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METHOD OF THE MEASUREMENT OF SUPERSMALL BEAM SIZE IN ELECTRON-POZITRON COLLIDERS BY COMPTON SCATTERING

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НОВОСИБИРСК

METHOD OF THE MEASUREMENT OF SUPERSMALL BEAM SIZE IN ELECTRON-POSITRON COLLIDERS BY COMPTON SCATTERING

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ABSTRACT

A new method of measurement of submicron transverse sizes of the electron beam is discussed. This method permits to define the electron-positron luminosity without taking into account the beam-beam interaction effects by using the information from the electron-photon Compton scattering. Analytic formulae and quantitative estimations are given for the most significant parameters of an experimental condition. As shown at this paper less then 10% accuracy can be easily achieved at the measurements of the electron beam sizes for FFTB experiment at SLAC and about 20% accuracy for VLEPP project at Protvino.

INTRODUCTION

One of the problems in an achievement of a high luminosity in cycle and linear superhigh energy colliders is associated with the obtaining of high density electron and positron bunches with small sizes at the interaction point because of the beam transverse dimension sizes are near one micron or even less [1-3]. The problem of the measurement of such supersmall beam sizes is staying opened now.* We consider a new method to measure so small beam sizes by using simple technique based on the Compton scattering of a laser light (see, for example,[4]).

MEASUREMENT METHOD

A general idea can be understood with a help of Fig.1. A linearly polarized standing electromagnetic wave is formed perpendicularly to the electron beam orbit. In this case a photon density distribution is given as:

$$n_p = \frac{2}{\lambda\,\Sigma_x\Sigma_y}\sin^2\left(\frac{2\pi z}{\lambda}\right), \qquad \int n_p dV = 1, \qquad (1)$$

where λ - the wavelength of the electromagnetic wave which is comparable with σ_z - the transverse size of the electron beam; Σ_x, Σ_y - photon beam sizes along X and Y axes accordingly. Charged particles passing through the standing wave structure, interact with photons and give rise to high energy gamma ray which travel along electron beam trajectory. The intensity of the gamma ray production depends on the standing wave point in which the electron beam interacts with the electromagnetic wave. Changing the locations of the standing wave density maxima by pulling them across electron beam orbit one can obtain a gamma ray intensity modulated according a periodic function. A modulation depth δ can be defined as:

$$\delta = \frac{\dot{N}_{max} - \dot{N}_{min}}{\dot{N}_{max} + \dot{N}_{min}} \qquad (2)$$

where $\dot{N}_{max}$ and $\dot{N}_{min}$ are a maximum and a minimum of the gamma ray intensity. This value δ depends on the shape and the size of the electron beam.

The gamma ray intensity is $\dot{N} = \sigma_c \cdot L_{ep}$, where σ_c -the Compton scattering cross section, L_{ep} - the electron-photon luminosity. The last one can be written as:

$$L_{ep} = N_e N_p f \iiint n_p n_e dx\, dy\, dz, \qquad (3)$$

where N_e, N_p - numbers of the particles in the electron bunch and the photon beam accordingly, f - the frequency of the electron-photon interaction, n_e - the distribution of the electron beam density. Because of the photon density is supposed to be unvariable along the X and Y axes in the interaction area we can integrate on this dimensions. So $\dot{N} \sim \int \rho_p \rho_e dz$, where ρ_p, ρ_e- linear density distributions of the photon beam and the electron bunch accordingly.

Let's to analyze of the behaviour of the intensity $\dot{N}$ at the different distributions of the electron beam density which

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*During preparing of the text of this article we received KEK Preprint 90-173 A (January 1991) by T.Shintake with the same method discussed independently.

are shown at Fig.2. The results of the calculations are given at the Table 1, where $k = \frac{2\pi\sigma}{\lambda}z$, z, z_0- coordinates along the normal of the standing wave front. One can see that in all cases the expression for the intensity of the gamma ray contains $\cos\left(\frac{4\pi z}{\lambda}{}_0\right)$, which represents the modulation of the intensity. The factor before cosine is the modulation amplitude - δ (modulation depth eq.2). In the second column of the Table 1 the expressions for the average beam transverse

Table 1.

Density distribution for particles in the electron beam	Expressions for modulation and size of the beam
1. Gauss	$\dot{N} \sim 1 - \exp(-2k^2)\cos\frac{4\pi z_0}{\lambda}$
$\rho_e = \frac{1}{\sqrt{2\pi}\,\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$	$\sigma_z = \frac{\lambda}{2\pi}\sqrt{\frac{1}{2}\ln\left(\frac{1}{\delta}\right)}$
2. Circle	$\dot{N} \sim 1 - \frac{2J_1(k)}{k}\cos\frac{4\pi z_0}{\lambda}$
$\rho_e = \begin{cases} \frac{8}{\pi\sigma_z^2}\sqrt{\frac{\sigma^2}{4} - z^2}, & \lvert z\rvert \le \frac{\sigma}{2} \\ 0, & \lvert z\rvert > \frac{\sigma}{2} \end{cases}$	$\sigma_z \approx \frac{\lambda}{2\pi}\sqrt{12 - 2\sqrt{48\delta} - 13}$
3. Exponential	$\dot{N} \sim 1 - \frac{1}{1+4k^2}\cos\frac{4\pi z_0}{\lambda}$
$\rho_e = \frac{1}{2\sigma_z}\begin{cases} \exp\left(\frac{z_0 - z}{\sigma_z}\right), & z \le z_0 \\ \exp\left(\frac{z - z_0}{\sigma_z}\right), & z \ge z_0 \end{cases}$	$\sigma_z = \frac{\lambda}{2\pi}\sqrt{\frac{1}{4\delta} - \frac{1}{4}}$
4. Square	$\dot{N} \sim 1 - \frac{\sin(k)}{k}\cos\left(\frac{4\pi z_0}{\lambda} + k\right)$
$\rho_e = \frac{1}{\sigma_z}\begin{cases} 1, & z \in [z_0, z_0 + \sigma] \\ 0, & z \notin [z_0, z_0 + \sigma] \end{cases}$	$\sigma_z \approx \frac{\lambda}{2\pi}\sqrt{10 - 2\sqrt{30\delta} - 5}$

size σ_z are given up to the third order at the k-parameter. The amplitude modulation δ of the intensity as the function of

the parameter k is performed on Fig.3. It's easy to see that δ has the similar qualitative dependence for the different distributions of the electron beam density.

It's important to mark that the "dispersed" distributions such as the "gauss" and the "exponential" have more strongly decreasing (curves 1 and 3) than the "bounded" ones - the "circle" and the "square" (curves 2 and 4).

From Fig.3 one can see that this method provides the highest sensitivity when the wavelength λ and beam transverse size correspond to the amplitude modulation of the intensity about $\delta \approx 1/2$, because of the derivative of the modulation amplitude is the highest at this point.

Here we can set the condition for the optimal wavelength of the initial photons. For the "gauss" and "exponential" distributions it is $\lambda \approx 4\pi\sigma_z$, for the "circle" and "square" - $\lambda \approx \pi\sigma_z$. The reason of such distinction is that the "dispersed" distributions destroy modulation of the intensity stronger than the "bounded" ones do. Moreover the correct comparison of the different cases require a new parameter $\sigma_{1/2}$ - a value of the electron beam transverse size containing a half of all particles of the electron beam. The Table 2 represents this value $\sigma_{1/2}$ in the column 2.

This parameter as shown in the Table 2 differs in 2.8 times at the "square" and "gauss" distributions and about 1.15 at the "gauss" and "exponential" for the same modulation amplitude of the intensity (δ=1/2).

An expression for the electron-positron luminosity $L_{e^+e^-}$ can be written as:

$$L_{e^+e^-} = N_{e^+}\cdot N_{e^-}\cdot f_0\cdot G, \qquad (4)$$

where N_{e^-}, N_{e^+} - numbers of the particles in the electron and positron bunches accordingly, f_0 - the electron-positron collision frequency. $G = \int \rho_{e^+}\,\rho_{e^-}\,dz$, - the geometric factor of the electron-positron luminosity connected with the shapes and sizes of the beam density distributions without taking into account the beam-beam interaction effect (ρ_{e^-}, ρ_{e^+} - the linear densities of the electron and positron beams). This parameter is shown in the third column of the Table 2. It's easy to calculate that for $\delta=1/2$ the values of the geometric

Table 2

Density distribution	$\sigma_{1/2}$	Geometric factor for $L_{e^-e^+}$
Gauss	$\frac{.67\,\lambda}{2\pi}\sqrt{\frac{1}{2}\ln\left(\frac{1}{\delta}\right)}$	$\frac{1}{\lambda}\sqrt{\frac{2\pi}{\ln\left(\frac{1}{\delta}\right)}}$
Circle	$\frac{.41\,\lambda}{2\pi}\sqrt{12-2\sqrt{48\delta-13}}$	$\frac{64}{\lambda\cdot 3\pi\sqrt{12-2\sqrt{48\delta-13}}}$
Exponential	$\frac{.69\,\lambda}{2\pi}\sqrt{\frac{1}{4\,\delta}-\frac{1}{4}}$	$\frac{\pi}{\lambda\sqrt{\frac{1}{\delta}-1}}$
Square	$\frac{.5\,\lambda}{2\pi}\sqrt{10-2\sqrt{30\delta-5}}$	$\frac{2\pi}{\lambda\sqrt{10-2\sqrt{30\delta-5}}}$

factors at all the distributions are practically the same. This is right for the wide variation range of the modulation depth δ. Fig.4 represents precise calculation of the parameter G as the function of the modulation depth δ for different

cases of the electron beam density distributions. So for calculation of the electron-positron luminosity $L_{e^+e^-}$ without beam-beam interaction effects it is enough to measure modulation depth δ in the electron-photon Compton scattering on the standing wave. The electron-positron luminosity can be defined without a knowledge about the shape of the electron beam density distributions and of the electron beam size.

ERROR SOURCES

Now we intend to study some effects decreasing the amplitude modulation of the intensity measured by the method which has been discussed above. It is very easy to estimate that in the case of $\sigma_z \ll \sigma_x$ the polar angle φ between the front of the electromagnetic wave and Z-axis (Fig.1) must be $\varepsilon\cdot\sigma_z/\sigma_x$, where ε is the required accuracy of the electron beam size measurement, σ_z/σ_x is about (10^{-2}- 10^{-3}) rad for the most of the collider projects. So for the 10% accuracy (ε = 0.1) the polar angle φ should be (10^{-3}- 10^{-4}) rad.

A nonparallelness of the front of the standing electromagnetic wave to the electron beam trajectory at the interaction point affects the intensity modulation depth. Let's introduce a new parameter α - a asimutal angle between the electron beam orbit direction and the electromagnetic wave front at the interaction point (Fig.1). Certainly $\alpha < \frac{\lambda}{\Sigma_y} \ll 1$, ($\Sigma_y$- the size of the electromagnetic wave region along the beam trajectory) because of no modulation could be observed if the maximum at the beginning and at the end of interaction field were displaced more than for a half of wavelength. Moreover α contributes to the absolute intensity value and influences on the gamma ray energetic spectra bound. At the first approach on the parameter α one can estimate that $\dot{N}(\alpha) = \dot{N}\cdot\left(1 - \frac{2\omega_p\gamma}{m_o c^2}\alpha\right)$, where ω_p - the energy of the initial photon, γ, $m_o c^2$ - gamma factor and the electron rest energy, $\omega_{max}(\alpha) = \omega_{max}(1 + \alpha)$, ω_{max} - the maximum of the gamma quanta energy distribution [4].

All expression for $\dot{N}$ in the Table 1 could be written as:

$$\dot{N} \sim 1-F(k)\cdot\cos\left(\frac{4\pi z}{\lambda}o + \psi\right), \qquad (5)$$

where F(k) is one of the functions shown at Fig.3 and ψ -some phase in the modulation of the intensity. Integral of the expression (5) with the written as (1) density distribution of the photon beam along Y-axis (the electron beam trajectory) one can obtain $\dot{N}$ as a function of α:

$$\dot{N}(\alpha) \sim 1 - F(k)\frac{\sin\left(\frac{2\pi\alpha\Sigma}{\lambda'}y\right)}{\frac{2\pi\alpha\Sigma}{\lambda'}y} \cdot \cos\left(\frac{4\pi z}{\lambda'}o + \frac{2\pi\alpha\Sigma}{\lambda'}y + \psi\right), \qquad (6)$$

where $\lambda' = \lambda\cos(\alpha)$. Hence, more strict condition for the α-parameter is (from eq.6):

$$\alpha = \varepsilon\,\frac{\sqrt{6}\,\lambda}{2\pi\Sigma_y} \qquad (7)$$

where ε - required accuracy of the electron beam size measurement.

As a result the radius of the wave front should be greater then:

$$R_{min} = \varepsilon\,\frac{\pi\,\Sigma_y^2}{\sqrt{6}\,\lambda} \qquad (8)$$

The photon beam has the flat front in the focusing region and this place is the most suitable for the electron-photon interaction because of the high density of the energy and the large wave front radius are there. As all the measurements are provided on the size about the photon wavelength, the variation of the photon density connected with the laser ray focusing is negligible.

Another significant factor decreasing the modulation amplitude is the fluctuations of the electron beam trajectory which produces the fluctuation of the location of the interaction point. It's necessary to examine two cases. Let A - a characteristic magnitude of the beam fluctuation in some vicinity of its average position. The first case corresponds to the "big" fluctuations $2A > \frac{\lambda}{2}$, the second to the "small" one $2A < \frac{\lambda}{2}$. In the first case there's no reason of use to pull the electromagnetic wave across the electron beam trajectory because of the beam can scatter the photons at any point of the standing wave, both at the maximum and minimum of the energetic density distribution. Nevertheless, this "big" fluctuation can be used for the modulation amplitude determination if much more then one gamma quantum are produced and registered per one electron-photon collision. In this case the equation (2) for δ should have the minimum and maximum quantity of the gamma quanta produced in one interaction act instead of intensities. It's easy to estimate the average gamma quanta quantity which is necessary to obtain σ_z with 10%

accuracy. For this we should measure δ with approximately the same accuracy (7%). The electron beam is considered to have the "gauss" density distribution and in this case the errors is given by equation:

$$\frac{\Delta\delta}{\delta} \approx \sqrt{8}\, k^2 \, \frac{\Delta\sigma}{\sigma} \qquad (9)$$

where $\frac{\Delta\delta}{\delta}$ and $\frac{\Delta\sigma}{\sigma}$ - the relative errors for δ, σ accordingly, $\left(\frac{\Delta\sigma}{\sigma} = \varepsilon\right)$ and k is about 0.5 when $\delta \approx 0.5$ (Fig.3).The relative error for δ is given by equation:

$$\left(\frac{\Delta\delta}{\delta}\right)^2 = \frac{4}{N_{max} - N_{min}} \cdot \frac{N_{max} \cdot N_{min}}{N^2_{max} - N^2_{min}} \qquad (10)$$

For the "gauss" electron beam density distribution we obtain from eq.(1),(9) and (10) expression for the average number of the gamma quanta per one electron-photon collision:

$$< N > = \frac{1 - \delta^2}{16\, \delta^2\, k^4\, \varepsilon^2}\,, \quad \text{where} \quad < N > = \frac{N_{max} + N_{min}}{2} \qquad (11)$$

To suppose $\varepsilon = 0.1$ and $\delta = 0.5$ we obtain the necessary average quantity of the photons about 300. So the registration of the 300 gamma quanta per one electron-photon collision allows us to measure σ_z with $\varepsilon = 10\%$ accuracy.

In the case of "small" fluctuations the measured beam transverse size will be lager than the real one because of the modulation amplitude is decreased, as the electron beam scatters the standing electromagnetic wave not only in the points of the maximum and minimum of the intensity but in some vicinity around them. Fig.5 represents how the measured beam size depends on the average trajectory fluctuation of the electron beam. One can see that the distinction is not significant when $A < \frac{\lambda}{8}$, and rapidly increases when $A \approx \frac{\lambda}{4}$.

Nevertheless in the case of the "small" fluctuations it is possible to subtract the systematic error connected with the beam trajectory fluctuations by measuring the fluctuation magnitude A. It's require the electron beam scattering the electromagnetic standing wave at the place where the gradient of the energy density is high. The difference between maximum and minimum quantity of the produced gamma quanta depends on the fluctuations magnitude so it could be measured.

SCHEME OF THE EXPERIMENT

The most profitable way to create the high density electromagnetic wave is to close the optical laser-like cavity with the inserted active element (Fig.6) where the linearly polarized standing wave is formed. The magnetic vector direction is collinear to the impulse of the electron which makes the electron scattering upon the magnetic field negligible and permits to obtain the modulation of the gamma ray intensity. The vacuum chamber with the electron-photon interaction area is placed into the optic cavity working in the pulse regime. Pickup - the electron beam position monitor and the electron beam trajectory corrector give some facilities for fixing the interaction point. Moreover it let us to scan the electron beam across the standing wave. The optical mirror upon the piezoelectric holders also gives us the possibility to pull the standing wave across the electron beam trajectory.

Due to the energy dissipation in the transparent and reflective elements the traveling wave appears. It's the contribution to the decreasing of the amplitude modulation of the intensity which could be simply calculated if the dissipation coefficients are known.

Now we intend to make some quantitative estimations. For example one needs to measure 50 GeV electron beam containing $5 \cdot 10^{10}$ particles with the transverse sizes about $\sigma_z = 0.06\ \mu m$ and $\sigma_x = 1\ \mu m$. These parameters are typical for the Final Focus Test Beam experiment (FFTB) at SLAC [5]. For the best sensitivity at $\delta \approx 1/2$ we choose the first harmonic of the Nd:YAG - laser at the wavelength $\lambda = 1.06\ \mu m$, the peak power - 2 MW inside optical cavity and the laser beam diameter about 2 mm which are quite typical for the industry manufactured lasers. For the high photon density we focus the laser beam into the spot $\Sigma_y = 0.2$ mm in the diameter. Under these conditions and for the $\varepsilon = 0.1$ the angle α must be 0.2 mrad (eq.7) which can be easily fulfilled. It's easy to calculate that we would have about 600 gamma quanta per one electron-photon collision with the highest energy 15 GeV. Thus the accuracy of the measured value σ_z would be $\varepsilon = 7\%$ as δ accuracy equals to 5% (eq.9-11) In the case of such quantity of the high energy photons is well enough to use the total absorption detector for the intensity modulation depth measuring. The requirement to this detector is that the accuracy of the total energy measurements connected with the statistical fluctuations must be less than the measuring accuracy of δ.

This method gives possibility to measure the nanometer

range transverse electron beam sizes. In this case it is difficult to obtain the best sensitivity of this method. For VLEPP project [1] the maximal electron energy 1 TeV, number of the particles in the bunch - $2\ 10^{11}$, $\sigma_x = 3\ \mu m$ and $\sigma_z = 10^{-3}\ \mu m$. In this case we can use the Ar_2- excimer laser with wavelength $0.126\ \mu m$ and e can obtain the modulation depth $\delta = 0.995$, $k = 0.05$ (see Table 1). From eq.11 for the 20% accuracy in the electron beam size measurement we need 2600 gamma quanta per one electron-photon collision.

CONCLUSIONS

As shown in this paper the suggested method can be easily realized for the measurements of the electron beam transverse sizes at existing and constructed high luminosity electron-positron colliders. Advantage of this method is the wide range of the supersmall transverse beam sizes can be measured as it has no principal physical limits. As shown in this paper for the calculation of the electron-positron luminosity $L_{e^+e^-}$ without taking into account the beam-beam interaction effects it is enough to measure the amplitude of the modulation of the gamma quanta intensity δ in the electron-photon Compton scattering on the flat standing wave. The electron-positron luminosity can be defined without information about the shape and size of the electron beam. It's important to mark that it doesn't require any unique equipment to except a widely used laser and detector systems.

The present device configuration also gives a possibility to measure two electron beam parameters - a sign and a degree of the electron longitudinal polarization. The intensity of the Compton gamma quanta output on the angle along the electron beam trajectory near zero depends on the electron beam longitudinal polarization if the laser beam has had the circle polarization. This can be achieved if laser resonator is turned on angle α about 1 rad. For the FFTB experiment [5] at the electron beam longitudinal polarization degree is 0.5, the values $\frac{d\sigma}{d\omega}c\cdot(\omega_{max})$ differ about 40% each to other for the right and left circle polarization of the laser photon.

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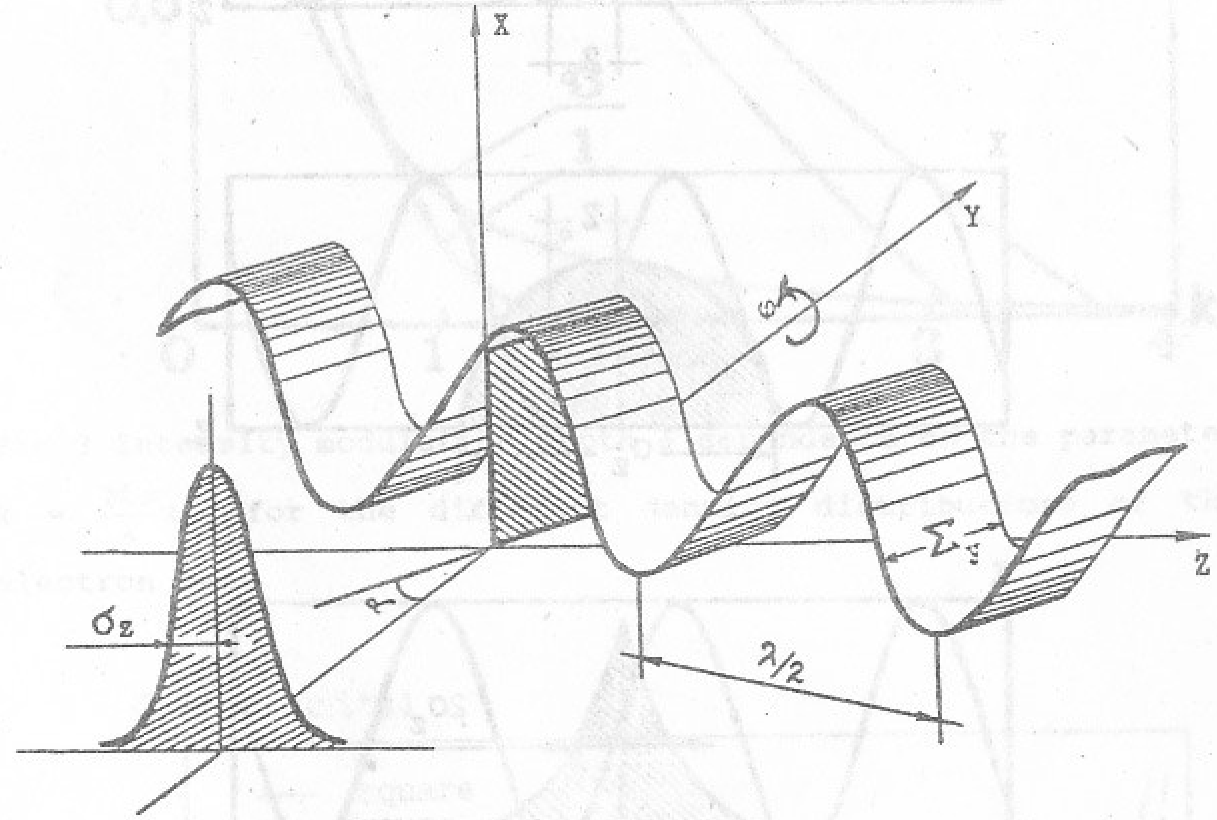

Fig. 1 The principal scheme of the interaction point.

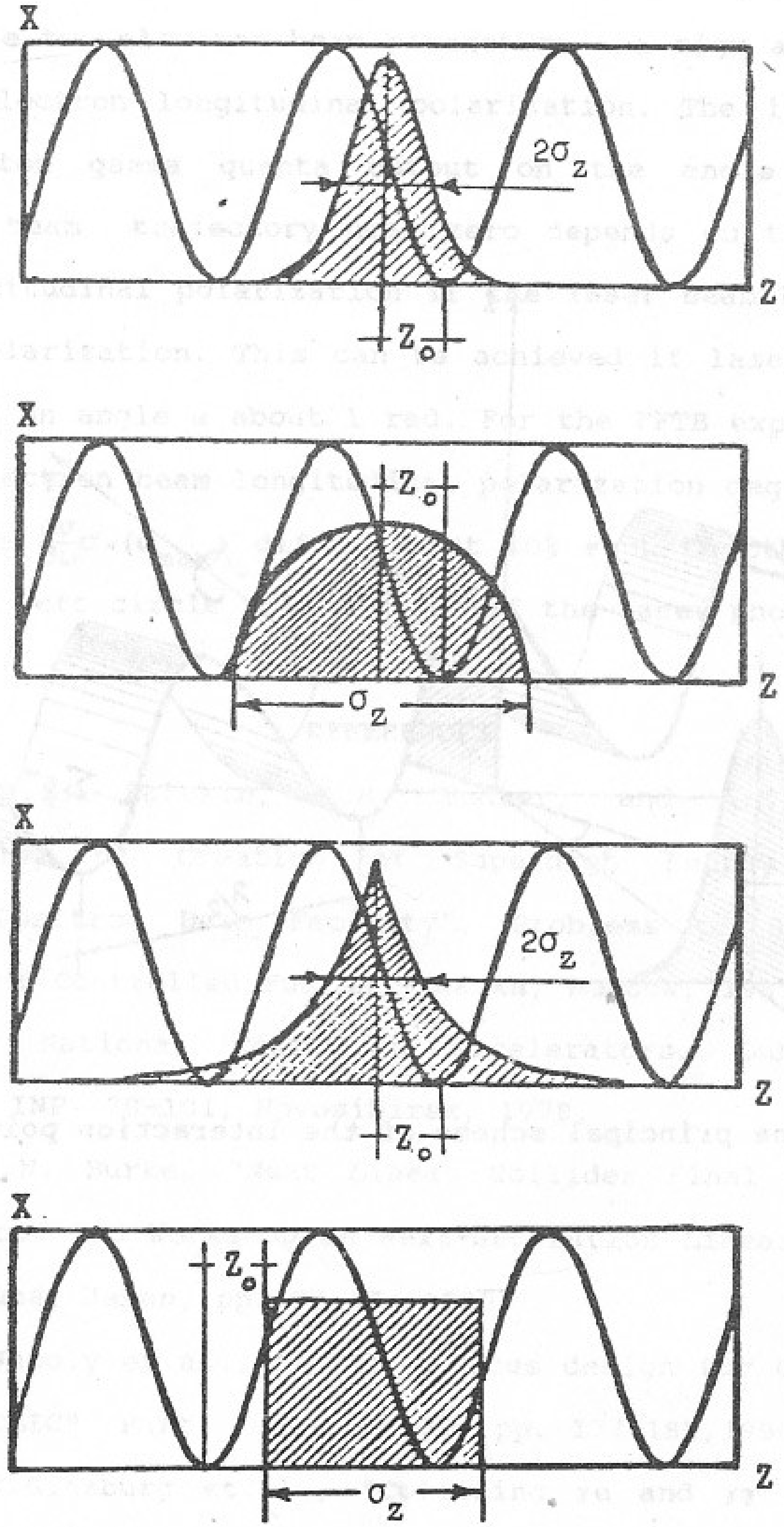

Fig. 2 Density distributions of the electron beam.

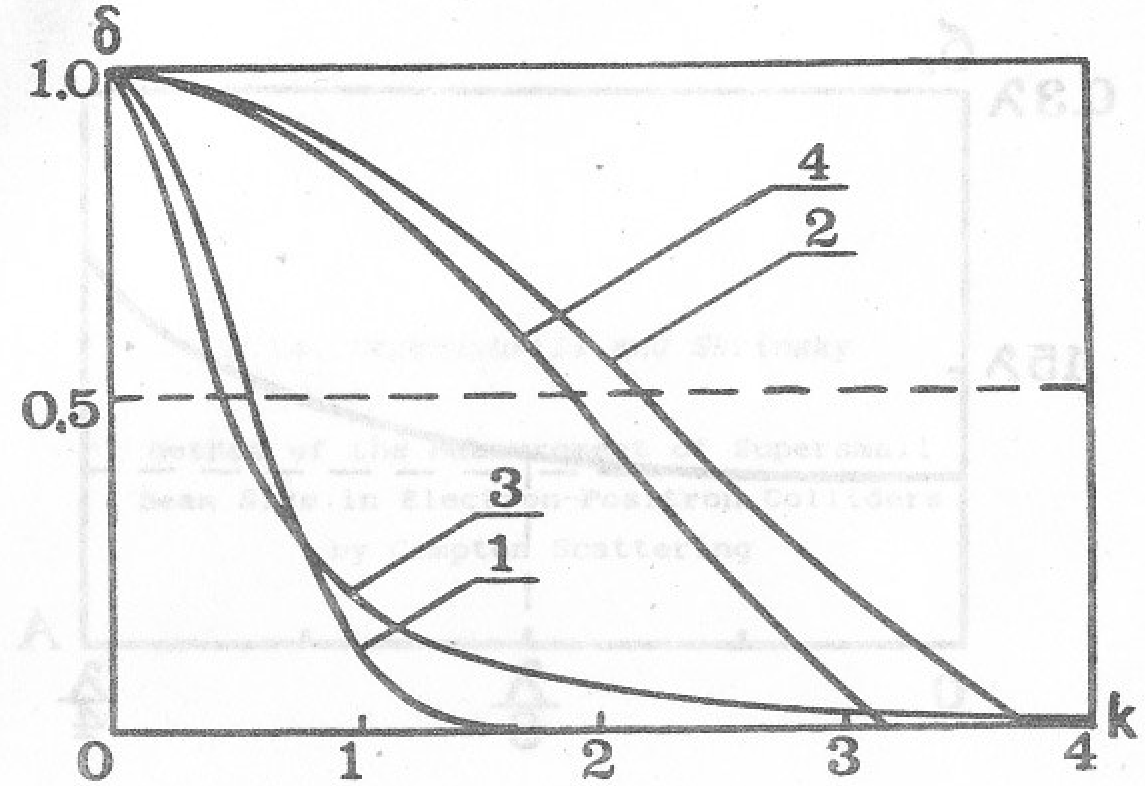

Fig. 3 Intensity modulation depth δ dependence on the parameter $k = \frac{2\pi\sigma_z}{\lambda}$ for the different density distributions of the electron beam.

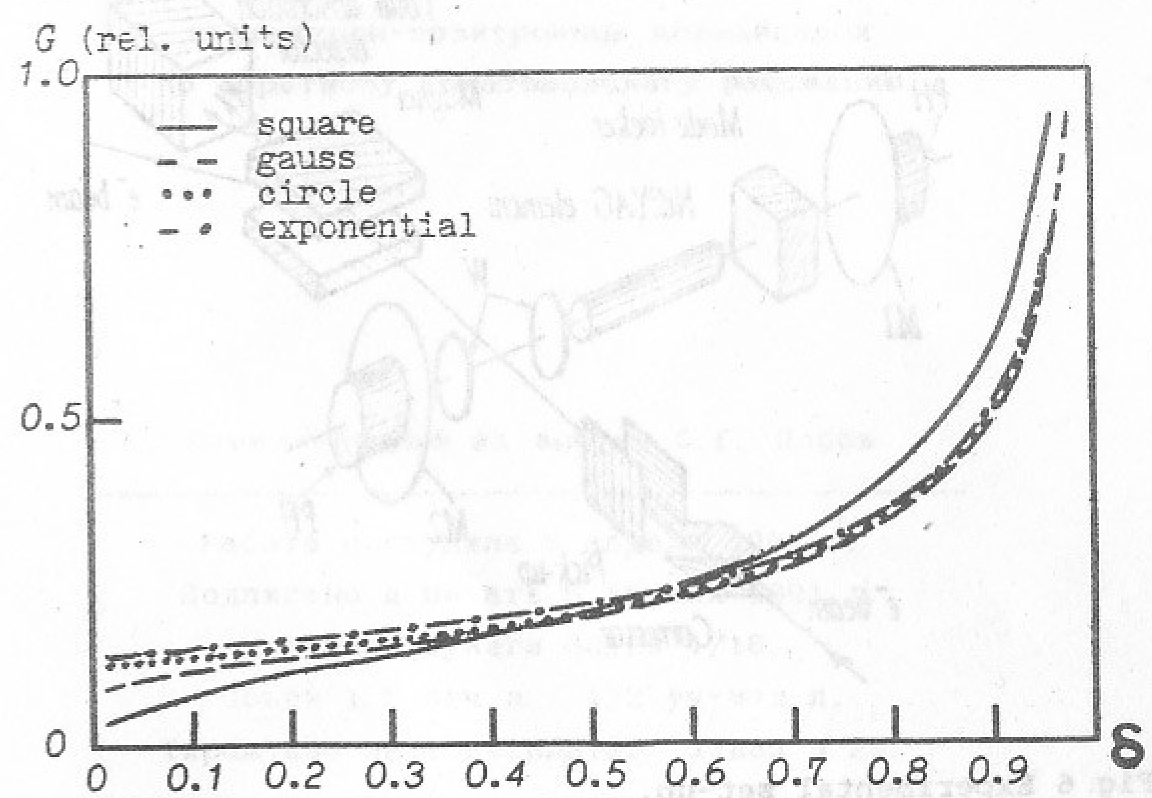

Fig. 4 The geometric factor G for the electron-positron luminosity as the function of the intensity modulation depth δ.

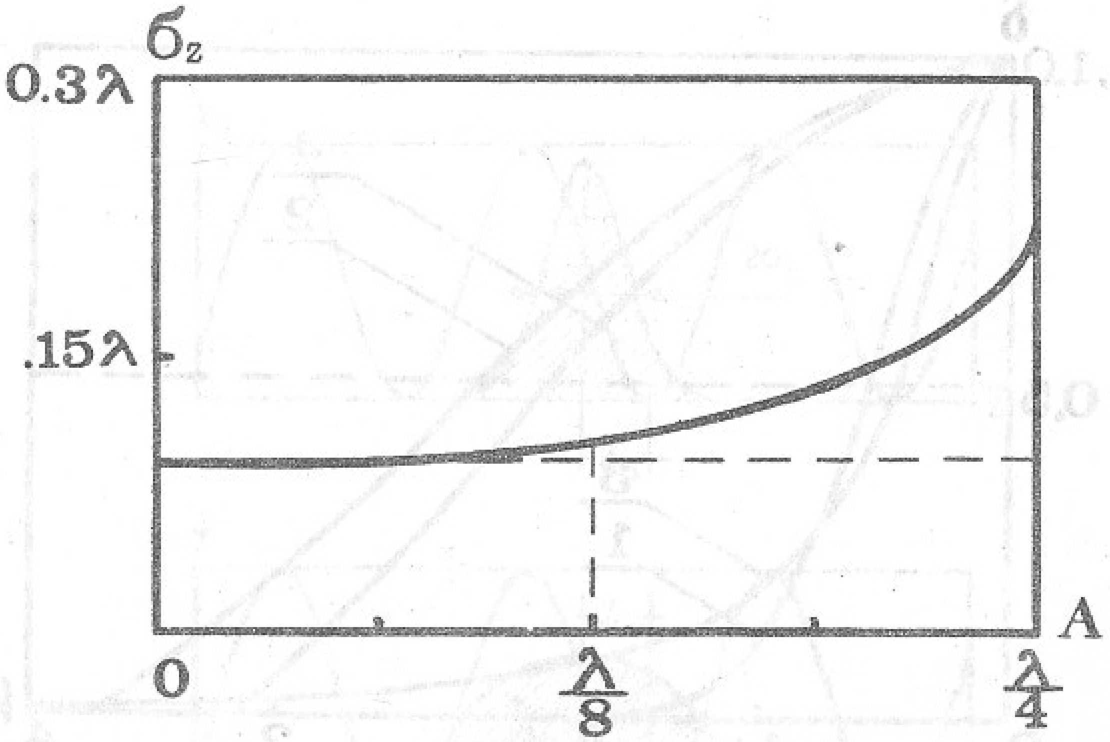

Fig. 5 Measured electron beam size as the function of the electron beam trajectory fluctuation magnitude A.

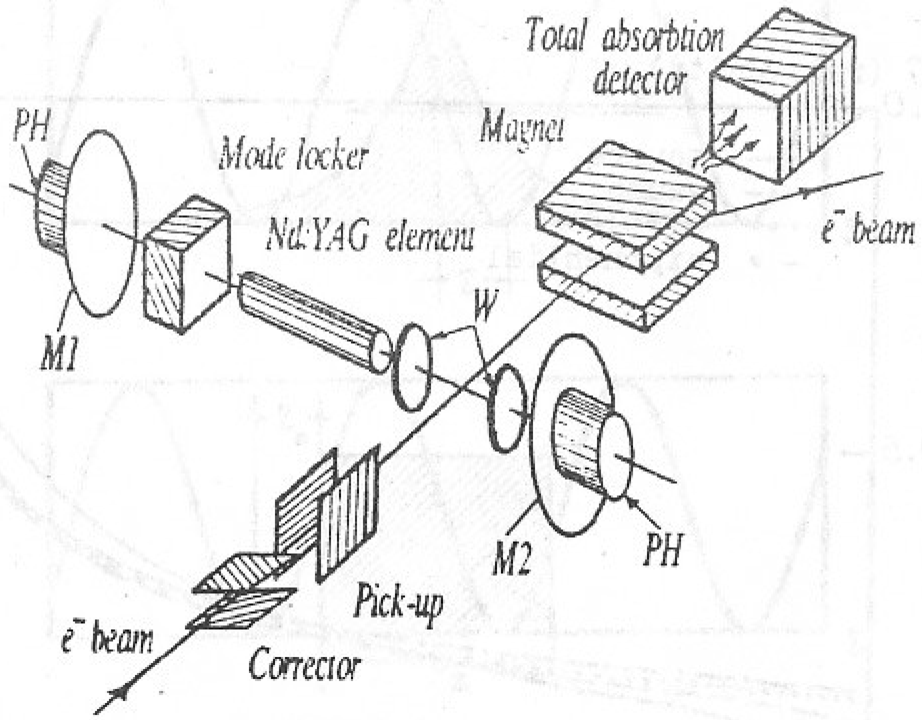

Fig. 6 Experimental set-up.

M1, M2 - optical mirrors, PH - piezoelectric holders, W - optical windows of the vacuum chamber.

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Method of the Measurement of Supersmall Beam Size in Electron-Positron Colliders by Compton Scattering

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Метод измерения сверхмалых размеров пучков в электрон-позитронных коллайдерах по обратному комптоновскому рассеянию

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