 (a/R)^3$ in the rest of the first Brillouin zone. It follows from this, for example, that $\sum_{\vec{k}} J_{\vec{k}}^2 \simeq J_0^2 (a/R)^3$. In case of nearest-neighbour interaction we have $(a/R)^3 = 1/z$, where z is the number of these neighbours, and for cubic lattices $1/z \leqslant 1/6$.

There are some effects that are correctly described in the picture of weakly interacting magnon gas. In that cases the representation (2.12) is less convenient than the explicit Anzatz by Holstein—Primakoff [13] or Dyson—Maleev [14], which ignore the finite-dimensionality of spin states space. Therefore in the present work we are concentrate upon the nonmagnon part of the magnet dynamics, more precisely, upon spin longitudinal component fluctuations dynamics.

2. It was obtained in [4] that the correlator $K_{ij} = \langle S_i^z(0)\, S_j^z(t) \rangle$ contains two terms of different nature. The first one arises due to the magnons, which take away the magnetization and, thereby, contribute to $K_{ij}(t)$. This dynamical contribution is proportional to a power of $(\beta R)^{-1}$. The second term corresponds to the «frozen-in» Ising-like fluctuations of the longitudinal spin component (i. e. of the S^z). It is nonanalytical in the temperature and does not have a smallness in R^{-1}. At low temperatures and not very large t this last static term in $K_{ij}(t)$ can be ignored. However, when the real time $t \to \infty$, the dynamical contribution decreases as t^{-1} (see [4, 5]), while the static one does not depend on t. The absence of the terms in perturbations series of [3, 4] providing the relaxation

of static contribution in $K_{ij}(t)$ means that this series are not complete. This remark applies equally to the equilibrium version of the diagram technique for the spin operators developed in [5]. It is shown below that in the functional representation (2.19), where the structure of spin degrees of freedom is taken into account correctly, the «frozen» fluctuations do revive. Their dynamics can be described by the neutral scalar field with the nonzero mass.

3. We are unable to perform the integration over $\psi^{\pm}$ in (2.19) exactly and, as usually, we divide the action into free part and perturbation:

$$Z(h) = \int D\rho\, D\psi^+\, D\psi^- \exp(-\tilde{\Gamma}),$$

$$\tilde{\Gamma} = \Gamma_0 + \Gamma_{int},$$

$$\Gamma_0 = \int_0^\beta dt \left(\frac{1}{2} \rho_i J_{ij}^{-1} \rho_j + 2\psi_i^+ J_{ij}^{-1} \dot{\psi}_j^- - \right.$$

$$\left. - 2\bar{\rho} J_{ij}^{-1} (\psi_i^- \psi_j^+ - \psi_i^+ \psi_j^-) \right) + \sum_i \int_0^\beta \rho_i\, dt -$$

$$- \int_0^\beta h_i J_{ij}^{-1} (\rho_j + 2\psi_j^+ \psi_j^-)\, dt, \tag{3.1}$$

$$\Gamma_{int} = \int_0^\beta dt (- 2\tilde{\eta}_i J_{ij}^{-1} (\psi_i^- \psi_j^+ - \psi_i^+ \psi_j^-) +$$

$$+ 2\psi_i^+ \psi_i^- J_{ij}^{-1} \psi_j^+ \psi_j^- - 2\psi_i^+ J_{ij}^{-1} (\psi_j^-)^2 \psi_j^+) - \Gamma_{np},$$

$$\Gamma_{np} = \sum_i \ln \left[1 + \exp \left(\int_0^\beta \rho_i\, dt \right) + \psi_i^-(\beta) \int_0^\beta \psi_i^+(t) \exp \left(\int_0^t \rho_i\, dt' \right) dt \right].$$

Here Γ_{np} denotes the nonpolynomial part of Γ_{int}, $\tilde{\eta}_i = \rho_i - \bar{\rho}$, $\bar{\rho}$ is the expectation value of the field ρ_i. We assume that $\vec{h}_i = (0, 0, h_i)$ since we are interested in the dynamics of z-component only. The saddle-point value $\bar{\rho}_0$ is defined by minimization of the bare effective potential, which is equal to (see (2.26)):

$$V_0(\bar{\rho}) = \frac{1}{2} J_0^{-1} \bar{\rho}^2 + \bar{\rho} + W_0(\bar{\rho}) = \frac{1}{2} J_0^{-1} \bar{\rho}^2 + \frac{1}{2} \bar{\rho}. \tag{3.2}$$

Then

$$\bar{\rho}_0 = (-1/2)\, J_0 \tag{3.3}$$

and the mean spin $\langle S^z \rangle_0 = -1/2$ corresponds to the ordinary ferro-magnetic vacuum. The bare propagator of the field ψ in $\vec{k}$, t-representation has the form:

$$G^0_{\vec{k}}(t_1, t_2) = \langle \psi^-_{\vec{k}}(t_1)\, \psi^+_{\vec{k}}(t_2) \rangle =$$

$$= \frac{1}{2} J_{\vec{k}}\, \theta(t_1 - t_2) \exp\left[\bar{\rho}\left(1 - \frac{J_{\vec{k}}}{J_0}\right)(t_1 - t_2)\right]. \tag{3.4}$$

The contributions of terms from Γ_{int} into various averages are small either in temperature or in the inverse radius of interaction and a perturbation theory can be developed.

4. The real-time correlation functions are obtained from representation (3.1) immediately by substitution of П-like contour in complex plane of t for the line segment $(0, \beta)$ (see [15], and, for example, [6]).

The correlator $K_{ij}(t)$ is given in our representation by the average:

$$K_{ij}(t) = J_{il}^{-1} J_{im}^{-1} \langle (\tilde{\eta}_l(0) + 2\psi^+_l(0)\, \psi^-_l(0))\, (\tilde{\eta}_m(t) +$$

$$+ 2\psi^+_m(t)\, \psi^-_m(t)) \rangle. \tag{3.5}$$

When $t \to \infty$, the dynamical contribution due to magnons disappears as t^{-1} (this may be verified by explicit calculations). What only remains is

$$K_{ij}(t) \xrightarrow[t \to \infty]{} J_{il}^{-1} J_{im}^{-1} D_{lm}(t)\,,$$

$$D_{lm}(t) = \langle \tilde{\eta}_l(0)\, \tilde{\eta}_m(t) \rangle. \tag{3.6}$$

After the integration over $\psi^{\pm}$ we come to the expression for $Z(h)$ of the form:

$$Z(h) = \int D\tilde{\eta} \exp\left(-\int_0^\beta dt\, \tilde{\eta} J_{ij}^{-1} \tilde{\eta}_j + \sum_i W[\tilde{\eta}_i] + \int_0^\beta dt\, \tilde{\eta}_i J_{ij}^{-1} h_j\right) \tag{3.7}$$

where the functional $W[\tilde{\eta}]$ is expressible as the series in $\tilde{\eta}$. This expansion starts from the second power of $\tilde{\eta}$: $W = W_2 + W_3 + \ldots$, since the linear terms can be eliminated by redefinition of $\bar{\rho}$. The behaviour of $K_{ij}(t)$ at $t \to \infty$ is determined by the infrared-singular

contributions in $W_2[\tilde{\eta}]$:

$$W_2[\tilde{\eta}] = \frac{1}{2} e^{\beta\bar{\rho}} \left(\int_0^\beta \tilde{\eta}\, dt\right)^2 + \int_0^\beta dt \left(\sum_{\vec{k}} G^0_{\vec{k}}(\beta, t)\, e^{t\bar{\rho}}\right) \left(\int_0^t \tilde{\eta}\, dt'\right)^2. \tag{3.8}$$

The terms, which are omitted in (3.8), are less than the kept one, or do not play the role in forming of asymptotics of $K_{ij}(t)$ at $t \to \infty$. The latter relates, for example, to the contribution from the vertex of local interaction of fields $\tilde{\eta}$ and ψ. Using in (3.8) the following form of $J_{\vec{k}}$:

$$J_{\vec{k}} = \begin{cases} J_0, & ka < a/R \\ J_\infty \ll J_0, & ka > a/R \end{cases} \tag{3.9}$$

we obtain in the first nonvanishing order of R^{-1}:

$$W_2[\tilde{\eta}] = \frac{1}{2} e^{\beta\bar{\rho}} \left(\int_0^\beta \tilde{\eta}\, dt\right)^2 +$$

$$+ J_0 \left(\frac{a}{R}\right)^3 \frac{4\pi}{3} \int_0^\beta dt\, e^{\bar{\rho} t} \left(\int_0^t \tilde{\eta}\, dt'\right)^2 +$$

$$+ J_\infty e^{\beta\bar{\rho}} \int_0^\beta dt \left(\int_0^t \tilde{\eta}\, dt'\right)^2.$$

Keeping the infrared-singular terms only we come to the expression:

$$W_2^{sing}[\tilde{\eta}] = \frac{1}{2} e^{\beta\bar{\rho}} \left(\int_0^\beta \tilde{\eta}\, dt\right)^2 + J_\infty e^{\beta\bar{\rho}} \int_0^\beta dt \left(\int_0^t \tilde{\eta}\, dt'\right)^2. \tag{3.10}$$

Substitute the contour C, coinciding with the line segment $(0, \beta)$ on the real axis and having the ends on $+i\infty$, instead of this segment $(0, \beta)$. (It is meaningful to say about the infrared singularity in this case only.) If the integral $\int_C \tilde{\eta}\, dt$ is not equal to zero for the given trajectory, the contribution of this trajectory to the action is infinitely large due to the second term in (3.10) and thus can be ignored. It means that the change of variables

$$\tilde{\eta} = \dot{\eta}$$

is admissible, where the field η is equal to 0 on the ends of the contour C. (More precisely: we change the variables from $\tilde{\eta}(t)$ to $\eta(t)$ and $\xi=\int_C \tilde{\eta}\,dt$. Quadratic in ξ part of the action has an infinite coefficient and the fluctuations of this mode do not contribute to the dynamics of other modes and to observable correlators). The generating functional (3.7) takes the form

$$Z(h)=\int D\eta \exp\Big(-\sum_{\vec{k}} \frac{1}{J_{\vec{k}}} \int_C dt(|\dot{\eta}_{\vec{k}}|^2 - -J_{\vec{k}} J_\infty e^{\beta\bar{\rho}} |\eta_{\vec{k}}|^2) + \sum_{\vec{k}} \frac{1}{J_{\vec{k}}} \int_C h_{\vec{k}} \dot{\eta}_{-\vec{k}}\, dt\Big). \tag{3.11}$$

It is clear from this expression that the terms of the next order of R^{-1} are inessential only for the fluctuations $\eta_{\vec{k}}$ having $ka > a/R$. This part of $\vec{k}$-space dominates and here $J_{\vec{k}}=J_\infty$. The functional $Z(h)$, restricted to this fluctuations, is equal to

$$Z(h)=\int D\eta \exp\Big(-\frac{1}{J_\infty}\sum_{\vec{k}} \int_C dt(|\dot{\eta}_{\vec{k}}|^2 - -m_\eta^2 |\eta_{\vec{k}}|^2) + \frac{1}{J_\infty} \sum_{\vec{k}} \int_C h_{\vec{k}} \dot{\eta}_{-\vec{k}}\, dt\Big), \tag{3.11'}$$

where

$$m_\eta^2 = J_\infty^2 e^{\beta\bar{\rho}}. \tag{3.12}$$

The expression for the propagator of η satisfying the zero boundary conditions on the far-away ends of C follows from (3.11′) immediately:

$$\langle \eta_{\vec{k}}(0)\, \eta_{-\vec{k}}(t) \rangle = \frac{J_\infty}{2m_\eta} e^{-m_\eta t}. \tag{3.13}$$

Thus, the asymptotics of the longitudinal correlator is

$$K_{\vec{k}}(t) \xrightarrow[t\to+\infty]{} \frac{1}{2} e^{\beta\bar{\rho}/2} e^{-m_\eta t} \tag{3.14}$$

for $ka > a/R$. Such $\vec{k}$ give the dominant contribution to, for example,

$$K_{ii}(t) = \sum_{\vec{k}} K_{\vec{k}}(t)$$

and the asymptotics of $K_{ii}(t)$ coincides with the right-hand side of (3.14) in the leading order of R^{-1}.

The exponential in β factor in the expression (3.12) for m_η^2 has a simple explanation. The step-by-step perturbation theory describes the shallow transversal choppines against a background of «frozen-in» longitudinal fluctuations. The relaxation of this fluctuations does not arise in any finite order of magnon perturbation theory. The destruction of longitudinal correlations is provided by the fast rolling of spins on lattice sites; the probability of such configurations is suppressed just by the factor $e^{\beta\bar{\rho}}$ since $-\bar{\rho}$ is the required energy. The similar symmetry restoration picture for the particle in the double well potential was described in detail in the work [16].

5. Our formalism is not homogeneous in time: for example, the averages $\langle \rho_i(t) \rangle$ and $\langle \psi_i^+(t)\, \psi_i^-(t) \rangle$ are nontrivial functions of t. But the observable value

$$\langle S_i^z \rangle = J_{ij}^{-1}(\langle \rho_j(t) \rangle + 2\langle \psi_j^+(t)\, \psi_j^-(t) \rangle) \tag{3.15}$$

does not depend on t in any order of the perturbation theory. It can be verified by direct calculations that the expansion of (3.1) over powers of R^{-1} leads for $\langle S_i^z \rangle$ to the result of [4]. The Green's function of the field ψ is defined in this computations by the bilinear part of action Γ together with the term (see (3.1))

$$\sum_i \psi_i^-(\beta) \int_0^\beta \psi_i^+(t)\, e^{\bar{\rho}t}\, dt. \tag{3.16}$$

Instead of (3.4) we obtain:

$$G_{\vec{k}}(t_1, t_2) = G_{\vec{k}}^0(t_1, t_2) + \frac{G_{\vec{k}}(\beta, t_2)\, F_{\vec{k}}(t_1)}{1 - F_{\vec{k}}(\beta)},$$

$$F_{\vec{k}}(t) = \int_0^\beta G_{\vec{k}}^0(t, t')\, e^{t'\bar{\rho}}\, dt'. \tag{3.17}$$

Using for the evaluation of $\langle \psi_i^+(t) \psi_i^-(t) \rangle$ the propagator (3.17) and taking into account the contribution to $\langle \rho_j(t) \rangle$ due to the term of Γ_{int}:

$$\sum_i \psi_i^-(\beta)\int_0^\beta \psi_i^+(t)\, e^{t\bar{\rho}}\left(\int_0^t \eta_i(t')\, dt'\right) dt . \tag{3.18}$$

We come to the expression for $\langle S_i^z \rangle$ of the work [4].

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