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CORRELATIONS OF PSEUDO-RANDOM NUMBERS OF MULTIPLICATIVE SEQUENCE

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Correlations of Pseudo-Random Numbers of Multiplicative Sequence

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ABSTRACT

In the paper there is suggested algorithm of the search with a computer in unit n-dimensional cube for the sets of planes, where all the points fall, whose coordinates are composed of n successive pseudo-random numbers of multiplicative sequence. This effect should be taken into account in Monte-Carlo calculations with definite constructive dimension. The parameters of these planes are obtained for three random number generators.

1. INTRODUCTION

The Monte-Carlo method applied in various calculations demands random number generators. Bad quality of a generator can decrease the accuracy of calculations, so it is important to know the statistical properties of generators being used.

At present the most investigated are the properties of the multiplicative generators because of their simplicity.

In the paper [1] the author has presented a brief report of the multiplicative generators properties and some new investigation of a serial correlation coefficient. In that paper it was also suggested to use the program realization of multiplication of integer numbers with a big number of binary bits in order to improve the statistical properties of multiplicative series.

All properties of these generators are suitable for solving most of the problems, but one type of correlation, pointed in first [2], is traditionally treated as a disadvantage of multiplicative series and makes difficult to use random numbers of this type in Monte-Carlo problems with large constructive hyperspace dimension. The mentioned effect is the following: if one considers every n successive numbers of multiplicative sequence as the coordinates of the point in n-dimensional cube, then all these points will get to the set of parallel equidistant hyperplanes, the upper limit for the number of planes in this set being obtained in [2]. In Table 1 these limits are cited for $m=32$ and $m=63$, where m is the number of bits in mantissa of random number (the choice of these values for m will be clear further).

Table 1.

The Number of Hyperplanes in n-Dimensional Hyperspace, to which All the Random Points Fall

n	3	4	5	6	7	8	9	10
$m=32$	2953	566	220	120	80	60	48	41
$m=63$	3811000	122000	16170	4335	1731	884	531	357

Existence of space volumes unattainable for the random vectors is not a property of multiplicative generators only. For other generators with the same period these «empty» volumes can be of some complicated shape, but in the general case it does not remove the possibility of a systematic error in Monte-Carlo calculations. The following statement seems to be indisputable. If for any generator of pseudo-random numbers there some type of correlation is found, preventing its use for some problems, there are no reasons to change it to another generator whose correlations of this type were not investigated or such investigations were not completed because of the difficulty of the task.

To the author's point of view the analysis of possibility to use this type of generators for some particular problems is hampered more not by the fact that these limits have appeared to be surprisingly low, but that these limits are the upper ones, and there are no reasons to think of some particular generator, that the number of planes in the minimum set will appear to be close to the theoretical limit.

In this paper the algorithm is suggested for the search of the sets of such planes and their characteristics are obtained for three multiplicative generators used in our Institute.

2. ALGORITHM OF HYPERPLANES PARAMETERS DETERMINATION

Multiplicative sequence of pseudo-random numbers r_i is quite determined by the generation constant k and the starting random number. All random numbers are integer without sign, in the range from 0 to $M=2^m$. Every next number can be obtained from the previous one according to the simple algorithm:

$$r_{i+1}=r_i \cdot k \pmod{2^m}. \tag{1}$$

Floating point random numbers in the range (0, 1) are obtained by division of corresponding integer numbers by M.

Attempts of a search for these sets of planes with the help of minimization of some aim function and multidimensional Fourier transformation failed.

For the suggested algorithm of the search for the planes it is essential that the points in n-dimensional space lie exactly on the hyperplanes looked for, and these hyperplanes are parallel and equidistant as well, that follows from the proof in [2]. The algorithm scheme is as follows.

1. A rather big number of points in n-dimensional space is to be stored. Coordinates of every random point are n successive random numbers of the generator studied. The number of points should be at least greater than space dimension n.

2. This array of points is arranged according to their distance from the zero point of coordinate frame.

3. The point closest to the center of coordinate frame is considered as the origin of new system of reference and its coordinates are subtracted from the coordinates of all other points. After this operation only vectors R_i thus obtained are considered.

4. At the next step one should choose n points R_i so that the determinant of the dimension n composed of these points coordinates was not equal to zero. In general this condition is sufficient for the search of the solution of the system of linear equations, but for saving computer time these points should be on the planes as close to each other as possible, but not on the same one, that corresponds to the choice of the smallest determinant not equal to zero.

5. In order to understand the next step of the algorithm let us imagine that the direction of the normal to hyperplanes and the gap between them are known. Let us form the normal vector X with a modulus equal to the inverse value of the gap between the hyperplanes. Then the scalar product of the vector X and any of the vectors R_i will be equal to an integer number. If we knew n these integer numbers for the set of vectors obtained at the previous step, then we could find the unknown vector X, solving the system of linear equations, because the system determinant is not equal to zero. Hence the obvious final step of algorithm follows—one should try all possible sets of n integer numbers (with + and — signs) and check whether all the points R_i fall to the planes well determined by the vector X. Apparently one should check every combination only until the first disturbance, and if any point does not get

exactly to one of the planes, then one should change the set of integer numbers. The number of combinations can be decreased by the factor of two if the first nonzero integer number has only positive value.

From the theoretical point of view there is a weak point in the described algorithm. Next set of planes obtained from the set of integer numbers is checked on the large array of random points, but not on all possible points. So there is some probability that after successful check on the limited array of points the set of planes is nevertheless a false one. One can guess that for the false sets of planes the distribution of random points over the projection to the normal vector is uniform (and this was observed during the test run of program). Then the probability to take a false set of planes for the true one decreases exponentially with increasing the number of checks.

Evidently there exists an infinite number of solutions of this problem. Actually all random numbers appear to be discrete and one can at least form the set of planes parallel to the coordinate plane and with the gap equal to the minimum distance between the random numbers. Having obtained one such a solution, one can transform it to the infinite number of new solutions putting new «false» planes between old ones.

If we are interested only in those solutions where the hyperplanes are put far apart, then we can use the result of the paper [2] that one can find at least one solution with the quantity of hyperplanes in the whole unit hypercube not more than

$$N=(n!\cdot 2^{m})^{1/n}. \qquad (2)$$

Starting from this formula one can obtain the upper limit for the modulus of vector X.

Using this or similar restriction it is possible to sort out all integer number combinations of interest. Unfortunately no effective way to decrease the number of combinations was found.

In our Institute three generators of the described type are widely used: RANDM, RNDM and DRNDM. Generator RANDM has been written in our Institute and is used at ES-type computers since 1978. The generation constant of random sequence in RANDM $k_1=1AFD498D_{16}=452807053$. Generator RNDM is copied from the CERN program library, the generation constant $k_2=$ $=10DCD_{16}=69069$. Generator DRNDM is a version of DRANDM generator, suggested in [1], where the instruction for multiplication

of integer numbers with 63 bits was simulated. In this case two times slowing of generator is compensated by the essential improvement of statistical properties of the sequence. The generation constant of DRNDM $k_3=400040010115_{16}=70369817985301$. Generator DRNDOM with the same generation constant k_3, but slightly different program organization, is included into the program library of SLAC (USA). Since the statistical properties of generator are determined completely by the generation constant, the following results for DRNDM generator are valid for DRNDOM generator as well.

It is curious that all three constants are not prime numbers and have the following expansion: $k_1=17\times 281\times 94789$, $k_2=$ $=3\times 7\times 11\times 13\times 23$, $k_3=29\times 43\times 53\times 971\times 1096541$. However the results of the presence of cited factors in the expansion of generating constants are unknown.

3. RESULTS

As a result of investigation it was noticed that for any of listed generators in n-dimensional space there exists many sets of planes with different orientation and different gaps between planes. A set of planes with a maximum gap between planes is of the greatest interest among all possible sets of planes. Let us designate this maximum gap as H_{max}. In some cases it can be useful to have the characteristics of sets of planes with less gaps as well, but it is impossible to describe all sets of planes in practice.

Main results of the search for the planes are included in Table 2. An obvious difficulty in the result presentation is the problem — how many sets of planes should be presented for every generator and each hyperspace dimension. For practical use of these data one should know the maximum gap H_{max} between the planes and the parameters of the sets of planes with a gap in the range $(0.5\cdot H_{max}, H_{max}]$. However in some cases even under these conditions the number of sets of planes appeared to be very big.

It should be explained why the left boundary of the range is chosen to be equal to $0.5\cdot H_{max}$. One can easily see that if there exists a set of planes with a gap H, then the set of planes with the gap $0.5\cdot H$, obtained from the initial set by combining this set with the one shifted by $0.5\cdot H$, also contains all the random points, and moreover not less than a half of the total number of planes will contain no one random point. Evidently these defective sets should not be included into results.

Finally it was decided to cite the detailed information about no more than ten sets of planes for every generator and hyperspace dimension from 3 to 10. Tables with the parameters of planes are presented in Appendix.

One can not choose the best generator of RANDM and RNDM on the grounds of data in Table 2. At one value of n generator RANDM is better, at another value of n generator RNDM is better. However at all values of n the generator DRNDM is much better than generators with ordinary precision, and in the case when usage of ordinary precision generators of multiplicative type gives rise to doubts in reliability of the results and the decrease of the generation rate of random numbers by two times is not critical for the problem being solved, one can use the generators with double precision.

If the theoretical limit for the number of planes and real characteristics of the generators are compared, then the difference by several times can be noticed. If the dependence of the number of planes in the minimum set on the generation constant k were known in analytic form, then one could choose the optimal value k for this parameter. However there are no grounds to expect that the theoretical limit (2) is reached at some value of k. Taking into account the last remark one can consider the characteristics of RANDM and RNDM generators with respect to the investigated effect rather close to the best ones.

During the calculation of the characteristics of the planes an interesting and not evident effect was found out. The parameters of the planes turned out not to vary with the change of an initial random number to any other belonging to the same random sequence (as is known, two random separate sequences with the same period correspond to any generation constant with maximum period). This effect immediately rejects the idea of struggling against the falling of random points to the planes by means of rejecting from time to time some amount of random numbers of sequence.

In conclusion I would like to repeat that the multiplicative generators in my opinion have a great advantage as compared with another type generator in the first place because the main characteristics of their random sequences are explored in details: period, uniformity, correlation properties. At the same time for any new

algorithm the determination of even the simplest sequence characteristic — period, is usually a rather difficult problem.

Table 2

Characteristics of the Generators RANDM, RNDM, DRNDM with Respect to the «Plane-Like» Correlation

n	Генератор	Максимальный шаг между плоскостями H_{max}	Количество плоскостей в наборе с H_{max} (теор. предел)	Количество наборов плоскостей с шагом в инт. $(0.5 \cdot H_{max}, H_{max}]$
3	RANDM	0.00142	1021 (2953)	4
	RNDM	0.00278	395 (2953)	1
	DRNDM	$9.22038 \cdot 10^{-7}$	$1.47(3.81) \cdot 10^{6}$	10
4	RANDM	0.0113	157 (566)	1
	RNDM	0.0102	156 (566)	3
	DRNDM	$5.44669 \cdot 10^{-5}$	$0.34(1.22) \cdot 10^{5}$	2
5	RANDM	0.0366	57 (220)	1
	RNDM	0.0230	88 (220)	11
	DRNDM	$2.91860 \cdot 10^{-4}$	$0.59(1.62) \cdot 10^{4}$	14
6	RANDM	0.0386	53 (120)	44
	RNDM	0.0643	35 (120)	3
	DRNDM	$1.75544 \cdot 10^{-3}$	$1.20(4.34) \cdot 10^{3}$	1
7	RANDM	0.0570	41 (80)	130
	RNDM	0.0767	30 (80)	24
	DRNDM	$2.29547 \cdot 10^{-3}$	$1.04(1.73) \cdot 10^{3}$	392
8	RANDM	0.1313	21 (60)	4
	RNDM	0.0767	30 (60)	378
	DRNDM	$5.97871 \cdot 10^{-3}$	382 (884)	142
9	RANDM	0.1387	19 (48)	35
	RNDM	0.1000	26 (48)	723
	DRNDM	$8.97809 \cdot 10^{-3}$	302 (531)	946
10	RANDM	0.1387	17 (41)	393
	RNDM	0.1387	19 (41)	424
	DRNDM	$1.38648 \cdot 10^{-2}$	166 (357)	2120

Appendix

PARAMETERS OF PLANES TO WHICH THE RANDOM POINTS FALL

Let us write the equation determining the system of parallel equidistant hyperplanes in the hyperspace of dimension n in the form:

$$(R\cdot W)=H\cdot(D+l), \qquad (3)$$

where R is a random vector, formed by n successive random numbers of multiplicative sequence: W is a unit normal vector to the planes; H is a gap between the neighbouring planes; D is a shift (some value in the range from -0.5 to 0.5); l is an arbitrary integer number with plus or minus sign.

As was mentioned, all the random numbers, generated by the given generating constant, are divided into two separate sequences with the same period, the lowest two bits of the first series forming the number of 1 and that of the second series forming the number of 3. It turned out that among all parameters of planes only the shifts are different. Later on these parameters will be designated as D_1 и D_2.

It has remained unknown whether the equality of W and H parameters of these two sequences is theoretically strict or the difference is negligible at the level of calculation accuracy.

The number of sets of planes, presented in Tables for three generators and for the hyperspace dimension from 3 to 10, is not constant from one variant to another. No more than 10 sets of planes with greatest gap H were selected under the condition that H gets into the interval $(0.5\cdot H_{max},\ H_{max}]$.

In the first column of the Tables the gap H between planes is presented, in the second and third columns there are the shifts D_1 and D_2, in the fourth column there is the number of planes for this set intersecting the unit hypercube, and in the last column the unit normal vector W is displayed. Because of the lack of space in the Table, when necessary the values of the coordinates of the vector W are written in several lines.

One should keep in mind that for the generators RANDM and RNDM the number of digits in the parameters of planes is sufficient for the points to be distributed correctly among the planes, but for DRNDM generator the accuracy of the presented parameters H and

W is insufficient. However, the accuracy of planes parameters in tables for DRNDM is quite enough to estimate the accuracy of the results of Monte-Carlo calculations. Moreover, when necessary one can easily get more exactly the values of the parameters, using the Table data as the starting point.

RANDM $n=3$

$H\cdot10^3$	D_1	D_2	N	W
1.41921	0.50	0.50	1021	−0.84159 0.53504 −0.07380
1.04959	0.00	0.00	1458	0.28234 0.89005 −0.35791
0.97665	0.50	0.50	1623	0.84187 0.46000 −0.28225
0.75377	0.50	0.50	1941	−0.24422 0.92337 −0.29623

RANDM $n=4$

H	D_1	D_2	N	W
0.01128	0.50	0.50	157	0.587 0.034 −0.519 0.621

RANDM $n=5$

H	D_1	D_2	N	W
0.0366	0.25	−0.25	57	−0.403 −0.366 −0.183 −0.366 −0.732

RANDM $n=6$

H	D_1	D_2	N	W			
0.0386	0.00	0.00	53	0.424	−0.039	−0.193	0.193
				0.386	−0.772		
0.0366	0.25	−0.25	57	−0.403	−0.366	−0.183	−0.366
				−0.732	0.000		
0.0366	0.25	−0.25	57	0.000	−0.403	−0.366	−0.183
				−0.366	−0.732		
0.0360	0.50	0.50	58	0.144	−0.108	0.576	0.252
				−0.324	−0.684		
0.0341	0.50	0.50	53	−0.239	−0.068	0.716	0.068
				−0.648	0.034		
0.0333	−0.25	0.25	53	−0.133	−0.266	−0.866	−0.400
				−0.033	−0.033		
0.0325	−0.25	0.25	61	0.812	−0.260	−0.162	−0.032
				−0.260	0.422		
0.0263	0.25	−0.25	80	0.763	−0.289	0.289	0.158
				−0.447	−0.158		
0.0259	−0.25	0.25	90	−0.389	0.182	−0.545	−0.441
				−0.285	0.493		
0.0254	−0.25	0.25	69	0.916	−0.229	−0.254	0.102
				0.051	−0.178		

RANDM $n=7$

H	D_1	D_2	N	W			
0.0570	−0.25	0.25	41	−0.342	−0.057	0.285	−0.627
				0.171	−0.570	−0.228	
0.0453	0.50	0.50	45	−0.498	−0.272	0.136	0.724
				−0.362	0.045	0.045	
0.0446	0.25	−0.25	53	−0.357	−0.446	0.223	0.045
				0.625	0.179	−0.446	
0.0444	0.00	0.00	50	0.355	−0.666	0.133	0.177
				−0.577	0.177	−0.133	
0.0430	0.25	−0.25	55	−0.730	−0.301	0.344	0.215
				−0.215	−0.387	−0.129	
0.0423	−0.25	0.25	55	0.085	−0.677	0.339	−0.296
				−0.423	−0.254	−0.296	
0.0420	0.50	0.50	59	0.545	−0.252	0.545	0.126
				0.420	−0.252	−0.294	
0.0415	0.50	0.50	53	−0.083	−0.373	0.000	0.497
				0.456	0.580	−0.249	
0.0408	0.50	0.50	48	0.082	−0.204	0.734	−0.082
				0.000	0.571	−0.285	
0.0406	0.25	−0.25	58	0.406	0.203	0.527	−0.122
				−0.568	0.406	0.122	

RANDM $n=8$

H	D_1	D_2	N	W			
0.1313	0.50	0.50	21	−0.263	−0.394	−0.394	−0.131
				−0.525	−0.131	0.394	0.394
0.0729	0.25	−0.25	29	0.073	0.438	0.000	−0.073
				0.000	0.729	0.365	−0.365
0.0729	−0.25	0.25	32	−0.073	0.219	−0.219	−0.146
				−0.292	0.656	0.583	−0.146
0.0690	0.25	−0.25	33	−0.690	0.414	−0.483	0.000
				−0.138	−0.138	−0.276	−0.069

RANDM $n=9$

H	D_1	D_2	N	W				
0.139	0.00	0.00	19	−0.277	−0.139	0.000	0.277	−0.416
				0.416	0.555	0.000	−0.416	
0.131	0.50	0.50	19	0.000	−0.263	−0.394	−0.394	−0.131
				−0.525	−0.131	0.394	0.394	
0.131	0.50	0.50	21	−0.262	−0.394	−0.394	−0.131	−0.525
				−0.131	0.394	0.394	0.000	
0.096	0.25	−0.25	29	−0.096	−0.385	0.289	0.385	0.096
				−0.577	−0.385	0.192	−0.289	
0.089	0.25	−0.25	29	0.178	0.178	0.535	−0.178	0.178
				−0.267	0.535	0.445	−0.178	
0.088	−0.25	0.25	31	0.439	0.088	0.175	−0.263	−0.614
				0.263	−0.351	−0.088	0.351	
0.087	0.25	−0.25	31	−0.261	−0.435	0.261	0.522	−0.174
				−0.261	0.000	0.174	−0.522	
0.083	−0.25	0.25	29	−0.083	−0.167	0.500	0.583	0.167
				−0.167	−0.250	−0.083	−0.500	
0.081	−0.25	0.25	23	0.243	0.000	0.162	−0.081	−0.811
				0.487	0.000	−0.081	0.081	
0.080	0.00	0.00	29	0.160	−0.320	0.400	0.240	−0.721
				0.000	−0.320	0.080	−0.160	

RANDM $n=10$

H	D_1	D_2	N	W				
0.139	0.00	0.00	17	0.277	0.139	0.000	−0.277	0.416
				−0.416	−0.555	0.000	0.416	0.000
0.139	0.00	0.00	19	0.000	0.277	0.139	0.000	−0.277
				0.416	−0.416	−0.555	0.000	0.416
0.131	0.50	0.50	19	0.000	0.000	−0.263	−0.394	−0.394
				−0.131	−0.525	−0.131	0.394	0.394
0.131	0.50	0.50	19	0.000	0.262	0.394	0.394	0.131
				0.525	0.131	−0.394	−0.394	0.000
0.131	0.50	0.50	21	0.263	0.394	0.394	0.131	0.525
				0.131	−0.394	−0.394	0.000	0.000
0.129	0.50	0.50	19	−0.258	−0.129	−0.258	−0.129	−0.775
				0.258	0.000	−0.129	0.000	0.387
0.109	−0.25	0.25	23	−0.546	−0.109	−0.436	0.000	0.436
				−0.436	0.000	0.000	−0.109	0.327
0.105	−0.25	0.25	27	0.105	0.316	−0.105	−0.211	0.316
				0.000	−0.316	0.422	−0.527	0.422
0.102	−0.25	0.25	25	−0.204	−0.408	−0.714	0.204	0.000
				0.000	−0.306	−0.102	0.204	−0.306
0.102	0.25	−0.25	27	−0.510	0.102	−0.102	0.306	0.510
				0.000	0.102	−0.306	−0.408	0.306

RNDM $n=3$

$H \cdot 10^3$	D_1	D_2	N	W		
2.77849	0.25	−0.25	395	0.06113	0.03612	−0.99748

RNDM $n=4$

H	D_1	D_2	N	W			
0.01016	−0.25	0.25	156	0.315	−0.203	0.914	0.152
0.00628	0.50	0.50	296	−0.722	0.295	0.559	−0.282
0.00583	−0.25	0.25	274	−0.851	0.391	−0.006	−0.350

RNDM $n=5$

H	D_1	D_2	N	W			
0.0230	0.25	−0.25	88	0.459	0.735	0.390	−0.184
				0.252			
0.0168	0.50	0.50	101	0.487	0.017	0.134	0.839
				−0.201			
0.0144	0.25	−0.25	153	−0.416	0.431	−0.402	0.574
				0.388			
0.0141	0.25	−0.25	130	0.127	−0.438	−0.127	0.819
				−0.325			
0.0141	0.25	−0.25	148	−0.661	−0.323	0.520	0.183
				0.394			
0.0138	0.50	0.50	133	−0.372	0.124	0.744	0.069
				0.537			
0.0130	−0.25	0.25	151	0.639	0.430	0.326	0.548
				−0.013			
0.0124	0.50	0.50	151	−0.112	0.768	−0.136	0.396
				0.471			
0.0120	0.00	0.00	160	−0.586	−0.024	−0.538	0.574
				0.191			
0.0119	−0.25	0.25	163	−0.214	−0.262	0.536	0.750
				0.190			

RNDM $n=6$

H	D_1	D_2	N	W			
0.0643	0.00	0.00	35	−0.193	0.707	−0.257	0.514
				−0.257	0.257		
0.0359	0.25	−0.25	53	0.574	−0.179	0.215	0.753
				−0.143	−0.072		
0.0329	−0.25	0.25	63	−0.626	0.527	−0.329	−0.428
				0.000	0.198		

RNDM $n=7$

H	D_1	D_2	N	W			
0.0767	0.00	0.00	30	0.230	−0.614	−0.537	−0.307
				−0.307	0.000	−0.307	
0.0643	0.00	0.00	34	0.000	−0.193	0.707	−0.257
				0.514	−0.257	0.257	
0.0643	0.00	0.00	34	0.193	−0.707	0.257	−0.514
				0.257	−0.257	0.000	
0.0524	0.00	0.00	44	0.524	0.419	0.577	−0.367
				−0.262	0.105	0.052	
0.0500	0.00	0.00	42	0.650	0.000	0.200	−0.550
				−0.450	0.100	−0.150	
0.0475	0.00	0.00	46	−0.475	−0.522	0.000	0.142
				0.617	−0.285	0.142	
0.0456	0.50	0.50	45	−0.091	0.320	0.046	−0.456
				−0.456	−0.046	0.685	
0.0455	0.50	0.50	48	0.545	0.045	0.455	0.136
				0.227	0.136	−0.636	
0.0426	0.50	0.50	48	0.043	−0.043	−0.255	−0.596
				−0.596	−0.043	0.468	
0.0422	0.25	−0.25	53	0.127	0.337	−0.506	0.084
				0.717	−0.169	−0.253	

RNDM $n=8$

H	D_1	D_2	N	W			
0.0767	0.00	0.00	30	0.000	−0.230	0.614	0.537
				0.307	0.307	0.000	0.307
0.0767	0.00	0.00	30	0.230	−0.614	−0.537	−0.307
				−0.307	0.000	−0.307	0.000
0.0729	0.00	0.00	30	0.219	−0.802	0.073	0.219
				0.000	0.292	−0.292	0.292
0.0684	0.00	0.00	33	−0.205	−0.137	−0.068	−0.479
				0.752	0.342	0.137	−0.068
0.0654	0.25	−0.25	35	−0.458	−0.196	0.065	−0.588
				0.000	0.131	−0.196	−0.588
0.0643	0.00	0.00	35	0.000	0.000	−0.193	0.707
				−0.257	0.514	−0.257	0.257
0.0643	0.00	0.00	34	0.000	−0.193	0.707	−0.257
				0.514	−0.257	0.257	0.000
0.0643	0.00	0.00	34	0.193	−0.707	0.257	−0.514
				0.257	−0.257	0.000	0.000
0.0640	0.25	−0.25	36	−0.256	−0.064	0.128	−0.128
				−0.704	−0.192	−0.320	−0.512
0.0635	0.25	−0.25	40	−0.254	−0.254	0.635	0.318
				−0.446	0.064	−0.318	−0.254

RNDM $n=9$

H	D_1	D_2	N	W				
0.100	0.50	0.50	26	0.200	0.300	0.000	0.200	−0.400
				−0.500	−0.400	−0.500	−0.100	
0.092	0.50	0.50	24	−0.092	0.092	−0.092	−0.460	0.644
				0.000	−0.184	−0.552	−0.092	
0.086	−0.25	0.25	29	−0.515	−0.343	0.172	−0.343	−0.600
				0.172	−0.086	−0.257	0.086	
0.083	0.25	−0.25	26	0.167	0.583	0.083	0.000	−0.167
				0.500	−0.083	0.000	0.583	
0.082	0.25	−0.25	33	0.493	0.329	0.082	−0.329	0.000
				−0.493	−0.247	0.247	−0.411	
0.082	0.50	0.50	26	0.163	0.000	−0.163	−0.163	0.000
				0.163	0.735	−0.572	0.163	
0.081	−0.25	0.25	30	0.324	0.081	0.081	−0.487	0.324
				−0.081	0.081	0.649	−0.324	
0.081	0.50	0.50	33	0.161	−0.242	0.725	−0.242	−0.242
				0.161	0.403	−0.242	−0.161	
0.078	0.50	0.50	29	−0.155	−0.233	0.233	−0.776	−0.233
				0.078	0.000	0.388	−0.233	
0.078	−0.25	0.25	35	0.621	0.310	−0.310	0.388	−0.310
				0.310	0.155	0.000	−0.233	

RNDM $n=10$

H	D_1	D_2	N	W				
0.139	0.50	0.50	19	−0.277	−0.416	0.000	0.139	−0.555
				−0.277	0.000	0.139	0.139	−0.555
0.116	0.00	0.00	21	0.233	0.116	−0.349	0.233	−0.697
				−0.116	0.116	−0.116	0.465	0.116
0.110	−0.25	0.25	24	0.552	−0.221	0.221	0.110	−0.110
				−0.110	0.110	0.552	0.442	−0.221
0.108	−0.25	0.25	23	−0.431	0.108	−0.108	0.431	−0.647
				0.108	0.108	0.108	−0.324	0.216
0.108	0.50	0.50	25	0.431	0.431	−0.324	0.108	−0.216
				0.108	0.108	−0.216	0.324	0.539
0.107	0.50	0.50	27	−0.107	0.107	−0.426	−0.426	−0.533
				−0.320	0.107	0.320	−0.320	−0.107
0.107	0.00	0.00	27	0.107	−0.320	0.426	−0.426	−0.426
				0.320	0.000	0.107	0.426	0.213
0.107	0.00	0.00	26	0.320	0.213	0.107	0.426	−0.320
				0.213	0.213	0.533	0.000	0.426
0.105	0.00	0.00	27	0.105	0.422	−0.422	−0.527	−0.105
				−0.105	0.105	0.211	−0.422	0.316
0.104	0.50	0.50	23	−0.104	−0.209	0.417	0.313	−0.104
				0.104	0.104	0.730	−0.313	−0.104

DRNDM $n=3$

$H\cdot10^7$	D_1	D_2	$N\cdot10^{-6}$	W		
9.22038	0.50	0.50	1.47	0.924661	−0.377194	0.052220
6.91108	−0.25	0.25	1.95	0.423631	−0.016571	−0.905683
6.78952	−0.25	0.25	2.14	0.264704	−0.261472	0.928205
6.16011	0.00	0.00	1.88	−0.165643	−0.986173	−0.004961
5.73357	0.50	0.50	2.53	0.729162	0.683336	0.037089
5.12772	−0.25	0.25	3.25	0.337798	0.623425	0.705148
4.78335	0.25	−0.25	3.30	0.772903	−0.207150	−0.599758
4.71554	−0.25	0.25	3.29	0.162251	−0.766218	−0.621759
4.67213	0.50	0.50	2.79	0.342910	−0.939094	0.022698
4.66380	0.25	−0.25	3.61	0.649536	−0.370399	0.664009

DRNDM $n=4$

$H\cdot10^5$	D_1	D_2	$N\cdot10^{-4}$	W			
5.44669	0.00	0.00	3.44	−0.69685	0.33312	0.26346	−0.57795
2.75915	0.25	−0.25	5.73	0.41255	−0.10110	−0.17932	−0.88737

DRNDM $n=5$

$H\cdot10^4$	D_1	D_2	$N\cdot10^{-3}$	W				
2.91860	−0.25	0.25	5.93	0.35461	0.34877	0.85457	0.02627	0.14710
2.75419	0.50	0.50	7.17	0.52275	0.51779	0.01928	0.59959	−0.31425
2.71489	0.25	−0.25	7.76	−0.56877	0.45339	0.23538	0.58995	0.25981
2.48717	−0.25	0.25	7.96	0.16987	0.17037	−0.71083	0.51907	−0.40914
2.23755	0.50	0.50	8.38	−0.74063	0.10628	−0.46116	0.46608	0.10136
2.18088	−0.25	0.25	7.10	−0.87083	−0.04580	0.17382	−0.00087	0.45755
1.84613	0.50	0.50	10.2	−0.51286	0.18184	0.68768	0.01588	0.48036
1.82196	0.00	0.00	10.7	−0.25726	0.42907	−0.36275	0.77615	−0.12535
1.80656	0.00	0.00	11.3	−0.15898	0.51758	0.68559	0.40883	0.26394
1.75477	−0.25	0.25	9.22	−0.03457	0.62294	0.16442	0.76333	−0.03229

DRNDM $n=6$

$H\cdot10^3$	D_1	D_2	$N\cdot10^{-3}$	W				
1.75544	−0.25	0.25	1.20	0.12464	−0.38971	0.61440	0.40726	−0.53716
				0.02633				

DRNDM $n=7$

$H \cdot 10^3$	D_1	D_2	$N \cdot 10^{-2}$	W				
2.29547	−0.25	0.25	10.4	0.50500	0.34662	0.17446	0.08952	−0.44073
				0.56010	−0.28005			
2.27882	0.50	0.50	9.48	0.07748	−0.33727	−0.58794	−0.15724	−0.05697
				0.64719	0.29645			
2.27000	0.50	0.50	6.54	0.29964	0.00908	−0.02724	−0.04313	0.94432
				0.04313	−0.11804			
2.26268	0.25	−0.25	10.1	0.19006	−0.05204	−0.50910	0.13802	0.33261
				0.50231	−0.56567			
2.22444	0.00	0.00	9.70	−0.59615	−0.39595	−0.64286	0.09120	−0.17128
				−0.09120	−0.16906			
2.19488	−0.25	0.25	10.4	−0.10535	−0.05926	−0.46751	0.17559	−0.59042
				0.44556	−0.43459			
2.16198	0.25	−0.25	9.52	−0.10810	−0.27025	−0.07135	−0.28106	−0.37186
				0.13404	0.82155			
2.13554	−0.25	0.25	10.5	0.17511	−0.25840	−0.09610	−0.31820	0.52107
				0.17298	0.70046			
2.08794	−0.25	0.25	11.1	0.35495	0.20253	−0.37374	0.54078	−0.27561
				0.00000	0.57001			
2.01833	−0.25	0.25	9.32	0.13119	0.79926	−0.09890	−0.51266	−0.06660
				0.01413	0.25835			

DRNDM $n=8$

$H \cdot 10^3$	D_1	D_2	N	W				
5.97871	0.00	0.00	382	0.03587	0.59189	−0.25708	0.07772	−0.20328
				−0.34676	0.62776	0.14349		
5.09224	−0.25	0.25	458	0.05601	−0.44812	−0.37173	−0.13240	−0.12221
				0.09166	0.57542	0.53469		
5.09066	0.25	−0.25	472	−0.41234	−0.44289	0.02036	0.31053	−0.11199
				−0.64142	0.17817	−0.28508		
4.66760	−0.25	0.25	508	0.52277	−0.54611	0.48076	0.08402	−0.18670
				−0.35941	0.03267	0.15870		
4.47590	−0.25	0.25	436	0.02238	−0.83699	−0.13428	−0.17456	0.04476
				0.34017	0.03581	0.36255		
4.13739	−0.25	0.25	606	0.48821	−0.07447	0.24824	0.12826	−0.30617
				−0.55855	−0.46339	0.23997		
4.12990	0.25	−0.25	538	−0.53689	−0.04130	0.21476	−0.05782	0.21063
				−0.46668	−0.06608	0.62776		
4.03541	0.50	0.50	550	0.77883	−0.12106	0.39951	−0.17352	−0.07264
				0.19774	−0.11299	0.36319		
4.01338	0.25	−0.25	592	0.32107	0.08428	0.64214	−0.14850	0.42140
				0.27291	0.44950	−0.03612		
3.99215	0.25	−0.25	598	−0.34732	−0.74254	0.18763	0.19162	0.04791
				−0.27147	−0.27945	−0.31937		

DRNDM $n=9$

$H\cdot10^3$	D_1	D_2	N	W				
8.97809	0.50	0.50	302	−0.08080	0.32321	−0.35015	0.13467	−0.22445
				−0.59255	−0.31423	−0.27832	−0.41299	
8.30913	−0.25	0.25	272	0.83091	0.10802	−0.14956	0.07478	−0.33237
				−0.16618	0.29082	−0.14956	0.15787	
7.92354	−0.25	0.25	342	0.41202	0.37241	−0.49126	−0.14262	0.03962
				−0.30109	−0.18224	0.31694	0.45164	
7.90866	0.00	0.00	302	0.00791	−0.29262	0.05536	0.03954	−0.11863
				−0.51406	−0.51406	−0.35589	0.49034	
7.82110	0.00	0.00	296	−0.28156	−0.79775	−0.17206	0.19553	0.25028
				−0.32066	−0.21899	0.03911	−0.03911	
7.77275	−0.25	0.25	362	−0.24873	0.38086	0.36532	0.38086	0.36532
				−0.39641	−0.00777	−0.34977	0.31868	
7.75077	0.00	0.00	370	0.37204	−0.26353	0.34103	0.20927	−0.34878
				0.13951	0.44954	−0.44954	−0.29453	
7.62715	0.50	0.50	324	−0.46526	0.00000	−0.49576	−0.30509	−0.42712
				−0.39661	0.04576	0.32034	0.01525	
7.57272	0.50	0.50	304	0.23475	−0.02272	0.00000	0.59067	0.40893
				−0.35592	0.53766	−0.09845	−0.05301	
7.45812	0.00	0.00	376	−0.27595	−0.48478	−0.21629	0.14916	0.35053
				0.17899	0.27595	0.37291	−0.49969	

DRNDM $n=10$

$H\cdot10^2$	D_1	D_2	N	W				
1.38648	0.50	0.50	166	−0.16638	0.29116	−0.23570	0.79030	0.02773
				0.15251	0.05546	0.05546	−0.40208	−0.12478
1.29445	0.50	0.50	212	−0.29772	−0.44011	−0.45306	0.19417	−0.12945
				0.18122	0.32361	−0.02589	−0.15533	0.54367
1.22831	−0.25	0.25	224	0.40534	−0.30708	−0.49132	0.25795	−0.27023
				0.51589	−0.14740	−0.07370	−0.25795	−0.02457
1.22776	−0.25	0.25	208	0.24555	0.63843	−0.35605	−0.15961	−0.08594
				0.01228	0.00000	−0.33149	0.35605	−0.36833
1.21716	0.00	0.00	222	−0.25560	0.09737	0.18257	0.25560	−0.66944
				0.06086	0.19475	−0.24343	0.34081	−0.40166
1.20860	0.25	−0.25	202	−0.03626	0.21755	−0.77350	0.02417	−0.20546
				0.18129	0.30215	−0.35049	0.20546	0.14503
1.20456	0.00	0.00	214	−0.49387	0.24091	−0.54205	−0.07227	−0.16864
				−0.04818	−0.54205	0.16864	−0.16864	0.13250
1.18896	0.50	0.50	204	−0.20212	0.46369	−0.13079	−0.65393	0.26157
				0.35669	−0.01189	0.00000	0.02378	0.32102
1.17250	0.00	0.00	202	−0.02345	0.04690	0.32830	−0.17588	−0.44555
				0.72695	−0.25795	0.03518	−0.09380	−0.23450
1.14693	−0.25	0.25	224	−0.66522	−0.05735	0.37849	−0.24085	−0.29820
				0.38996	0.17204	0.08028	0.01147	0.27526

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A.D. Bukin

Correlations of Pseudo-Random Numbers of Multiplicative Sequence

А.Д. Букин

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