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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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INSTANTONS IN QCD II. CORRELATORS OF PSEUDOSCALAR AND SCALAR CURRENTS

PREPRINT 88-63

НОВОСИБИРСК

Instantons in QCD II. Correlators of Pseudoscalar and Scalar Currents

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ABSTRACT

We calculate the instanton-induced contributions to correlation functions in the QCD vacuum using numerical data on the ensemble of pseudoparticles (*PPs*) obtained previously. We show how quarks, «hopping» from one *PP* to another, do form the pseudoscalar mesons, with parameters close to the experimental ones. The hierarhy of the π, K, η, η' masses are explained, as well as the sign and (approximately) the magnitude of the $\eta-\eta'$ mixing. All octet members have about the same coupling constants, while that for η' seems to be larger by about 50%. Our results for the $I=1$ scalar channel is consistent with the meson mass around 1 GeV and the coupling close to that of the pion.

1. INTRODUCTION

In the first paper of this series [1] (below referred as CI) we have obtained the numerical data for properties of the ensemble of interacting instantons in the QCD vacuum (for brevity, the «instanton liquid»). By this paper we start applications of this theory to hadronic phenomenology, making the measurements of corresponding correlation functions.

As the most reasonable starting point we have chosen the correlators of various pseudoscalar and scalar quark-antiquark operators. The reason for it is that the corresponding lowest excitation of the QCD vacuum, the pseudoscalar and scalar mesons, are in many respects rather exceptional members of the family of hadrons. Through the whole history of hadronic physics, from the «naive» quark models to such modern approaches as numerical lattice calculations, the puzzles related to these particles have remained in the center of attention and, in spite of multiple efforts made, we still do not have their satisfactory explanation.

The most striking observation is, of course, the fact that the pion is extraordinary light, $m_\pi=138$ MeV, but this fact was actually understood when it was realized that the $SU(3)_f$ chiral symmetry is spontaneously broken in QCD, and the pion is nothing else but the *nearly massless Goldstone mode.* It is so light because *m_π^2 is proportional to the sum of quark masses* (m_u+m_d), which is only of the order of 10 MeV (see details and references e. g. in [2]). However, then another observation becomes puzzling: *the pion* *appears to be extraordinary heavy*, because these 10 MeV is multiplied by some *large factor* of the order of 1700 MeV (we discuss this point in the next section in details). We formulate this question as our problem number one: (i) *why masses of the pseudoscalar octet mesons are so sensitive to small quark masses*?

The second problem is the «U(1) problem», first noticed by S. Weinberg [3]: (ii) *why* η' *is extraordinary heavy*, $m_{\eta'}=958$ MeV? It was in principle solved when it was realized that the U(1) chiral symmetry is explicitly broken by the axial anomaly [4]. However, the quantitative description of the η' mass is still missing.

The third problem we address in this work is also not new: (iii) *why the strange sector in the pseudoscalar multiplet is not separated from the nonstrange one, as in other multiplets, but, on the contrary, is strongly mixed with them*? This fact was «in principle» understood when G.'t Hooft has discovered specific flavor-mixing mechanism induced by instantons [5], but, again, no quantitative theory of such mixing phenomena was so far developed.

Not going into detailed account for all efforts of the theorists also addressing these problems, let us only mention the most important observations related with the latest development. Not speaking about various model-dependent approaches, we comment on two the most powerful and most fundamental approaches toward the hadronic physics: the so-called *QCD sum rules* [6] and *numerical simulations on the lattice* (see e. g. reviews [7]).

It was found in [8] that the ordinary operator product expansion (OPE), which works so well for the vector and the axial currents, give wrong results for the correlators of the pseudoscalar currents. We discuss this point in Section 4 and show that, formally speaking, that is due to nonsingular at small distances ($x\to 0$) effects, missing in that form of this theory. From physical point of view the way out was suggested in our work [9], (referred below as AIV, as it was the fourth paper in the series) in the framework of the «instanton liquid» model. (In fact, in the present work we actually repeat the same calculations as in [9], but on the new level: instead of the model-dependent estimates and the single-instanton approximation we now have a detailed theory of the instanton-induced phenomena.)

Lattice numerical experiments have demonstrated chiral symmetry breaking, but they still have not produced convincing quantitative data capable to shed light on three problems mentioned

above. However, indirectly, they have produced some hints showing that there is strong sensitivity of the problem to small quark masses, even below 100 MeV: for example, even the order of chiral symmetry restoration phase transition was shown quite sensitive to quark masses. Unfortunately, studies of light quarks needs very large and still unaccessible lattices. Let us also mention that instantons were demonstrated to be there, in the ensemble of lattice fields. Unfortunately, it is difficult to see them, for they are strongly masked by quantum fluctuations. The corresponding methods are not yet very well developed and, respectively, the data are still very uncertain. What was found is more or less consistent with our approach, see BI [10].

Comparing our approach to the lattice-based calculations, one should say that, although both share a lot of common technical points, they have quite different status and potential applications. In lattice studies one in principle integrates over all possible gauge field configurations, while we concentrate on only a subclass of them (and therefore may miss a lot). However, our strategy have paid back by the tremendous simplification: the number of variables needed for the description of gauge fields per unite space-time volume is decreased by about 4 orders of magnitude! Therefore, with very modest computer power used, we are able to discuss a lot of quantities still unaccessible on the lattice.

Concluding our comments on current literature, we have to mention also studies of the instanton-induced interaction between quarks made by its introduction as *ad hoc, as some effective multifermion interaction.* Such studies were actually initiated by the classical Nambu paper, which has first emphasized connections between the chiral symmetry breaking and superconductivity. They are reviewed recently, in particular, in the paper [14] many questions overlapping with this paper are considered, to which we refer below. The principle problems with such approach is that such effective interaction is, unfortunately, of the unrenormalizable type, so it is hopeless to make a consistent theory out of it. Our approach deals with the same physics, but without these difficulties.

Let us now describe in «general physics» terms the essence of the effects studied in our work. We developed a detailed theory of how light quarks, «hopping» from one pseudoparticle to another, may travel to large distances.

As it was repeatedly emphasized in the previous works, the physics of the quark condensate is analogous to that of conducting electrons in metals: it is based on existence of many potential wells with nearly degenerate «energy levels». In metals degeneracy of the atomic energy levels is caused by identical physical conditions for various atoms in the ordered lattice. (A disordered matter may be a conductor too, e. g. liquid metals, but in this case random displacemets of atomic levels should not be too strong.) In our case the ensemble of pseudoparticles is disordered, but rather dilute. Approximate degeneracy of the eigenvalues of the Dirac operator is caused by the topological reasons, due to the Atyah-Singer theorem. With proper account for «hopping» of quarks, the rather *narrow zone of collectivized quark states* [11] with eigenvalues near zero is formed. The width of this zone is proportional to relatively small «hopping» amplitude.

In this paper we study excitations of the vacuum, and show that at least the pseudoscalar octet of mesons is made entirely out of such collectivized quark states. Small width of the zone explains why the vacuum parameters are so sensitive even to small quark masses (thus explaining the problem (i)), see Section 5.

Another part of the story is «short distance» manifestation of instantons, based mainly on strong flavor-mixing effects induced by them. These effects are also studied in details below, and we have succeded to reproduced the points (ii), (iii) below, see Sections 6, 7.

Complecting the introduction let us add few words about scalars, for which the situation is very unclear, even from the phenomenological side. The lowest isovector and isoscalar particles with such quantum numbers are (nearly degenerate) $S^*(975)$ and $\delta(980)$, but they are unusual in some respects and were susspect to be a four-body (two quark—two antiquark) mesons. Other isosinglet candidates are $\varepsilon(1300)$ and also the low mass broad enhancement seen in the $\pi\pi$ scattering, a «sigma meson». No bright isovector candidates other then δ particle are on the market. Thus, phenomenology does not actually suggest how the corresponding correlation function should look like, and therefore their measurements cannot really test a new theory. But, as the measurements of a scalar currents are made in paralell with the pseudoscalar ones, we also present these data, see Section 8.

Discussion of the results obtained in this paper is made in the concluding Section 9.

2. CORRELATORS AND HADRONIC PHENOMENOLOGY

The correlation functions we are going to measure are the time-ordered current product avaraged over the QCD vacuum

$$\Pi_{AB}(x) = \langle 0 | T\{j_A(x)\, j_B(0)\} | 0 \rangle . \quad (1)$$

The current subscripts indicate the names of the lowest mesonic state excited by them, e. g.

$$j_{\pi^+} = \bar{d}(i\gamma_5)\, u, \quad j_{\pi^0} = \frac{i}{\sqrt{2}} (\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)\,;$$

$$j_\eta = \frac{i}{\sqrt{6}} (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s), \quad j_{K^+} = i(\bar{s}\gamma_5 u); \quad (2)$$

$$j_{\eta'} = \frac{i}{\sqrt{3}} (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s)\,.$$

The correlators are calculated in the Euclidean space-time, so the distance x between the points is space-like. The Fourier transform of a correlator obeys standard dispersion relation

$$\Pi^F(q^2) = i \int e^{iqx} \Pi(x)\, d^4x\,,$$

$$\Pi^F(q^2) = \frac{1}{\pi} \int ds\, \mathrm{Im}\, \Pi^F(s)/(s+q^2) \quad (3)$$

which can be rewritten back in space-time representation as

$$\Pi(x) = \frac{1}{\pi} \int \mathrm{Im}\, \Pi^F(s)\, D(\sqrt{s}, x)\, ds\,,$$

$$D(m, x) = \frac{m}{4\pi^2 x} K_1(mx) \quad (4)$$

(where K_1 is Bessel function of imaginary argument). The function $D(m, x)$ here is just the propagator of a mass m particle to distance x, so relation (4) is selfexplanatory. The logical structure of the present work (as well as of the lattice studies or of the QCD sum rules) is as follows: one calculates the l.h.s. of (4) from the theory and compare it with the r.h.s., derived from the experiment. However, there are some differences between all three approaches in details, on which we have to comment.

On the lattice people usually measure the correlator not of the local currents, but rather by corresponding «charges», or currents integrated over the space. Therefore the correlators depend on the Euclidean time only, and the propagator D in (4) is substituted by its one-dimensional version, the simple exponential.

More precisely, due to periodic boundary conditions expression like $\exp(-mx)+\exp(-m(L-x))$ is used, where L is the box size. We also work with the piece of the QCD vacuum on the torus, so we also have to account for existence of nontrivial paths on this manifold. We have done it as follows. Dispersion relation (3) holds for any momenta q, and if one takes them to be discretized, $q_n=2\pi n/L$, with integer n, and then perform the discrete Fourier transform he gets relation like (4), but with the modified $D_T(m,x)$, with the propagator on the torus:

$$\Pi_T(x)=\frac{1}{\pi}\int \mathrm{Im}\,\Pi^F(s)\,D_T(\sqrt{s},x)\,ds,$$

$$D_T(x)=\frac{1}{L^4}\sum_{n_1n_2n_3n_4=-\infty}^{\infty}\exp\left(i\frac{2\pi}{L}n_ix_i\right)\Big/\left[m^2+\left(\frac{2\pi}{L}\right)^2 n^2\right]. \qquad (5)$$

Let x be along one of the axis (e. g. the temporal one), then one corresponding sum can be calculated

$$D_T(x)=\frac{1}{L^3}\sum_{n_1n_2n_3=-\infty}^{\infty}\frac{\cosh\left(x-\frac{1}{2}\right)\omega}{2\omega\sinh(\omega L/2)};\quad \omega^2=m^2+\left(\frac{2\pi}{L}\right)^2(n_1^2+n_2^2+n_3^2). \qquad (6)$$

Evaluating the remaining sum numerically we get results shown in Fig. 1, as the ratio of D_T/D. For mL close to unity corrections are strong enough, and at $m\to 0$ they are even infinite due to «zero mode» for $n_i=0$. However, for $mL\gg 1$ the only deviation of D_T/D from unity is at $x=L/2$, where it approaches 2 because there are two paths of the same length here. Our lightest particle, the «pion» (see Section 5) is such that $m_{«\pi»}L\sim 3$, our D_T/D ratio is actually different from unitity only at x around $L/2$.

(Few words about the theoretical status of our trick. We do not pretend that the use of the propagators on the torus give us the exact correlator on the torus by the eq. (5). In fact, if one puts QCD on the torus, in principle the physical spectral density is to be modified as well. Again, we hope that our $mL\geqslant 3$ is large enough, and one may neglect such «spectrum rearrangement». On the lattice people do the same, even in worse conditions.)

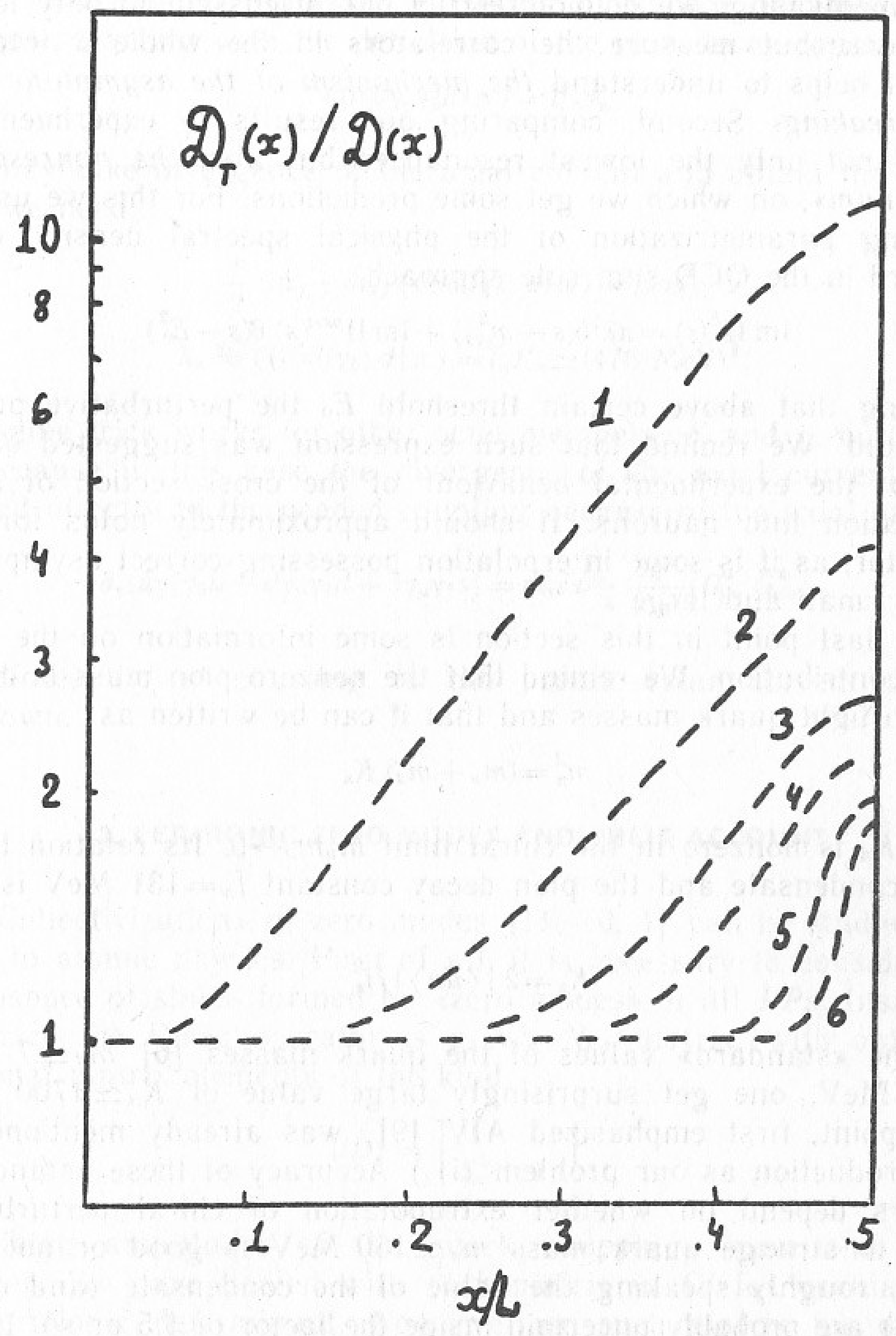


Fig. 1. The ratio of the propagator on the torus to that for ordinary space as the function of the distance, normalized to the torus period L. The numbers near the curves are the particle mass m times L.

Another difference between our approach and the lattice one is worth mentioning: we do not restrict our discussion to only lowest excitations, but measure the correlators in the whole x interval. First, it helps to understand *the mechanism of the asymptotic freedom breaking*. Second, comparing our results to experiment we include not only the lowest resonances but also *the nonresonant «continuum»*, on which we get some predictions. For this we use the following parametrization of the physical spectral density (it is standard in the QCD sum rule approach):

$$\mathrm{Im}\,\Pi^F(s) = \pi\lambda^2\delta(s - m_{res}^2) + \mathrm{Im}\,\Pi^{pert}(s)\,\theta(s - E_0^2) \qquad (7)$$

assuming that above certain threshold E_0 the perturbative predictions hold. We remind that such expression was suggested on the basis of the experimental behaviour of the cross section of e^+e^- annihilation into hadrons. It should approximately holds for any correlator, as it is some interpolation possessing correct asymptotics both at small and large x.

The last point in this section is some information on the resonance contribution. We remind that the nonzero pion mass is due to nonzero light quark masses and that it can be written as

$$m_\pi^2 = (m_u + m_d)\,K_\pi \qquad (8)$$

where K_π is nonzero in the chiral limit $m_u m_d \to 0$. Its relation to the quark condensate and the pion decay constant $f_\pi = 131$ MeV is well known

$$K_\pi = 2\,|\langle \bar{u}u\rangle|/f_\pi^2. \qquad (9)$$

With the «standard» values of the quark masses [6] $m_d \simeq 7$ MeV, $m_u \simeq 4$ MeV, one get surprisingly large value of $K_\pi \simeq 1700$ MeV. (That point, first emphasized AIV [9], was already mentioned in the Introduction as our problem (i).) Accuracy of these «standard» numbers depend on whether extrapolation of chiral perturbation theory to strange quark mass $m_s \simeq 150$ MeV is good or not (see below), roughly speaking the value of the condensate (and quark masses) are probably uncertain inside the factor of 1.5 or so. Let us also remind the reader, that in any calculations from first principles the quark masses are some external input parameters. Thus, one actually calculates not, say, the pion mass but *rather the value of the parameter* K_π. This may be done using any values for quark masses, provided they are small enough.

The coupling constants of the mesons to the pseudoscalar currents also can be expressed in terms of known parameters. For example, starting with the definition of the pion decay constant

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 d|\pi\rangle = i f_\pi p_\mu \qquad (10)$$

one may take divergence of the axial current and obtain the expression we need

$$\frac{1}{2}(m_u + m_d)\langle 0|\bar{u}(i\gamma_5)\,d|\pi\rangle = f_\pi m_\pi^2\,,$$

$$\lambda_\pi \stackrel{def}{=} \langle 0|\bar{u}(i\gamma_5)\,d|\pi\rangle = f_\pi K_\pi \simeq (476\ \mathrm{MeV})^2. \qquad (11)$$

The same trick works for other octet members, K and η, but not for η', because in this case the divergence of the axial current is not related directly to the needed coupling because of the axial anomaly

$$\partial_\mu(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s) = m_s\bar{s}s + \frac{3g^2}{16\pi^2}(G^a_{\mu\nu}\tilde{G}^a_{\mu\nu})\,. \qquad (12)$$

Therefore, the η' coupling to the pseudoscalar current remains unknown.

3. FERMIONIC ZERO MODES AND THEIR ACCOUNT

«Collectivization» of zero modes [11, 10, 1] can be studied similarly to atomic physics. First of all, it is necessary to consider only a subspace of states formed by «zero modes» of all PPs. Inside this subspace the Dirac operator is a $N_{PP}\cdot N_{PP}$ matrix, with only nondiagonal matrix elements of the kind

$$(i\hat{D}) = \begin{vmatrix} 0 & T_{IA} \\ T_{AI}^+ & 0 \end{vmatrix} \qquad (13)$$

describing amplitudes of the quark «hopping» from one PP to another. We have dealed with such matrix in CI for evaluation of the fermionic determinant. Now we use such matrix for deriving quark propagator, which, roughly speaking, is just an inverse to it.

Let us introduce a set of states $\varphi_i(x)$ as the linear combinations of PP zero modes ψ_0 which diagonalize the (zero mode part) of the operator iD

$$\varphi_j = \sum_{K=1}^{N_{PP}} U_{jK}\, \psi_0(x - z_K)\,,$$

$$(i\hat{D})\, \varphi_j(x) = \varepsilon_j \varphi_j(x)\,. \qquad (14)$$

In their terms the corresponding part of the propagator looks as follows

$$s^{(zero\ modes)}(x, y) = \sum_j \frac{\varphi_j(x)\, \varphi_j^+(y)}{\varepsilon_j + im} \qquad (15)$$

where we have also introduced a nonzero quark mass, acting as a regulator in case of too small eigenvalues ε_j.

We have expicitly found $\varphi_j(x)$ for each configuration of our *PP* ensemble, and, while measuring VEV of some operators or their correlation function, express them in the φ-representation. This is done in three subsequent steps. For example, consider measurements of the expectation value of the scalar operator $\bar{\psi}\psi$ at some point x. At the first step, one writes down this operator in the «zero mode basis», substituting the zero modes of the two *PPs* $I1$ and $I2$

$$(\bar{\psi}\psi)_{I1,I2} = \mathrm{Tr}\,[\,\psi_0^+(x - z_{I1})\, \psi_0(x - z_{I2})] \qquad (16)$$

(by chirality properties, this particular quantity is nonzero either if both $I1$, $I2$ are instantons, or both are anti-instantons).

The second step is rotation of this matrix $(\bar{\psi}\psi)_{I1,I2}$ to the eigenstates of the operator $(i\hat{D})$, or to the φ-representation:

$$(\bar{\psi}\psi)_{ij} = \sum_{I1,I2} U_{i,I1}^+ (\bar{\psi}\psi)_{I1,I2}\, U_{I2,j}\,. \qquad (17)$$

Finally, the third one is the summation over all states, including the propagators

$$\langle\, \bar{\psi}\psi(x)\,\rangle = \sum_j \frac{(\bar{\psi}\psi)_{jj}}{\varepsilon_j + im} \qquad (18)$$

Performing this sum we get the value of the scalar current at the point x, to be used for evaluation of VEVs, correlators etc.

In particular, just averaging over x the quantity $\bar{\psi}\psi$ we may obtain the value of the quark condensate. Such averaging can also be done analytically, because the zero modes $\psi_0(x)$ and their (nor-

malized) combinations $\varphi_i(x)$ obey the normalization condition

$$\int \psi_0^{+I}(x)\, \psi_0^J(x)\, d^4x = \delta^{IJ} \qquad (19)$$

which leads to the well known expression

$$\langle 0 |\, \bar{\psi}\psi\, | 0 \rangle = \frac{1}{V_4} \sum_i \frac{1}{\varepsilon_i + im} \qquad (20)$$

used in *CI* (and elsewhere). We have used this relation as a test of the efficiency of numerical averaging over points, and it has shown that random selection of points on the torus is «too expensive», the number converges to the value (20) too slowly. Of course, it happens because the scalar current has rather complicated distribution, thus it is desirable to select points more effectively.

Solution to this problem is rather standard. We have generated ensemble of points distributed with the weight function $W(x)$

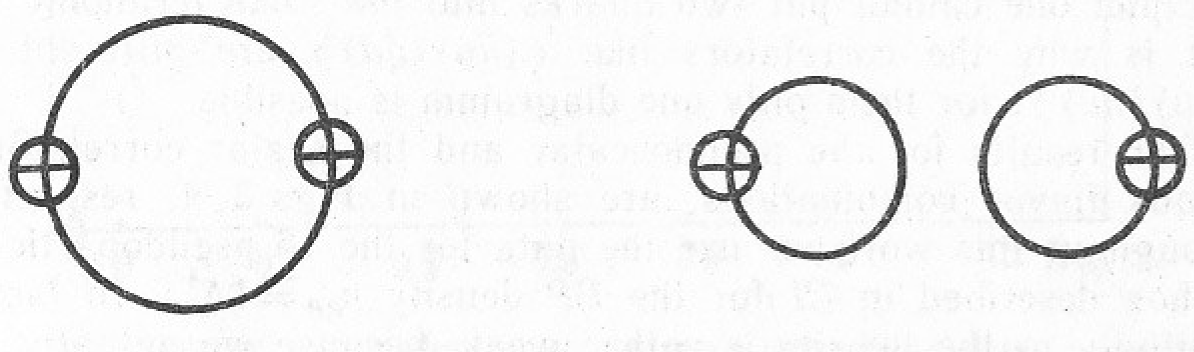
$$W(x) \sim \sum_{I,A} \frac{\rho_{I,A}^2}{|(x - z_{I,A})^2 + \rho_{I,A}^2|^3} \qquad (21)$$

which mimics behaviour of the scalar current in the dilute instanton gas, and then make measurements by the expression

$$\langle J(x)\,\rangle = \int J(x)\, dx = (\int W(x)\, dx\ \langle J/W\rangle_W\,;$$

$$\langle J/W\rangle_W \stackrel{def}{=} \int (j/W)\, W dx / \int W dx \qquad (22)$$

In this case convergence to the true result was much better, and usually few thousands of points give the average with about 20% accuracy.



a *b*

Fig. 2. Two diagrams for the correlators, see text.

Another test is provided by measurements of the pseudoscalar current $\bar{\psi}(i\gamma_5)\psi$. Of course, its average value is zero. In our model this happens because instantons and anti-instantons give now contributions of the opposite sign. Again, random selection of points x leads to inefficient calculation, but the method described above gives more satisfactory results, with $\langle\bar{\psi}(i\gamma_5)\psi\rangle/\langle\bar{\psi}\psi\rangle$ typically of the order of only few percent for few thousands of points.

Before we proceed to the correlation functions let us add few technical remarks. In the local limit $x\to 0$ the correlators are just the currents squared, and in order to measure these quantities the most natural (and the most economic) calculation is made if the ensemble of points is taken with another weight function

$$W'(x)\sim\sum_{I,A}\frac{\rho_{I,A}^4}{[(x-z_{I,A})^2+\rho_{IA}^2]^6} \tag{23}$$

which mimic the behaviour of the currents squared. We have used it in the calculation of the correlators

$$\langle j(x)\,j(0)\rangle = \; dy\, j(x+y)\, j(y) = \; [j(x+y)\, j(y)/W(y)]\; W(y)\, dy = \int (W dy)\langle j(x+y)\, j(y)/W(y)\rangle_W . \tag{24}$$

Our second comment is that if two quarks are of the same flavor there exist two diagrams (shown in Fig. 2*a*,*b*), contributing with the opposite sign (due to fermionic nature of anticommuting quark operators):

$$\langle j(x)\,j(0)\rangle = \sum_{k,l}(j_{kl}(x)\,j_{lk}(0) - j_{kk}(x)\,j_{ll}(0))/(\varepsilon_k+im)(\varepsilon_l+im). \tag{25}$$

In the φ representation one may say that (25) just obeys the Pauli principle: one cannot put two quarks into *the same* fermionic state. That is why the correlators like $\langle(\bar{u}u)(\bar{d}d)\rangle$ are different from $\langle(\bar{u}u)(\bar{u}u)\rangle$: for them only one diagramm is possible.

Our results for the pseudoscalar and the scalar correlators, in various flavor combinations, are shown in Figs 3, 4, respectively. Throughout this work we use the data for the 16 pseudoparticles in the box described in *CI* for the *PP* density $n_{pp}=1\Lambda_{PV}^4$. In fact, the sensitivity to the density is rather weak, because we actually make relative measurements, of the correlators divided by $\langle\bar{\psi}\psi\rangle^2$. (By such analysis we also essentially reduce fluctuations from one configuration to another, as well as those due to particular choice of the coordinate points.)

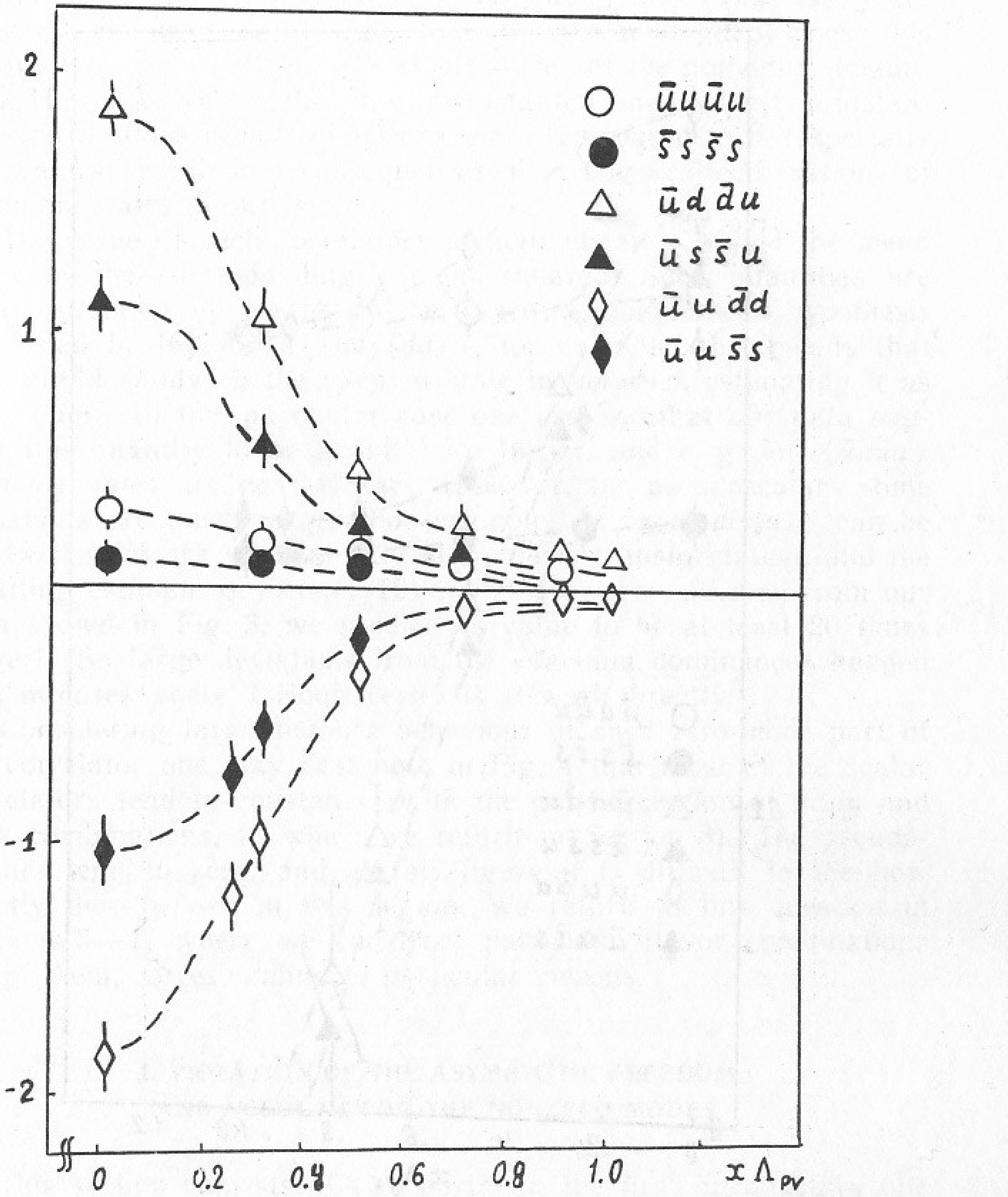


Fig. 3. Contributions of zero modes into the correlation functions of the pseudoscalar currents, divided to $\langle\bar{u}u\rangle^2$. Various points correspond to different flavor combinations. Note, that $i\gamma_5$ are omitted in the notations but are actually implied. The dashed lines are just for guiding the eye.

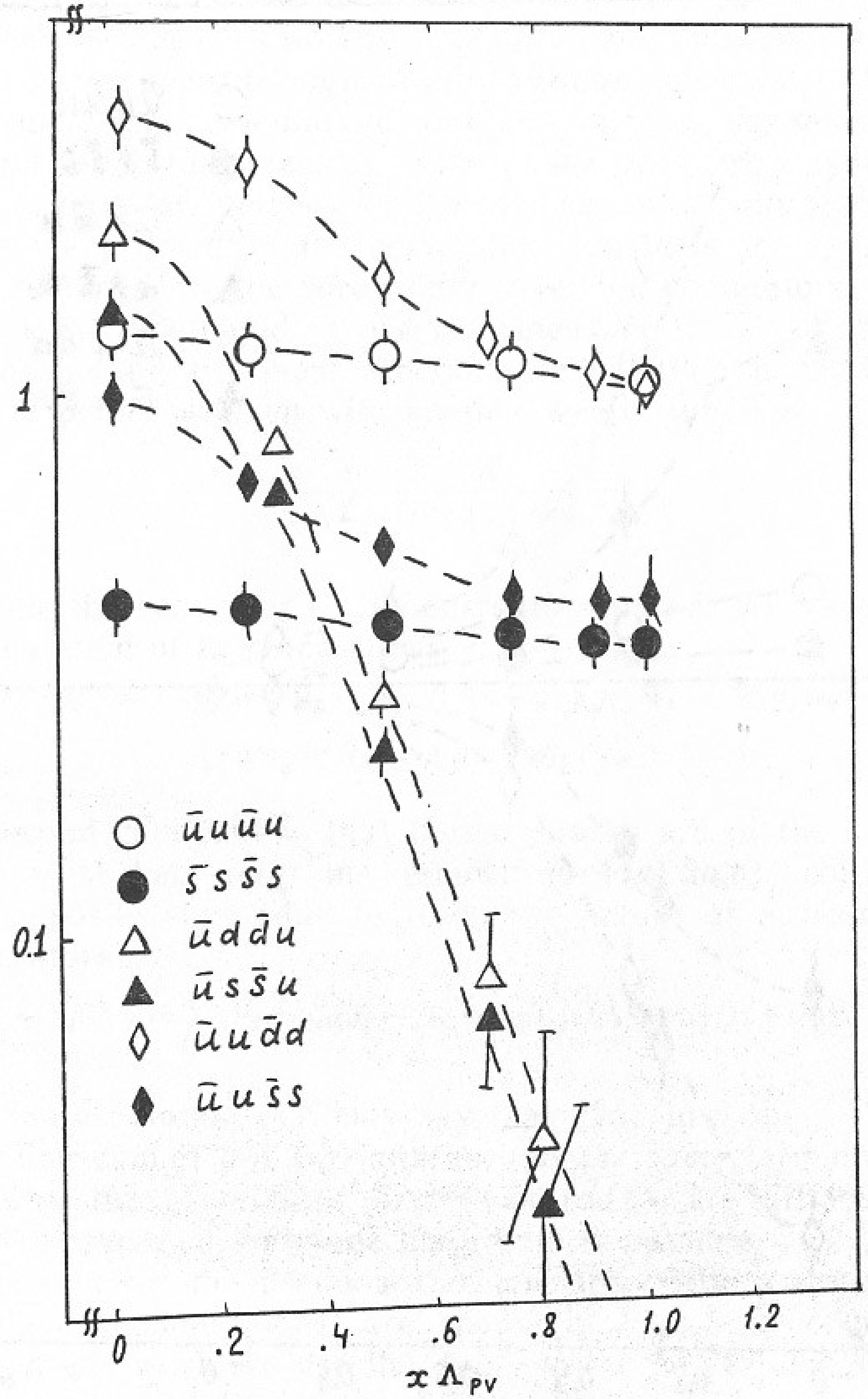


Fig. 4. The same as in Fig. 3, but for the scalar currents.

Looking at the data points at both Figs 3, 4 one may note, that the flavor-changing combinations $\langle (\bar{u}u)\ (\bar{d}d) \rangle$ and $\langle (\bar{u}u)\ (\bar{s}s) \rangle$ are at small distances even larger than the flavor-diagonal ones: this the trace of the structure of 't Hooft vertex of the individual instantons. However, we see that in our «instanton liquid» the two-instanton contributions is not really very small compared to it (especially for scalars) which is a consequences of strong «collectivization» of fermionic states in our liquid.

The value of such correlators at zero distance is just the mean value of the corresponding current squared. Such quantities are often estimated by means of the «vacuum dominance» hypothesis suggested in [6]. For $\langle (\bar{u}u)(\bar{d}d) \rangle$, for example, this means that one should sandwich the vacuum state in between, estimating it as $\langle \bar{u}u \rangle \langle \bar{d}d \rangle$. In this particular case one can see that our data suggest this quantity to be about twice larger, and e. g. for $\langle \bar{u}u\bar{u}u \rangle$ such estimates are nearly exact. However, for pseudoscalars some deviations are much larger. For example, $\langle (\bar{u}i\gamma_5 d)(\bar{d}i\gamma_5 u) \rangle$ can be sandwiched by the vacuum only after Fiertz transformation, and the resulting estimate is then $(1/12)\langle \bar{u}u \rangle^2$. However, as seen from our data shown in Fig. 3, we predict its value to be at least 20 times larger! (So large deviations from the «vacuum dominance» happen only in cases where 't Hooft vertex is at work directly.)

Considering large-distance behaviour of such zero-mode part of the correlator one may first note in Fig. 4 that most of the scalar correlators tend to constants (with the only exception of $\bar{u}d\bar{d}u$ and $\bar{u}s\bar{s}u$ combinations, to which we return in Section 8). The pseudoscalars tend to zero, and at this figure it is difficult to see how exactly they behave in this region. We return to this question in Sections 5—7, where we construct particular flavor combinations out of them, corresponding to particular mesons.

4. VIOLATION OF THE ASYMPTOTIC FREEDOM AND ACCOUNT FOR THE NONZERO MODES

This section consists of two parts. In the first introductory one we discuss the asymptotic freedom violation by nonperturbative phenomena, suggested by the formulae based on the operator product expansion (OPE) in its form used in the QCD sum rules [6]. Then we turn back to our «instanton liquid», and consider the contribution of the nonzero modes.

The OPE version suggested in [6] is based on the expansion in powers of $1/q^2$ (q is the momentum transfer flowing trough the current). Returning to space-time representation one gets (e. g. for the pion correlator)

$$\Pi_\pi(x)=\frac{3}{\pi^4x^6}-\frac{\langle 0|(gG^a_{\mu\nu})^2|0\rangle}{128\pi^4x^2}+\frac{\langle 0|A|0\rangle}{16\pi^2}\ln\left(\frac{1}{x\mu}\right)^2+O(x^2\ln x^2);$$

$$A\overset{def}{=}\frac{\pi\alpha_s}{2}[(\bar u\sigma_{\mu\nu}t^a u)^2+(\bar d\sigma_{\mu\nu}t^a d)^2]+\frac{\pi\alpha_s}{3}(\bar u\gamma_\mu t^a u+\bar d\gamma_\mu t^a d)\times \tag{26}$$

$$\times\Big(\sum_{u,d,s}\bar q\gamma_\mu t^a q\Big)-\pi\alpha_s(\bar u\sigma_{\mu\nu}t^a u)(\bar d\sigma_{\mu\nu}t^a d).$$

Note, that the first term $O(1/x^6)$ is nothing else but *the free quark propagator squared*, so it really describes free propagation of two quarks in agreement with the «asymptotic freedom». Note also, that only terms possessing a singularity at $x\to0$ are obtained, while the finite ones and those proportional to positive powers of x^2 are absent. (In this respect, the original OPE suggested by Wilson was different: it has included expansion in powers of x). The question we are going to address is as follows: which corrections, the singular or the nonsingular ones, do in fact dominate, describing deviations from the asymptotic freedom. (We return to it in Sections 5, 6.)

The next $O(1/x^2)$ term also is of interest to us, because it is related to the so called «duality-based» estimates [6] for the threshold parameter E_0 of our parametrization of the spectral density (7). Consider the correlation function written as an integral over the physical spectral density (7) at $x\to0$. The particle propagator is $O(1/x^2)$, therefore our resonance give such a singular contribution. However, the nonresonant continuum also produce such $O(1/x^2)$ term, which is most easily seen if one writes it as

$$\int_{E_0}^{\infty}\mathrm{Im}\,\Pi^{(pert)}(s)\,D(\sqrt{s},x)\,ds=\int_0^{\infty}\mathrm{Im}\,\Pi^{(pert)}D\,ds-\int_0^{E_0}\mathrm{Im}\,\Pi^{(pert)}D\,ds=$$

$$=\frac{3}{\pi^4x^6}-O(1/x^2) \tag{27}$$

where the second negative term can be considered as the contribution of the «missing states» with $E<E_0$. Assuming that $O(1/x^2)$ contributions are equal at both sides, one gets a relation

$$-\frac{\langle(gG^a_{\mu\nu})^2\rangle}{128\pi^4}=\frac{\lambda_\pi^2}{4\pi^2}-\frac{3}{64\pi^4}E_0^4. \tag{28}$$

With the «standard» numbers for $\langle(G^a_{\mu\nu})^2\rangle$, λ_π it suggests $E=1.3$ GeV . Note however, that the $\langle gG)^2\rangle$ term in (27) is actually small and unimportant, and, without it, such relations were suggested earlier. The idea was that *the resonance contribution is «dual» to the gap «eaten up» from the non-resonant continuum*, if their integral contributions are considered.

The next OPE terms contain certain four-fermion operators, and, by dimensional reason, the corresponding singularity is only logarithmic. In the preceeding section we have discussed the contribution of the zero modes, which obviously give finite contributions, at $x\to0$ tending to VEVs of some four-fermion operators. As log (x) (more precizely, it is log $(x\mu)$ where μ is the normalization point scale) is not actually large in practice, all depends on relative magnitude of the corresponding VEVs. Those which are connected with zero modes are so much enhanced, that they are much larger than the OPE ones, which we disregard in what follows.

Let us now turn to the contribution of the nonzero modes. We remind that for the one-instanton background field the contribution of the nonzero modes to quark propagator was explicitly found in Ref. [12]:

$$S(x,y)=S^{zero\ modes}(x,y)+\hat D_x\Delta(x,y)\left(\frac{1+\gamma_5}{2}\right)+\Delta(x,y)\,\hat D_y\left(\frac{1-\gamma_5}{2}\right) \tag{29}$$

where Δ is the scalar propagator in the same background field

$$\Delta(x,y)=\frac{1}{4\pi^2(x-y)^2}\,\frac{1+\rho^2(x_\mu\tau_\mu^-)(y_\nu\tau_\nu^+)/x^2y^2}{[(1+\rho^2/x^2)(1+\rho^2/y^2)]^{1/2}} \tag{30}$$

where x and y are counting from the instanton center and the 4-dimensional Pauli matrices are

$$\tau_\mu^\pm=(\vec\tau,\mp i).$$

It can be used for the evaluation of the correlators, as a (color and spin) trace of the propagator (29) squared. After somewhat combersome calculations we get the result

$$\Pi(x,y)=\frac{3}{\pi^4(x-y)^6}\,\frac{A(\Delta,\sigma,\cos\theta,\rho)}{B(\Delta,\sigma,\cos\theta,\rho)};$$

$$A=(\sigma^2+\rho^2)^4+\Delta^2[(\sigma^2+\rho^2)^2\sigma^2\sin^2\theta+2\rho^6-2\rho^2\sigma^4]+$$

$$+\Delta^4\Big[\sigma^4\Big(\frac{5}{16}-\cos^2\theta\Big)+\rho^4\frac{77}{16}+\frac{\sigma^2\rho^2}{8}\Big]+\frac{\Delta^6}{8}(\rho^2+\sigma^2\sin^2\theta/2)+\Delta^8/256\,;$$

$$B=(\sigma^2+\rho^2)^4+\Delta^2(\sigma^2+\rho^2)^2(\sigma^2+\rho^2-2\sigma^2\cos^2\theta)+\Delta^4\Big[\frac{3}{8}\sigma^4+\frac{3}{8}\rho^4+\frac{3}{4}\sigma^2\rho^2-$$

$$-(\sigma^4\rho^2+\sigma^2)\cos^2\theta\Big]+\frac{\Delta^6}{16}(\sigma^2+\rho^2-2\sigma^2\cos^2\theta)+\Delta^8/256\,. \qquad (31)$$

where

$$\sigma=\frac{x+y}{2}+z_{I,A};\quad \Delta=x-y;\quad \cos\theta=(\sigma_\mu\Delta_\mu)/\sigma\Delta\,.$$

Our first comment to it is that $O(\Delta^2)$ corrections are present here, but absent in the OPE expansion (26). This is because only in the vacuum we have to use only the scalar operators, while in some external field we should also include operators like $G^a_{\mu\nu}\eta^a_{\mu\nu}$ of dimension 2. (In average, such effects should disappear.)

Our second comment: as (31) is the rational function, no logarithms of x are present and therefore *no OPE terms but the* $\langle(gG)^2\rangle/\Delta^2$ *one is actually present* (this is a consequence of the known Dubovikov — Smilga theorem). On the contrary, the finite and power terms in Δ^2 are present, unlike in the $1/Q$ OPE expansion.

We do not have similar formula for the propagator in the complicated multi-instanton background, but, as we are going to consider only small-distance effects in this section, we use some simple approximation based on this expression. We just take the product of correction factors (31) over our instantons

$$\Pi(x,y)=\frac{3}{\pi^4(x-y)^6}\prod_{I,A}\frac{A(\sigma_{I,A},\Delta,\rho_{I,A})}{B(\sigma_{I,A},\Delta,\rho_{I,A})}\,. \qquad (32)$$

It was shown that for most of them it is just unity, and only «the most important» one gives the main correction. Obviously, we have averaged (32) over the random point pairs and over our data for the *PP* configurations.

The last question we address in this section is the interference term of zero and non-zero modes, arising when we square the propagator in order to get the correlation function.

Our first observation is that in this case chirality of both quark and antiquark zero modes are identical, thus both of them cannot be provided by the same pseudoparticle and we need both an

instanton and an anti-instanton. This fact alone shows that the effect cannot be very strong, for instanton and anti-instanton are typically separated in space-time, and the product of their zero modes is never large.

The second observation is that the relative orientation matrix enters the interference term linearly. Thus, if it is more or less random, this term should be small in average.

The third point is that for the pion correlator the interference term cannot be very important because the x dependence of zero and non-zero mode contribution is quite different. As a result, either the former or the latter strongly dominate, with small «window» in between where they are comparable: only here one may see noticeable interference term.

Due to all these reasons the interference term can be disregarded. An exception may be the η, η' channels (see section 6), because in this case the zero mode contribution decays nearly as strongly as that of the nonzero ones and our third remark does not hold. As it contains the contribution of the strange quark, one may also avoid the first point: in the term proportional to m_s there is no chirality problem, the zero modes have opposite chiralities. Moreover, if they belong to the same pseudoparticle, the relative orientation matrix is just unite one, and the «random angles» argument drops too.

For this case we present the formulae for such effect

$$\Pi(x,y)=\frac{3}{\pi^4(x-y)^6}\Big[1+\frac{\pi^2(x-y)^4}{36}m_s|\langle\bar{s}(x)\,s(0)\rangle|\Big] \qquad (33)$$

which can be estimated from above by the substitution of constant $\langle\bar{s}s\rangle$. The resulting correction to the free correlator looks as follows

$$\Pi(x,y)=\frac{3}{\pi^4(x-y)^6}\Big[1+0.23\Big(\frac{x-y}{fm}\Big)^4\Big] \qquad (34)$$

which is too small at distances we consider to be relevant.

5. CORRELATORS FOR THE PSEUDOSCALAR OCTET

Now we are in the position to compare our results with the experiment, which we do it in this section it in two steps. First we plot the data emphasizing large distance limit and the light meson contributions, and then we turn to «intermediate x» at which the asymptotic freedom is violated.

The correlation functions rapidly decay with distance, and in order to see whether the measured points indeed follow the expected behaviour at large x it is convenient to divide the correlator by some standard function possessing similar behaviour. In Fig. 5 we show pionic correlator divided by the massless propagator, or $4\pi^2x^2\Pi_\pi(x)$. (As in Section 3, $\Pi_\pi(x)$ is a dimensionless combination $\langle j_\pi(x) j_\pi(0)\rangle / \langle \bar{u}u\rangle^2$, which makes data much less sensitive to uncertanties.) The points show the contribution of the zero modes, the dotted line gives that of the nonzero modes and the shaded region is their sum.

The dashed line corresponds to the «expected behaviour», which we have to comment on. Taking the ratio of the correlator to $\langle \bar{u}u\rangle^2$ we have made it dimensionless, but the distance x in absolute unites, in fermis, should now be put in our standard length unites Λ_{PV}^{-1}. For definiteness, we have used the best measurements of deep inelastic (muon) scattering [13]

$$\Lambda_{\overline{MS}} = \begin{cases} 230\pm40\ (\text{stat})\pm80\ (\text{syst})\ \text{MeV} & (\text{BFP Coll.}) \\ 220\pm20\ (\text{stat})\pm60\ (\text{syst})\ \text{MeV} & (\text{BCDMS Coll.}) \end{cases} . \qquad (35)$$

This quantity $\Lambda_{\overline{MS}}$ practically coincide with our Λ_{PV}. We use $\Lambda_{PV}=220$ MeV, thus our standard length unite is $\Lambda_{PV}^{-1}=0.9$ fm.

Normalization of the correlator to $\langle \bar{u}u\rangle^2$ may appear not very reasonable from the experimental side, because experimental value of this quantity is not accurately known. However, as we are now mainly interested in the pion contribution, we note that just for such normalization the uncertain quark condensate value drops in the «expected pion» line because the coupling constant λ_π is also proportional to the condensate and

$$\lambda_\pi^2/\langle \bar{u}u\rangle^2 = 4/f_\pi^2 , \qquad (36)$$

so that only the (accurately known) pion decay constant remains.

Our last comment on the «expected» curves in Fig. 5 is that, using the first order chiral perturbation theory, we have actually shifted the experimental pion mass to that, corresponding to the quark masses really used. (Larger masses are needed because, as discussed in *CI* [1], due to the finite-volume effects the eigenvalue spectrum is distorted at their small values and it is dangerous to work with too small quark masses.) Thus, both the «theoretical» shaded region and the «expected» dashed curve in Fig. 5 in fact

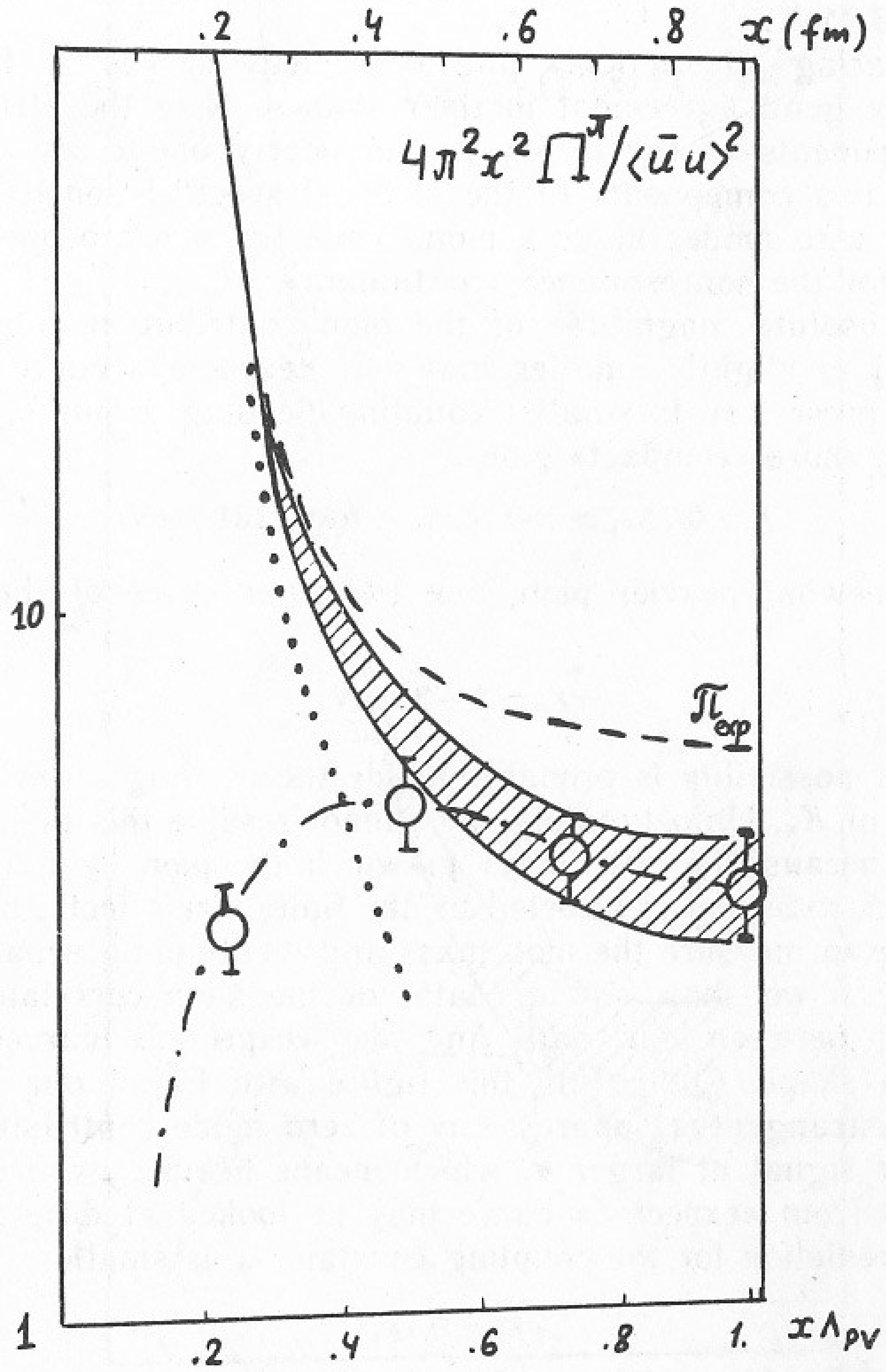


Fig. 5. The correlation function of the pionic currents versus distance. The quantity plotted is actually $4\pi^2x^2\Pi_\pi/\langle \bar{u}u\rangle^2$ and the distance x is measured either in Λ_{PV}^{-1} or in fermis (lower and upper scales, respectively). The dotted line is the contribution of the nonzero modes, while the points stand for that of zero modes. Their sum is shown by the shaded area. The dashed line is the expected pion signal, with experimental f_π and with $E_0=1600$ MeV (as suggested by Fig. 7).

correspond to the nonstrange quark masses $m_u = m_d = 0.1\Lambda_{PV} \simeq 22$ MeV. The «expected» pion mass is then $m_{\pi^-} \simeq 276$ MeV.

Comparing «theoretical» and «expected» curves in Fig. 5 one finds very good agreement in their shapes. Note the striking fact: two components of the theory are in nearly one-to-one correspondence to two components of the physical spectral density. Roughly speaking, zero modes make a pion, while the nonzero ones are responsible for the nonresonance «continuum».

The absolute magnitude of the pion contribution suggested by the theory is slightly smaller than our «expected» curve. This can be either prescribed to smaller coupling, leading to our «prediction» of slightly more «compact» pion

$$f_\pi \simeq 0.7\Lambda_{PV} \simeq 160 \text{ MeV} \quad (\text{exp.: } 131 \text{ MeV}) \tag{37}$$

or to somewhat heavier pion, due to larger value of the constant K_π:

$$K_\pi \simeq 2-3 \text{ GeV}. \tag{38}$$

This latter possibility is probably inside the existing uncertainties of the value of K_π. Unfortunately, we cannot resolve this dilemma with our data because the «window» in which the pion sygnal is dominant is not wide and is affected by the finite size effects, so that we are unable to measure the pion mass and its coupling separately.

In Fig. 6 we show similar data for the kaon correlator. Again agreement between our data and the «expected» curve is good enough in shape. Comparing this figure with Fig. 5 one may note that the «strangeness suppression» of zero mode contribution leads to smaller signal at larger x, which means heavier particle. Again, deviations from «expected» curve may be looked at differently: either our prediction for the coupling constant λ_K is smaller

$$\lambda_K \simeq 0.7\lambda_\pi \tag{39}$$

or we predict slightly heavier kaon, $m_K \simeq 600$ MeV. Similar data were obtained for eta meson, which we do not show because they are very similar to kaonic ones.

Now we turn to Fig. 7, which again shows the pion correlator, but plotted in different way. The correlator is now divided by the free propagation one, being its small-x asymptotics. Again very good agreement is found in shape with the «expectations», shown by

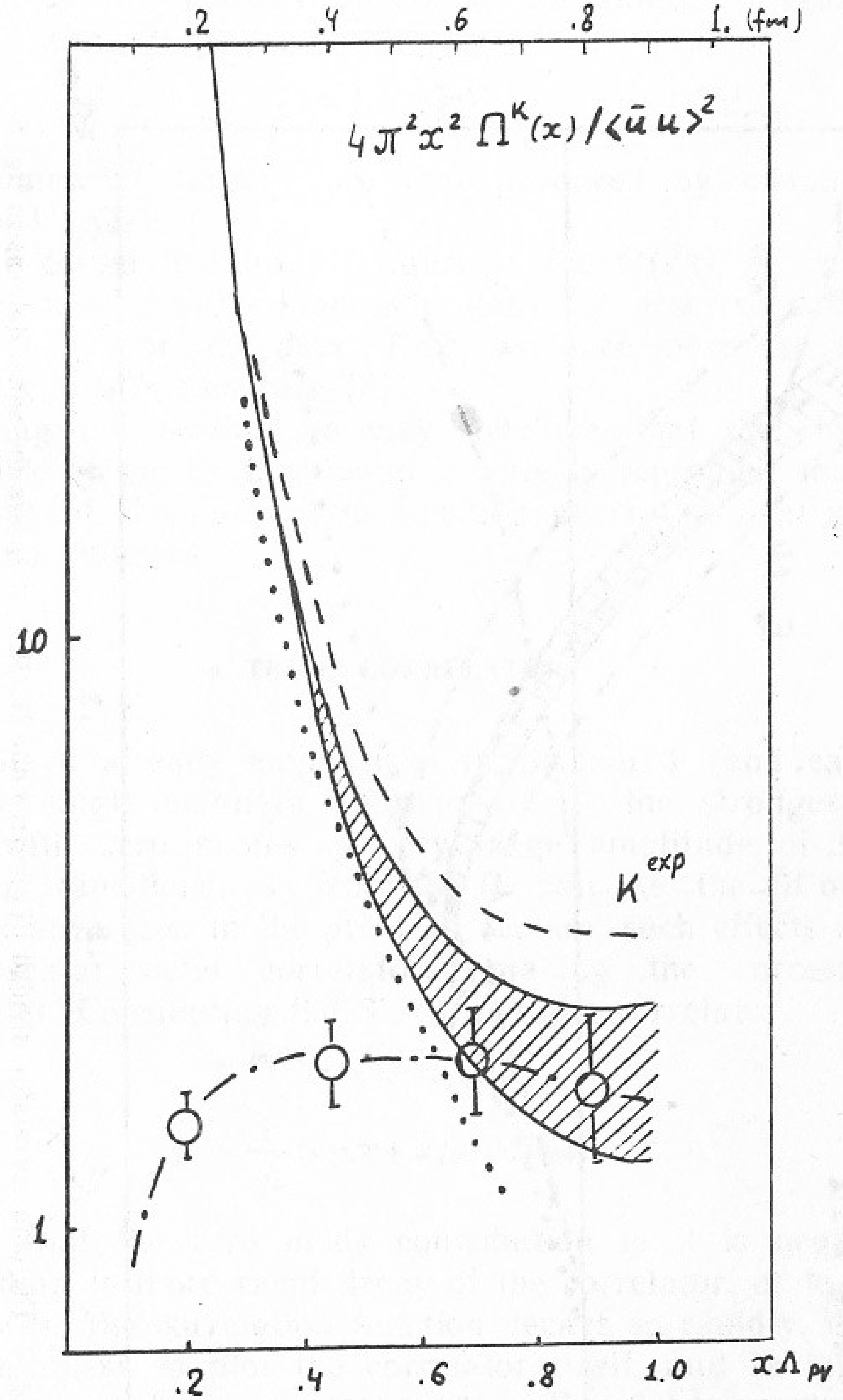


Fig. 6. The same as in Fig. 5, but for the kaon. Note that the zero mode contribution has dropped significantly compared to the pion case, which shows that the kaon is much heavier.

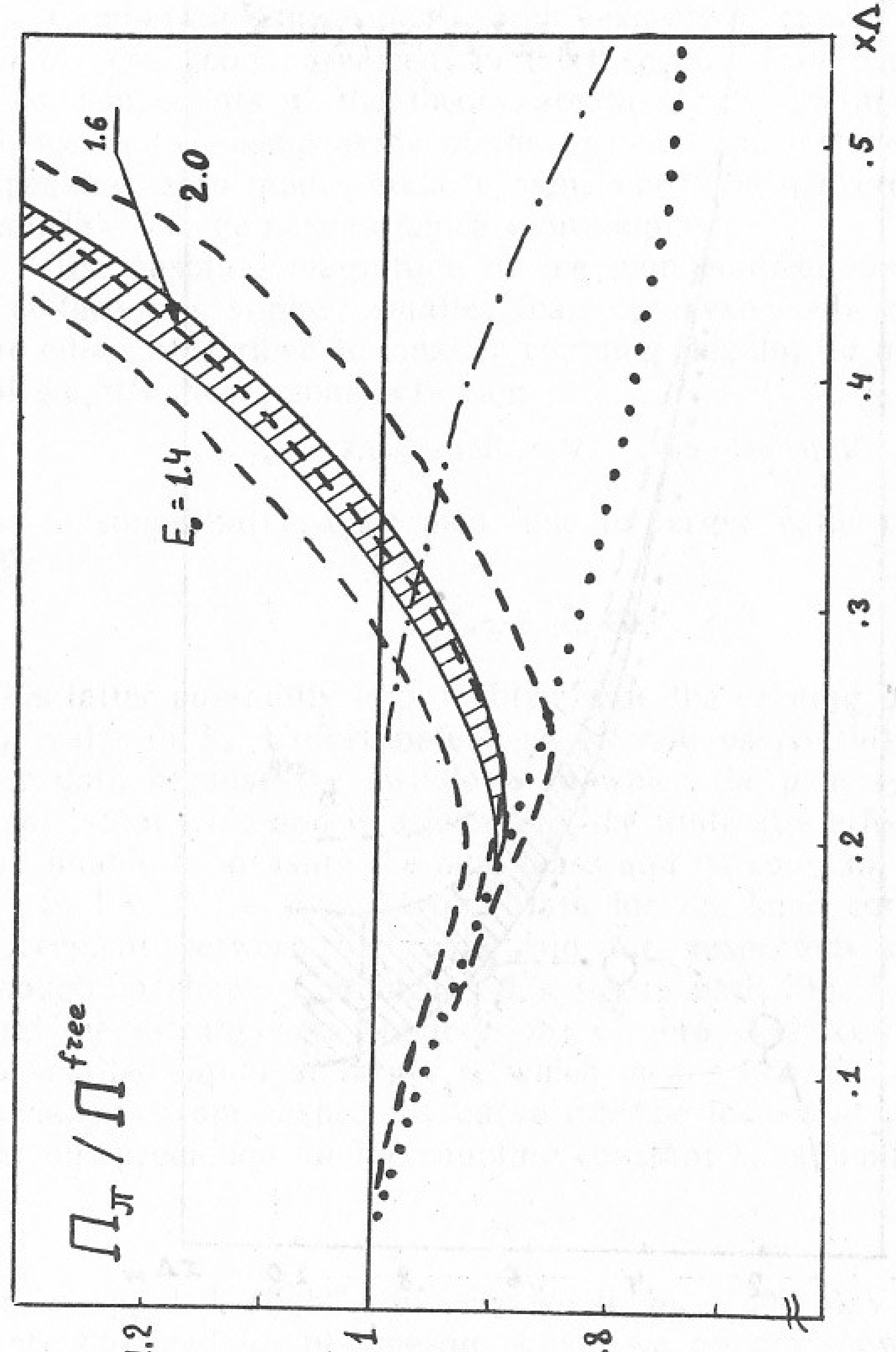


Fig. 7. The pion correlator divided by the free one Π_π/Π^{free}, it shows the same data as in fig. 5 but with emphasis on deviations from the asymptotic freedom at smaller x. The dotted line is the contribution of the nonzero modes, while the dashed region is the sum of all contributions. Three dashed lines stand for various values of the threshold parameter E_0 given in the figure. The dash-dotted line corresponds to the OPE $O(\langle (gG^a_{\mu\nu})^2\rangle)$ corection, which is too small to be important.

the dashed lines for various values of the «continuum threshold» E_0. The best one seems to be

$$E_0 \simeq 1.6 \text{ GeV}$$

which is somewhat larger than that produced by «duality estimates», $E_0 = 1.3$ GeV.

The dash-dotted line in this figure is the $O(\langle G^2 \rangle)$ correction. Note that even at small distances it does not describe neither the «expected» curve nor our data. Thus, we once more see that *the OPE analysis fails in this case* [8].

Completing this section we may conclude, that the «instanton liquid» picture of the QCD vacuum is able to reproduce the expected behaviour of the correlation function of two probes with the octet quantum numbers.

6. THE η' CORRELATOR

As we have already emphasized in Section 3 (and earlier in AIV, in the single-instanton approximation), the strongest effect associated with zero modes is very large amplitude of the flavor-changing transitions, say from the $\bar{u}u$ pair into the $\bar{d}d$ or the $\bar{s}s$ ones. As we have seen in the previous section, such effects increase the pseudoscalar octet correlators, making the corresponding mesons lighter. Considering the $SU(3)_f$ singlet correlator

$$\Pi^{singlet} = \langle j_{\eta'}(x)\, j_{\eta'}(0) \rangle,$$

$$j_{\eta'} = \frac{i}{\sqrt{3}}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s) \qquad (41)$$

we observe that the zero mode contribution to it is negative at small x leading to more rapid decay of the correlator, or to heavier physical states. The correlation function decays so rapidly, that it is rather meaningless to plot the correlator itself, and in Fig. 8 we show its ratio to the free correlator, (as in Fig. 7).

The non-trivial behaviour of shaded region (which is our prediction) is due to sign-changing contribution of zero modes. At small x it is negative due to the flavor-changing amplitude, while at larger x it changes sign because the flavor-diagonal ones become dominant. (In order to emphasize the importantance of such behaviour let us remind the reader, that in the single-instanton approximation

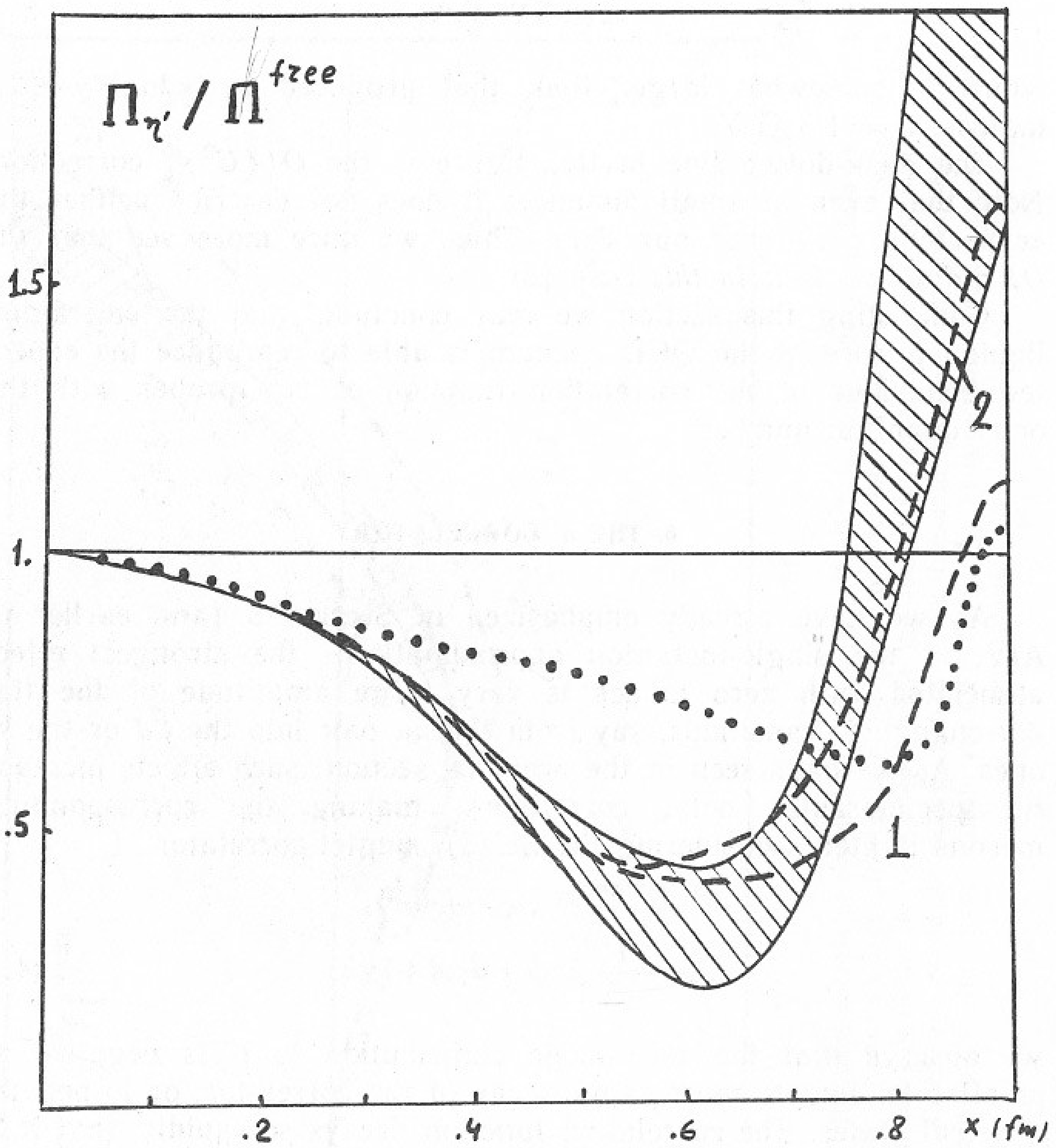


Fig. 8. The same as in Fig.7, but for the flavor-singlet (or η') current. Note that now the zero mode contribution is changing sign, so that the shaded region is first below the contribution of the nonzero ones (the dotted line) and then it rapidly rizes. Two shaded lines correspond to $E_0=2400$ MeV both, but with $\lambda_{\eta'}^2/\lambda_\pi^2=1$ and 2, respectively.

considered in AIV only the negative flavor-changing contribution was present. Therefore, at large enough distances the total correlator became negative too, which is not possible. That was a sign that something important was missing, and now it is seen that such missing ingredient is the flavor-diagonal terms generated by the whole multi-instanton «liquid».)

With such negative zero mode correction, deviations from the asymptotically free behaviour are significant even at rather small x. In terms of the physical spectral density it means a very large «gap» up to the threshold E_0, our fit gives it in this case as high as

$$E_0 \simeq 12\Lambda_{PV} \simeq 2.4 \text{ GeV} \tag{42}$$

to be compared with the octet value 1.6 GeV. It may be that rather large mass scale of asymptotic freedom violation expected in gluonic channels [8] show up in this correlator as well.

It is not easy to get data on the η' mass, although certainly it is not lighter than, say, 1 GeV. Even on its coupling we got only bad data, because η' signal dominates only at distances where it is already very weak. We present some information on it because, as we have already discussed in Section 2, *there is no «expected» value of the η' coupling to the pseudoscalar current.* Our points have large arrors at largest x (see Fig. 8), but they prefer the value

$$\lambda_{\eta'} \simeq 1.4\lambda_\pi . \tag{43}$$

We hope the conclusion (43) is true also because there are two other arguments in its favor. First, duality arguments usually reasonably relate the resonant coupling to the thereshold E_0, and from (43) one gets

$$E_0^{(duality)} \simeq 2.6 \text{ Gev}. \tag{44}$$

The second argument is based on our data on the $\eta\eta'$ mixing, which we now are going to consider.

7. THE $\eta\eta'$ MIXING

We remind the reader that «mixing» of the SU(3) singlet and octet states is the well known consequence of the strange quark mass, and traditionally it is considered by means of the diagonalization of the Hamiltonian written as a 2×2 matrix in the subspace made by the pair of «mixing» mesons. The following numbers for the mixing angles for various hadronic multiplets

J^P	θ	
0^-	$\approx -20°$	[15]
1^+	$+39°$	
2^-	$+28°$	
3^-	$+29°$	

(45)

show that the pseudoscalar case is indeed exceptional. (All other angles are close to the «ideal mixing» value $\theta = 35.3°$, which just means that *strange states are decoupled* from the nonstrange ones.)

In order to investigate this phenomenon we consider the singlet-octet correlation function

$$\Pi_{\eta\eta'}(x) = \langle T j_\eta(x)\, j_{\eta'}(0) \rangle \quad (46)$$

which is sensitive to the mixing angle and to the difference of physical spectral densities in both channels. For not too small x it can just be written as

$$\Pi_{\eta\eta'}(x) = \cos\theta \cdot \sin\theta \,[\lambda^2_{\eta'} D(m_{\eta'}, x) - \lambda^2_\eta D(m_\eta, x)]. \quad (47)$$

Our data are shown in Fig. 9, and it has the typical behaviour with the varying sign. At large distances η should dominate over η' because it is lighter: looking at our data we see that they definitely imply that the angle is negative, as it is in experiment.

Further, the shape of the curve is sensitive to the ratio $\lambda_{\eta'}/\lambda_\eta$: the fit gives quite narrow window for it

$$\lambda_{\eta'}/\lambda_\eta \simeq 1.4 \pm 0.2 . \quad (48)$$

As eta coupling λ_η was expected (and actually obtained, within statistical uncertainties) to be close to pionic and kaonic ones, this implies that $\lambda^2_{\eta'}$ is about twice larger than λ^2_π. Such statement is consistent with the conclusion reached in the preceeding section.

Unfortunately, due to large enough errors in absolute values of λ_π, we cannot say what is our result for the theta value with good accuracy. If $\lambda_\eta = \lambda_\pi$ then it magnitude is $\theta \simeq -25°$ (this corresponds to the line in Fig. 9), but the *uncertainty in factor 2 is obviously there.* Note by the way, that in some simplified model mentioned in the Intoroduction [14], all features of the mesons were fitted and then θ was found to be around $-30°$.

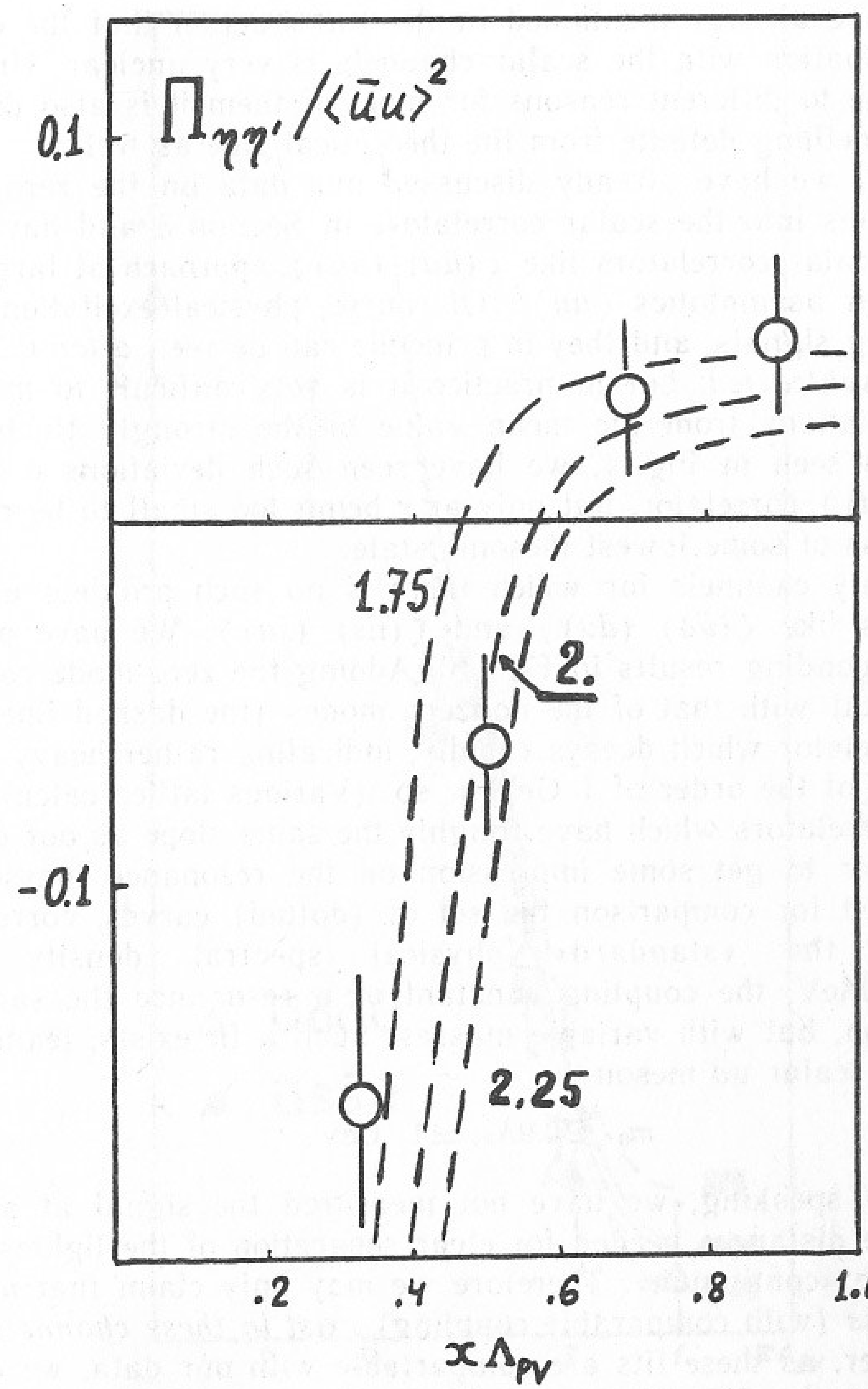


Fig. 9. The singlet-octet nondiagonal correlator. The points are zero mode contribution measured, the dashed lines are for different $\lambda^2_{\eta'}/\lambda^2_\eta$ ratios, with numbers given in the figure.

8. THE SCALAR CORRELATORS

We have already mentioned in the Introduction that the experimental situation with the scalar channels is very unclear. Unfortunately, due to different reasons for most of them it is also difficult to say something definite from the theoretical side as well.

Indeed, we have already discussed our data on the zero mode contributions into the scalar correlators in Section 3 and have seen how the scalar correlators like $\langle(\bar{u}u)\ (\bar{u}u)\rangle$ approach at large distances their asymptotics $\langle\bar{u}u\rangle^2$. Of course, physical excitations lead to decaying signals, and they in principle can be seen *after this constant be subtracted*, but in practice it is very difficult to measure small deviations from the mean value of the strongly fluctuating signal. As seen in Fig. 4, we have seen such deviations e. g. for $\langle(\bar{u}u)\ (\bar{d}d)\rangle$ correlator, but only at x being too small to be related to the mass of some lowest mesonic state.

The only channels for which there is no such problem are the correlators like $\langle(\bar{u}d)\ (\bar{d}u)\rangle$ and $\langle(\bar{u}s)\ (\bar{s}u)\rangle$. We have plotted the corresponding results in Fig. 10. Adding the zero-mode contibution (points) with that of the nonzero modes (the dashed lines) we get a correlator which decays rapidly, indicating rather heavy states with mass of the order of 1 GeV or so. (Various lattice calculations lead to correlators which have roughly the same slope as our one.)

In order to get some impression on the resonance masses we also plotted for comparison the set of (dotted) curves, corresponding to the «standard» physical spectral density with $E_0=1600$ MeV, the coupling constant of a resonance the same as for the pion, but with variable masses. Such a fit exists, leading to masses of scalar ud meson

$$m_{0^+}\simeq 0.9\Lambda_{PV}\simeq 1\ \text{GeV}. \tag{49}$$

Strictly speaking, we have not measured the signal at sufficiently large distances needed for clear separation of the lightest state from the «continuum». Therefore we may only claim that *no lighter mesons* (with comparable coupling) *exist in these channels*.

However, as these fits are compartable with our data, we consider it quite plausible that the $\delta(980)$ meson is indeed the particle seen in this correlator. If so, we find no unusual properties associated to it: both its coupling and thereshold parameter E_0 are roughly the same as for the pseudoscalar octet.

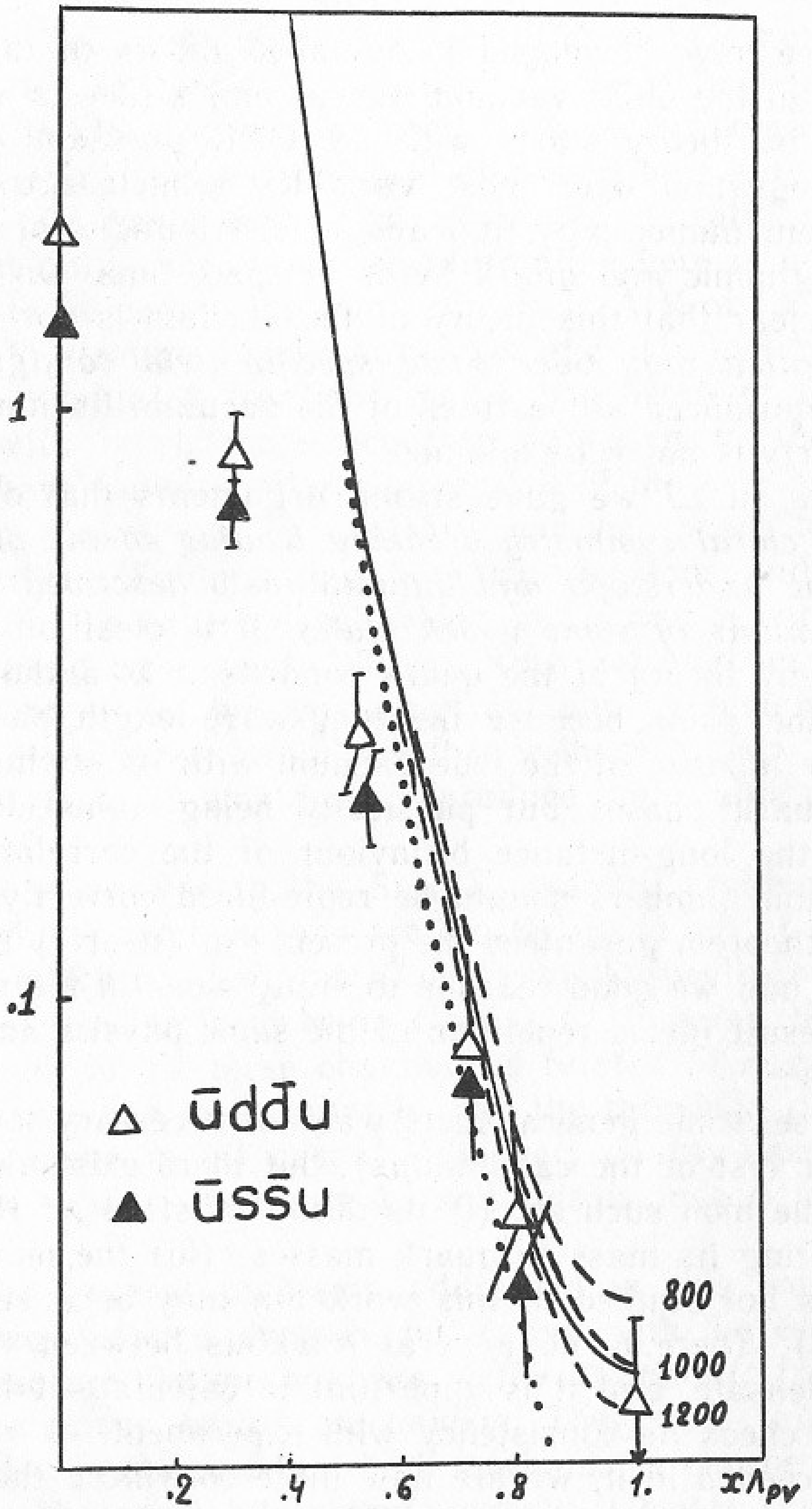


Fig. 10. The scalar correlation function (solid line) is compared with the «expected» behaviour with different meson masses (the dashed curves). The dotted line and the points stand, as above, for nonzero and zero mode contributions.

9. DISCUSSION OF THE RESULTS

In *CI* we have developed the detailed theory of the topological phenomena in the QCD vacuum. Let us emphasize its «microscopic» character: the theory starts with the QCD partition function and, making integration over most variables semiclassically and over some of them numerically, it leads to distributions of the (nonperturbative) gluonic and quark fields in space-time. On general grounds it is clear that this theory of the vacuum is not complete: *we have integrated only over some specific field configurations* and have not reproduced all features of the vacuum: its famous confinement property is not yet explained.

However, in *CI* we gave strong arguments that one important aspect, *the chiral symmetry breaking leading to the quark condensate, can be understood and quantitatively described by means of «collectivization» of some quark states.* It is clear on general grounds that any theory of the quark condensate is authomatically the theory of the pions because the long-wave-length pion is nothing else but the mixture of the true vacuum with its «twins», posessing different quark phases but physically being quite identical to it. Therefore, the long-distance behaviour of the correlators with the pion quantum numbers should be reproduced correctly: the famous Goldstone theorem garantees the presence of (nearly) massless pion pole. If so, had we good reasons to study pions in the present work, aren't its result just a repetition of the same physics as in *CI* in different terms?

Of course, some general facts was not necessary to check (maybe, only for test of the calculatons), but there exist also other properties of the pion such as: (i) its decay constant f_π; (ii) parameter $K_\pi(8)$, relating its mass to quark masses; (iii) the pion formfactor, (which was not studied in this work but may be a subject for future works). There is no general relations between them and the quark condensate, and it is important to calculate it from our theory and to check its consistency with experiment. as we have more or less succeeded in it, we are now more convinced that this theory of the vacuum structure is reasonable.

Another general problem addressed above is *the role of the strange quark mass in the QCD vacuum.* The question of whether it can or cannot be treated as small parameter is very controversial. As our theory of quarks in the instanton liquid suggests *the presence*

of a narrow peak in the spectrum of eigenvalues of the Dirac operator around zero, with the width of the order of 100 MeV, our answer is that, generally speaking, $m_s = 150$ MeV is not actually a small parameter and many applications of chiral first-order perturbation theory to strange hadrons need to be reconsidered.

Such important statement should of course be carefully checked. Our data of *CI* have shown that $\langle \bar{s}s \rangle / \langle \bar{u}u \rangle$ ratio significantly deviates from unity, but empirical information here is not yet accurate enough to be convincing. Our studies in AIV of the correlators connected with strange quarks have used the «strangeness suppression factors» evaluated from the model and being of the order of 1/2. Now, with much more accurate approach to the instanton physics, we have found similar *suppression of the zero-mode effects.* The non-zero modes, on the contrary, can be shown to be rather insensitive to *ms*. Summarising both contributions and comparing their sum with the expected behaviour for K and η correlators we have found good agreement. Without such «strangeness suppression» of the zero modes we would not get any agreement with their masses. (In other terms, we claim that without narrow «eigenvalue zone» around zero we would not reproduce high value of the parameter K (8))

Finally, our answer to the SU(3) breaking problem is as follows. The lightest octet mesons are mostly made out of *collectivized zero modes, which are very sensitive to* m_s. Respectively, masses of these particles are far from being close, etc. However, the non-resonant continuum in all these channels is related to *non-zero modes which are insensitive to* m_s: that is why the threshold parameter E_0 is about the same in all cases. Moreover, duality-type arguments demand then similar couplings λ_i, and this is indeed found in our data.

Two other general problems addressed in this work deal not with the «long distance physics», but rather with the «small distance» one. These are *the mechanisms of the asymptotic freedom breaking and the flavor mixing mechanism.* They were related in AIV [9], where it was claimed that both mechanisms can be quantitative explained by *the same* interaction, namely 't Hooft instanton-induced multifermion interaction of small-size instantons.

Our present data clearly show that this interpretation was correct. The flavor-changing correlator is indeed the strongest one at small distances, leading e. g. to $\langle (\bar{u}i\gamma_5 d) \cdot (\bar{d}i\gamma_5 u) \rangle$ value by the factor 20 larger than that suggested by the «vacuum dominance» hypothesis [6]. That is why such effect dominates over, say $\langle (gG)^2 \rangle$ effect in Fig. 7 (in spite of the fact that the latter has singular $O(1/x^2)$ behaviour). Thus, the failure of the OPE-based sum rules for the pseudoscalar currents (noticed in [8]) is explained.

One manifestation of strong flavor-mixing amplitude is the unusual singlet-octet mixing of the pseudoscalars, which is reproduced by our data in sign and (roughly) by magnitude. Even more interesting is our conclusion of the nesessary *existence of unusually large gap in the spectral density of the* η' *correlator.* Let us remind in this connection, that existence of such a new large scale was suggested in [8] for the gluonic channels, in particular for the pseudoscalar operator $G\tilde{G}$. As is believed to be somehow mixed with gluonium, appearence of such effect in this correlator is probably related to this fact.

Thus, concluding this work we may say that we are mostly satisfied by its results, in the sence that *most of the suggestions of our model-dependent treatment in AIV are reproduced by the present «microscopic theory».* We are now much more convinced that the lightest pseudoscalar mesons are mainly «made out of zero modes», while the nonresonant continuum just represent the nonzero modes. Such relation between the eigenvalue spectrum of a Dirac operator in the one-instanton background field and the mesonic spectrum in QCD is, of course, very intriguing. Is it true for other light hadrons, say for vector mesons or the baryons, are they also made out of the collectivized zero modes? Such questions we hope to answer in our subsequent publications.

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и скалярных токов

Ответственный за выпуск С.Г.Попов

Работа поступила 19 апреля 1988 г.
Подписано в печать 5.05. 1988 г. МН 08300
Формат бумаги 60×90 1/16 Объем 3,0 печ.л., 2,7 уч.-изд.л.
Тираж 250 экз. Бесплатно. Заказ № 63

Набрано в автоматизированной системе на базе фотонаборного автомата ФА1000 и ЭВМ «Электроника» и отпечатано на ротапринте Института ядерной физики СО АН СССР,
Новосибирск, 630090, пр. академика Лаврентьева, 11.