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E.V. Shuryak

INSTANTONS IN QCD IV. VECTOR AND AXIAL MESONS

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НОВОСИБИРСК

Instantons in QCD IV.
Vector and Axial Mesons

E.V. Shuryak

Institute of Nuclear Physics
630090, Novosibirsk, USSR

ABSTRACT

Correlation functions of vector and axial currents (with flavor content *ud* and *us*) are calculated in the «instanton liquid» model. Results are compared with the data on $e^+e^-\to$hadrons ($I=1$) and $\tau\to\nu_\tau+$hadrons, respectively. Very good agreement is found, especially in the vector case, in which the theory does reproduce a «fine tuning» of $\rho(770)$, $\rho(1450)$, $\rho(1700)$ and the nonresonance contributions existing in data.

1. PHENOMENOLOGY OF THE CORRELATION FUNCTIONS OF VECTOR AND AXIAL CURRENTS

After the theory of the «instanton liquid» was developed so that straightforward numerical simulations became possible C1, we have first applied these data to the «exceptional» cases of the hadronic spectroscopy, the famous pseudoscalars like the pion and η' mesons in C2. (Here and below C1, C2 etc. refer to subsequent papers of this series [1].) The reason for doing so was the long-standing suspicion that just instantons make these particles so exceptional. As a step toward «more regular» channels, we have considered in C3 some hydrogen-atom-like hadrons, the mesons made of a heavy quark and a light antiquark. They are simpler from the theoretical side, but, unfortunately, the experimental information in this case remains so far very limited.

In the present paper we come to the correlation functions of vector and axial currents. Although theoretically this case is much more complicated (strong cancellations of various effects appear, which makes predictions difficult, see below), in principle *it is very good test for the theory*. Indeed, such currents (in contrast to many others invented by theorists) really exist in nature as (a part of) electromagnetic and weak currents. The spectral density of the corresponding correlation functions is directly measurable, and using dispersion relations *one may obtain complete correlation functions*. In this way we get much more detailed information on the correlation functions and can test our ideas about the QCD vacuum structure much better. (By the way, such excellent tests were not actually used for lattice calculations yet.)

In this paper we focus only for the *flavor nonsinglet currents* (of the $\bar u\Gamma d$ and $\bar u\Gamma s$ types, where Γ stands for some gamma matrices). This case is simpler in calculations: there is no the «two-loop» diagrams (in which the quark emitted by the current is absorbed back by it) but only the one-loop diagram (quarks travel from one current to another) contribute. Of course, additional and very interesting experimental information is available for the $I=0$ vector channels (ω and φ ones) as well , and it can also be used for further tests of the theory. Wa are planning to do this in our subsequent publications.

Now let us consider the «ρ-meson» and «A_1-meson» currents

$$j_\mu^{(\rho^0)} = \frac{1}{\sqrt{2}}(\bar u\gamma_\mu u - \bar d\gamma_\mu d)\,,$$
$$j_\mu^{(\rho^-)} = \bar u\gamma_\mu d\,,\quad j_\mu^{(A_1)} = \bar u\gamma_\mu\gamma_5 d\,. \tag{1}$$

In e^+e^--annihilation experiments one deals, of course, with ρ^0, while we work with the $\rho^\pm$ case. Obviously, all their properties are the same, apart of small isospin-breaking effects, which are ignored anyway. (Note also, that for consitency our ρ^0 current is by $\sqrt{2}$ larger than that in [3].)

The spectral density of its correlation function is related to the cross section of e^+e^--annihilation into hadrons ($I=1$) as follows

$$\Pi_{\mu\nu}(x) = \langle 0|\, Tj_\mu(x)\, j_\nu(0)\,|0\rangle\,,$$
$$\Pi_{\mu\nu}(q) = i\int dx \exp(iqx)\,\Pi_{\mu\nu}(x) = (q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi(q^2)\,,$$
$$\operatorname{Im}\Pi(q^2) = \frac{1}{6\pi}\,R_{I=1}(q^2)\,,$$
$$R_{I=1}(s) \equiv \sigma(e^+e^-\to \text{hadrons}, I=1)/\sigma(e^+e^-\to\mu^+\mu^-)\,. \tag{2}$$

The previous analysis of these data [2] was made using the so-called «Borel-transformed» representation suggested by Shifman, Vainshtein and Zakharov [3]. We do it directly in (Euclidean) space-time representation (as it was first suggested in Ref. [4])

$$\Pi_{\mu\nu}(x) = (g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\left(\frac{1}{12\pi^2}\right)\int^{\infty} ds\, R_{I=1}(s)\, D(\sqrt{s}, x)\,,$$

$$D(m,\tau) = \frac{m}{4\pi^2\tau}\, K_1(m\tau)\,. \tag{3}$$

Here $D(m, x)$ is just the propagator of a scalar particle with mass m to distance x, so the physical meaning of it is more or less self-evident. The distance x is below the space-like, $x^2=-\tau^2$, where τ is «Euclidean time» (and $D(m, x)$ is $\exp(-m\tau)$ at large τ).

Experimental selection of the $I=1$ channel is made via selection of *even* number of the pions in the final state. The data quality on the low energy e^+e^--annihilation have been essentially improved during the last decade, passed since the SVZ original work. Roughly speaking, there are three different component in this spectral density: (i) the prominent rho-meson resonance; (ii) complicated mixture of (at least) two «primed» resonances, $\rho(1450)$ and $\rho(1700)$, seen mainly in the 4 pion channel as a wide bump; (iii) the nonresonance «continuum» above 2 GeV, where $R(E)$ roughly follow the parton model prediction $R=3/2$. An approximate expression for $R(E)$ is as follows

$$R_{I=1}(E) = \frac{A}{4(E-m_\rho)^2/\Gamma^2+1} + \frac{A'}{4(E-m')^2/\Gamma'^2+1} +$$
$$+\theta(E-E_0)\,\frac{3}{2}\,(1+\alpha_s(E)/\pi)\,,$$
$$A=6.5\,;\quad \Gamma=160\text{ MeV}\,;\quad A'=2.5\,;\quad \Gamma'=300\text{ MeV}\,,$$
$$E_0=2\text{ GeV}\,;\quad m'=1.6\text{ GeV}\,. \tag{4}$$

The correlators considered are strongly decreasing function of x. At small x they are governed by the asymptotic freedom, for massless quarks they are (by dimensional reasons) $\Pi_{\mu\nu}(x)\sim x^{-6}$. At large x $\Pi(x)$ the decreases exponentialy. Therefore, it is inconvenient to plot the correlators themselves, and *we systematically use below the ratios* $R(x)=\Pi_{\mu\mu}(x)/\Pi_{\mu\mu}^{free}(x)$, where $\Pi_{\mu\mu}^{free}(x)=6/\pi^4x^6$ corresponds to the free quark propagation. At small x such ratios should tend to unity due to the «asymptotic freedom».

In Fig. 1 we show $R_V(x)$ dependence calculated from the e^+e^--data. The contributions of all three components of the spectral density mentioned above are shown separately.

The striking observation (as far as I know, never noticed before) is that in this case there exists some «fine tuning» of all contributions, so that resulting $R_V(x)$ keeps constant (close to 1.1) up to very large distances of the order of 1.5 fm! Nothing like this is seen in other channels. For example, for pseudoscalar and scalar currents (see C2) such ratio strongly deviates from unity at much smaller $x \simeq 0.2 \div 0.3$ fm.

Due to these features the $I=1$ vector correlator is unique. Theoretically it means, that here there are strong cancellation of all corrections, leading to «much more freedom» for the quark—antiquark propagation in this channel.

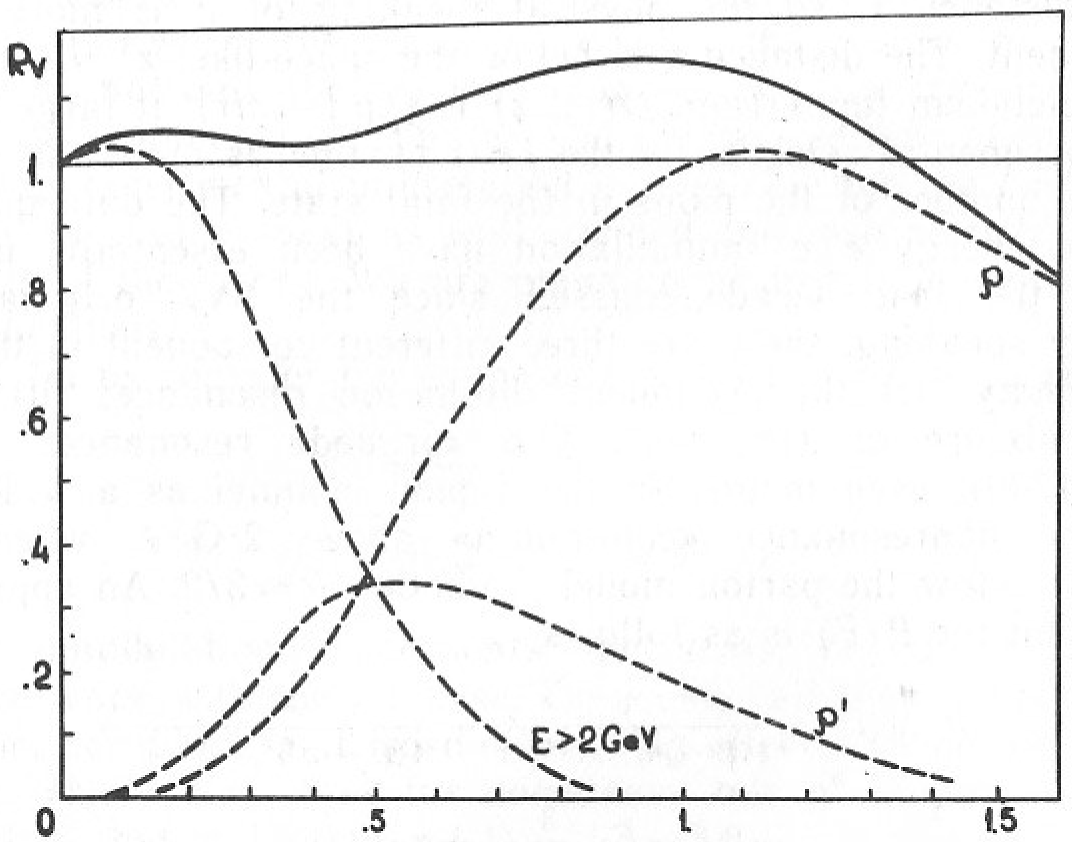

Fig. 1. The ratio $R_V(x)=\Pi_{\mu\mu}(x)/\Pi_{\mu\mu(x)}^{free}$ (of the correlator to its «asymptotically free» version) versus the distance x (in fermi). Three parts of the spectral density discussed in the text are shown separately by the dashed lines, as well as their sum (solid line). One may see that this ratio remains approximately constant in the whole x region considered.

Now we come to the axial current $j_\mu^A=\bar{u}\gamma_\mu\gamma_5 d$. All general formulae written above for the vector correlator are valid for it too. The corresponding data are now available from the τ lepton decay into odd number of pions. (Even number of pions corresponds to the vector part of the weak current, the corresponding data are consistent with what was said above, but they are still less accurate than the e^+e^--data.) Decay into ν_τ *plus one pion* does not provide new information: the corresponding coupling constant f_π is well known from the $\pi\to\mu\nu_\mu$ decay. In this paper we are not going to consider the pion properties (considered in different but related context in C2) but the A_1 meson. It is easy to get rid of the pion signal

in the correlation function: simple convolution of indexes μ, ν leads to

$$\partial_x^2 D(m=0,x)=\partial_x^2\left(\frac{1}{4\pi^2x^2}\right)=-\delta(x)\,. \qquad (5)$$

Thus, in the chiral limit the pion signal is delta-function like and *at finite x* (to be considered) *it is absent.*

There are two channels with three pions, $(\pi^-\pi^-\pi^+)$ and $(\pi^0\pi^0\pi^-)$, they have branching ratios $(6.8\pm.6)\%$ and $(7.5\pm0.9)\%$, respectively. (All numbers are from 1988 Particle Data). So, inside uncertainties they are equal. Invariant mass distribution is strongly peaked around 1.2 GeV (see review [5]). Both facts suggest dominance of the A_1 resonance.

Let us introduce the coupling constants f_ρ, f_A for ρ and A_1 mesons

$$\langle 0|\,\bar{d}\gamma_\mu u\,|\rho\rangle=f_\rho m_\rho\varepsilon_\mu\,,$$
$$\langle 0|\,\bar{d}\gamma_\mu\gamma_5 u\,|A_1\rangle=f_A m_A\varepsilon_\mu\,, \qquad (6)$$

where ε_μ stands for the polarization vector of a meson. (Note: what we call f_A is f_A/m_A in [5], but our notations are more standard, similar to f_π definition etc.)

From e^+e^--data one get $f_\rho\simeq 200$ MeV. From the experimental ratio

$$\frac{r(\tau\to\nu_\tau A_1)}{r(\tau\to\nu_\tau\rho)}\simeq 0.59\pm0.10 \qquad (7)$$

and its theoretical expression

$$\frac{r(\tau\to\nu_\tau A_1)}{r(\tau\to\nu_\tau\rho)}=\left(\frac{f_A}{f_\rho}\right)^2\frac{(1-m_A^2/m_\tau^2)^2(1+2m_A^2/m_\tau^2)}{(1-m_\rho^2/m_\tau^2)^2(1+2m_\rho^2/m_\tau^2)} \qquad (8)$$

one finds that

$$f_A/f_\rho=1.\pm0.15\,. \qquad (9)$$

Resulting A_1 contribution to the correlator is shown in Fig. 2. Comparing it with the ρ meson one given in fig. 1, one may see that the A_1 contribution is larger at small x (because it is proportional to $f_A^2 m_A^2$, and $m_A > m_\rho$), but it drops rapidly already at $x > 0.6$ fm.

(Unfortunately, our conclusion (9) is not as solid as it may appear, and further analysis is needed here. The data for τ decay show essentially wider A_1 [5], with $\Gamma_A > 400$ MeV, while in hadronic relations $\Gamma_A \simeq 300$ MeV. There certainly is some admixture here, and therefore we actually consider some «effective A_1» resonance, including this admixture. The A_1 curve in Fig. 2 should also be understood in this sense.)

Production of hadronic states heavier than A_1 in τ decays is suppressed, for its mass $m_\tau = 1784$ MeV and a cutoff is too close. No strong 3-pion production above A_1 is seen, and the 5-pion branching

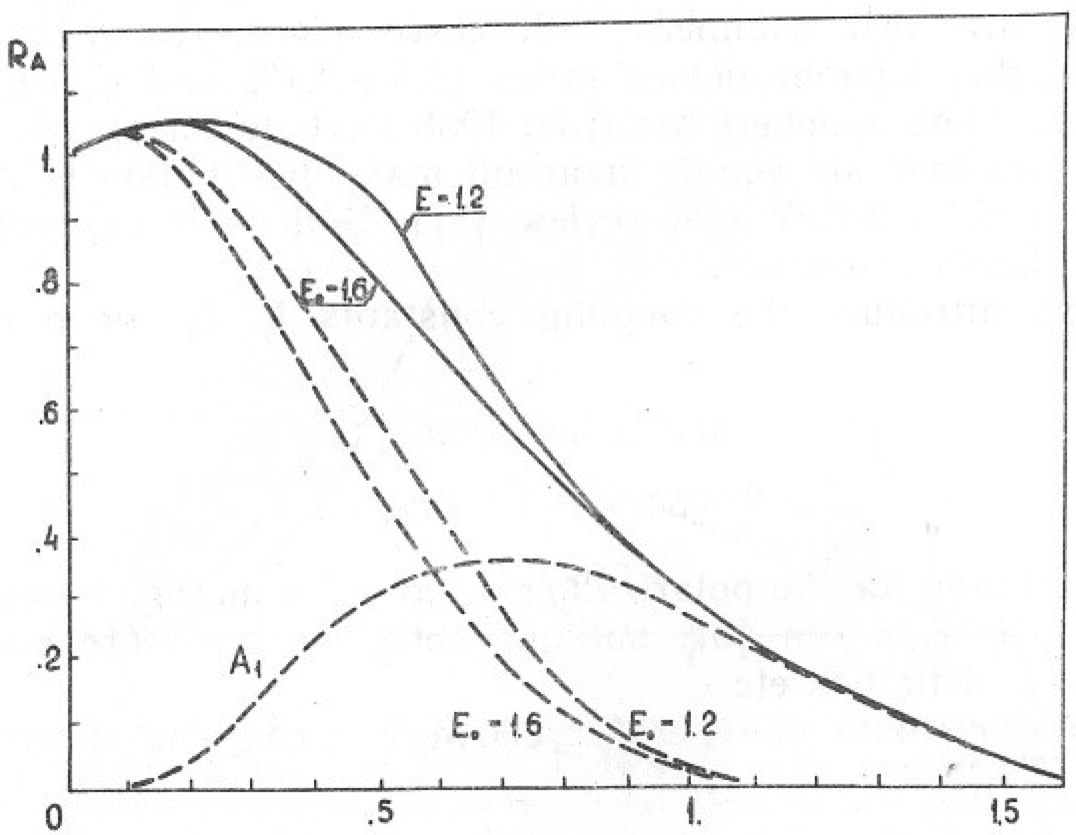

Fig. 2. The same as in Fig. 1, but for the axial current.

is small, of the order of 0.1%. We do know theoretically, that at large energies hadronic production from the axial and vector currents should become equal, so we approximate such nonresonance «continuum» contribution by the standard one-parameter expression

$$R_A(s) = \theta(s - E_0^2)\ \frac{3}{2}\ (1 + \alpha_s(s)/\pi)\ . \qquad (10)$$

We do not know E_0 value and can only claim that it is not smaller than m_{A_1}. Taking it as a lower bound, we get the curve for the total axial correlator function shown in Fig. 2. One more curve, with $E_0 = 1.6$ GeV (the threshold of the «invisible region» in τ decay) is shown for comparison, demonstrating existing uncertainty in our knowledge of this correlator.

We also consider strange ($\bar{u}\Gamma s$) channels in this paper. This is mainly done because it is simple change of our formulae. Phenomenologically in these cases we know only masses of the resonances. In vector case we have $K^*(892)$ (to be compared to $\rho(770)$), and two excitations, at 1415 and 1715 MeV, also having their nonstrange partners with close masses. Thus, there are reasons to assume that both correlators are similar. In the axial case there are two mixed states $K_1(1270)$ and $K_1(1400)$ (to be compared to $A(1260)$ and $B(1235)$). Roughly speaking, their mean is also shifted up by about the strange quark mass m_s. But the pseudoscalar and scalar cases are different: here we have $K(492)$ (to be compared to $\pi(138)$) and $K_0^*(1430)$ (to be compared to $A_0(980)$). We have already discussed in C2 why the $\pi - K$ mass difference is so large. As for the scalar resonances, probably both particles mentioned do not significantly contribute to the correlation functions considered (see below), so we cannot say anything on the origin of their large mass difference.

2. THE PREVIOUS OPE-BASED ANALYSIS OF THE CORRELATORS

Analysis of $I=1$ *vector and axial currents* was historically *the groundstone of the QCD sum rules* approach [3], based on Wilson *operator product expansion* (OPE). Its application has produced very impressive results [3] including «predictions» of masses and coupling constants for ρ, π and A_1 mesons. Now, after a decade of experimental work (briefly summarized above) and in view of new theoretical development (to be discussed below), it is necessary to comment on the present status of this approach and its correspondence to what follows.

Presenting the OPE analysis we deviate from the fundamental SVZ paper in one point: instead of their «Borel transform» representation we use the space-time one. Let us remind the reader why they have used it. The reason was twofold: it both *suppresses contribution of larger dimension operators* compared to momentum representation, and also it better *suppresses the contribution of heavy nonresonance states.* It was a clever trick, aiming to increase the «window» in which one tries to match the small-distance OPE with the large-distance hadronic asymptotics.

Of course, we use the space-time representation because we calculate the correlators *numerically*, we have no choice. But even if we have, our aim is now quite different. We want to *test our model of the QCD vacuum*, predicting certain deviations from the asymptotic freedom. Thus, *there is no reason to suppress these deviations* (the objest of investigation) by any tricks. We are also sure, that physical transparency of the space-time representation is now more important that some (if any) increase of the «window» for confronting theory with experiment.

After this preamble, let us present the OPE formulae [3] under consideration. We omit the $m\bar\psi\psi$ operators and the lengthy expressions for higher dimension operators (see Refs [6]) because they are not actually used in applications. In Euclidean time ($\tau=\sqrt{x^2}$) our ratios $R_{V,A}(\tau)$ are predicted to be

$$R_{V,A}=1+\frac{\alpha_s(\tau)}{\pi}-\frac{\langle(gG^a_{\mu\nu})^2\rangle}{3\cdot 2^7}\tau^4-\langle O_{A,V}\rangle\frac{\pi^2}{16}\tau^6\ln\left(\frac{1}{\tau\mu}\right)+\dots \qquad (11)$$

(here μ is the normalization point).

The complicated operators $O_{V,A}$ are different for vector and axial channels:

$$O_V=-\frac{\pi\alpha_s}{2}(\bar u\gamma_\mu\gamma_5 t^a u-\bar d\gamma_\mu\gamma_5 t^a d)^2-\frac{\pi\alpha_s}{9}(\bar u\gamma_\mu t^a u+\bar d\gamma_\mu t^a d)\left(\sum_{u,d,s}\bar q\gamma_\mu t^a q\right),$$

$$O_A=O_V-2\pi\alpha_s(\bar u_L\gamma_\mu t^a u_L-\bar d_L\gamma_\mu t^a d_L)\,(\bar u_R\gamma_\mu t^a u_R-\bar d_R\gamma_\mu t^a d_R)\,, \qquad (12)$$

$$\langle O_V\rangle\simeq-\frac{7\cdot 2^4}{3^4}\langle\bar\psi\psi\rangle^2\pi\alpha_s\,;\qquad \langle O_A\rangle\simeq+\frac{2^5}{3^4}\langle\bar\psi\psi\rangle^2\pi\alpha_s\,.$$

Estimates of their vacuum average values are given according to the so-called «vacuum dominance» hypothesis, suggested by SVZ [3]. (Its validity we are not going to discuss here.) The resulting curves are shown in Fig. 3, where we have also used other «standard» SVZ numbers such as

$$\Lambda^{(e^+e^-)}=100\ \text{MeV}\,,$$
$$\alpha_s\,|\langle\bar\psi\psi\rangle|=(250\ \text{MeV})^3\,, \qquad (13)$$
$$\langle(g\,G^a_{\mu\nu})^2\rangle=0.5\ \text{GeV}^4\,,$$

Comparing these (solid) curves with those obtained from experiment (the dotted ones) one can see, that their behaviour is indeed reproduced correctly, up to distances of about 1/2 fm. (Small displacement of the curves is only few percent effect, which is just about the size of experimental uncertainties.)

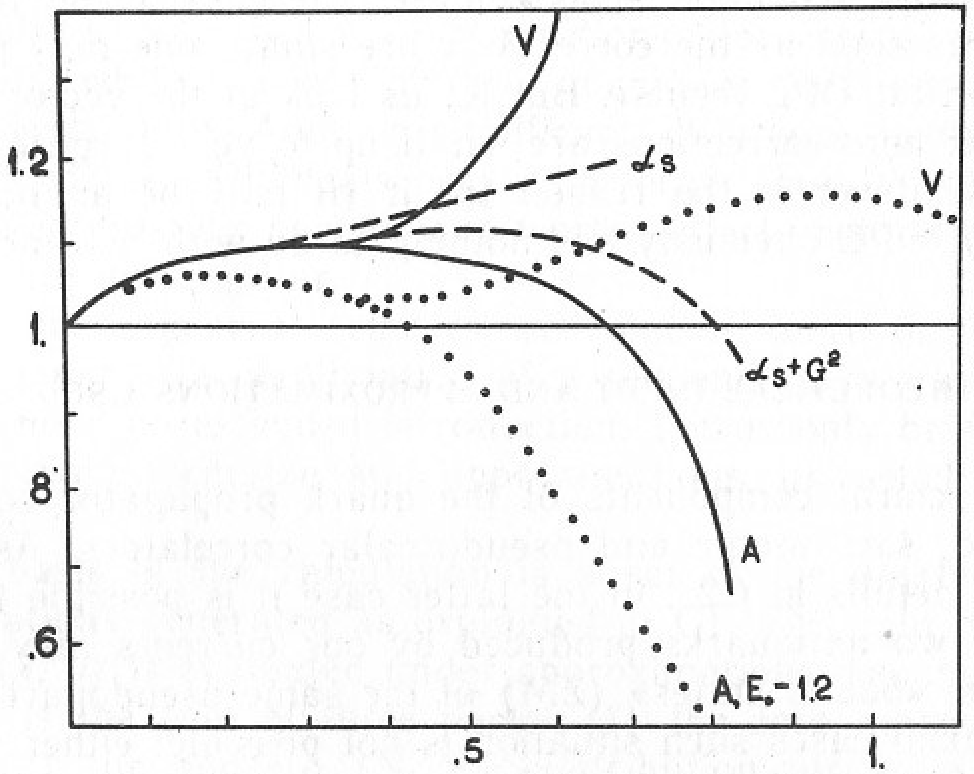

Fig. 3. R according to the operator product expansion [3] (solid lines). The dashed lines show correction due to radiative correction (α_s), as well as its combination with the «gluonic condensate» (α_s+G^2). Two dotted lines are «experimental» ones, shown in Figs 1 and 2.

Assuming (but not predicting!) certain shape of the curve (resonance plus continuum), it was shown in [3] that, for example, parameters of ρ meson can be obtained from the fit in this region. (Such wonderful reconstruction of the whole curve from its small known part reminds me a reconstruction of the dinosaurus by using only one bone, done by experienced paleontologists.)

It should be noted that many other applications of such OPE-based calculations have produced very good results. As the most nontrivial example of such predictions let me mention determination of the so called «wave functions» for various mesons and baryons, which nicely correspond to what we know from exclusive reactions (see review [11]). By the way, let me mention two particular predictions to be mentioned below: the coupling constants for A meson $f_A/f_\rho=0.9$ and for the strange vector resonance K^* $f_{K^*}/f_\rho=1.05$, which we will mention below.

But keeping all this in mind, one still has to ask *whether the OPE-based analysis is really justified?* After all, it is but small distance expansion. Should one trust it at distances of the order of 1/2 fm?

We will discuss this issue below, and now let us make only one comment on the «common sence» level. There exist the following argument: «as soon as the corrections are small, one may probably use only the first OPE terms». But let us look at the vector channel at this angle: here corrections are small up to very large distances, $\tau \sim 1.5$ fm. Whatever is the reason for it (it may be an occasional cancellation), OPE obviously has nothing to do with this fact.

3. THEORETICAL INPUT AND APPROXIMATIONS USED

Different chiral components of the quark propagator contribute differently to, say, vector and pseudoscalar correlators. As it was discussed in details in C2 , in the latter case it is possible that two quarks and two antiquarks produced by our currents may directly jump into the «bound states» (ZM) of the same pseudoparticle. For vector and axial cases such situation is not possible: either we have to use zero modes of one istanton and one anti-instanton, or we have to consider interference of zero and nonzero modes (proportional to the quark mass, see below). Therefore, there exist a general reason explaining why the instanton-induced effects are not so strong in vector and axial channels as they are in pseudoscalar and scalar ones.

Our next general comment is related with the validity region of the OPE. Our calculations are based on the theory of instantons, which develops its own scales of length. The smallest one is the typical instanton size $\rho_0 \simeq 1/3$ fm. As soon as x is comparable to it, one cannot treat x as a small parameter, so our theory suggests that the OPE should not be trusted for $x \gtrsim 1/3$ fm. If so, it is practically useless, because here all effects are too small to be visible.

It was demonstrated in C2 that such situation really takes place for pseudoscalar and scalar correlators: the OPE predictions for $x > 1/3$ fm neither agree with the data, nor with the instanton theory. But, as we have just demonstrated, for vector and axial cases OPE works well. However, as we are going to show shortly, our instanton-based theory also reproduces these data, even in much wider range of distances. Thus, we are inclined to the opinion that successful applications of OPE in the vector and axial cases is more or less ocasional and has no real theoretical justification.

The difference between our formulae and OPE is far greater

than just the fact that we do not expand in powers of x. There is essential difference in physics. In particular, we do not include in our analysis any radiative corrections (processes with gluonic exchange), while two out of three dominant OPE corrections (11) are of this nature. Another significant difference is the following: this form of the OPE deals only with terms singular at $x \to 0$, while the most interesting part of our work — effects connected with zero modes — are nonsingular.

After these general remarcs we come to more technical points. As this paper is a continuation of a sequence of works, we do not actually need an extended introduction. Let us only briefly recapitulate the main formulae and approximations, on which our results are based.

The basis of the calculation is a set of the «instanton liquid» configurations generated as explained in C1. The light quark propagator $S(x, y)$ is evaluated under approximations discussed in C3. In short, it is the sum of three parts: (1) the zero mode contribution (ZM); (2) the nonzero (NZM) contribution for massless quarks; (3) the term proportional to the quark mass.

The zero modes are «collectivized», so we perform explicit diagonalization of the Dirac operator in the corresponding subspace and then use the general relation

$$S^{ZM}(x, y) = \sum_{\lambda} \frac{\psi_\lambda(x)\, \psi_\lambda^\dagger(y)}{i\lambda + m}. \qquad (14)$$

The second component is evaluated as follows: it is assumed that *one of all pseudoparticles gives the dominant correction to the free quark propagator*. When this pseudoparticle is found for a given configuration and given x, y points, this correction is calcluated by the single instanton formulae due to Brown et al. [7]. (See details in the preceeding work C3.) By definition, the dominant pseudoparticle is the one which gives the maximal deviation of $\mathrm{Tr}[S^{NZM}(x-y)_\mu \gamma_\mu]$ from its free value. We have checked, that when such corrections are significant, the «dominant» one indeed essentially exceed all the others in most cases. Certainly it is desirable to improve our calculations at this point, but at the moment it is the best we can do.

The third component, a term proportional to the quark mass, was not previously considered. Naively in the chiral limit ($m \to 0$) it can be omitted, but one should be careful here. It was shown by

Andrei and Gross [8] (who have considered the vector correlator in the one-instanton background field) that the ZM term (which is O(1/m) in this case) can be multiplied by such $O(m)$ correction in another propagator, leading to finite (and numerically important) correction. In the multipseudoparticle background the denominator in (14) ZM part is $1/(i\lambda+m)$, but the limit $m\to 0$ is not smooth because there are arbitrarily small eigenvalues λ of the Dirac operator. (That is why the chiral symmetry is spontaneously broken.) It prevents us from naive omission of the $O(m)$ terms.

The general expression for this term can be written as follows

$$S^{(m)}(x,y)=m\int d^4z\, s^{\mathrm{NZM}}(x,z)\, s^{\mathrm{NZM}}(z,y)\,,$$

$$s^{\mathrm{NZM}}(x,y)=\hat{D}_x\Delta(x,y)\,\frac{1}{2}(1+\gamma_5)+\Delta(x,y)\,\hat{D}_y\,\frac{1}{2}(1-\gamma_5)\,, \qquad (15)$$

where S^{NZM} is the NZ mode propagator, D_x is the covariant derivative and $\Delta(x,y)$ is the propagator of a scalar particle in the instanton field. Such nonlocal expression is inconvenient to use in practice. Note however, that for one quark chirality it can be written in the local form due to the following identity

$$\Delta(x,z)\,\frac{1}{2}(1+\gamma_5)\,(\hat{D}_z)^2\,\Delta(z,y)=$$

$$=-\Delta(x,z)\,\frac{1}{2}(1+\gamma_5)\,D_z^2\Delta(z,y)=\Delta(x,z)\,\delta(z-y)\left(\frac{1}{2}\right)(1+\gamma_5)\,. \qquad (16)$$

One can check that for one-pseudoparticle backround only this chiral component contributes to the correlators of vectors and axials [2]. For the collectivized zero modes another chirality part contributes too. We also take it in the form (16), so that our total $O(m)$ correction looks like

$$S^{(m)}(x,y)=m\Delta(x,y)=\frac{m}{4\pi^2(x-y)^2}\,\frac{[1+\rho^2(\sigma_\mp x)(\sigma_\pm y)/x^2y^2]}{(1+\rho^2/x^2)^{1/2}(1+\rho^2/y^2)^{1/2}} \qquad (17)$$

($\pm$ for the instanton, $\mp$ for antiinstanton, x and y are here counted from the pseudoparticle center.) Physically this approximation means that (in this correction) we ignore the interaction of the spin-induced quark gluomagnetic moment with the field ($\sigma_{\mu\nu}\sigma_{\mu\nu}$ terms).

A number of approximations just discussed significantly simplify the problem and makes it treatable, but we have to pay for them

high price: some general relations do not stricly hold. In particular, electromagnetic current conservation demands that

$$\partial_\mu\Pi_{\mu\nu}(x)=\partial_\nu\Pi_{\mu\nu}(x)=0\,. \qquad (18)$$

Using the general expression for the propagator in terms of the eigenfunction of the Dirac operator

$$S(x,y)=\sum_x\frac{\psi_\lambda(x)\,\psi_\lambda^+(y)}{\lambda}\,;\qquad \hat{D}\psi_\lambda=\lambda\psi_\lambda \qquad (19)$$

one may easily prove (18), providing the set of ψ_λ is complete; $\sum_x\psi_\lambda(x)\,\psi_\lambda^+(y)=\delta(x-y)$. Indeed:

$$\partial_\mu\Pi_{\mu\nu}=\mathrm{Tr}\sum_{\lambda,\lambda'}\left(\frac{1}{\lambda\lambda'}\right)\partial\psi_\lambda(x)\,\psi_\lambda^+(y)\,\gamma_\nu\psi_{\lambda'}(y)\,\psi_{\lambda'}^+(x)=$$

$$=\sum_{\lambda,\lambda'}\mathrm{Tr}\left[\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)\psi_{\lambda'}^+(x)\,\psi_\lambda^+(y)\,\gamma_\nu\,\psi_{\lambda'}(y)\right]=0\,. \qquad (20)$$

We use (19) for the incomplete set of states (that of zero modes), while the NZ ones are treated differently. As a result, electromagnetic current is not *exactly* conserved. Whether this defect of our approximations is numerically large or small we plan to consider in subsequent more detailed publications. It should be small if the «instanton liquid» is *sufficiently dilute*.

4. OUR RESULTS

As we discuss in this work only the flavor-nonsinglet cases, we may consider only the «one-loop» diagram for the correlation function

$$\Pi_{\mu\nu}(x)=\langle\mathrm{Tr}[\,\Gamma_\mu S(x,0)\,\Gamma_\nu S(0,x)]\,\rangle\,;\quad \Gamma_\mu^V=\gamma_\mu\,;\quad \Gamma_\mu^A=\gamma_\mu\gamma_5\,, \qquad (21)$$

which is evaluated by averaging over points x, y (see C2 for details) with the propagator $S(x,y)=S^{\mathrm{NZM}}+S^{\mathrm{ZM}}+S^m$ discussed above. Typically our measurements are based on about 1000 pairs of points at any $|x-y|$ for each of 10 recorded configurations. It is natural to use all types of gamma matrices Γ simulteneously, and, as a test, we have checked that for pseudoscalars the results agree with those obtained in C2, where somewhat different account for the NZ mode part was made.

It is instructive to start with the results of the incomplete calculation, in which instead of S only the NZM part S^{NSM} is used. The corresponding curves are shown in Fig. 4. Thus, if only quark scattering on the color field is taken into account, the correlation func-

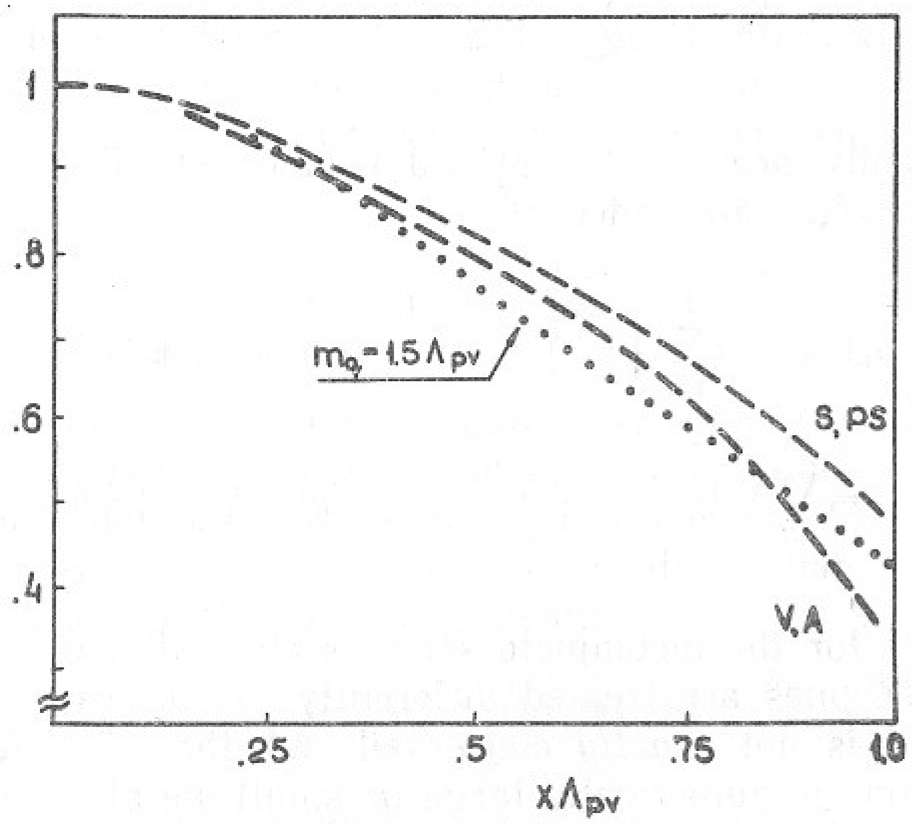

Fig. 4. The dashed curves represent our measurements for the scalar (S), the pseudoscalar (PS), the vector (V) and and the axial (A) correlators if only nonzero modes are taken into account. (There are only two curves because they are degenerate in this approximation.) The dotted curve (shown for comparison) corresponds to free propagation of quarks with the mass $1.5\Lambda_{PV}$ (about 330 MeV).

tion drops faster than for the free quarks. Roughly speaking, it looks similar for all channels, and its behaviour can be understood in terms of quark effective mass (see the dotted line in Fig. 4, shown for comparison).

The results of a complete calcluation display drastically different behaviour for different channels, see Fig. 5. Six dashed curves in Fig. 5 are the «delta plus theta functions» fits

$$\Pi^{V,A}_{\mu\mu}(x) = 3 f_{V,A}^2\, m_{V,A}^2\, D(m_{V,A}, x) + \frac{3}{4\pi^2}\int_{E_0}^{\infty} dE\, E^3 D(E,x)\left(1+\frac{\alpha_s(E)}{\pi}\right), \quad (22)$$

$$\Pi^{s,ps}(x) = \lambda^2 D(m,x) + \dots$$

to our data (except for the scalar case S, which is fitted without any resonance, see below). The values of the parameters (in Λ_{PV} unites are as follows:

(π)	$m_\pi = 0.\pm 0.7$	$\lambda_\pi^2 = 30.\pm 1.$	$E_0 = 9.9 \pm 1.2$	
(K)	$m_K = 3.\pm 1.$	$\lambda_K^2 = 32.\pm 1.$	$E_0 = 9.8 \pm 0.8$	
(K^*)	$m_{K^*} = 6.4 \pm 1.$	$f_{K^*} = 1.3.\pm 0.1$	$E_0 = 9.8 \pm 0.8$	
(ρ)	$m_\rho = 6.7 \pm 1.$	$f_\rho = 1.2 \pm 0.1$	$E_0 = 13.6 \pm 0.7$	(23)
(A_1)	$m_{A_1} = 9.3 \pm 1.2$	$f_A = 0.7 \pm 0.3$	$E_0 = 10.0 \pm 0.7$	
(S)			$E_0 = 10.6 \pm 0.9$	

First of all, both pseudoscalar curves rapidly go up. The pion becomes massless, and the correlator decays at large x as $1/x^2$ (see details in C2), so our ratio $R(x)$ grows as x^4. The same rising tendency is observed for the kaon, although it is less rapid (respectively, we get the nonzero kaon mass (23).)

New element in this figure are our results for vector and axial channels. If one compares them with the experimental curves (Figs 1 and 2) discussed in section 1, he notes that the shapes of both V and A curves are reproduced correctly, including in the vector case even such details as the shallow minimum at $x \simeq 0.2$ and the striking tendency of R_V to remain close to unity in wide x range. Our data are systematically about 10% below the experimental curve, but this is obviously due to the fact that we have not included perturbative quark-antiquark interaction (the α_S/π correction). So, accuracy of our predictions of this correlator is really remarcable: we claim that we have reproduced it on the level of 10% effects, while the correlator falls from $x\Lambda_{PV} = 1/3$ to 1 by about three orders of magnitude!

High sensitivity of the shape of this curve to resonance parameters is also seen from the K^* (strange vector) curve: it looks different in Fig. 5, but actually our fit shows that it is only due to slightly larger coupling constant: $f_{K^*}/f_\rho = 1.1$. Such trend is consistent with the sum rules estimates [11].

Our data for the axial channel are also qualitatively consistent with the phenomenological curve shown in Fig. 2. The fitted mass values give $m_A/m_\rho = 1.40 \pm .25$, and, comparing it to the experimental ratio 1.6, we get nice agreement. For the coupling constants we

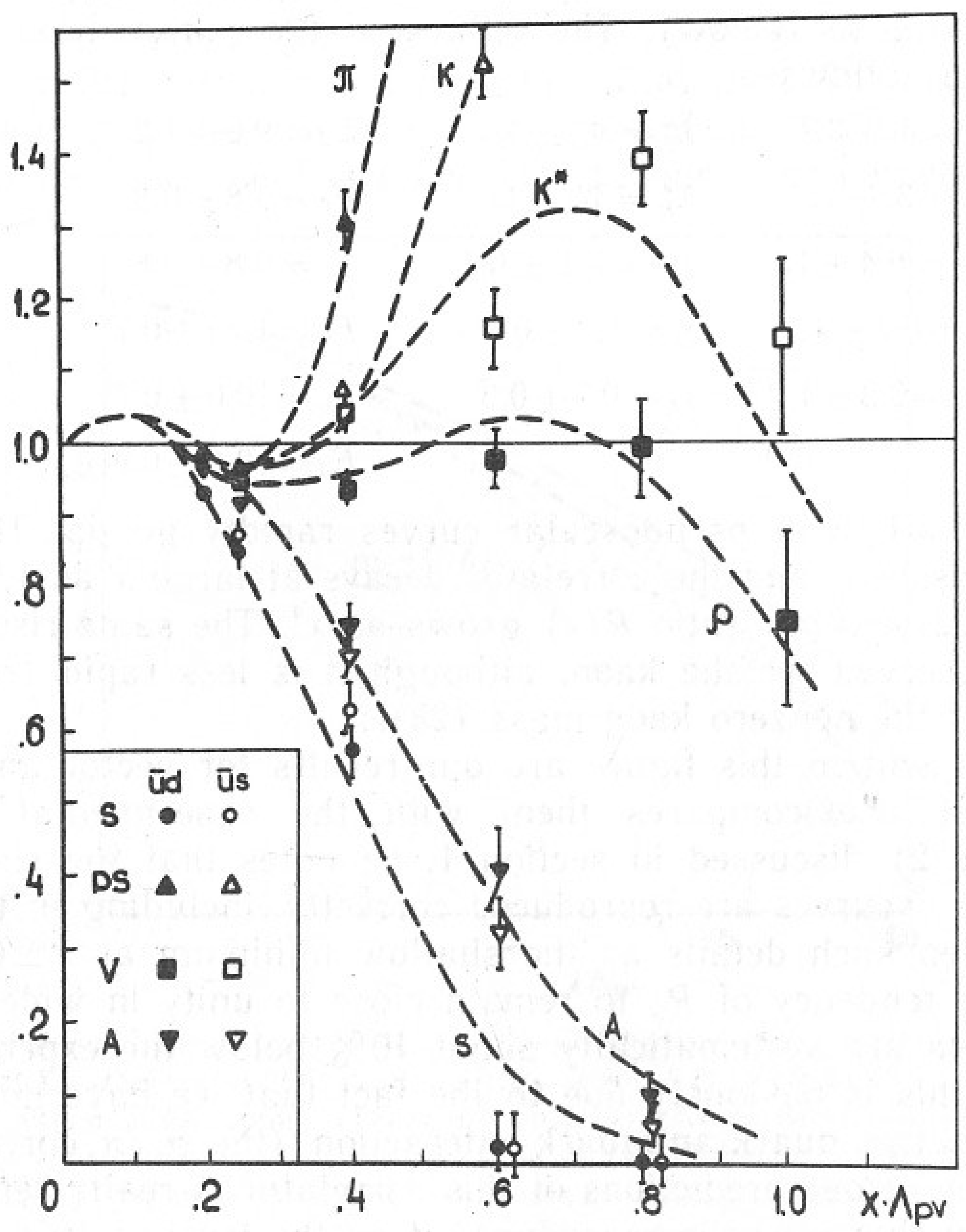

Fig. 5. The same as in Fig. 4 but for the complete propagator. We have included here both *ud* and *us*-type currents (see notations in the left lower corner). The dashed lines are the fit discussed in the text.

get $f_A/f_\rho=0.6\pm0.35$, to be compared to the phenomenological ratio 1.0 obtained in section 1. Although there is no formal contradiction here, we may claim that even deviations seen are in reasonable direction: experiment is for the «effective A_1» (all 3π-states in this mass region), while our «theoretical A_1» does not include the nonresonance states nearby (note that E_0 found is close to the A_1 mass).

One comment concerning the scalar channels. We have found that our scalar correlator decays rapidly, and it is unlikely that the only experimental scalar $I=1$ resonance $a_0(980)$ significantly contributes to it. (Thus, we add new arguments in favour of the long-standing suspicions [9] that $A_0(980)$ is not a normal $\bar{q}q$ meson but a $\bar{q}\bar{q}qq$ or $\bar{q}gq$ state.) Good fit to our data is given by the nonresonance continuum only, with the thresholds at $E_0/E=1.5$. (Even larger «gap» to the lowest physical states in this channel, about 1.7 GeV, was suggested in Ref. [10] in the QCD sum rule context.)

Finally, few words about the absolute scale of our predictions. In the preceeding works we have postulated that $\Lambda_{PV}=220$ MeV, as suggested by fits to deep inelastic data. Now we see, that all predictions (23) suggest another scale $\Lambda_{PV}\simeq150$ MeV. Although such Λ_{PV} value is still far from being definitely excluded experimentally, we have to remind the reader here that the absolute density of the «instanton liquid» was also not actually firmly determined, thus this observation most probably just indicates that our liquid is «a little bit too dense» in these calculations. We have found good qualitative description of different correlators, but the absolute normalization can only be fixed in future more detailed works.

5. SUMMARY AND DISCUSSION

Our main results can be summarized as follows: all mesonic correlators are correctly reproduced in the «instanton liquid» model for the QCD vacuum. The most striking points are the following:

1. Calculations of the flavor-nonsinglet vector current correlator have produced the results, similar to the phenomenological curve even in slightest details. Most remarkably, this theory does reproduces cancellation of all corrections to the free quark propagation in wide region of distances, which experimentally is seen as some «fine tuning» of the parameters of three rho-type mesons and the nonresonant continuum.

2. Theory predicts quite different behaviour of the axial correlator, suggesting the A_1 meson to be essentially heavier than ρ. The mass ratio is roughly consistent with the phenomenological one, while the A coupling constant is predicted a little bit smaller than it is observed in τ decays.

3. We have found that our data for the scalar channel do not suggest any strong resonances: they can well be fitted without them. As the Particle Data Tables contain no suitable $\bar{u}d$ scalar resonances ($A_0(980)$ is too light, and it was long suspected [9] not to be a $\bar{q}q$ state), this conclusion is phenomenologically wellcomed.

Let us also make a parting comment on the relation between our theory and the OPE-based sum rules. They are not just different formulations of the same physics: OPE ignores contributions which are nonsingular at $x \to 0$, while we have ignored radiative corrections (on which the OPE analysis is significantly based). If our theory is true, we see no justification for application of the OPE formulae at distances of the order of 1/2 fm where they are actually used. Thus, although in vector and axial channels the OPE formulae do reproduce data well enough, we think that any conclusions about the vacuum structure based on them should only be taken as qualitative indications.

REFERENCES

1. *Shuryak E.V.* Instantons in QCD I, II and III. (or C series). Preprints INP 88-49, 88-63, 89-1 and Nucl. Phys., B, in press.
2. *Eidelman S.I., Kurdadze L.M. and Vainstein A.I.* Phys. Lett., 82B (1979) 278.
   Launer G., Narison S and Tarrach R. Z. Phys., C26 (1984) 433.
   Grosin A.G. and Pinelis Yu.F. Preprint INP 87-50, Novosibirsk.
3. *Shifman M.A., Vainstein A.I. and Zakharov V.I.* Nucl. Phys., B147 (1979) 385, 448, 519.
4. *Shuryak E.V.* Phys. Lett.136B (1984) 269.
5. *Kiesling C.* τ Decays. In: High Energy e^+e^--Physics, Ed. A.ALI and P. Soding, WCPC, Singapore 1988.
6. *Broadhurst D.J. and Generalis S.C.* Phys. Lett. 165B (1985) 175.
   Grosin A.G. and Pinelis Yu.F. Phys. Lett. 166B (1986) 429.
7. *Brown L.S., Carlitz R.D, Creamer D.B. and Lee C.* Phys. Rev. D17 (1978) 1583.
8. *Andrei N., Gross D.J.* Phys. Rev. D18 (1978) 498.
9. *Jaffe R.L.* Phys. Rev. D15 (1977) 267,285, D17 (1978) 1444.
   Martin A.D. et al. Nucl. Phys. B121 (1977) 514.
   Achasov N.N. et al. Phys. Lett. 96B (1980) 168.
10. *Zhitnitsky A.R. and Zhitnitsky I.R.* Yad. Fiz.37 (1983) N 6.
11. *Chernyak V.L. and Zhitnitsky A.R.* Phys. Rep. 112 (1984) 173.

E.V. Shuryak

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Э.В. Шуряк

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