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VIOLATION OF THE SCHIFF THEOREM FOR UNSTABLE ATOMIC STATES

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НОВОСИБИРСК

Violation of the Schiff Theorem for
Unstable Atomic States

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ABSTRACT

We discuss the screening of the external static electric field on the nucleus of the neutral atom. It is shown that for the excited atomic states the screening is not complete.

Due to the well known Schiff theorem [1] for the neutral atom the external static homogeneous electric field is exactly screened on the nucleus by the polarization of the electronic shells. The theorem is valid for the relativistic electrons [2, 3] as well as with accounting of the radiation corrections [2]. One can easy understand this theorem: The homogeneous electric field does not accelerate the neutral atom. Therefore the field acting on the nucleus is equal to zero.

The physical arguments as well as formal prove of the theorem are valid only for an atom in the stationary state. For the excited states which decay due to the photon emission the situation is not obvious. This problem is connected with the radiation correction to the energy levels.

First of all let us demonstrate the simple physical arguments in favor of the Schiff theorem violation for unstable states*). In the present work we will consider the Hydrogen atom with infinitely heavy nucleus to avoid the recoil. Let us consider $2s_{1/2}$- state which decays via M1-transition to the $1s_{1/2}$ (in the present work we are interested in one-quantum transition only). In the external electric field there is the mixing with $2p_{1/2}$- state which decays via E1-transition (Fig.1). We neglect the mixing with $2p_{3/2}$-state. The decay amplitude corresponding to Fig.1 equals

$$f_1 = \langle 1s_{1/2}|h_\gamma|2s_{1/2}\rangle + \frac{\langle 1s_{1/2}|h_\gamma|2p_{1/2}\rangle\langle 2p_{1/2}|-e\vec{\mathcal{E}}\vec{r}|2s_{1/2}\rangle}{E_{2s1/2} - E_{2p1/2} + i\Gamma_p/2} . \quad (1)$$

Here $\vec{\mathcal{E}}$ is the external electric field, Γ_p is the radiation width of the $2p_{1/2}$-state.

$$h_\gamma = \sqrt{2\pi/\omega}\; e\; \vec{\alpha}\vec{\varepsilon}\; e^{i\vec{k}\vec{r}} \quad (2)$$

is the operator of the radiation of the photon with momentum $\vec{k}$ and polarization $\vec{\varepsilon}$, $\vec{\alpha}$ is the Dirac matrix. The using of relativistic notations is technically convenient. Simple calculation in linear in $\vec{\mathcal{E}}$ approximation gives the angular distribution of the photons averaged over the polarizations of atomic states (see Ref.[4]).

*) We are grateful to V.V. Flambaum in discussion with these arguments were formulated.

$$dW(\vec{k}) = \Gamma_{2s}(1+\lambda\vec{k}\vec{\mathcal{E}})\frac{d\Omega}{4\pi},$$

where

$$\lambda = \frac{1}{\omega}\frac{E1}{M1}\frac{D\Gamma_p}{(E_s - E_p)^2 + \Gamma_p^2/4} . \quad (3)$$

Here $E1$ and $M1$ are the amplitudes of γ-transitions $2p_{1/2} \to 1s_{1/2}$ and $2s_{1/2} \to 1s_{1/2}$. $D = \left\langle 2p_{1/2}, \frac{1}{2}\right| -ez \left| 2s_{1/2}, \frac{1}{2}\right\rangle$ is the amplitude of $2s$-$2p$ mixing, $\Gamma_{2s} = \frac{4}{3}\omega^3|M1|^2$ is the one photon width of the $2s_{1/2}$- state. Let us stress that correlation of flight direction with electric field $\vec{k}\vec{\mathcal{E}}$ in (3) is T-odd. Just therefore λ is proportional to Γ_p. With angular distribution (3) the photon takes away the average momentum directed along the electric field. The recoil force is equivalent to unscreened electric field at the nucleus.

$$\vec{\mathcal{E}}_N = \frac{1}{e}\frac{\lambda}{3}\omega^2\Gamma_{2s}\vec{\mathcal{E}}. \quad (4)$$

If nucleus has an electric dipole moment $\vec{d}$ then one may suppose that correction to energy level should arise.

$$\delta E_{2s} = -\vec{\mathcal{E}}_N\vec{d}. \quad (5)$$

Now we would like to understand what the energy shift (5) means and how it can be observed. Let us stress once more that $\delta E \sim \Gamma_p\Gamma_{2s}$, i.e. it arises just due to instability of the levels. At infinite mass of the nucleus the only probe of an electric field at the origin can be the electric dipole moment of the nucleus. The interaction of the nucleus dipole moment with the external field and with the electron is equal to

$$H_d = e\, \vec{d}\vec{r}/r^3 - \vec{d}\vec{\mathcal{E}} = \frac{i}{e}\ \vec{d}[\vec{p},\ H], \qquad (6)$$

where

$$H = \vec{\alpha}\vec{p} + \beta m - e^2/r - e\,\vec{\mathcal{E}}\vec{r} \qquad (7)$$

is the Dirac electron Hamiltonian. This Hamiltonian takes into account the external electric field exactly. To stress this point we will denote its eigenstates by the bar: $H|\bar{n}\rangle = \bar{E}_n|\bar{n}\rangle$. The eigenstates of the total Hamiltonian $H+H_d$ we will denote by tilde: $(H+H_d)|\tilde{n}\rangle = \tilde{E}_n|\tilde{n}\rangle$. Due to the Schiff theorem $\tilde{E}_n = \bar{E}_n$. Treating H_d as perturbation one can easy calculate the matrix element of the radiation operator (2) between the tilde states.

$$\langle\tilde{m}|h_\gamma\ \tilde{n}\rangle = \langle\bar{m}|h_\gamma|\bar{n}\rangle + \frac{i}{e}\ \vec{d}\langle\bar{m}|[h_\gamma,\ \vec{p}]|\bar{n}\rangle = (1 - i\,\frac{\vec{k}\vec{d}}{e})\langle\bar{m}|h_\gamma|\bar{n}\rangle \qquad (8)$$

We have used the representation of H_d in the commutator form (6).

Now we can ask the question: Does the correction (5) mean the shift of 2s-energy level which can be observed in the resonant scattering of light on 1s-state of Hydrogen? In the leading order in h_γ the amplitude of the resonant scattering is shown at Fig. 2, and due to the Eq.(8) it equals

$$f = (1 + i\vec{k}_1\vec{d}/e)(1 - i\vec{k}_2\vec{d}/e)\ \frac{\langle 1\bar{s}|h_\gamma(k_2)|2\bar{s}\rangle\langle 2\bar{s}|h^+_\gamma(k_1)|1\bar{s}\rangle}{\omega + E_{1\bar{s}} - E_{2\bar{s}} + i0}, \qquad (9)$$

where $\vec{k}_1$, $\vec{k}_2$ are the momenta of the initial and final photons. The amplitude (9) depends on $\vec{d}$, but this dependence is not connected with any shift of energy. Moreover $|f|^2$ is independent of $\vec{d}$ However it is obvious beforehand that in the leading order in h_γ the shift δE (5) can not arise, since $\delta E \sim \Gamma_{2s}\Gamma_{2p}$. One should take into account at least the Lamb shift (Fig.3a), and even second order in the Lamb shift (Figs.3b,c,d). Nevertheless one can easy verify that in any order in the radiation correction there are no dependence on $\vec{d}$ in the scattering amplitude except the trivial dependence (9). Actually, let us consider for example the amplitude Fig.3a. The insertion is the self energy operator

$$\Sigma(E) = \sum_n \int \frac{d^3q}{(2\pi)^3}\ \frac{\langle 2\tilde{s}|h_\gamma(q)|\tilde{n}\rangle\langle\tilde{n}|h^+_\gamma(q)|2\tilde{s}\rangle}{E - \tilde{E}_n + i0}, \qquad (10)$$

but due to the Eq.(8) dependence on $\vec{d}$ in the matrix element of h_γ exactly compensates that in the matrix element h^+_γ. In the same way we can prove the independence of $\vec{d}$ of the insertions in the diagrams presented at Figs.3b,c,d. Thus, there are no proportional to $\vec{d}$ energy shift which can be observed in the resonant scattering of the light on the atomic ground state. Then the question arises: What the formula (5) means?

To answer this question let us first of all to answer to a more simple one: What is the usual pressure of light? Thus without any external static electric field the laser shines on an atom in the resonance with transition $1s_{1/2}\to 2p_{1/2}$. The interaction of an electron with the classical electromagnetic wave is of the form (cf. with Eq.(2)).

$$\hat{V} = V^{(+)} + V^{(-)},$$
$$V^{(+)} = e/2\, \vec{A}\vec{\alpha}\, e^{i\vec{k}\vec{r}-i\omega t}, \qquad V^{(-)} = (V^{(+)})^{+}. \qquad (11)$$

$\vec{A}$ is the wave vector potential. The rescattering is isotropic and therefore the light pressure arises. This is the static electric field acting on the nucleus. Due to balance of momentum at small saturation parameter

$$<2p|V|1s>/\Gamma_p \ll 1$$

$$\vec{\mathcal{E}}_N = -\frac{1}{e}\vec{k}\,\frac{\Gamma_p}{(\omega-\omega_0)^2+\Gamma_p^2/4}\,\frac{1}{2}\sum_{\alpha,\beta}|<2p_{1/2},\ \alpha|V^{(+)}|1s_{1/2},\ \beta>|^2. \qquad (12)$$

$\omega_0 = E_{2p1/2} - E_{1s1/2}$, α, $\beta = \pm 1/2$ are the projections of angular momentum. Similar to (5) the shift of energy proportional to $\vec{d}$ must arise. However we argue above (Eq.(10)) that there are no corrections to the photon scattering amplitude proportional to $\vec{d}$. Thus we can conclude that the photon which prepares the unstable quantum state can not measure by itself the recoil electric field (4), (12) on the nucleus. However the different experiment is possible. Let the laser field (11) prepares the unstable quantum state and the other field probes the atom. Say using the radio frequency field one can search the dependence of the nuclear magnetic resonance (NMR) frequency on the nucleus electric dipole moment $\vec{d}$. Just in such an experiment the recoil electric field (4), (12) can be measured and exactly in this sense the Schiff theorem is violated for the unstable quantum states.

The shift of the NMR frequency due to the nucleus electric dipole moment is equal to $\delta E = \mathrm{Tr}(H_d\rho)$. Here ρ is the density matrix of an atom in the laser field (11) and H_d is defined by Eq.(6). The equation for the density matrix we will solve by iterations in perturbation $\hat{V}$.

$$\left(i\frac{\partial}{\partial t} - \omega_{ik} + i\Gamma_{ik}\right)\rho_{ik} = [\hat{V},\rho]_{ik}. \qquad (13)$$

In the zero approximation there are only independent of t components of ρ which correspond to the equal population of the states $|1s_{1/2},\ \pm 1/2>$. In the first approximation the positive and negative frequency components of ρ arise.

$$\rho^{(1\pm)}_{ik} = \frac{[\hat{V}^{(\pm)},\ \rho^{(0)}]_{ik}}{\pm\omega - \omega_{ik} + i\Gamma_{ik}}. \qquad (14)$$

The effect we are interested in arises in the second approximation. Time independent components of $\rho^{(2)}$ are equal to

$$\rho^{(2)}_{ik} = -\frac{1}{\omega_{ik}}\left([V^{(+)},\ \rho^{(1-)}]_{ik} + [V^{(-)},\ \rho^{(1+)}]_{ik}\right). \qquad (15)$$

The further calculation is straightforward.

$$\delta E=\mathrm{Tr}(H_d\rho^{(2)})= \frac{i}{e}\,\vec{d}\sum_{ik}[\vec{p},\,H]_{ki}\rho^{(2)}_{ik}=$$

$$=-\frac{i}{e}\,\vec{d}\;\mathrm{Tr}\left([\vec{p},\,V^{(+)}]\rho^{(1-)}+[\vec{p},\,V^{(-)}]\rho^{(1+)}\right)=$$

$$=-\frac{i}{e}\,\vec{d}\vec{k}\;\mathrm{Tr}\left(V^{(+)}\rho^{(1-)}-\rho^{(1+)}V^{(-)}\right). \qquad (16)$$

After substitution of $\rho^{(1)}$ from Eq.(14) we really get $\delta E = -d\vec{\mathcal{E}}_N$ with $\vec{\mathcal{E}}_N$ from Eq.(12).

We now return to the Schiff theorem (formulae (4), (5)). Here the situation is very similar to consideration of the light pressure. However in this case the indexes i, k in density matrix ρ_{ik} numerate not only the states of an atom but the states of a photon as well. This is rather unusual situation and therefore stress it once more. Usually in the density matrix description one averages over all photon states and keeps explicitly the electron degrees of freedom only. To catch the Schiff theorem violation (Eqs.(4), (5)) we should keep explicitly the states of an atom with one photon and average over the states with more then one photon.

Let laser (11) is tuned to the transition $1s_{1/2}\to 2s_{1/2}$. It produces some population of the $2s_{1/2}$-level which corresponds to stationary density matrix $\rho^{(0)}$. Say at saturation the populations of the all four states $|1s_{1/2},\pm\frac{1}{2}\rangle$, $|2s_{1/2},\ \pm\frac{1}{2}\rangle$ are equal. We will solve equation

(13) starting from $\rho^{(0)}$. First of all let us take into account the interaction with the external static electric field $U=-e\vec{\mathcal{E}}\vec{r}$ which mixes $2s_{1/2}$-and $2p_{1/2}$-levels.

$$\rho^{(1)}_{ik}=-\frac{[U,\,\rho^{(0)}]_{ik}}{\omega_{ik}-i\Gamma_{ik}}. \qquad (17)$$

More explicitly,

$$\rho^{(1)}_{sp}=\frac{\langle s|U|p\rangle}{\omega_{sp}-i\Gamma_p/2}\,\rho^{(0)}_{ss}, \qquad \rho^{(1)}_{ps}=(\rho^{(1)}_{sp})^{+}. \qquad (18)$$

Here s,p denotes $2s_{1/2}$ and $2p_{1/2}$, and $\rho^{(2)}_{sp}$ still is a matrix in the projections of angular momenta.

Interaction with the photon with momentum $\vec{q}$ and polarization $\vec{\varepsilon}$ due to Eq.(2) is of the form

$$H_\gamma(\vec{q},\vec{\varepsilon}) = H^{(+)}_\gamma + H^{(-)}_\gamma,$$
$$H^{(+)}_\gamma = h_\gamma a_{q,\varepsilon}, \qquad H^{(-)}_\gamma = h^{+}_\gamma a^{+}_{q,\varepsilon}. \qquad (19)$$

Here a^{+} and a are the creation and annihilation operators of the photon. In the second approximation

$$\rho^{(2\pm)}_{ik}=[H^{(\pm)}_\gamma,\,\rho^{(1)}]_{ik}\Big/(\pm\omega-\omega_{ik}+i\Gamma_{ik}). \qquad (20)$$

Similar to Eq.(15) time independent part of $\rho^{(3)}$ is equal to

$$\rho^{(3)}_{ik}=-\frac{1}{\omega_{ik}}\left([H^{(+)}_\gamma,\,\rho^{(2-)}]_{ik}+[H^{(-)}_\gamma,\,\rho^{(2+)}]_{ik}\right). \qquad (21)$$

Analogously to Eq.(16) correction to the energy is of the form

$$\delta E=\mathrm{Tr}(H_d\rho^{(3)})= - \frac{\mathrm{i}}{\mathrm{e}}\ \mathrm{Tr}\left(\vec{d}\vec{q}\left(H_\gamma^{(+)}\rho^{(2-)}-\rho^{(2+)}H_\gamma^{(-)}\right)\right) =$$

$$=- \frac{i}{e}\int\frac{d^3q}{(2\pi)^3}\ \vec{d}\vec{q}\sum_{ik}\left\{\frac{h_{ki}[h^+,\rho^{(1)}]_{ik}}{-\omega-\omega_{ik}+i0} - \frac{[h,\rho^{(1)}]_{ki}\,h^+_{ik}}{\omega-\omega_{ki}+i0}\right\}. \quad (22)$$

$\omega_{ik}=-\omega_{ki}$ and therefore one can easy verify that at denominators in this expression only δ-function survives: $(\omega-\omega_0-i0)^{-1}\to i\pi\delta(\omega-\omega_0)$. Using Eq.(18) δE can be transformed to the form

$$\delta E = \frac{4\pi}{\mathrm{e}}\ \mathrm{Re}\int\frac{d^3q}{(2\pi)^3}\ \vec{d}\vec{q}\ \delta(\omega-\omega_0)\ \mathrm{Tr}\left(\rho_{sp}^{(1)}\langle p|h|1s\rangle\langle 1s|h^+|s\rangle\right). \quad (23)$$

Trace in this formula is carried out over the all projections of angular momentum. Comparing with Eqs.(1)-(5) we see that expression (23) identically coincides with the energy shift (5) which is derived from the balance of momenta.

In the conclusion we formulate the results of the present work. The Schiff theorem (screening of the external static homogeneous electric field on the nucleus of the neutral atom) is violated for the excited (unstable) atomic states. As a matter of principle this violation can not be observed in the scattering of a photon at the ground state of an atom. In other words there is no effect if one uses as a probe the photon which itself prepares the unstable quantum state. The violation takes place if the photons (laser field) are used to prepare the unstable quantum state

and the other field probes the atom. Say using the radio frequency field one can observe the dependence of nuclear magnetic resonance frequency on the nucleus electric dipole moment $\vec{d}$. Just in this sense the Schiff theorem is violated for the unstable quantum states.

We would like to pay attention of the reader to an interesting principal possibility. We mean the using of the light pressure static electric field (12) for the experimental search of the nucleus electric dipole moment in the nuclear magnetic resonance experiments. The advantage of this method could be the absence of an any external static electric field. This method is more suitable for the light atoms.

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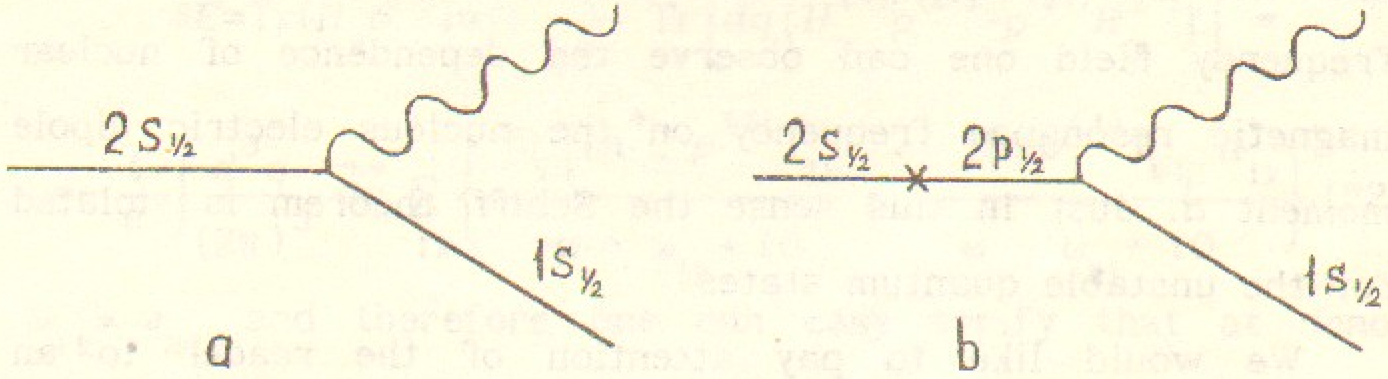

Fig. 1. Amplitude of the $2s_{1/2}$—state decay in the external electric field. The cross corresponds to the states mixing in the field.

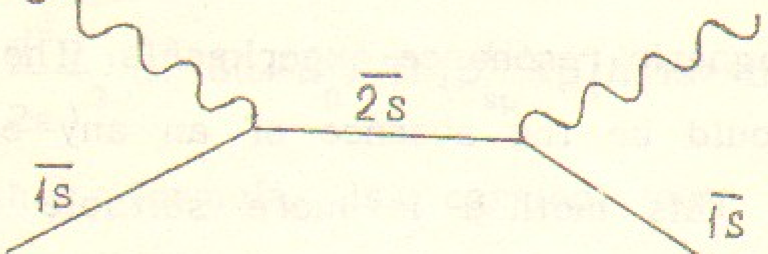

Fig. 2. Amplitude of the photon resonant scattering in the leading order in H_γ.

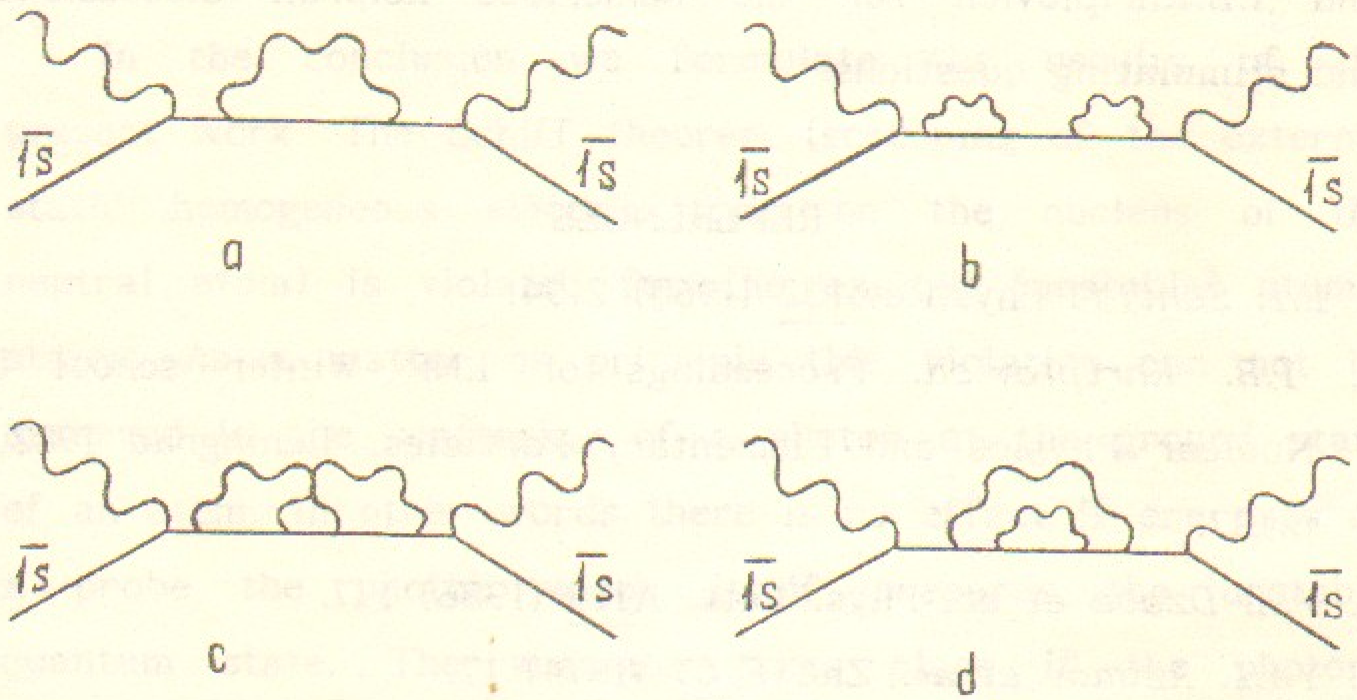

Fig. 3. Amplitude of the photon resonant scattering with the Lamb shift taken into accoutt.

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